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Abstract

We propose a method for estimating household income uncertainty that does not impose restrictions on the underlying income shocks or assumptions about household behaviors. We measure income uncertainty as the variance of linear projection errors at various future horizons, up to 25 years ahead, conditional on only the information available to households when the projection is made. Our uncertainty estimates change substantially over the life cycle. We calibrate an income process to match our estimates, allowing the variances of both transitory and persistent shocks to change over the life cycle. Relative to previous studies, we find lower and less persistent income uncertainties that call for a life cycle consumption profile with a less pronounced hump.

JEL Classification: D12, D91, E24

Keywords: income uncertainty, projection errors, life cycle dynamics, precautionary saving, consumption hump

1 Introduction

Income uncertainty plays an important role in household decisions regarding consumption, saving, and investment. First, in the presence of incomplete financial markets, households facing uninsurable income risk have to save for precautionary reasons (Aiyagari (1994)). How much they need to save crucially depends on their level of future income uncertainty. Second, people choose their investment portfolios to achieve optimal exposure to risks. How much people should invest in risky assets, such as stocks, also depends on their idiosyncratic exposure to income risks (Viceira (2001)). Finally, choices of financial contracts and durable goods purchasing decisions are critically affected by household income uncertainty. For example, Campbell and Cocco (2003) show that homeowners with more risky income should choose fixed-rate mortgages over adjustable-rate mortgages.

Many of these household financial and economic decisions exhibit strong life cycle patterns. It is well known that nondurable goods consumption has a hump-shaped life cycle profile that closely tracks income (e.g., Gourinchas and Parker (2002)). Meanwhile, stock market participation rates are particularly high among middle-aged households. Homeownership and durable goods consumption also tend to vary along the life cycle (Fernández-Villaverde and Krueger (2010)). Despite its pivotal importance, the existing literature has not yielded a concrete, model-independent measure of income uncertainty, and to the best of our knowledge, even less work has been done to characterize how income uncertainty evolves and affects consumption choices over the life cycle. This paper helps bridge that gap.

The most frequently used approach to characterizing income uncertainty assumes income shocks can be decomposed into a permanent (random walk) and a transitory (typically i.i.d.) component.[1] The identification of the variances of income shocks is obtained by examining either the variances of income growth over periods of different lengths in lon-

[1]For a recent example of an exception, see Storesletten, Telmer, and Yaron (2004), which modeled the nontransitory shocks as an AR process.

gitudinal data (e.g., Carroll and Samwick (1997)) or the changes of income distributions of the same cohort over time in repeated cross-sectional data (e.g., Deaton and Paxson (1994)).

Deviating from this paradigm, we measure income uncertainty as the variance of projection errors of future income conditional on the information available to the households when the projection is made. We impose essentially no restrictions on the statistical properties of the projection errors. Instead, we construct the projection errors and estimate their variances for each horizon up to 25 years ahead. Then, we study how these projection errors are correlated across different horizons and how their variances evolve over the life cycle. We find smaller and less persistent income uncertainty than previously documented. We also find significant variation in uncertainty at fixed horizons as one progresses through the life cycle.

Another innovation of this paper is to solve a life cycle consumption optimization model that explicitly takes into account the age-dependent variations in income uncertainty. To the best of our knowledge, the existing literature has not provided a solution to such a model. This is partly due to the complexity of constructing such a model within the rational expectations framework, which requires a consumer to update his beliefs about the probability distribution of future incomes conditional on the current state using Bayes' rule at each stage of the life cycle. Indeed, specifying such a data generating process for income that matches our estimates of the second moments is a technically difficult problem that we defer to future work. Instead, we calibrate separate income processes as perceived by households at various ages, without imposing that these perceptions are time consistent in a Bayesian sense.

Specifically, for each age we specify an income process that is consistent with the estimates of the variances and correlations of projection errors at each future time horizon. The consumer solves for his optimal consumption function in the present via a backwards recursion while assuming this posited income process governs his income dynamics in all future periods. However, in each ensuing period the consumer will posit a different income process governed by the variance and correlation matrix estimates for that age. Thus the consumer has time-inconsistent expectations about the second-order moments, though we assume he has time-consistent expectations about the mean of income. Solving such a calibrated partial equilibrium model yields a life cycle consumption profile with a much less pronounced hump relative both to the existing literature and the profile obtained if one ignores the age-dependence of income uncertainty.

Broadly speaking, our empirical method is similar in spirit to using cross-sectional variations to characterize income riskiness. However, relative to the existing literature, our approach has three important advantages. First, it allows us to estimate the persistence of income shocks without resorting to any *a priori* parameterizations of these shocks, especially the unit root process that is typically assumed in previous studies. Indeed, examining the correlations of income shocks over different horizons, we find that the typical income process is more accurately characterized as a persistent autoregressive (AR) process rather than a unit-root process. Consequently, our approach generally implies substantially lower cumulative income risks over the life cycle compared to existing results in the literature.

Second, our approach allows us to study how income uncertainty evolves over the life cycle. For a given age in the future, we consistently find, as one would expect, that uncertainty about income diminishes as the consumer approaches this age. More strikingly, for a fixed future horizon, income uncertainty demonstrates a U-shaped profile. When consumers are young, income uncertainty at a fixed future horizon gradually declines with age, presumably as decisions on career, human capital development, and fertility are resolved. Income uncertainty reaches and stays at its low levels during middle age. Afterwards, uncertainty rises again—potentially due to uncertainty about working hours and health risks. This dynamic of income uncertainty is consistent with Jaimovich and Siu (2009) who find that the volatility of the business cycle component of hours worked exhibits a U-shaped pattern with respect to age.

Third, the existing methodology typically filters the predictable component of income conditional on demographic and work-related information observed concurrent to income. Essentially, the filtered out part is what is ex post accountable to econometricians, and not necessarily what is ex ante predictable to households when they assess the riskiness of their future income. In contrast, our approach adopts more realistic and flexible information specifications. Our baseline specification assumes that households in year t project their income at year $t + s$ conditional on only the demographic and work-related information about the household in year t. This assumption is rather restrictive and in some sense allows for only the information available to econometricians, not to households, in year t. Conceivably, households may know more about their future selves in year $t + s$. For example, Cunha, Heckman, and Navarro (2005) point out individuals may have private information that is relevant to predicting their future income. In this spirit, in augmented specifications, we assume that households in year t possess some demographic and work-related information about their future selves in year $t + s$. Not surprisingly, we find that expanding the information set further reduces the estimated income uncertainties, especially at farther horizons.

A different paradigm for studying income uncertainty is to infer household perceived income risks using the observed choices of households, such as consumption and savings. We view this strategy as a direction that has much potential. Indeed, this research program has yielded results that are rather promising (see, for example, Cunha, Heckman, and Navarro (2005) and Guvenen and Smith (2010)). In this paper, we choose not to infer income uncertainty from decision variables because we want to make our inference model-independent, thus preserving its nonparametric characteristics.

2 A Critical Review of the Existing Approaches to Studying Income Uncertainty

Students of household level data often study income uncertainty via other aspects of variability, such as income inequality and volatility. This is not surprising. On the one hand, uncertainty per se is difficult to observe. On the other hand, greater uncertainty will often lead to greater inequality or volatility.[2]

More direct results are obtained using longitudinal surveys. Much of this work has explicitly focused on the volatility and inequality, instead of uncertainty, of household income. For example, Dynan, Elmendorf, and Sichel (2007) examine the standard deviation of biennial income changes across households and conclude that household income growth has become more volatile over the past several decades.[3] An alternative approach takes the mean income of a household over a given period as a proxy for permanent income and treat the difference between the observed income and the mean as the transitory component of income (see, for example, Gottschalk and Moffitt (1994)). In general, it is hard to make an inference about income uncertainty (and its changes) from these results because the variability they studied may include income changes that are predictable to households.

More sophisticated models often postulate some parsimoniously parameterized income process such as

$$y_t = p_t + \varepsilon_t, \tag{1}$$

where y_t is the logarithm of income, ε_t is a transitory shock and p_t is a permanent income shock that follows

$$p_t = g_t + p_{t-1} + \eta_t, \tag{2}$$

in which g_t is a predictable component and η_t is the shock to permanent income. Although

[2] For example, in a seminal early contribution, Deaton and Paxson (1994) examine the repeated cross-sectional data and document that within-cohort inequality of consumption and income increases with age. They argue that this increased inequality is the "cumulative differences in the effects of luck."

[3] Dynan, Elmendorf, and Sichel (2007) also provide a survey of the literature studying household income volatilities using household data. More recent studies using similar approaches include Kopczuk, Saez and Song (2010) and Shin and Solon (2011).

estimating this specification comes closer to the goal of measuring income uncertainty, we argue this methodology can be refined in the following respects.

First, the predictable component of income is often constructed by regressing y_t on household demographics and work-related variables observed in the same year (See, for example, Carroll and Samwick (1997) and Gourinchas and Parker (2002)). The fitted value, $\widehat{y_t}$ is then interpreted as the predictable component of income. To the extent that households do not have perfect foresight about all information used in the regression, $\widehat{y_t}$ is essentially the component ex post explainable to econometricians rather than the component ex ante predictable to households. Put differently, constructing the predictable income by conditioning on information concurrent with the realization of income potentially assumes too much information, and consequently may lead to underestimation of the income uncertainty perceived by the household.

Second, if we fix the calendar time of a given future income, the predicted value should converge to the realized value as the household progresses in time and uncertainties are resolved. However, because the predictable component is not constructed specifically for any projection horizon in the standard method, this method does not directly capture this evolution of income uncertainty.

Third, the above parametrization assumes the income process is subject to permanent shocks. As we show later in this paper, household income shocks do have a persistent component. However, this component is best characterized as a fairly persistent, but not permanent (or unit-root), process. This distinction has a substantial effect on the level of cumulative uncertainty over the life cycle and in turn affects the implied precautionary saving motives (in particular for younger consumers).

Finally, apart from a few exceptions (e.g., Baker and Solon (2003) and Karahan and Ozkan (2009)), previous studies have not fully characterized how household income risks evolve over the life cycle. It is both theoretically and empirically appealing to study whether household income uncertainty stays constant, or, if not, how it evolves over the life cycle.[4]

3 A Projection-Based Approach

We propose an approach to estimating household income uncertainty that attempts to address the concerns outlined in the previous section. This approach is flexible enough to incorporate various assumptions about the household's information set and can be used to characterize the evolution of income uncertainty over the life cycle. Our key insight is that greater income uncertainty should make future income more difficult to forecast. Accordingly, we use projection accuracy as a metric of the underlying income uncertainty or riskiness. The larger the variances of projection errors are, the greater uncertainty a household faces regarding its future income.

For a household whose head's age is t, let the logarithm of its income s years ahead be $y_{t,s}$, which can be decomposed as

$$y_{t,s} = E\left[y_{t,s}|\mathcal{I}_t^H\right] + \varepsilon_{t,s}, \tag{3}$$

where $E\left[y_{t,s}|\mathcal{I}_t^H\right]$ is the mathematical expectation of age $t+s$ income, $y_{t,s}$, conditional on age-t household information, $\mathcal{I}_t^H$, and $\varepsilon_{t,s}$ is an error term orthogonal to $\mathcal{I}_t^H$. We characterize life cycle income uncertainties using a variance matrix, Ω, and a sequence of correlation matrices, Θ^q. The element $\omega_{t,s}$ of the Ω matrix is the variance of the s-year-ahead projection errors of age-t households, i.e.,

$$\omega_{t,s} = Var[\varepsilon_{t,s}] = Var[y_{t,s} - E(y_{t,s}|\mathcal{I}_t^H)]. \tag{4}$$

Elements of the Θ^q matrix, $\theta_{t,s}^q$, are the correlation coefficients between the s-year-ahead projection errors, $\varepsilon_{t,s}$, and the q-year-ahead projection errors, $\varepsilon_{t,q}$, of age-t households, i.e.,

$$\theta_{t,s}^q = Corr(\varepsilon_{t,s},\ \ \varepsilon_{t,q}). \tag{5}$$

[4]Heuristically, a single 22-year-old college graduate entering the labor market should have more uncertainty about his income five or ten years down the road than a 40-year-old man with a family and a settled career path over the same time horizon.

For example, elements of Θ^1 are the correlation coefficients between the s-year-ahead projection errors and the 1-year-ahead projection errors. Apart from the fact that the projection errors are constructed using standard linear projections, we do not presume that the income shocks, $\varepsilon_{t,s}$, follow any specific process. In examining the unrestricted Ω and Θ^q matrices, we can infer the persistence of income uncertainty as well as its dynamics over the life cycle. In the remainder of the paper, we refer to year t, the year in which the projection is made, as the base year.

One hurdle to implementing this strategy is that we do not know the joint distribution of $y_{t,s}$ and $\mathcal{I}_t^H$. As a result, we cannot compute $E[y_{t,s}|\mathcal{I}_t^H]$ directly. Indeed, we do not even know exactly what $\mathcal{I}_t^H$ includes. To establish a benchmark and to assess the bias introduced by ignoring any additional (superior) information potentially possessed by households, we experiment with two specifications. First, in what we label as the restricted information specification (RIS), we project $y_{t,s}$ conditional on $\mathcal{I}_t^R$, the information set that includes only what an econometrician can observe regarding a household as of age t. This information is what households *certainly* possess at age t. Second, in what we label as the augmented information specification (AIS), we project $y_{t,s}$ conditional on the augmented information set $\mathcal{I}_t^A$, where

$$\mathcal{I}_t^A = \mathcal{I}_t^R \cup \mathcal{I}_t^F. \tag{6}$$

The augmenting information set, $\mathcal{I}_t^F$, contains the household's future demographic and work related characteristics as of year $t+s$. This information is what households *likely or possibly* know at age t. To fix the idea, we estimate the following RIS equation,

$$y_{i,t,s} = \alpha + \beta_0 y_{i,t} + \beta_1 y_{i,t-1} + \beta_2 y_{i,t-2} + \gamma Z_{i,t} + \xi Trend_{i,t,s} + \varepsilon_{i,t,s}, \tag{7}$$

and AIS equation,

$$y_{i,t,s} = \alpha + \beta_0 y_{i,\,t} + \beta_1 y_{i,t-1} + \beta_2 y_{i,t-2} + \gamma Z_{i,t} + \delta Q_{i,t,s} + \xi Trend_{i,t,s} + \varepsilon_{i,t,s}. \tag{8}$$

In the above specifications, $Z_{i,t}$ is a vector of variables that belong to $\mathcal{I}_t^R$ for household i.

We only include information about the head of household.[5] This vector contains race, educational attainment, marital status, family size, a currently laid-off or unemployed dummy, a currently self-employed dummy, and a vector of occupation and industry dummies, all evaluated at age t. In addition, $Z_{i,t}$ includes a fourth-order age polynomial, evaluated at age $t+s$. $Q_{i,t,s}$ is a vector of variables that belongs to the augmenting information set, $\mathcal{I}_t^F$. We assume $Q_{i,t,s}$ includes family size, marital status, a retirement dummy, a part-time dummy, a self-employed dummy, and a vector of occupation and industry dummies, all evaluated at age $t+s$.[6]

In addition to $\mathcal{I}_t^R$ and $\mathcal{I}_t^A$, our specification deviates from most previous specifications in that we include both current and lagged income in the projection equations. In principle, if we have a very long income history for a given household, a univariate time series model could potentially have decent forecasting power. Such a long time series of household income is also useful for identifying the income process heterogeneity that Lillard and Weiss (1979) and Guvenen (2007) have stressed. Including some recent income history in the projection equation can help tease out information about recent income shocks and capture part of the individual-specific information of income growth that is not revealed by current income and other observable characteristics. In practice, our model includes two lags to preserve degrees of freedom. Finally, we added a simple calendar year trend to control for aggregate economic growth.[7]

Several important caveats apply to our specifications. First, as most households make plans about their family and career ahead of time, it is not unreasonable to assume households know several years ahead of time what their family size and marital status will be, whether they will be working, retired, or self-employed, or whether they will change occu-

[5] Our data source does not have information for spouses as complete as heads. Including the available spousal information does not qualitatively change the results.

[6] We do not include $t+s$ educational attainment because the data we use do not regularly update household education attainment information.

[7] We also estimate the model with a vector of year dummies, assuming households have perfect foresight on future aggregate growth and business cycle fluctuations. The results are very much similar to the model with a linear trend.

pation and industry. However, it is less likely that households know all this information when s is large.[8] That said, the information we have is limited to what is collected in surveys. There are other information elements ι such that $\iota \in \mathcal{I}_t^H$, but $\iota \notin \mathcal{I}_t^R \cup \mathcal{I}_t^F$. Therefore, it is generally not true that the AIS estimates yield a lower bound on household income uncertainty. However, the comfortably large value of R^2 of Eq. (8) reassures us that the unobservable information only has a limited effect.

Second, it is worthwhile to point out that Eqs. (7) and (8) are merely statistical equations, and they are estimated solely on the basis of maximizing R^2. We are not estimating a structural model, so the coefficients estimated for Eqs. (7) and (8) should not be interpreted as structural parameters. Indeed, some of the coefficients of control variables, such as occupation, industry, and self-employment dummies, are potentially estimated with a bias due to selection-related endogeneity. Such a bias could then influence the estimated variances of projection residuals. As a part of the robustness analyses, we will show that such a bias does not qualitatively alter our results.

Third, because we let the households use the estimated coefficients to project their future income, we implicitly assume that either these coefficients are stable over time or that households have foresight on the future values of these coefficients (e.g., the household would know in advance the return to schooling and experience). We follow Carroll (1994) and resort to the first assumption in the baseline analysis. As a robustness check, we alternatively allow for time-varying coefficients and estimate the coefficients using a lagged sample. By this exercise, we found that the so-estimated income uncertainty over all projection horizons is somewhat higher than in our baseline specifications. However, all key results are qualitatively unaltered.

Finally, although we allow income uncertainty to vary with age, we compute these projection errors by applying the same coefficients of the model to all households. As another robustness check, we also estimated the model separately for each age group.

[8] Our study projections future income up to twenty-five years ahead.

Our results are qualitatively preserved, which reassures us that the age profile of income uncertainty is not driven by any age dependence in the accuracy of estimating the model.

4 Data Description and Sample Construction

We use the 1968 to 2005 data of the Panel Study of Income Dynamics (PSID), a nationwide longitudinal survey of households. Before 1997, the PSID was an annual survey; after 1997, it became a biennial survey. The longitudinal structure of the PSID permits us to link the information collected for the same household in two waves that are s years apart. For instance, consider a household that was surveyed for ten years over 1971 through 1980. When we project the five-year-ahead income, this household renders five current and future income pairs $(t, t+s) = (1971, 1976)$, ... , (1975, 1980). This structure is essential to estimate the projection equation and to experiment with alternative information specifications.

We estimate Eqs. (7) and (8) for each horizon, s. Accordingly, the samples are constructed separately for each horizon. Several rules apply when we construct the sample. First, we include only the nationwide representative sample and exclude households in the low-income supplemental sample.[9] Second, to focus on working-age households, we restrict household heads to be older than 23 in the base year and younger than 65 in the year to be projected. For example, in the sample we use to project five-year-ahead income, we restrict the heads of sample households to be younger than 60. Consequently, the sample we use to study projection errors at farther horizons is typically smaller than the sample used for a closer horizon. Third, we remove households that reported zero or negative income in the base year, the two lagged years, or the year to be projected. Fourth, we remove households whose heads were disabled or retired, were primarily keeping house, or were students in the base year. Fifth, we remove households whose heads reported zero

[9] The core PSID sample consisted of two independent samples: a nationwide representative sample and a sample of low-income families. In the first wave of the survey, the nationwide representative sample has about 3,000 households and the low-income sample has about 2,000 households.

Table 1: Number of Observations for Each Sample

s	1 Year	2 Years	3 Years	4 Years	5 Years
N	43,313	47,109	37,583	39,782	32,926
s	6 Years	7 Years	8 Years	9 Years	10 Years
N	33,421	28,761	27,740	24,947	22,736
s	11 Years	12 Years	13 Years	14 Years	15 Years
N	20,553	18,372	16,500	14,640	12,970
s	16 Years	17 Years	18 Years	19 Years	20 Years
N	11,331	9,957	8,572	7,452	6,293
s	21 Years	22 Years	23 Years	24 Years	25 Years
N	5,342	4,376	3,603	2,834	2,244

working hours in the base year and the two lagged years.[10] Finally, in order to minimize the bias caused by outliers and measurement errors, we trim off households with very high- or very low-income levels and growth rates between age t and $t + s$.[11]

To explore the precautionary saving implication of the estimated income uncertainty, we focus on the most relevant income definition, family non-asset income. Because Carroll and Samwick (1997) also study family non-asset income, examining the same income definition also facilitates comparison with their results. In our study, we estimate the variance of projection errors of future family non-asset income up to 25 years ahead. For illustration purposes, Table 1 lists the number of observations used in the estimation for each horizon. We notice that the sample constructed for the two-year-ahead projection has the largest number of observations (more than 47,000) because the PSID became a biennial survey after 1997. Our sample sizes largely decline with future horizons. However, even at the 25-year-ahead horizon, we maintain a decent sample size that is above 2,200.

[10] We do not require positive hours in the year to be projected because we allow that the head may be retired at age $t + s$.

[11] We trim off the top and bottom 1% of lagged, current, and future income level distributions and the distribution of income growth between year t and $t + s$. This strategy may appear to be aggressive and lead to an underestimate of income risks. However, when comparing the results to other methods, we apply the previous methods on a sample after the same trimming and find that our estimates of income uncertainty remain substantially lower than the estimates of other methods.

5 Main Empirical Results

We present five-year centered moving averages of the estimated variances of projection errors to smooth out noise in the series. Recall that we have from one-year-ahead to twenty-five-year-ahead projections, and our sample household heads were between the age of 23 to 64 in the base year. The centered moving average uncertainty matrix Ω therefore has a dimension of 38×25, with each row corresponding to the same age and each column corresponding to the same projection horizon. To demonstrate how income uncertainty evolves over the life cycle, for each model specification (RIS and AIS), we plot uncertainty age profiles at four future horizons (four columns of Ω): one-year and two-year ahead, representing the near future, as well as five-year and ten-year ahead, representing the medium and more remote future. To illustrate the statistical significance of our results, we also plot the 95% confidence interval band for each profile .

5.1 Income Uncertainty Changes over the Life Cycle

5.1.1 Family Non-asset Income

We begin by examining the uncertainty of total family non-asset income estimated under the more restrictive information specification, the RIS equation. The results are shown in the top four panels of Figure 1. Consistent with our intuition that uncertainty regarding income at a fixed future date resolves as the date approaches, the level of uncertainty regarding income two years ahead is considerably higher than the income uncertainty one year ahead. The confidence intervals associated with both uncertainty profiles indicate this gap is statistically significant. Notably, both profiles share a similar U-shaped pattern over the life cycle. Income uncertainty is high when the head is young; it declines during his late twenties and early thirties. Afterward, uncertainty fluctuates within a narrow range at relatively low levels before rising in the mid-fifties, as households approach retirement. The ratios between the maximum and minimum levels of uncertainty over the life cycle (the max-min ratio) are both about 1.5 for one-year- and two-year- ahead future income. The plotted 95% confidence intervals suggest that both profiles are rather tightly estimated, which reinforces the statistical significance of the U-shaped pattern. A similar convex profile of income uncertainty is documented in Baker and Solon (2003). They decompose the stochastic part of income into a transitory and a permanent component and find that the age profile of the variance of the transitory innovations exhibits a pronounced U-shaped pattern.[12] Our results, in contrast, do not rely on the dichotomy of transitory and permanent shocks.

Profiles of similar shape repeat in the left panel on the second row, which shows income uncertainties at the medium-term (five-year-ahead) future. The standard errors of the estimated profiles are somewhat larger than in the top two panels because we have a smaller sample over these longer projection horizons. The uncertainty associated with income five years ahead hits bottom in the mid-thirties and rises gradually through the early fifties. After that, income uncertainty rises more rapidly as households approach retirement ages. The profile of income uncertainty ten years ahead does not exhibit a pronounced decline during young ages. The level of uncertainty stays low through the mid-thirties before rising in the mid-fifties. Comparing with the near future, the variations of the five-year-ahead income uncertainty over the life cyle are about as pronounced, with the max-min ratio equal to 1.5. However, the variations of income uncertainties at the more remote future are less pronounced, with the max-min ratio equal to merely 1.3 for the ten-year horizon. Moreover, although the level of uncertainty in the medium future is much higher than in the near future, the increase is not proportional to the future horizon. The two-year-ahead uncertainty is on average 60% higher than the one-year-ahead uncertainty. If these uncertainties rose proportionally with the horizon length, the five-year-ahead uncertainty, for example, should be 240% higher than the one-year-ahead uncertainty. However, it is only 140% higher. As we will further elaborate later in the paper, this nonlinearity suggests the persistent component of income shocks is not permanent.

[12] A similar pattern was also noticed by Gordon (1984).

Income-uncertainty profiles at the very remote future (up to 25 years) are also estimated (not shown). Because of smaller sample sizes at these horizons, these profiles are estimated with wider confidence intervals. Like the ten-year-ahead uncertainty age profile, the very remote income uncertainty age profiles all show a pronounced upward trend throughout the life cycle but do not exhibit clear declines during young ages. Finally, for overlapping ages, the twenty-five-year-ahead profile is on average 450% higher than the one-year-ahead profile, which is much less than what proportional increases would imply.

Now, we turn our attention to the augmented information specification, AIS, in which we allow households to have foresight about some of their future demographic and work-related characteristics. The results are shown in the bottom four panels of Figure 1. Two features are noteworthy. First, because these future income projections are conditioned on a larger information set, the AIS projection error variances are significantly smaller than the variances estimated using the RIS equation. This pattern holds for all future horizons and for households of all ages. Moreover, the discrepancy between the RIS and AIS results widens with the projection horizon. The estimated one-year-ahead uncertainty under the RIS is only 6% higher than that estimated under the AIS, whereas the gap is about 25% at the ten-year horizon. It widens to 36% at the 25-year horizon. This widening trend is consistent with our intuition because the difference between $\mathcal{I}^R$ and $\mathcal{I}^A$ is what households are assumed to know about themselves at age $t+s$ as of age t. Given that these characteristics typically change only gradually, if s is small, the correlation between the elements in $\mathcal{I}^R$ and $\mathcal{I}^F$ will be high, discounting the net value of $\mathcal{I}^F$. Conversely, if s is large, $\mathcal{I}^R$ should have less predictive power on $\mathcal{I}^F$. Consequently, augmenting with $\mathcal{I}^F$ adds more new information and significantly improves model performance. Second, the U-shaped contours at the near and the intermediate future horizons are similar to those shown in the top panels. However, the max-min ratios are somewhat lower for the AIS estimates, especially for the profiles of more remote future horizons. This result is not surprising because a good portion of the rise in income uncertainty after middle age comes from retirement and related risks, of which information is provided in the augmented information set.

5.1.2 Male Labor Earnings

We have shown that our estimates of family non-asset income uncertainty exhibit significant variability over the life cycle. Regarding the pronounced increase of uncertainty beyond age 55, We would like to point out that much of this increase is due to labor hour variations, which are particularly significant for this age group. To see this, we estimate life cycle uncertainty profiles of male labor earnings and male hourly wages.[13]

The red curve in Figure 2 plots the estimated 1-year-ahead uncertainty profile for male labor earnings. Like family non-asset income, the uncertainty profile for male labor earnings also declines moderately when the individual is young. However, unlike family non-asset income, the increase of labor earnings uncertainty over the latter part of life cycle is much more pronounced. Notably, the sharp increase of the estimated uncertainty is largely following the share of males working part-time for each age group (the purple curve). Furthermore, if we assume that working part-time reflects more of an individual's preference change than a shock and control for *future* part-time status in the AIS specification, so-estimated profile (the green curve, labelled as AIS-plus) is markedly lower than that estimated without controlling for part-time status and the difference is most pronounced beyond age 55.[14] The lower panel of Figure 2 presents the estimated uncertainty-profile for male average hourly earnings. Similarly to male labor earnings, this profile declines moderately over younger ages and rises sharply beyond age 55, suggesting that the increase of income uncertainty beyond age 55 is attributable to both heightened hourly earnings variability and variability of labor hours itself, which in turn may be due to some type of preference changes rather than pure unanticipated shocks.[15] Finally, because male earnings

[13] Focusing on male labor earnings also helps isolate income variability from family compositional changes.

[14] Uncertainty profiles estimated for farther horizons share qualitatively same characteristics.

[15] Interestingly, the average hourly earnings uncertainty profile essentially does not change when future part-time status is controlled for.

uncertainty profile share very similar dynamics as family income uncertainty, it is not likely that the life cycle variations of the latter are drive by household compositional changes.

5.1.3 Information Merits of Health-related Variables

What is perhaps conspicuously excluded from the information set for projecting future income is households' health and disability status.[16] From 1972, the PSID started asking survey participants whether and the extent to which their current disability conditions limited them from doing a certain amount of work. Later, from 1984, the PSID began collecting self-reported health status information. The values of these variables are categorical. For example, one can report that he was completely, severely, moderately, or only slightly limited from doing work and has excellent, good, fair or bad health conditions. Conceivably, these variable should carry information that helps predict future income and an individual may have fairly good expectations of their future disability and health status, which is not contemporaneously observed by an econometrician.

That said, when we experiment with including health and disability related variables in projection, we find that these variables do not significantly lower variances of projection residuals. Specifically, we find that including time t values of health and disability variables in the RIS specification yields essentially the same income uncertainty profiles as the one estimated without including these variables.[17] Further, even including future $t+s$ values of health and disability variables does not appear to meaningfully lower the estimated income uncertainty profiles.

These results are somewhat puzzling because the estimated coefficients of disability/health variables are typically statistically and economically significant. For example, for 5-year ahead projection, relative to individuals who do not have disability-related work limits, one with severe limits would have 20% lower labor earnings and one with moderate

[16]We thank Fatih Guvenen, the editor, for suggesting exploring this topic.

[17]Because the health and disability information was not collected over the entire PSID sample period, our comparison uses data collected after 1984, a period when health and disability variables were collected.

limits would have 8% lower labor earnings. Similarly, relative to individuals who reported having excellent health conditions, one with good conditions and fair conditions has 5% and 12% lower labor earning, respectively. All coefficients are significant at the 99.9% level.

Why do these apparently relevant variables not beef up the accuracy of future income projections? For disability related information, the subdued contribution could be partly due to the fairly small number of observations in our sample that reported disability having limited their work ability (fewer than 15% of individuals in our sample reported to have some disability related work limits). Moreover, it is possible that the rather coarse categories defined for health and disability variables in the PSID data have limited the extent to which they can account for income variability.

5.2 Correlations of Projection Errors

Besides the levels of income uncertainties over the life cycle, to completely characterize the income uncertainty, we must know how the stochastic components of income at different horizons are correlated. Figure 3 presents the correlations between the one-year-ahead projection errors and the projection errors at other horizons. To keep the graph readable, we only plot the correlations for households whose heads are 30, 40, and 50 years old, using the projections of the RIS equation. The AIS results are very similar. We note that, on the one hand, because the correlations are positive at all future horizons, income shocks must have a persistent component. On the other hand, the correlations decline substantially with future horizon. The correlation between the one-year-ahead and the two-year-ahead projection errors is about 0.5, whereas the correlation between the one-year-ahead and ten-year-ahead projection errors is only about 0.2. This rapid decline suggests that the persistent component of income shocks is more likely to be an AR process than a unit-root process. This is consistent with Alan and Browning (2010) and Gustavsson and ´Osterholm (2010), which also find lower income risk and lower persistence in income shocks compared to the traditional approaches. Although we cannot rule out the existence of a unit-root component, if one exists its innovations should have a relatively small variance. Indeed, some recent studies, such as Karakan and Ozkan (2009) and Hryshko (2010) work with an income process that includes both a permanent and an AR(1) component in addition to a transitory shock. Finally, the chart also reveals that the correlations of households of various ages are very similar, suggesting little change in the persistence of income shocks over the life cycle, in contrast to the evolution of shock variances.

5.3 Robustness Tests

First, because some of the variables (such as the occupation, industry and self-employment dummies) that we included in the projection equations are potentially endogenous to an individuals' unobserved characteristics, coefficients of these variables could be estimated with a bias, which could in turn affect projection residuals. We find that the effects of such biases on our estimated income uncertainty profiles are quantitatively small. Estimating a projection equation that includes only current income, race, educational attainment and an age-polynomial, i.e., restricting the coefficients of all possibly endogenous variables to zero, the dynamics of uncertainty estimated for family non-asset income and male labor earnings are qualitatively the same and, on average, only 5% higher than the RIS estimates.[18]

Second, recalling that we project the future income of households at different ages using the same set of coefficients, we examine whether changes in uncertainty over the life cycle are due to such a potential model misspecification. Specifically, if the projection-equation coefficients should be age specific and the coefficients we use are closer to the true parameters for the middle-aged households than for the younger and older households, the one-size-fits-all approach will reduce fitness for younger and older households and artificially increase the estimated income uncertainties for these age groups. We divide our sample into five subgroups by household head age and estimate Eqs. (7) and (8) separately for each

[18]These results are available from the authors upon request. Note that this specification does not include lagged income that is part of the RIS and AIS equations, suggesting that including lagged income in projection does not materially change the results.

subgroup. Then, we calculate variances of projection errors constructed using the revised projection coefficients. We find that both the uncertainty age profiles and correlation curves are similar to what Figures 1 and 3 show.

Third, in the same spirit, we examine the effects of allowing for time-varying projection coefficients that households are assumed to have no foresight on. We split the sample into two parts and estimate the coefficients using the earlier sample. We then use the estimated coefficients to project future income in the later sample. This method assumes that households are backward looking in figuring out the coefficients applicable to the future and can be exceedingly restrictive since it implies one is inferring returns to college education in the 1990s using the returns of the 1970s. The purpose of the analysis is to illustrate how much bias can potentially be introduced if we ignore the time dependence of the projection coefficients. We only do this analysis up to ten years ahead because we need long enough subsamples to estimate the coefficients. We find that the contours of the life cycle profiles of income uncertainty and the correlations of the projection errors estimated using this method are very similar to those estimated assuming time-invariant coefficients. Not surprisingly, the levels of income uncertainty are somewhat higher. However, this gap narrows consistently with projection horizons. At the near term, the backward-looking estimates of variances of projection errors are about 17% higher than in the baseline specification, and at the ten-year ahead horizon, the gap becomes 7%.

Finally, we examine whether changes in the sample size as we vary future horizons (as given in Table 1) might drive the shape of the uncertainty profile. We estimate the projection equations using a smaller common sample that spans at least ten years and reconstruct the income uncertainty measures up to ten years ahead. Again, all results are qualitatively unchanged.

6 Comparison with Earlier Results

How substantive are the innovations we have introduced with the new measures of income uncertainty? How different are our results compared to previous studies? We answer these questions by contrasting our results to the income uncertainty estimates in the influential work of Carroll and Samwick (1997). Three reasons lead us to choose these results to compare with. First, their specific interest was precautionary saving, so their estimates focus explicitly on income uncertainty rather than inequality, volatility, or other types of income variability. Second, they also use the PSID data, making a comparison and interpreting the differences easier. Third, apart from the specified innovations, our models share many similarities with theirs.

As we summarized in Eqs. (1)-(2), Carroll and Samwick (1997) decompose y_t into a permanent component, p_t, and a transitory shock, ε_t. The permanent component, p_t, is further assumed to follow a random walk—(2). Let σ_η^2 and σ_ε^2 be the variance of the permanent shock, η_t, and transitory shock, ε_t, respectively. Define VAR_d as the variance of the d-year income difference. Filtering out g_t, it is easy to show that

$$VAR_d = Var[y_{t+d} - y_t] = d\sigma_\eta^2 + 2\sigma_\varepsilon^2, \tag{9}$$

noting that the econometrician does not know how either y_t or y_{t+d} decomposes into their permanent and transitory parts. The innovation variances σ_η^2 and σ_ε^2 can be estimated by evaluating $Var[y_{t+d} - y_t]$ at various difference lengths, d. Using a PSID sample from 1981 to 1987, Carroll and Samwick (1997) report $\sigma_\eta^2 = 0.022$, and $\sigma_\epsilon^2 = 0.044$. Gourinchas and Parker (2002) report almost identical results. These parameters are changed noticeably when we estimate their model using an extended PSID sample that covers a longer period, from 1968 to 2005. The updated estimates call for a significantly larger variance of the transitory income shocks, $\sigma_\varepsilon^2 = 0.054$, and a significantly smaller variance of permanent shocks, $\sigma_\eta^2 = 0.012$.[19]

[19] We will use these updated estimates in Section 7 to study the life cycle consumption profile.

Figure 4 contrasts the total family non-asset income uncertainty at various future horizons implied by the original and updated Carroll and Samwick estimates with our estimates (pooled across all ages). The σ_η^2 and σ_ε^2 reported in Carroll and Samwick (1997) imply a higher and steeper linear profile, whereas the σ_η^2 and σ_ϵ^2 that we estimated using the same methodology with the extended PSID sample imply a lower and flatter profile. The linearity between income uncertainties and future horizons arises because of the random walk assumption imposed. The slope of the linear profile is equal to the variance of permanent shocks, and the intercept is equal to two times the variance of transitory shocks. The higher concave curve is the profile estimated using the RIS equation, while the lower concave profile is estimated using the AIS equation.

Several features of this chart are noteworthy. First, we notice that the profile for the original Carroll-Samwick estimates is substantially higher than the profiles estimated using our approach even under the restrictive information specification, the RIS equation. At near horizons, the gap is about 20%, whereas at remote horizons, the wedge widens substantially to about 60%. Second, the updated Carroll-Samwick profile has a higher intercept but a flatter slope due to the larger σ_ϵ^2 but smaller σ_η^2. Third, the updated Carroll and Samwick profile is higher than the RIS profile in the near term but coincides with the RIS profile beyond five years. The updated Carroll and Samwick profile remains uniformly and substantially higher than the AIS profile. This gap is more striking, taking into account that the predictable component of income is constructed using age $t+s$ information in the Carroll and Samwick (1997) approach, whereas our approach conditions on a mix of age t and age $t+s$ information. Moreover, not shown in the chart, the updated Carroll-Samwick profile is also higher than the profile allowing for time-varying projection coefficients and assuming backward-looking households (the second robustness check in Section 5.3).

Why are the uncertainty profiles implied by the original and updated Carroll and Samwick estimates so different? We notice that in Carroll and Samwick (1997), the maximum difference length is six, which is relatively small. Using a longer panel not only adds data collected in more years, but also allows us to study the difference in income over longer intervals. To assure the comparability of our PSID sample with theirs, we first set $max[d] = 6$ and found the estimated transitory and permanent shocks variances to be 0.036 and 0.023, respectively. These estimates are very close to Carroll and Samwick (1997). Subsequently, we update these estimates for various choices of $max[d]$ to examine whether the estimates are sensitive to variation in d.

Figure 5 contrasts how the estimated variances of transitory and permanent shocks, identified by the specification of Eq. (9), vary with $max[d]$. Because we project future income up to 25 years ahead, we choose the largest value of $max[d]$ to be 25. The permanent shock variance decreases, whereas the transitory shock variance increases monotonically with $max[d]$.[20] This phenomenon implies either that the variance of the permanent shocks—if the persistent shock process is indeed a random walk—is estimated with a significant upward bias using a short panel, or that the persistent shocks do not follow a random walk process. To see this, suppose the true model of the shocks to "permanent" income is

$$p_t = \rho p_{t-1} + \eta_t, \tag{10}$$

instead of a unit-root process $p_t = p_{t-1} + \eta_t$. After some algebra, we can show that VAR_d is not equal to $d\sigma_\eta^2 + 2\sigma_\epsilon^2$ as in Eq. (9). Rather, we have

$$VAR_d = Var[y_{t+d} - y_t|y_t] = \frac{1-\rho^{2d}}{1-\rho^2}\sigma_\eta^2 + [1+\rho^{2d}]\sigma_\varepsilon^2. \tag{11}$$

Using L'Hôpital's Rule, it is easy to verify that (9) is the limiting case when $\rho \to 1$. If Eq. (9) holds, σ_η^2 can be calculated by taking the difference $VAR_{d+1} - VAR_d$. The Carroll and Samwick estimates can be viewed as a weighted average of such variances

[20]We do not restrict the samples used for different $max[d]$ to contain the same set of households. In panel data, the number of households participating continuously in the survey declines over time. Moreover, the sample attrition is likely non-random if households experiencing greater income shock are more likely to leave the sample. We find that when estimating the model for various $max[d] < 25$ using the same sample of households for $d = 25$, the estimated variances of permanent and transitory shocks are considerably smaller than shown in Figure 5 but permanent shock variance remains decreasing with $max[d]$ where as transitory shock variance remains increasing with $max[d]$.

across $d \leq max[d]$. However, if Eq. (11) is the true model, taking the difference between VAR_{d+1} and VAR_d, we get the presumed estimate of the variance of permanent shocks,

$$VAR_{d+1} - VAR_d = \rho^{2d}[\sigma_\eta^2 + (\rho^2 - 1)\sigma_\varepsilon^2]. \tag{12}$$

Assuming ρ is close to 1 (e.g., $\rho = 0.9$) and σ_η^2 and σ_ε^2 are of the same order of magnitude, then $VAR_{d+1} - VAR_d$ will be a decreasing function of d. Therefore, when we increase $max[d]$, the average of $VAR_{d+1} - VAR_d$ over $d \leq max[d]$ also decreases. Meanwhile, a downward biased estimate of the permanent shock variance leads to an upward biased estimate of the transitory shock variance.

Finally, Carroll and Samwick (1997) also estimate the variance of the permanent and transitory income shocks by age. However, their estimates do not imply a U-shaped life cycle pattern. Indeed, their transitory shock variance exhibits a hump-shaped pattern over the life cycle, peaking in the early forties, whereas their permanent shock variance demonstrates more irregular life cycle dynamics.

To summarize, in contrast to Carroll and Samwick (1997) and later work using similar methods (e.g., Gourinchas and Parker (2002)), the estimates of income uncertainty introduced in this paper reveal a uniformly lower level of income uncertainty over all future horizons, and suggest that the persistent component of income shocks is likely not a permanent shock. Our approach also implies life cycle dynamics of income uncertainty that are more consistent with layman's intuition and the results of Baker and Solon (2003) and Gordon (1984).

7 Implications for Consumption and Precautionary Saving over the Life Cycle

As we discussed at the beginning of the paper, a battery of household decisions critically depend on households' uncertainty about their future income. We revisit one such question, the optimal quantity of consumption and precautionary saving over the life cycle, as an example that illustrates how household decisions under the income uncertainty estimates introduced in this paper differ from those derived using preexisting income uncertainty estimates.

7.1 Background

The canonical Rational-Expectations Life Cycle/Permanent-Income Hypothesis (RE-LCPIH) predicts that rational consumers should allocate consumption over the life cycle in such a way as to maximize lifetime utility, which in turn implies a monotonic consumption profile over the life cycle. However, since Thurow (1969), many empirical studies have documented a hump-shaped pattern of life cycle consumption. In light of this apparent inconsistency, a large volume of literature has added various features to the standard RE-LCPIH model to account for a hump-shaped consumption profile.[21]

We focus on one of the main factors raised to explain the hump-shaped consumption profile—precautionary saving—for it directly speaks to the effect of income uncertainty. Nagatani (1972) first suggested that precautionary saving would reduce consumption early in the life cycle, and Skinner (1988) and Feigenbaum (2008a) have fleshed out how the growth rate of mean consumption from one period to the next increases with income uncertainty. If income uncertainty decreases over the life cycle, this can lead to a concave consumption profile. It has been shown extensively that, in partial-equilibrium models calibrated against the measures of uncertainty described in Section 2, precautionary motives can induce a sizable hump (see for example, Carroll and Summers (1991); Hubbard, Skinner, and Zeldes (1994); Carroll (1997); Gourinchas and Parker (2002); and Feigenbaum (2008b)).

However, it is not likely that precautionary saving single-handedly causes the hump, for other factors, such as leisure and consumption substitution (Heckman (1974); and Becker and Ghez (1975); and Bullard and Feigenbaum (2007)) or time-varying mortality

[21] For a more detailed review of this literature, see Browning and Crossley (2001).

risk (Feigenbaum (2008c) and Hansen and Imrohoroglu (2008)), can also account for the hump. This insight imposes a challenge to the existing parametric estimates of income uncertainty as the models calibrated with these estimates often yield a consumption hump that is larger than observed in data. We introduce

$$S = \frac{E[c_{t^*}]}{E[c_0]}$$

as a measure of the magnitude of the consumption hump, where t^* is the age that maximizes $E[c_t]$. The data constructed in Gourinchas and Parker (2002) suggest $S = 1.17$. However, as we will see below, under plausible choices of other parameters, the income risks reported in Gourinchas and Parker (2002) call for a much larger value of $S \approx 1.5$. Conceivably, adding other factors to these models would further exaggerate the size of the hump.

In contrast, here we show that with the income uncertainty estimates introduced in this paper, precautionary saving typically yields a substantially smaller hump than is observed in the data. The size of the hump is further dampened when we allow for age-dependent income uncertainty. This is because our estimates call for smaller variances and lower persistence, reducing the lifetime income uncertainty households face, in particular at a young age.

7.2 A Life Cycle Consumption-Saving Model with Age-Dependent Income Uncertainty

We wish to construct a life cycle model that incorporates the empirical results described in Section 5. For each age group, we have estimated variances of and correlations between projection errors at different time horizons. Note that the correlations are particularly important for inducing precautionary saving. If incomes at different ages are independent of each other, the variances of these incomes would have to be orders of magnitude larger than we observe in order to get a hump-shaped consumption profile with reasonable preference parameters.

It is not just the immediate uncertainty in the instant prior to receiving income that induces precautionary saving. Rather, Feigenbaum (2008b) shows that saving is induced by the household's uncertainty about all intervening shocks between the present age t and the future time $t+s$, when the income is earned, that reveal information about the value of this income. Thus the learning process plays an essential role in the precautionary saving mechanism.

However, accounting for these correlations introduces a major complication. The majority of the literature on precautionary saving discussed in Section 7.1 assumes that consumers have rational expectations. Consumers know the actual probability distribution for every possible sequence of income realizations. Given the history they observe as of age t, they use Bayes' Law to determine the conditional distribution of future incomes. It is straightforward to specify a probability distribution over continuation histories starting at age t that replicates our estimates of the second-order moments for age-t projection errors. However, there is no reason to expect that the age-$(t+1)$ probability distribution that matches these moments will be consistent with the probability distribution that we would obtain via Bayes' Law if we condition the age-t moment-matching probability distribution on the observation of age-$(t+1)$ income. Instead of imposing additional restrictions on estimates of the variances and correlations that would ensure consistency of income forecasts across time, we diverge from the rational-expecations paradigm and allow the consumer to have time-inconsistent expectations.

It is known that time-inconsistent consumption rules can produce a hump-shaped consumption profile (Caliendo and Aadland (2004)). We have also found that time-inconsistent expectations about the mean of income can produce a nonmonotonic consumption profile, so we impose time-consistent beliefs regarding the mean to eliminate this effect. Without also studying a time-consistent model where income uncertainty is not varying with age, we cannot separate the effects of precautionary saving and time-inconsistent expectations about second-order moments. However, because the volatility and correlation matrices vary smoothly with respect to both age and projection horizon, we think it is unlikely that the time-inconsistency of the model is playing a significant role in its own right since the expectations do not change abruptly from one period to the next.

We consider a partial-equilibrium model with a consumer who lives for T_w working periods and T_r retirement periods. Income is the only source of uncertainty in the model.[22] For each age $t = 0, ..., T_w - 2$, the consumer believes future income is determined by a stochastic process that matches the moments obtained from age-t income forecasts, and we do not require these beliefs to be consistent with Bayes' Law. Let $E^{(t)}$ be the expectation operator with respect to the consumer's beliefs at age t. An age-t consumer then maximizes

$$E^{(t)}\left[\sum_{s=t}^{T_w-1}\beta^{s-t}u(\widetilde{c}_s(t);\gamma)+\beta^{T_W-t}V_{T_w}(\widetilde{x}_{T_w}(t))\right],$$

where $c_s(t)$ is consumption planned for period s as of age t,

$$u(c;\gamma)=\begin{cases}\frac{c^{1-\gamma}}{1-\gamma} & \gamma\neq 1\\ \ln c & \gamma=1\end{cases} \tag{13}$$

for $\gamma > 0$, $x_{T_w}(t)$ is financial wealth at retirement, and $V_{T_W}(x_{T_W})$ is the retirement value function. Tildes denote random variables.

Let us suppose an age-t consumer believes income at age $s \geq t$, $y_s(t)$, can take on one of n values $Y_s^1(t) < Y_s^2(t) < Y_s^3(t) < \cdots < Y_s^n(t)$. We assume the probability distribution is a first-order Markov process with transition probabilities

$$\Pr{}^{(t)}[y_{s+1}(t)=Y_{s+1}^j(t)|y_s(t)=Y_s^i(t)]=\Pi_s^{ij}(t)>0 \tag{14}$$

for $i, j = 1, ..., n$. The expectation operator $E^{(t)}$ computes expectations with respect to this probability distribution.

We assume, for $0 \leq t < T_w$, that actual income at t, $y_t = y_t(t) \in \{Y_t^1, ..., Y_t^n\}$, where

$$Y_t^i = Y_t^i(t)$$

[22]Uncertainty about health and preferences can also generate precautionary saving and have an effect on the lifecycle profile. We focus on income uncertainty because data on income is more readily available.

for $i = 1, ..., n$ and $t = 0, ..., T_w - 1$. For $0 \leq t < T_w$, we denote the actual transition matrix by

$$\Pr[y_{t+1} = Y_{t+1}^j | y_t = Y_t^i] = \Pi_{ij} > 0 \tag{15}$$

for $i, j = 1, ..., n$. Denoting the invariant probability distribution of Π by π, we assume the actual initial probability distribution is

$$\Pr[y_0 = Y_0^i] = \pi_i.$$

Households can reallocate income across the life cycle using the one intertemporal asset in the economy, a risk-free bond that pays the fixed gross interest rate R. Let $b_{s+1}(t)$ denote the quantity of bonds an agent at age t plans to purchase at age s that would then pay $Rb_{s+1}(t)$ at age $s+1$. Thus the budget constraint an agent at age t expects to face at age s is

$$c_s(t) + b_{s+1}(t) = y_s(t) + Rb_s(t) \equiv x_s(t),$$

where $x_s(t)$ is cash on hand as defined by Deaton (1991).

Since our intent is to focus on the ability of uncertainty and precautionary saving to explain the consumption hump, we allow borrowing but with full commitment to debt contracts. Since consumption is required to be nonnegative, the consumer faces the endogenous borrowing limit (Aiyagari (1994)) that he would never borrow more than the minimum possible present value of income he might earn in the future, i.e. the minimum that he could possibly pay back in the future. However, there is potentially a disconnect between what the consumer believes this borrowing limit is and what the borrowing limit actually is. At age t an agent will believe at age s that the borrowing limit he faces is

$$B_{s+1}(t) = \sum_{i=s+1}^{T_w-1} \frac{Y_i^1(t)}{R^{i-s}}.$$

At age t, the actual borrowing limit should be

$$B_{t+1} = \sum_{i=t+1}^{T_w-1} \frac{Y_i^1}{R^{i-t}}.$$

To prevent the consumer from borrowing more than he actually can pay back, we impose the exogenous borrowing constraint[23]

$$b_{t+1}(t) \geq -B_{t+1}.$$

Thus, given y_t and b_t, the consumer's problem at age t is

$$\max_{\{c_s(t)\}_{s=t}^{T_W-1}, \{b_{s+1}(t)\}_{s=1}^{T_W-1}} E_t \left[\sum_{s=t}^{T_w-1} \beta^{s-t} u(\widetilde{c}_s(t); \gamma) + \beta^{T_W-t} \upsilon_{T_w}(R\widetilde{b}_{T_w}(t)) \right] \tag{16}$$

subject to

$$\begin{aligned} c_s(t) + b_{s+1}(t) &= y_s(t) + Rb_s(t) \quad s = t, ..., T_w - 1 \\ b_{t+1}(t) &\geq -B_{t+1}. \end{aligned}$$

7.3 Income Process Calibration

In this section, we calibrate a flexible but parsimonious income process that we can use to solve the model while replicating the second-order moments of the income process estimated in Section 5. Suppose that at age t, the consumer believes for $s \geq t$ that

$$y_s(t) = a_s p_s(t) z_s(t), \tag{17}$$

where a_s is an age-dependent factor, $p_s(t)$ is a persistent shock, and $z_s(t)$ is a temporary shock. Specifically, we assume that $p_s(t)$ and $z_s(t)$ are independent, unconditionally unit-mean processes such that

$$corr(ln(p_{s+1}(t)), ln(p_s(t))) = \rho < 1, \tag{18}$$

$$V[\ln p_{s+1}(t) - \rho \ln p_s(t) | p_s(t)] = \sigma_p^2(t), \tag{19}$$

[23] While imposing this constraint is necessary to ensure the model has a well-defined solution, the distinction between $B_{t+1}(t)$ and B_{t+1} is usually small, so the overwhelming majority of households will not be affected by the constraint.

and

$$V[\ln z_s(t)] = \sigma^2_{z,s-t}(t). \tag{20}$$

Note that the income specification of (17) differs from the standard specification of Gourinchas and Parker (2002) in two important respects. First, the permanent shocks do not follow a unit-root process, although this modification is less remarkable since many papers in the literature have considered an AR(1) income process, including Feigenbaum (2008a), Huggett (1996), and Storesletten, Telmer, and Yaron (2004). The second and more important difference is that the variance of the persistent shocks depends on the age t when projection occurs while the variance of the temporary shocks depends both on t and the projection horizon $s-t$.[24]

We assume that a_t is consistent across different ages since we are focusing on changes in the perception of the variance of income rather than on changes in the perception of the mean. We also assume that the autocorrelation ρ is consistent across ages as it is difficult to construct a Markov process that satisfies (18)-(20) if ρ is not constant.

We have

$$\ln y_s(t) = \ln a_s + \ln p_s(t) + \ln z_s(t).$$

For $h \geq 1$,

$$\begin{aligned} \ln y_{t+h}(t) &= \ln a_{t+h} + \ln p_{t+h}(t) + \ln z_{t+h}(t) \\ &= \rho^h \ln y_t + \ln a_{t+h} - \rho^h \ln a_t + \sum_{i=1}^{h} \rho^{h-i} \left(\ln p_{t+i}(t) - \rho \ln p_{t+i-1}(t)\right) \\ &\quad + \ln z_{t+h}(t) - \rho^h \ln z_t(t). \end{aligned}$$

Thus

$$V[\ln y_{t+h}(t)|y_t] = \frac{1-\rho^{2h}}{1-\rho^2}\sigma^2_p(t) + \sigma^2_{zh}(t) + \rho^{2h}\sigma^2_{z0}(t) \tag{21}$$

[24]There is no simple Markov process that can represent a persistent shock with a time-varying variance or autocorrelation.

For $h > 1$, the correlation between $\ln y_{t+h}(t)$ and $\ln y_{t+1}(t)$ conditional on y_t is

$$\begin{aligned} & corr(\ln y_{t+h}(t), \ln y_{t+1}(t)|y_t) \\ = & \frac{\rho^{h-1}\sigma_p^2(t) + \rho^{h+1}\sigma_{z0}^2(t)}{\sqrt{\left(\frac{1-\rho^{2h}}{1-\rho^2}\sigma_p^2(t) + \sigma_{zh}^2(t) + \rho^{2h}\sigma_{z0}^2(t)\right)\left(\sigma_p^2(t) + \sigma_{z1}^2(t) + \rho^2\sigma_{z0}^2(t)\right)}}. \end{aligned} \tag{22}$$

We calibrate ρ, $\sigma_p(t)$, and $\sigma_{zh}(t)$ so as to minimize the distance of the estimated correlation and variance matrices relative to their predicted values from (21) and (22). Specifically, we parameterize the permanent-shock standard deviation as a d_p-degree polynomial

$$\sigma_p(t) = \sum_{i=0}^{d_p} D_i^p t^i \tag{23}$$

and the temporary-shock standard deviation as a tensor product of d_z^t and d_z^h-degree polynomials

$$\sigma_{zh}(t) = \sum_{i=0}^{d_z^t} \sum_{j=0}^{d_z^h} D_{ij}^z t^i h^j. \tag{24}$$

Then we set ρ, $D_0^p, \ldots, D_{d_p}^p, D_{00}^z, \ldots, D_{0d_z^h}^z, D_{10}^z, \ldots, D_{1d_z^h}^z, \ldots, D_{d_z^t 0}^z, \ldots, D_{d_z^t d_z^h}^z$ to minimize the sum of the squares of the deviations between the predicted values of the matrix elements and their measured values. Note that, as is standard in this literature, the Bellman Eq. (28) implies that the household precisely knows its income state. Thus, in solving the model, we assume the consumer has additional information that the econometrician does not have when estimating the volatility and correlation matrices, for the consumer knows how his current income breaks down into permanent versus temporary income shocks.

Recall that we measure the volatility and correlation matrices for horizons up to $H_{\max} = 25$. For $t < T_w - H_{\max} - 2$ we also need to specify $\sigma_{zh}(t)$ for $h \in \{H_{\max}+1, ..., T_w-t-1\}$, but these standard deviations are not identified by the available data. Therefore, we linearly extrapolate (24) for $h > H_{\max}$. We also do not have information about $\sigma_{z0}(T_w - 1)$ since we have no data on projections in the last working period. However, we do have the volatility matrix element $V[\ln y_{T_w-1}(T_w - 2)|y_{T_w-2}]$, so it is reasonable to assume (24) will still be valid at $t = T_w - 1$. Likewise, we simply extrapolate (23) to obtain $\sigma_p(T_W - 1)$.

We consider the estimates obtained with both the AIS and RIS specifications of Section 5 with a cubic approximation that sets $d_p = d_z^t = d_z^h = 3$. To assess the importance of the age and horizon dependence of uncertainty we also consider a calibration of the income process where σ_p and σ_z are constants independent of t and h, so $d_p = d_z^t = d_z^h = 0$.

The age-dependent and age-independent calibrations of $\sigma_p(t)$ are plotted as a function of age t in Fig. 6 for both the AIS and RIS specifications. Likewise, the four calibrations of $\sigma_{zh}(t)$ are plotted as a function of age for representative horizons in Fig. 7. The correlation for each calibration is given in Table 2. The standard deviation of persistent income shocks is uniformly larger for the age-independent calibration than the age-dependent calibration and for some ages is twice as large. The correlations for the age-independent calibrations are modestly smaller than the corresponding age-dependent calibrations. At short time horizons the variance of temporary income shocks is comparable between the age-dependent and age-independent calibrations, but the variance of temporary income shocks increases with the projection horizon in the age-dependent model while necessarily remaining constant in the age-independent model. Thus the persistent income shocks will have greater emphasis in the age-independent models whereas temporary income shocks will have more emphasis in the age-dependent models.

As indicated by Fig. 7, the variance of the temporary income shocks increases with the projection horizon. The fact that the temporary shock variance for income at a fixed age in the future decreases as households approach this age implies that consumers learn a great deal about this income from other variables uncorrelated with income (such as the variables contained in the vectors $Z_{i,t}$ and $Q_{i,t+s}$ of Eqs. (7)-(8)). Since persistence in shocks can magnify precautionary saving, this raises the possibility that if we model the dynamics of these informative nonincome variables then we might get more precautionary saving than arises in the present model.

Fig. 8 compares the fitness of the income processes calibrated under different assumptions. We present the variances of projection errors at various future horizons constructed

Table 2: Calibrated Auto-correlation Coefficients

Model	ρ
AIS age-indep.	0.906
AIS age-dep.	0.946
RIS age-indep.	0.918
RIS age-dep.	0.953

Note: Correlation ρ for the income processes calibrated against the AIS and RIS estimates of the volatility and correlation matrices, with and without allowing for income uncertainty to be age-dependent.

using the income processes calibrated against the empirical life cycle income uncertainty profiles estimated using the RIS model.[25] Not surprisingly, allowing for age-dependent income uncertainty vastly improves the fitness of our calibration.

In addition to the income process, we also have to choose plausible values for the preference parameters β and γ, and since this is a partial-equilibrium model the gross interest rate R.[26] We set the interest rate to a common value from the literature: $R = 1.035$. Most estimates of the risk aversion γ that do not depend on equity premium data lie in the range 1 to 3. Since precautionary saving depends on risk aversion and we wish to see how much precautionary saving can account for the consumption hump, we settle on the upper bound of 3, which was also used by Feigenbaum (2008b). Note that the slope of the consumption profiles after any uncertainty has been revealed will be $(\beta R)^{1/\gamma} - 1$. If we set the discount factor to the reasonable value of $\beta = 0.933$, we obtain life cycle consumption profiles that run parallel to the data at retirement, though they are not constrained to replicate any other aspect of the life cycle consumption data.

7.4 Model Predictions for Consumption Hump

Virtually every theoretical explanation for the hump described in Section 7.1 can quantitatively account for the hump if both the interest rate R and discount factor β are free

[25]The AIS comparisons are very similar.

[26]We must also discretize the income process. Details are given in Appendix A.

parameters to be calibrated. Bullard and Feigenbaum (2007) and Feigenbaum (2008a) have emphasized the importance of studying the consumption hump with general-equilibrium models that put some discipline on the choice of β and R. However, because of the time-inconsistent nature of our calibrated income process, we do not endogenize R. The purpose of this theoretical exercise is only to see whether, for any plausible values of β, γ, and R, there is enough income uncertainty to induce a consumption hump.

Fig. 10 shows life cycle profiles for mean consumption for the age-dependent and age-independent calibrations under both the AIS and RIS specifications along with Gourinchas and Parker's empirical measurements of the mean consumption profile.[27] Since we have estimated the mean income profile differently from Gourinchas and Parker (2002), it is not informative to compare the absolute values of consumption in their data to the predictions of our model, which crucially depend on the absolute value of income. Consequently, we follow the procedure of Bullard and Feigenbaum (2007) and plot mean consumption relative to age-30 mean consumption. For comparison with the previous literature, we also include the consumption profile implied by the estimates of income shock variances used in Gourinchas and Parker (2002), $\sigma_p^2 = 0.021$ and $\sigma_z^2 = 0.044$, which are similar to what Carroll and Samwick (1997) report. We label this consumption profile as GP 2002. In Section 6 we reestimated the Carroll and Samwick (1997) specification using a longer panel, obtaining a larger temporary shock variance $\sigma_z^2 = 0.071$ and a smaller permanent shock variance $\sigma_p^2 = 0.009$. The consumption profiles that result with these parameters are also included in Fig. 10 (labelled as GP updated).[28] In Table 3, we also report the peak to initial consumption of the life cycle profile of mean consumption and the age of peak consumption for all consumption profiles presented in Fig. 10.

Since $\beta R < 1$, in the absence of uncertainty the life cycle consumption profile should

[27]The graphs in Fig. 10 and ensuing figures were obtained by simulating one million life cycle paths per age group. Appendix B describes the details of the computational procedure.

[28]Note that in the Gourinchas and Parker framework the nontransitory shock is assumed to be permanent, i.e., $\rho = 1$. This is not nested within the framework of Section 7.2. Consequently, these models have to be solved using the method described in Feigenbaum (2008b).

Table 3: Peak-to-Initial Consumption Ratios

Model	$\frac{E[c_{t_{\max}}]}{E[c_0]}$	$t_{\max}$
GP DATA	1.17	45
AIS age-dep.	1.03	38
RIS age-dep.	1.05	41
AIS age-indep.	1.05	49
RIS age-indep.	1.15	51
GP 2002	1.54	54
GP UPDATED	1.22	52

Note: Ratio of peak to initial consumption and peak age for the lifecycle consumption profile as measured by Gourinchas and Parker (2002) and as predicted by the model with income processes that have age-dependent and age-independent uncertainty, calibrated for both the RIS and AIS specifications. For comparison, we also include income processes similar to Gourinchas and Parker's (2002) baseline estimate and an updated estimate of their process using a longer sample.

be monotonically decreasing with a peak at the initial age of 25, in which case the peak to initial consumption ratio would simply be 1. If there is a consumption hump, it will be great than 1. Thus the peak to initial consumption ratio can be viewed here as a measure of how much precautionary saving causes these models to deviate from the LCPIH. Since we are also holding the degree of risk aversion fixed, the variation in precautionary saving over different income processes should only reflect the amount of uncertainty faced over the life cycle under each process.

First note that the original Gourinchas and Parker (2002) income uncertainty estimates call for a substantially larger peak to initial consumption ratio than the four models we estimate since the former implies the highest cumulative income uncertainty over the life cycle.[29] On the other hand, the GP updated profile exhibits much less uncertainty. The hump it produces with this preference calibration is only slightly larger than what is exhibited in the data, with a peak size of $S = 1.22$ as opposed to 1.17 in the data. For most

[29]Indeed, for $\beta = 0.96$ and $R = 1.035$, Gourinchas and Parker (2002) have to dial the risk aversion all the way down to one half to get a life cycle consumption profile that resembles the data using their estimates of the income process.

of the life cycle, this profile nearly coincides with the largest of the profiles obtained by our new procedure—the age-independent RIS model.[30]

Not surprisingly, since the AIS specification assumes households have more information than the RIS specification, the two AIS models have smaller peak to initial consumption ratios than their corresponding RIS models. For both the RIS and AIS specifications, we find that the age-dependent model has a significantly smaller peak to initial consumption ratio than the corresponding age-independent model. This can be explained in terms of Figs. 6 and 7. Because the age-independent model assumes a constant variance that does not vary with age and projection horizon, it needs a large persistent shock variance to best match the volatility and correlation matrices. In contrast, the age-dependent model has a smaller persistent shock variance and exploits its ability to increase the temporary shock variance at longer projection horizons to better match the projection-error moments. As Constantinides and Duffie (1996) argued, permanent shocks will have a substantially larger impact on the behavior of consumers, and Feigenbaum (2008a, 2008b) confirm that more persistent income shocks lead to greater precautionary saving, an intuition further corroborated here. Thus, failing to account for the time and projection-horizon dependence of uncertainty may bias upward estimates of the importance of precautionary saving.

If we account for the age-dependence of uncertainty, the RIS specification gives a consumption profile with a peak to initial ratio of 1.05, which is much smaller than what is evident in the data. The AIS specification gives an even smaller hump with $S = 1.03$. Since we have chosen the largest plausible risk aversion parameter, our analysis suggests that there is not enough income uncertainty for precautionary saving generated by income shocks to explain the observed consumption hump. Other factors such as leisure-consumption substitution must also be contributing to the hump.

[30]As is shown in Section 6, the updated Carroll and Samwick model perform about as well as our RIS model at all but the shortest projection horizons.

8 Conclusion

We propose a novel approach to measuring household income uncertainty and study the dynamics of income uncertainty over the life cycle. Our estimates of income uncertainty are typically smaller than previous studies have documented and imply less persistence in income shocks. Moreover, we find that income uncertainties evolve noticeably over the life cycle. Young and old consumers on average have more risky future income relative to middle-age consumers. In a companion paper, Feigenbaum and Li (2011) study how income uncertainty has changed over time using the same uncertainty measure. We found that household income uncertainty rose noticeably in the last three decades and the increase was widespread across demographic and income groups.

A wide variety of theoretical questions related to household financial decisions over the life cycle can be revisited using the metric of income uncertainty we have introduced. For example, we study the optimal consumption over the life cycle in this paper. Deviating from the standard rational expectations framework, we calibrate a sequence of income processes corresponding to consumers' belief about future income that match with the second moments of our estimated income process, not requiring these beliefs to be time consistent in a Bayesian sense. We then show that, in contrast to the existing literature, the life cycle profile of optimal consumption implied by our estimates exhibits a hump that is much smaller than observed in the data, in particular when income uncertainty is allowed to be age dependent. Our results suggest that precautionary saving by itself can account for, at the best, a fraction of the life cycle consumption dynamics. Our future research will focus on constructing a time-consistent income process that matches with our estimates of age-dependent income shock variances, which is necessary for analyzing the related effects in a general equilibrium model as well as quantifying the effects of time-inconsistent expectations.

A Discretizing the Income Process

To discretize the income process, we restrict $\ln p_s(t)$ to take on values such that

$$\frac{\ln p_s(t)}{\sigma_p(t)} \in \{P^1, ..., P^{n_p}\}$$

and $\ln z_s(t)$ to take on values such that

$$\frac{\ln z_s(t)}{\sigma_{z,s-t}(t)} \in \{Z^1, ..., Z^{n_z}\}.$$

Defining

$$P_s(t) = \frac{\ln p_s(t)}{\sigma_p(t)}$$

and

$$Z_s(t) = \frac{\ln z_s(t)}{\sigma_{z,s-t}(t)},$$

we specify an i.i.d. probability distribution for $Z_s(t)$,

$$\Pr[Z_s(t) = Z^i] = \pi_i^z \tag{25}$$

for $i = 1, ..., n_z$, and a Markov distribution for $P_s(t)$,

$$\Pr[P_{s+1}(t) = P^j | P_s(t) = P^i] = \Pi_{ij}^p \tag{26}$$

for $i, j = 1, ..., n_p$ and $s \geq t$. Thus the total number of income states at age t and horizon s is $n = n_p n_z$. For

$$k = n_z(i-1) + j, \tag{27}$$

where $i = 1, ..., n_p$ and $j = 1, ..., n_z$, the kth income state is

$$Y_s^k(t) = a_s \exp(\sigma_p(t) P^i(t)) \exp(\sigma_{z,s-t}(t) Z^j).$$

Note that this specifies the time-inconsistent probability distribution as perceived by the household. We must also specify the Markov process of the actual probability distribution for realized income y_t. Because the subjective expectations of the consumer are time-inconsistent, they do not necessarily imply a consistent probability distribution for the actual realizations of income. However, the specification of the actual probability distribution should not have any great effect on the shape of the consumption hump.[31] Consequently, we assume the objective income process between t and $t+1$ corresponds to the subjective process.

Since we assume that the household's perceptions of its current temporary and permanent shocks are correct, we have

$$y_t = y_t(t) = a_t \exp(\sigma_p(t) P_t) \exp(\sigma_{z0}(t) Z_t),$$

where

$$\begin{aligned} P_t &= P_t(t) \\ Z_t &= Z_t(t). \end{aligned}$$

This implies the set of actual income states for y_t coincides with the perceived states at t, so

$$Y_t^k = Y_t^k(t)$$

for $k = 1, ..., n$.

Let π_i^p denote the invariant distribution of Π^p. We then assume the actual probability distribution of P_0 is

$$\Pr[P_0 = P^i] = \pi_i^p$$

for $i = 1, ..., n_p$ and the actual probability distribution of Z_0 is

$$\Pr[Z_0 = Z^i] = \pi_i^z$$

[31] The shape of the mean consumption profile is ultimately determined by the mean rate of consumption growth. Feigenbaum (2008b) and Skinner (1988) have shown that the expected rate of consumption growth depends nonlinearly on wealth, so the mean rate of consumption growth will depend on the distribution of wealth. However, this wealth effect is proportional to the variance of income and so is second-order in terms of deviations of income from the mean. Since the first-order effect of deviations from the mean will, by definition, vanish after taking expectations, the lowest order effect of any dispersion in the wealth distribution is also second order. Thus the aggregate effect of deviations of wealth from mean wealth is a second-order correction to a second-order effect, making it a fourth-order effect. The effect of the wealth distribution on the shape of the life cycle consumption profile should therefore be negligible.

for $i = 1, ..., n_z$. For $0 \leq t < T_w - 1$, we assume that (25) and (26) correctly specify the probability distribution for P_{t+1} and Z_{t+1}, so

$$\Pr[Z_{t+1} = Z^i] = \pi_i^z$$

for $i = 1, ..., n_z$ and

$$\Pr[P_{t+1} = P^j | P_t = P^i] = \Pi_{ij}^p.$$

Note that this implies the unconditional distribution of P_t is π^p for all t.

For the income process, we assume both the permanent and temporary shocks are governed by two-state processes. The temporary shocks are parameterized by $Z^2 = -Z^1 = 1$ and $\pi_1^z = \pi_2^z = 1/2$. Likewise, the permanent shocks are parameterized by

$$P^2 = -P^1 = \frac{1}{\sqrt{1-\rho^2}}$$

and $\pi_1^p = \pi_2^p = 1/2$. The transition matrix for the permanent shocks is

$$\Pi^p = \frac{1}{2}\begin{bmatrix} 1+\rho & 1-\rho \\ 1-\rho & 1+\rho \end{bmatrix}.$$

B Computational Procedure

We can write the problem (16) as a recursive sequence of Bellman equations as follows. Let us denote the state variable $I_s(t)$ such that $y_s(t) = Y_s^{I_s(t)}(t)$. For each base age $t \in \{0, ..., T\}$, the consumer solves for the age-t perceived value functions $V_s(x_s, I_s(t); t)$ at current and future ages $s = t, ..., T_w - 1$. For $s = t, ..., T_w - 1$, the Bellman equation is

$$\begin{aligned} V_s(x_s, I_s(t); t) \quad = \quad & \max_{c_s(t), b_{s+1}(t)} u(c_s(t); \gamma) \\ & + \beta E^{(t)}[V_{s+1}(Y_{s+1}^{\widetilde{I}_{s+1}(t)}(t) + R b_{s+1}(t), \widetilde{I}_{s+1}(t); t) | I_s(t)] \end{aligned} \tag{28}$$

subject to

$$c_s(t) + b_{s+1}(t) = x_s$$

For the case when $s = t$, we impose the additional constraint

$$b_{t+1}(t) \geq -B_{t+1}.$$

The retirement value function that terminates the sequence of value functions is simply the perfect-foresight utility of an agent who will live for T_r periods with no further income. With CRRA utility (13), this is

$$V_{T_w}(x_{T_w}) = \left(\frac{\phi^{-T_r} - 1}{\phi^{-1} - 1} \right)^{\gamma} u(x_{T_w}; \gamma), \tag{29}$$

where

$$\phi = (\beta R^{1-\gamma})^{-1/\gamma}$$

is the inverse marginal propensity to consume in the limit of large lifetimes. The sequence of Bellman equations (28) can easily be translated into the sequence of Bellman equations solved in Feigenbaum (2008b), and the computational procedure we use here is described in the technical appendix for that paper.[32] Note that only the policy functions $c_t(x_t, I_t; t)$ for $t = 0, ..., T_w - 1$ are actually relevant to the behavior of the model.

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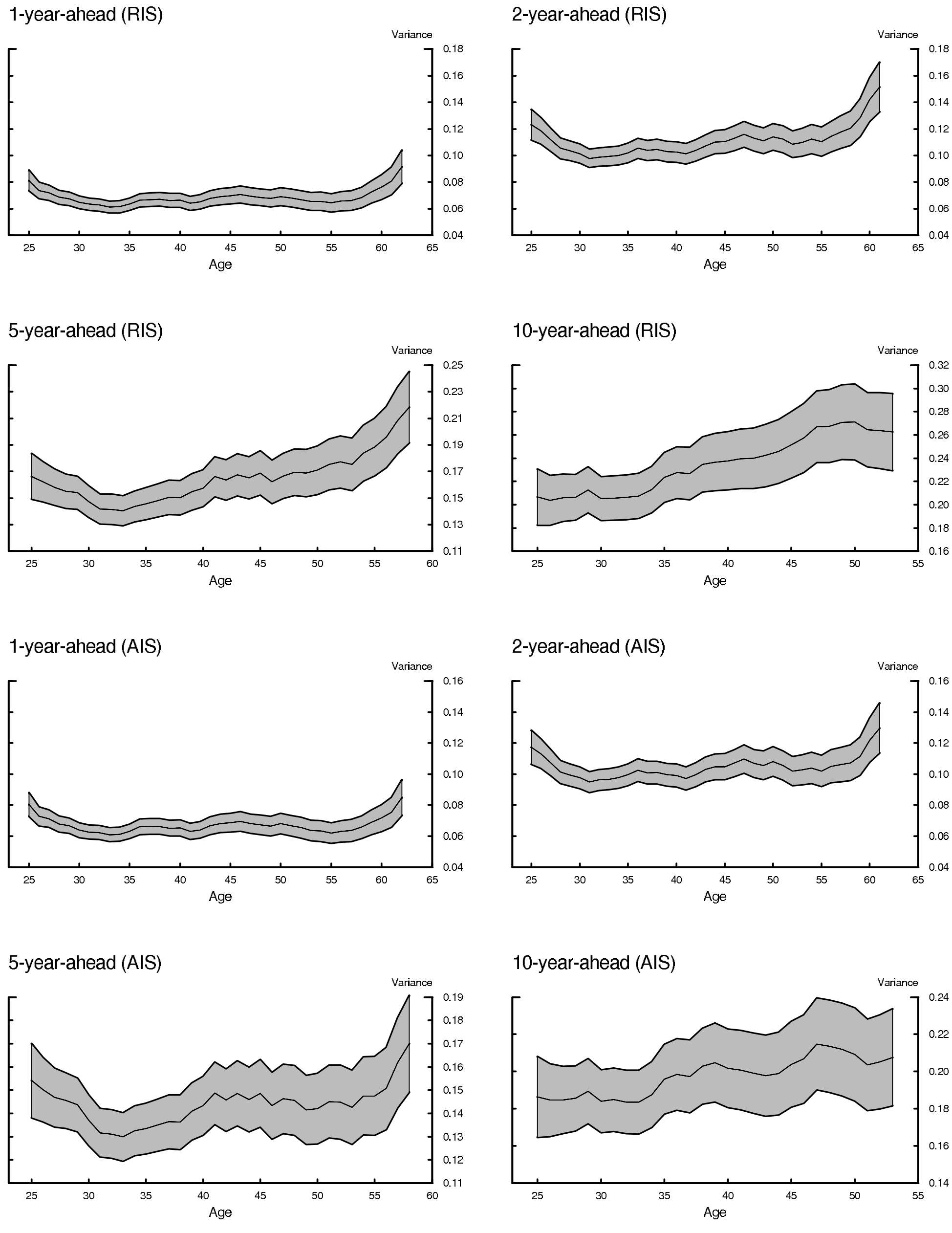
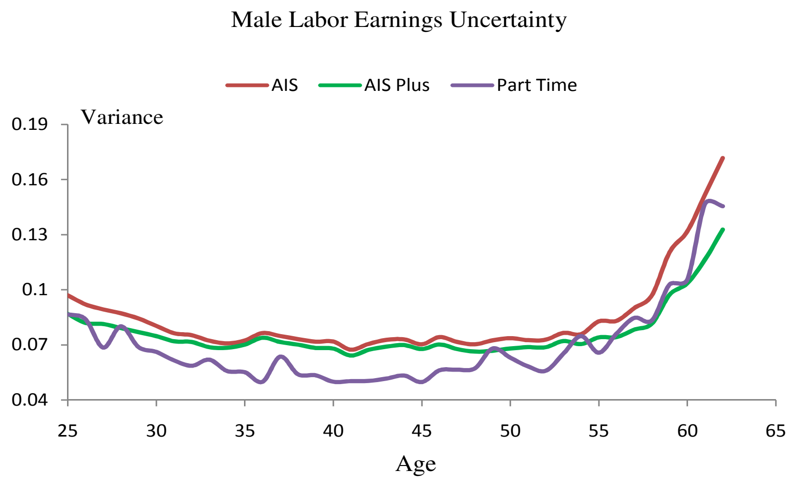
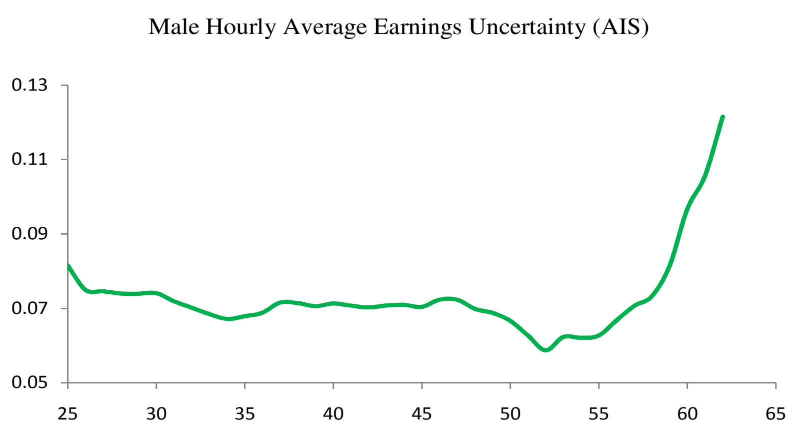
Figure 1: Income uncertainty over the life cycle—family noncapital income



Note: The upper four panels plot the age-profiles of family noncapital income uncertainty estimated using the RIS specification (Eq. 7) for 1-year, 2-year, 5-year, and 10-year-ahead, respectively. The lower four panels plot the corresponding age-profiles estimated using the AIS specification (Eq. 8). The shaded areas are the 95% confidence intervals.

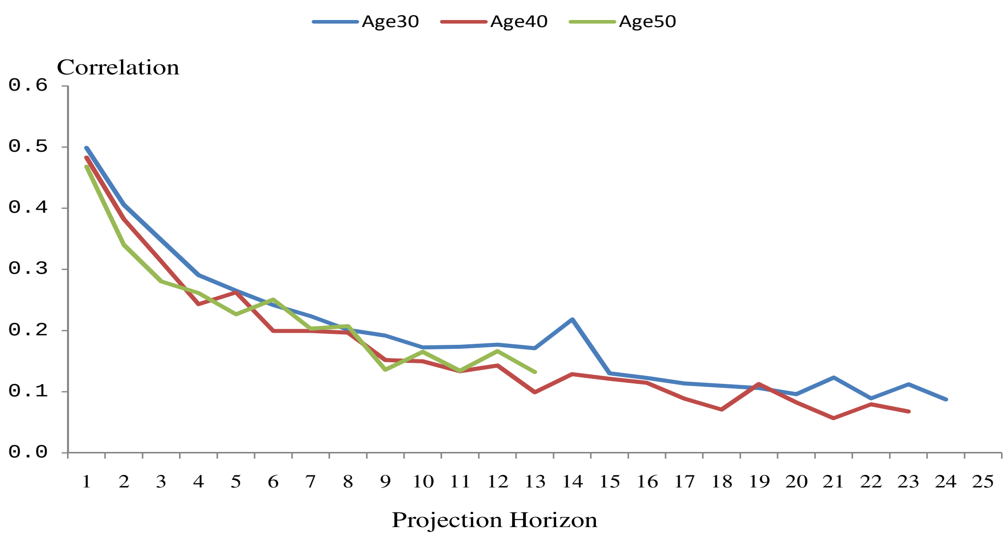
Figure 2: Income uncertainty over the life cycle—male labor earnings and male average hourly earnings





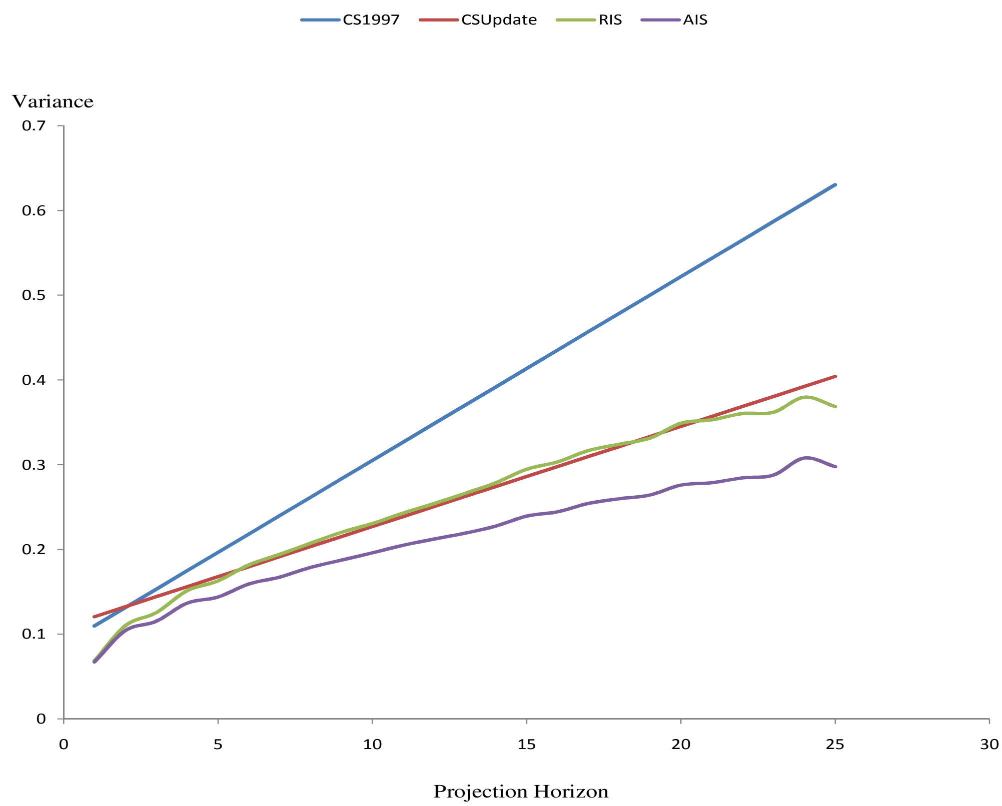
Note: The green and curves in the upper panel plot the age-profiles of male labor earnings 1-year-ahead uncertainty estimated using the AIS specification (Eq. 7) with and without controlling for future part-time status, respectively. The purple curve in the upper panel shows the share of males in our sample working part-time at each age. The lower panel plots the age-profile of male average hourly earnings uncertainty estimated using the AIS

Figure 3: Correlation structure of projection errors—family noncapital income



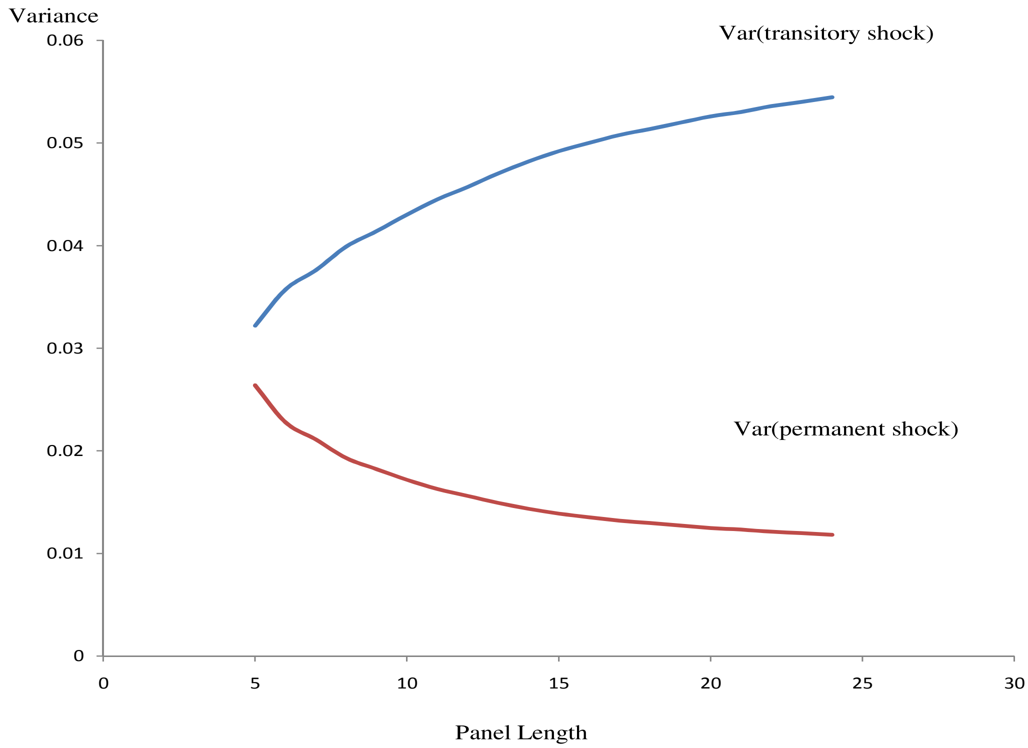
Note: The figure presents how projection residuals for various forecasting horizons beyond 1-year-ahead are correlated with the 1-year-ahead projection residuals of the same individual. The blue curve shows the correlation series for individuals 30-year old at the base year; the red and green curves for 40 and 50-year old individuals, respectively.

Figure 4: Comparison of the new estimates of income uncertainty with preexisting estimates (pooled across all ages)



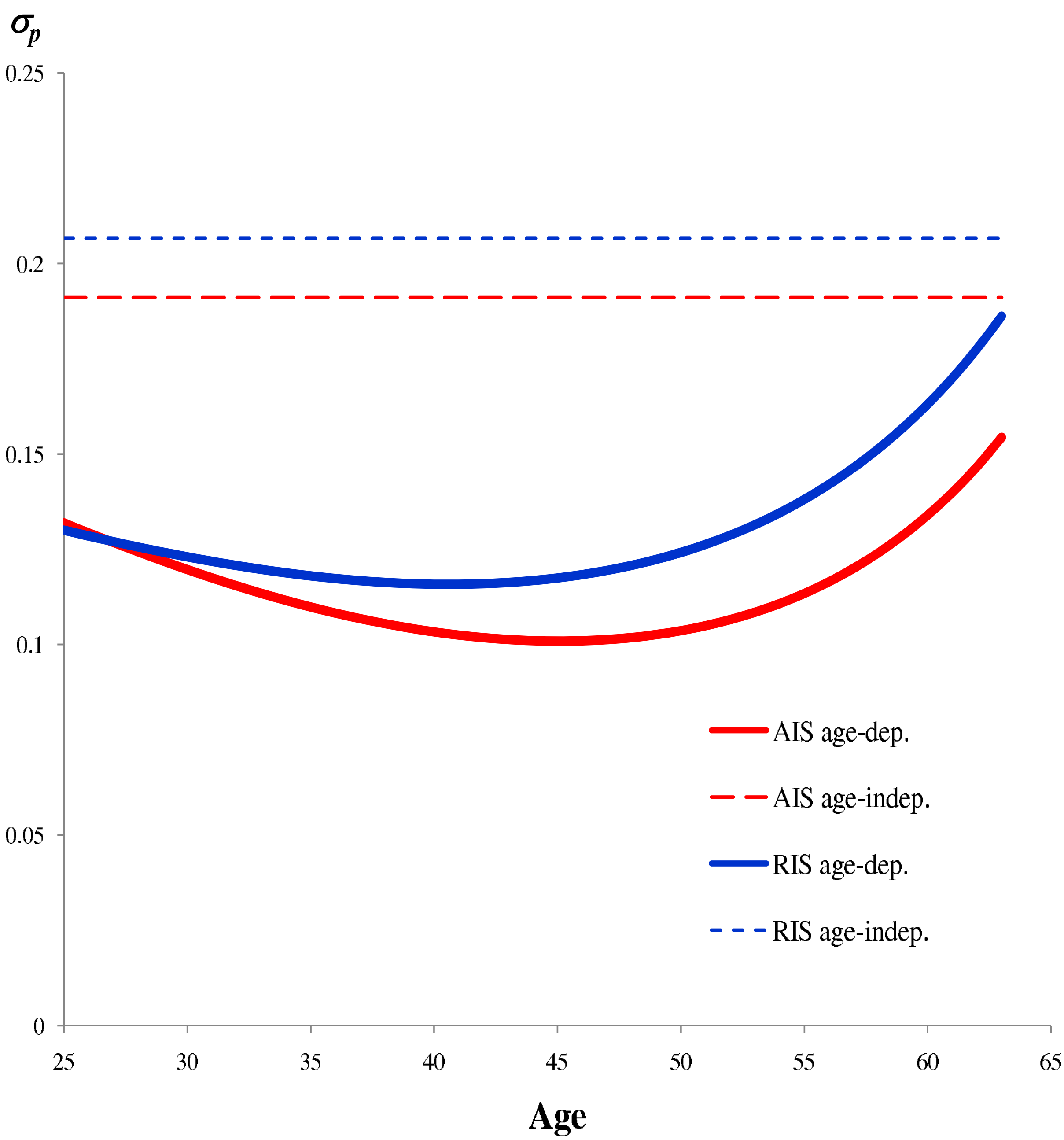
Note: The figure compares the income uncertainty estimated using the RIS and AIS specifications introduced in the current paper (Eq. 7 and 8) with those estimated in Carroll and Samwick (1997) and an updated estimate using the Carroll-Samwick method and an extended PSID sample.

Figure 5: Panel length and the estimated variances of permanent and transitory shocks



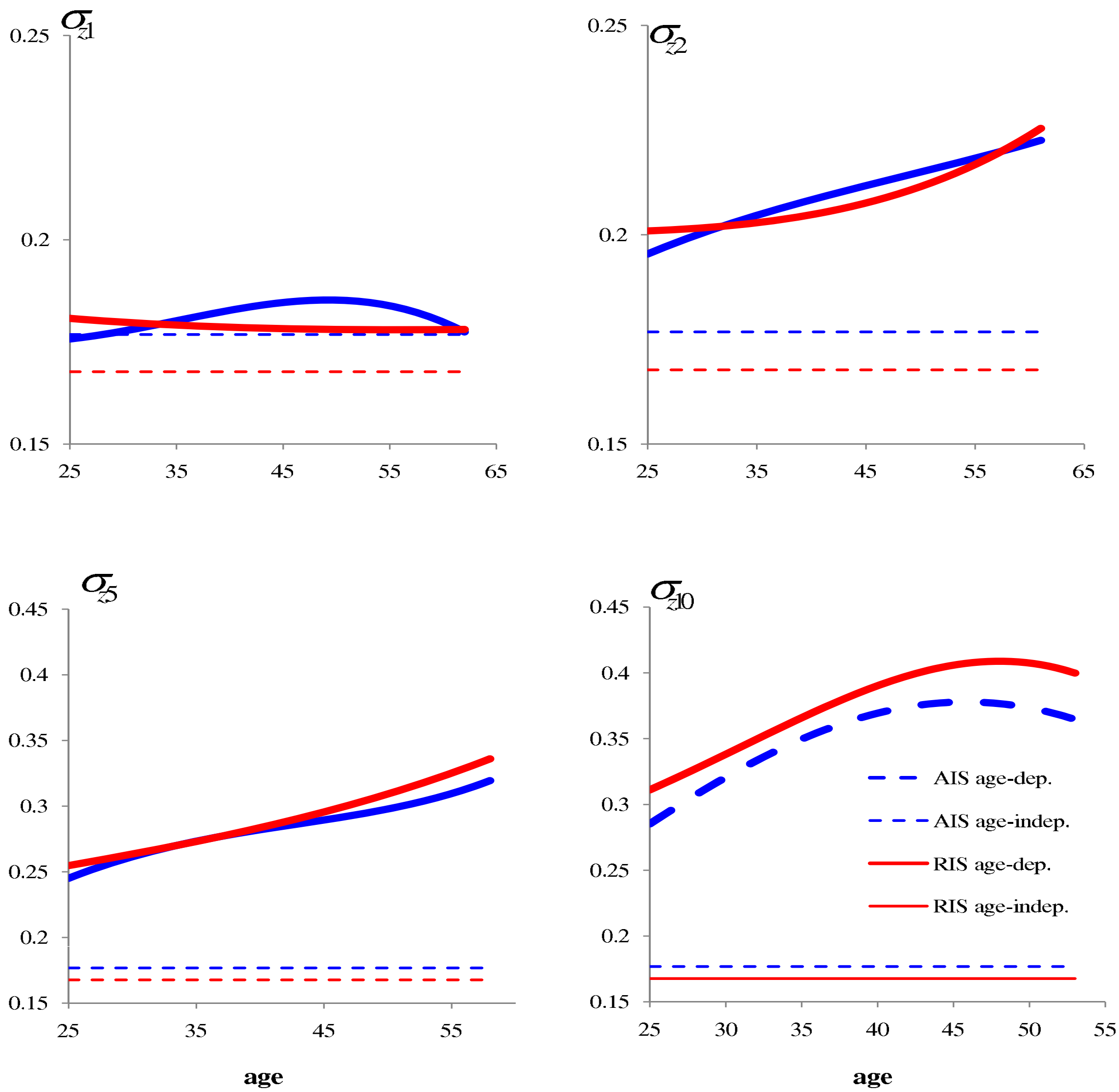
Note: The figure plots how the estimates of permanent and transitory income shock variances vary with the time span of the panel used for estimation. Permanent and transitory shocks are defined as in Eq (1-2). The blue curve shows the estimated transitory shock variance increases with panel length, whereas the red curve shows the estimated permanent shock variance decreases with panel length.

Figure 6: Calibration of the Variances of the Persistent Income Shocks over the Life Cycle



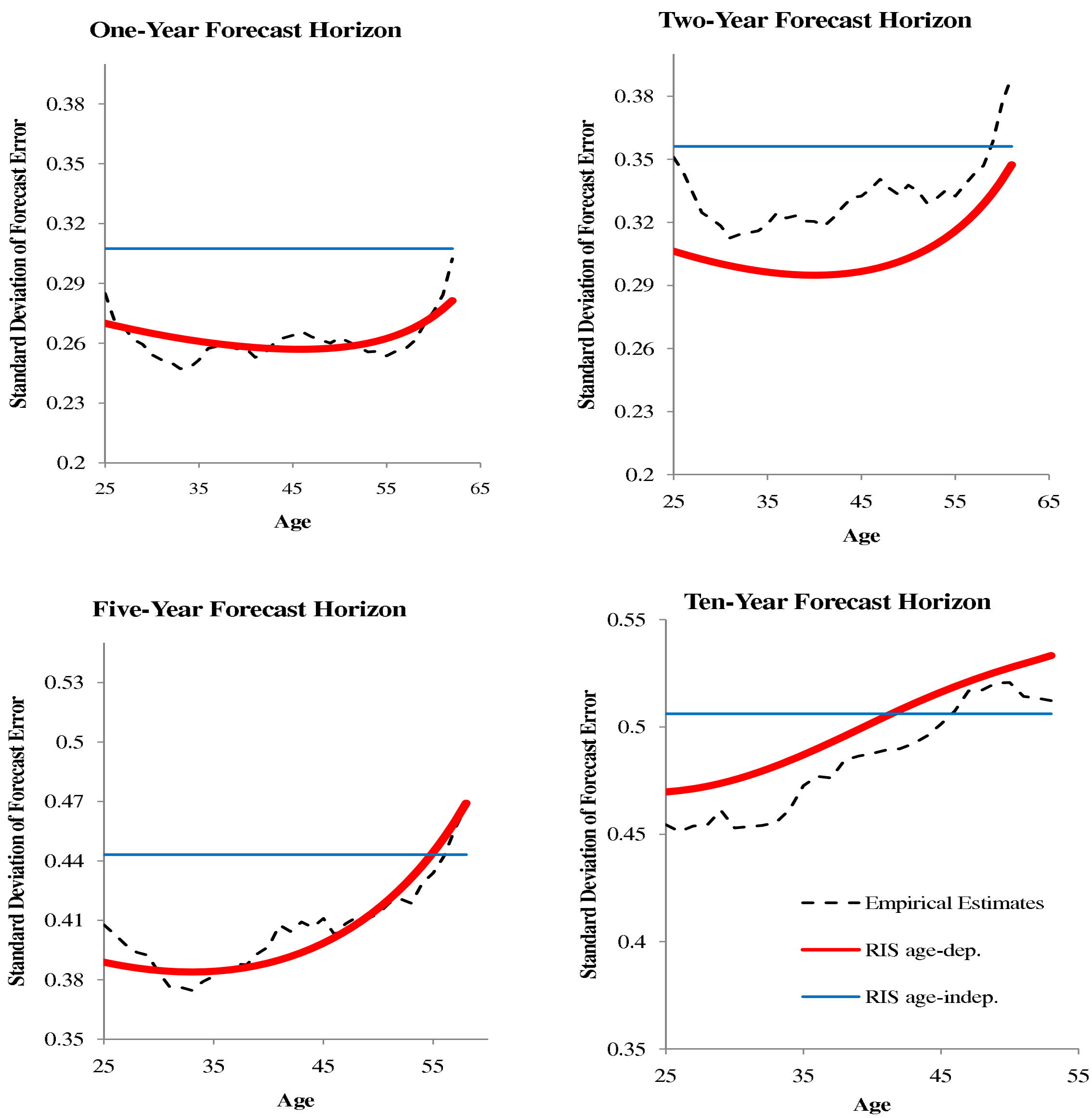
Note: Persistent shock standard deviation $\sigma_p(t)$ as a function of age t, calibrated against the empirical income uncertainty estimated under both the AIS and RIS models. The calibrations are implemented with and without allowing for age-dependent income uncertainty.

Figure 7: Calibration of the Variances of the Transitory Income Shocks over the Life Cycle at Different Horizons



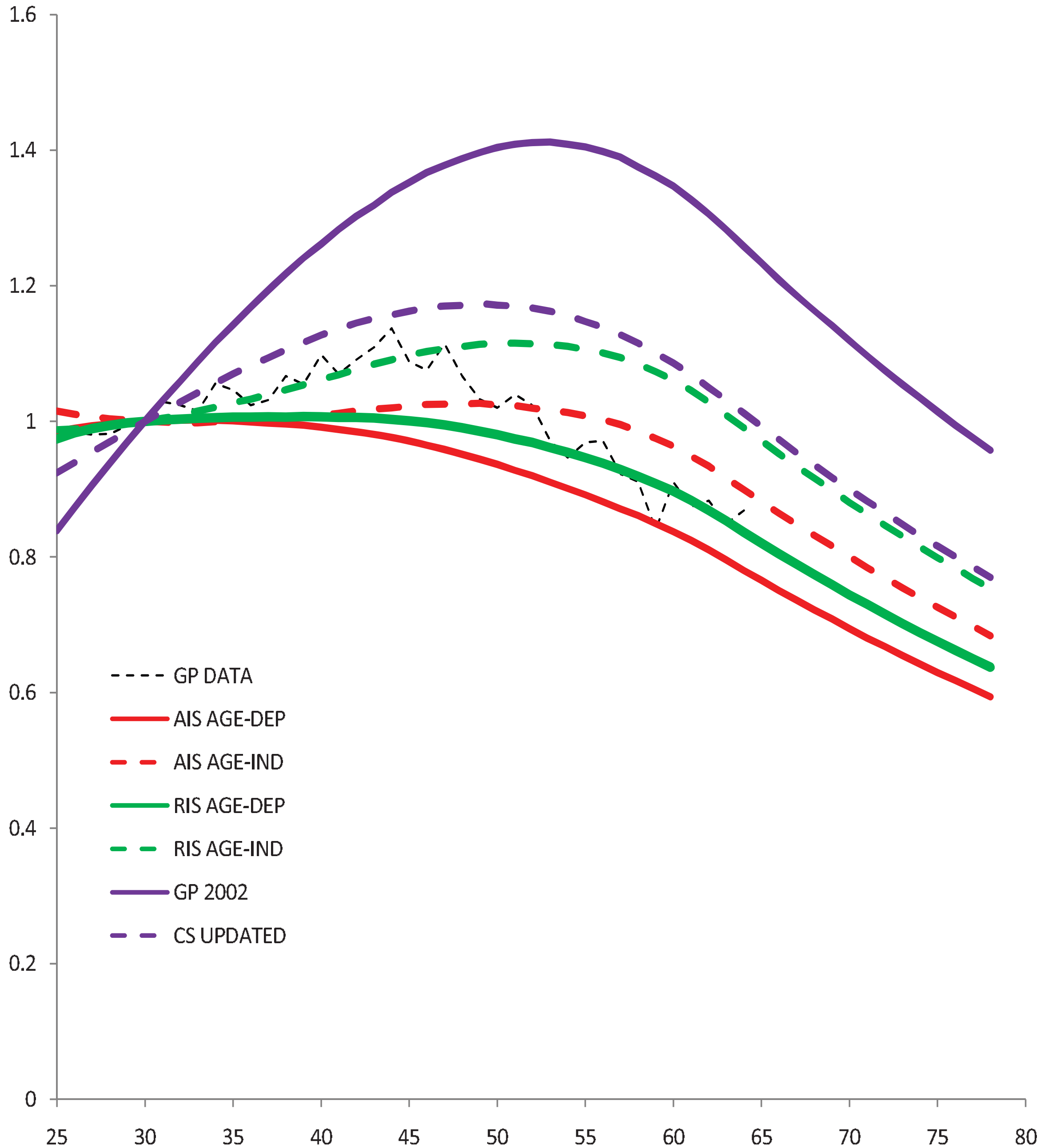
Note: Temporary shock standard deviation $\sigma_{zh}(t)$ as a function of age t for horizons h of one year, two years, five years, and ten years, calibrated against the empirical income uncertainty estimated under both the AIS and RIS models. The calibrations are implemented with and without allowing for age-dependent income uncertainty. Legends are shown in the lower right panel.

Figure 8: Contrasting the Variances of Projection Errors of the Calibrated Income Process with Data



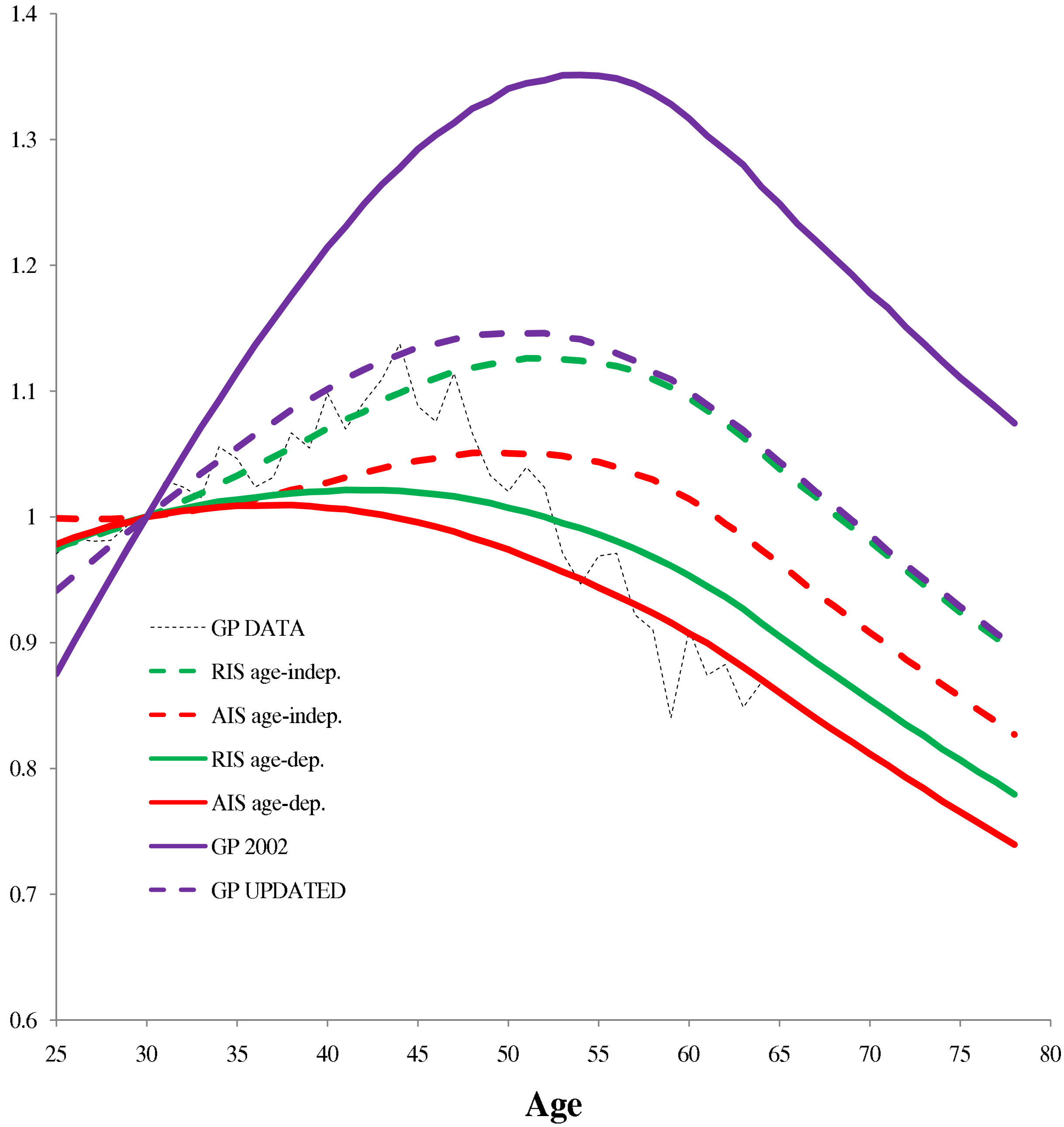
Note: The panels compare, at various horizons, the variances of projection errors constructed using the income processes calibrated using the RIS model with and without allowing for age-dependent income uncertainty. Legends are shown in the lower right panel.

Figure 9: Jim Add Title



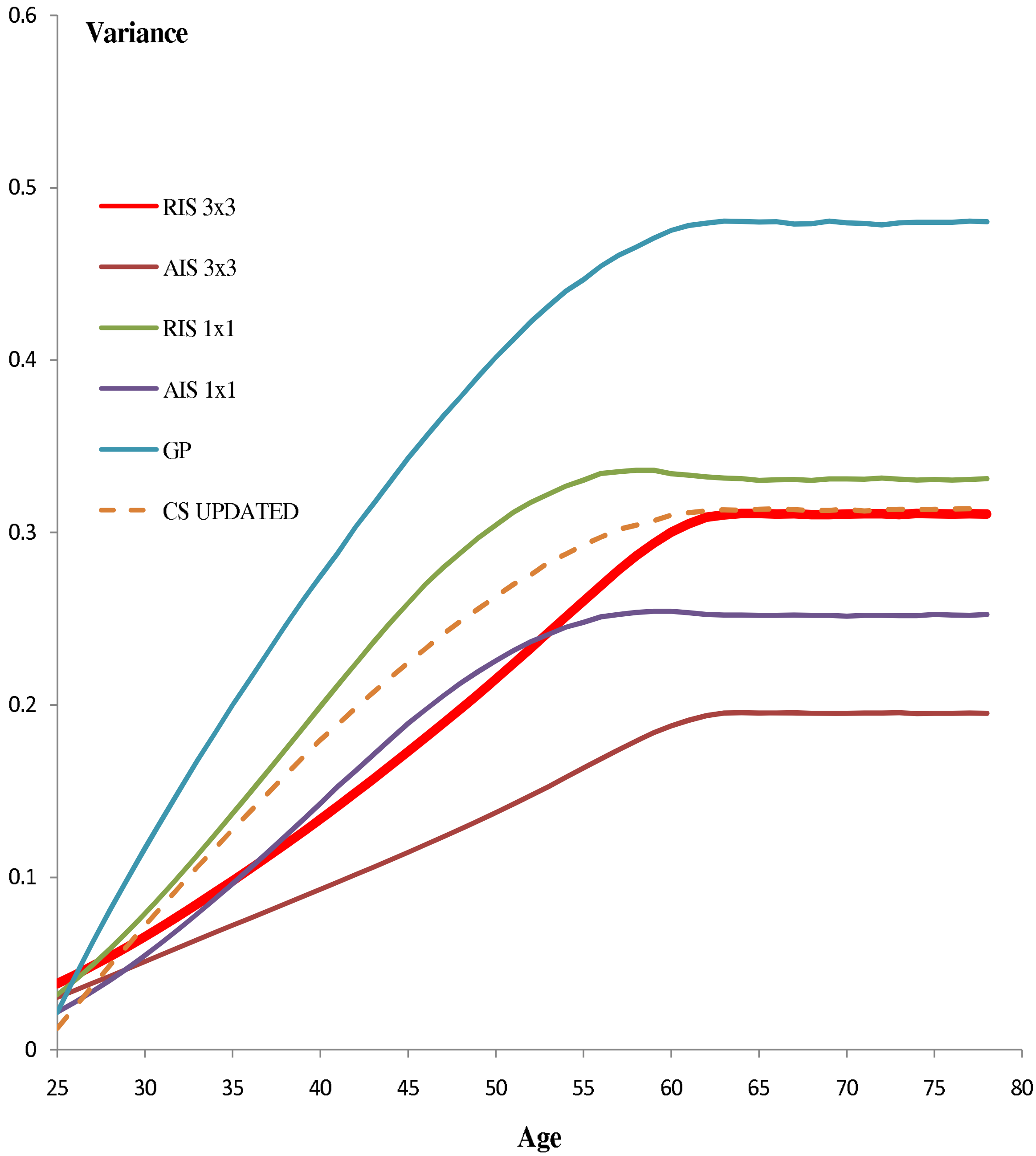
Note: Jim Add Note.

Figure 10: Comparison of the Life Cycle Consumption Profiles Derived from Various Income Processes



Note: Mean consumption as a function of age, normalized so age-30 mean consumption is 1 in all models and the data.

Figure 11: Cross-section Variances (Inequality) of Consumption over the Life Cycle Derived from Various Income Processes



Note: Jim Add Note.