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The Investment Decision of the Post-Keynesian Firm: A Suggested Microfoundation for Minsky's Investment Instability Thesis

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Hyman Minsky may have contributed more to our understanding of the financial determinants of investment instability than anyone since Keynes himself. In numerous articles and books Minsky (1975,1982,1986) has argued that in an environment of Keynesian uncertainty, expectations will be subject to endogenous cyclical instability and, as a result, investment will be cyclically unstable as well.

Though Minsky has chosen not to develop mathematical models to embody his theoretical insights, substantial progress along these lines has been made in recent years. In particular, formal models of Minsky cycles incorporating the interaction of investment and financial variables at the macro or general equilibrium level have been developed by Delli Gatti and Gallegati (1990), Jarsulic (1989), Semmler (19871, Semmler and Franke (1991), Skott (1991) and Taylor and O'Connell (1985). However, work on a theory of the enterprise investment decision that can provide a microeconomic foundation for such Keynes-Minsky macromodels has been relatively neglected.

One reason for this lack of progress at the micro level may be Minsky's adoption of Tobin's q-theory in which there is no independent firm decision-making process. Another is that the core assumptions of a Keynesian world make the process of formalization difficult. As explained in Section I of this paper, a Keynesian investment theory requires not only that the future be unknowable, but that investment be substantially irreversible as well. Either assumption considered in isolation presents substantial analytical difficulty; taken together they are formidable.' But real-world firms do accumulate illiquid capital in conditions of Keynesian uncertainty. Our challenge, then, is to construct a tractable theory of the investment decision that incorporates these two Keynesian assumptions. This paper offers one attempt to meet this challenge.

Section I presents an overview of the model, which is fully specified in Section II. Section III discusses the firm's optimal investment strategy, while Section IV discusses the comparative static properties of that policy. Section V then explains how the model can be used to micro found a Minsky cycle.

I. An Overview of the Model

This section presents an overview of the firm's investment decision. For convenience, all notation is defined in Table I.

We turn first to the characterization of the enterprise as a behavioral agent. Keynes and Minsky have taken opposite positions with respect to owner-manager relations.' Minsky generally accepts a variant of Tobin's q-theory in which owners and managers are assumed to be identical economic agents: there is no independent enterprise decision-making process. Keynes, on the other hand, insists on the qualitative differentiation of stockholders (and financial investors generally) and enterprise managers.

We follow Keynes's lead here. His approach is, in fact, consistent with the spirit of Minsky's model. Since Minsky rejects the neoclassical approach to uncertainty, it would be logical for him to assume that distinct agents such as owners and managers have both incomplete and asymmetric information. As Keynes stressed, managers know more about the firm and its environment than do the firm's stockholders.[3] Moreover, as discussed in Crotty (1990), there are compelling reasons to assume that owners and managers have qualitatively different objective functions as well as different planning horizons.[4]

The theory of the semiautonomous firm is most highly developed in the managerial and behavioral theoretical tradition. We accept the standard assumption of this literature that management seeks the reproduction, growth and security of the enterprise itself, and through these goals, its own income, status and job security.' Stockholder and creditor interests are not objectives pursued by management. Rather, they represent a potential threat to management's decision-making autonomy and a constraint on the pursuit of its objectives. To protect its control of the enterprise, the firm must pay dividends sufficient to prevent a share holders' revolt or a corporate raid and interest payments that prevent creditors from constraining managerial autonomy.

More formally, we assume that the firm maximizes a preference function $O(G,S)$ where G reflects the growth-profit objectives of the enterprise and S embodies management's concern for the financial security of the firm and thus for its own decision-making autonomy -- it financial <u>security-autonomy</u> objective. Both G and S are functions of the capital stock trajectory over management's long-term planning horizon. We make G a function of two subgoals: R', the present value of the future earnings the firm expects its capital stock to generate (which depends on the pattern of future demand and cost conditions it expects); and K', the average size of the capital stock over the planning horizon -- an index of the size-status of the firm.[6]

S is an index of the likelihood that management will experience a threat to its autonomy. This threat exists because growth can only be obtained through the accumulation of illiquid capital and capital accumulation must be financed. Debt finance creates explicit, legally-binding cash-flow commitments to creditors. But even internal funding and stock floatation create implicit cash-flow commitments to shareholders. When investment is irreversible, these financial commitments are irreversible as well. When expectations are disappointed, the firm cannot simply repay its creditors with the proceeds from the resale of the assets they financed. If commitments to stockholders cannot be met out of the future operating profits generated by invested capital, management may experience a threat to its decision-making autonomy; if commitments to creditors are not met, the firm might go bankrupt.

In a Keynesian-Minskian world, financial commitments to creditors are relatively certain while expected profits are not, To make sensible decisions about the accumulation of long-lived illiquid capital, then, the firm must form expectations of cash flows well into the future. But about such matters, Keynes told us, "We simply do not know". <u>When investment is irreversible and the future is unknowable, irreversible mistakes of serious magnitude are Possible</u>. It is the irreversibility of investment which creates the "legacy of past contracts" (Minsky, 1982, p. 63) that constrain current investment and threaten managerial autonomy. Thus, while accumulation is <u>necessary</u> (to achieve growth), it is simultaneously <u>danserous</u> for management. To specify S more concretely, let X_t be defined as the interest plus dividend payments necessary to preserve managerial autonomy (i.e., as the costs of autonomy) and let π_t be defined as the ex post gross profits available to meet these payments in period t. A threat to autonomy will arise when $(\pi_t - X_t)$ is expected to be small; a crisis will occur if $(\pi_t - X_t)$ is expected to be negative. The firm will want to avoid investment decisions which cause expectations of $(\pi_t - X_t)$ to become uncomfortably low. Suppose that we provisionally adopt the neoclassical assumption that the firm can, with complete confidence, form subjective probability distributions relating future profit flows to the size of the capital stock in each future period. Denote the expected distribution of π given K (or $\pi|K$) in any period as f,. The perceived likelihood of an autonomy crisis would then be given by F_t, the cumulative probability that π_t < X,, and $S(F_t)$ would then represent the firm's index of expected safety in t. Under this treatment of uncertainty, management's estimate of the likelihood of an autonomy crisis in period t would depend on: (1) the financial structure of the firm at the end of period (t-l) -- Minsky's "legacy of past contracts"; (2) f,, management's subjective probability distribution for π_t given its choice of K_t; and (3) the value of K_t selected by the firm. Since today's investment decision affects future expected net revenues, the future costs of autonomy and the future financial structure of the firm, <u>it inevitablv alters the relation of K to both G and S in future Periods</u>. Thus, the capital accumulation problem confronting management is inherently dynamic.

There are two reasons why this formulation of the problem is inadequate for our purposes. First, if we were to include the complete set of functions $S(F_t)$, $S(F_{t+1})$,... $S(F_T)$ in the objective function, the dynamic effects of today's investment decision would be extraordinarily complex because the firm's future financial commitments would be a complicated function of the K trajectory with an exponentially increasing number of stochastic terms: this formulation is analytically intractable. Second, as noted, it incorporates a neoclassical treatment of uncertainty.

In Appendix B (which should be consulted after reading Section II and Appendix A), we show that there does exist a tractable static variant of the firm's investment decision under Keynesian uncertainty that is equivalent to the full dynamic model just enumerated under a set of three assumptions that are both realistic and consistent with a Keynesian worldview.'

First, we assume that the firm adopts a sequential decision-making process in which it tentatively chooses an optimal capital stock trajectory each period but only orders the first period's capital goods at that time. It then updates its forecasts of future demand and cost conditions using data generated during the first period and re-evaluates the G(K) and S(K) functions before repeating the process. When errors can be extremely costly and when the forecasts on which beyond-period optimal capital stock decisions are based may be dramatically revised in the light of data generated in the current period, management will not want to commit itself beyond its "next best step" (Vickers, 1987, p. 8) on the basis of current data.

Second, the assumption of Keynesian uncertainty suggests that as the planning horizon lengthens, the firm's confidence in its ability to predict the precise form of the effects of today's investment on future growth and safety declines dramatically. We incorporate this phenomenon in our model by assuming: (1) that the firm can construct a neoclassical-type subjective probability distribution describing the effect of K on expected gross profits for the coming three to five year corporate planning cycle that we take as the length of a period*; and (2) that the firm does not believe that it has enough reliable information to fully specify all future $S(F_t)$ functions, so it cannot optimize over the beyond-first-period effects of current investment on the S function.

Third, we confine our analysis to the case where, in the G function, the firm expects a constant and non-negative time rate of growth of its product demand curve. Note carefully that assumptions two and three imply that the optimal stock is expected to grow each period and that there is no incentive in the model to over-invest now in anticipation of future growth. Thus, the optimal stocks of capital in future periods are <u>independent</u> of this period's stock and of each other.

These assumptions simplify the problem, but they also imply that S is a function of F_t alone: the firm is oblivious to the <u>existence</u> and not just to the precise form of the beyond-first-period effect of investment on safety.[9] To insure that the firm takes the existence and potential significance of the "legacy" of future financial commitments created by current investment fully into account and to guarantee that it does not blindly pursue <u>short-term</u> growth and safety at the expense of its <u>lons-term</u> financial security objectives, we respecify S as $S(F,D')$ where D' is an index of the firm's current perception of its long-term financial vulnerability. We define D' as $(D-\hat{D})$, where D is the current level of debt and $\hat{D}$ is the maximum debt level that management is comfortable carrying into its uncertain future.[10]

The inclusion of D' in S forces the firm's current investment decision to be consistent with its long-term safety objective. $\hat{D}$ reflects managerial optimism or pessimism. The brighter the firm's expectations of the long-term future, the larger the debt burden it is willing to accept and, ceteris paribus, the more it is willing to invest now. Conversely, when $\hat{D}$ is low and D' is high, the firm will be less likely to take on the additional long-term financial commitments associated with current investment even if short-term investment prospects as reflected in F do not look immediately threatening. Note that the use of a conventional, rule-of-thumb variable such as $\hat{D}$ constitutes a very Keynesian solution to this long-term aspect of the uncertainty problem.

This specification of the S function is ideally suited to underpin Minsky's theory of investment instability because both F and D' are subjective, conventionally-constituted variables that can shift endogenously as managerial optimism and management's confidence in its ability to forecast the future ebb and flow with the business cycle. To use Minskian terms, we might say that F and D' represent, respectively, the firm's short-term and long-term perceptions of financial fragility.[11]

Thus, in a world characterized by our assumptions,

Maximize

$$\Theta\,[G(R(I;K^0),\ K(I;K^0)),\ S\,(F(I;K^0),\ D'\,(I;K^0))] \qquad (1)$$

(where R is expected net revenue, I is net investment, K^0 is the initial capital stock, the relation between I and K is treated implicitly as $K(I;K^0)$, and all variables are current) is a sensible Keynesian-Minskian specification because the firm has: a long-term planning horizon; is aware that its current investment decision may have important future effects on safety and growth;

Table I. Summary of Mathematical Notation

A	present discounted value of costs of autonomy
a	lower limit of Π
b	uppper limit of Π (interest + dividends)
D	expected net debt
$\hat{D}$	perceived maximum safe debt Level
D'	$D-\hat{D}$
F	psuedo-probability of an autonomy crisis
f	probability density function of IT
G	utility from growth objectives
I	net investment
I^g	gross investment
K	capital stock
L	perceived maximum safe debt-equity ratio
O	manager's utility function
P	output price
p^K	price of capital
Q	output
r	interest rate
R	expected net revenue
S	utility from security-autonomy objective
t	time
U	unit labor costs
X	current cost of autonomy
Y	cash flow
α	mark-up on unit variable cos
β	dividend payout rate
π	ex post gross profits
π^g	expected gross profits
δ	constant rate of depreciation
σ^2	variance of Π
K^0, D^0, π^0, X^0	inital values

and believes that only the short-term future can be forecast with any degree of confidence,

This brings us to the core of management's decision-making problem: the <u>growth-safety tradeoff</u>. Were the firm to undertake only those investment projects with very high expected profits, it might be able to raise G and S simultaneously. But as it pushes capital accumulation to the point which maximizes G, it will accept projects with decreasing expected profitability and/or higher risk. It will, at the margin, lower S by raising F (the likelihood that π will fall short of X in the coming period) and increasing D' (and thus the likelihood a long-term autonomy crisis).[12] On the other hand, if the firm minimizes its vulnerability to autonomy crises by accepting only the safest projects, it will forego.expected growth opportunities. We explore this growth-safety tradeoff in detail below.

Investment, then, is a function of: (1) the determinants of the relation between expected profits and K; (2) the determinants of the relationship between expected safety and K; and (3) management's relative preference for growth versus safety. All three of these relations are subject to Keynesian-Minskian endogenous instability. After deriving the comparative static properties of the optimal investment decision in Section IV, we demonstrate that Minsky's stylized facts describing the typical business cycle will indeed induce cyclical investment instability in the model described here and fully specified in Section II.

II. The Model

We complete our modelling of the firm's investment problem by more fully specifying the R, K, F and D' functions in equation (1). In the remainder of the paper, the superscript zero refers to end-of-last period values.

Expected net revenues, $R(I;K^0)$, is specified as the difference between expected gross profits, π^g, and the present discounted value (PDV) of debt payments (costs of autonomy abstracting from dividend payments for the moment), A, associated with current gross investment:

$$R=\pi^g(K(I;K^0)) - A(K(I;K^0)) \qquad (2)$$

The level of A is determined by the financing mechanism of the firm. We assume that the representative firm is a net debtor with no liquid assets and with only two sources of investment finance: the internal funds or cash-flow carried over from last period and new debt in the event that gross investment expenditures exceed cash flow. When cash flow exceeds gross investment outlays, the residual goes to debt reduction. All debt is assumed to be variable rate consoles.

Since gross investment expenditure, $P^K I^g$, is either debt financed or financed from cash-flow residuals which would otherwise be used for debt reduction, the additional interest cost, direct and/or opportunity, associated with $P^K I^g$ is $rP^K I^g$ where P^K is the price of a unit of capital. A is the present discounted value of $rP^K I^g$. A formal specification of the financing and dividend payout mechanisms and thier implications for the debt structure and R function is contained in Appendix A. In order to avoid the reintroduction of future effects into Problem (1), dividends are defined as a percentage, β, of current gross profits minus <u>last</u> period's interest payments (see Appendix A). Further, to simplify the expression of the optimizing conditions associated with (1), dividends are treated as a seperate entity from the other costs of autonomy -- A, the PDV of debt payments.

$\pi^g(K(I;K^0))$ can be further decomposed as follows. We assume a fixed-coefficient, constant variable cost production function and a downward-sloping demand curve: $Q = Q(K)$, $P = P(Q(K))$ and $U=\bar{U}$ where Q is output, P is expected output price, U is expected unit variable cost and $\bar{U}$ is a constant. We further assume that $P_Q < 0$ and $P_{QQ} \leq 0$. Given, these assumptions we can alternatively specify $\pi^g(K(I;K^0))$ as $\alpha(K(I;K^0))Q(K)$ where $\alpha = (P - U)$ is the firm's markup on unit variable cost. Thus, $\pi^g_I = \alpha Q_I + Q\alpha_I$ and $\alpha_I = P_Q Q_I$.

Given that $P_Q < 0$, π^g_I is sign indeterminate. Moreover, the inclusion of the K subobjective in G and the S objective in 0 implies that the first order conditions for the maximization of 0 are not capable of restricting $R_I > 0$ by equating π^g_I and A, (or marginal revenue and marginal cost) as in the neoclassical treatment. While size considerations may drive investment to the point where marginal gross profits are less than the marginal costs of autonomy, financial security objectives may limit investments such that $(1-\beta)\pi^g_I > A_I$ where β is the dividend payout rate. Thus, in equilibrium $R_I \gtrless 0$ as $(1-\beta)\,\pi_I^g \gtrless A_I$.

The firm's size subobjective is simply specified by $K(I;K^0)$, the size of the firm's real capital stock, and thus needs no further elaboration. In addition, $K_I = 1$.

We turn now to the S subobjectives, F and D'. F is the probability that $\pi < X$ -- the likelihood of an autonomy crisis. F can be expressed as:

$$F = \int_a^X f\ (\pi;\ \pi^g(K\ (I;K^0)),\ \sigma^2)\ d\pi$$

where: f is the firm's subjective pseudo-probability distribution of π, the uncertain gross profit flows; $\pi^g(K(I;K^0))$ and σ^2 are the mean and variance of π; a is the lower limit of π, X is the firm's current financial obligations (costs of autonomy) as distinct from the present value of the costs of autonomy, A. For ease of exposition, we assume that π is distributed uniformly.[13] Given that f is uniform, a = $a(\pi^g(K);\sigma^2)$ with $\frac{\partial a}{\partial \pi^g}=1$ and $\frac{\partial a}{\partial \sigma} < 0$.

The sign of $F_I=(X_I-\pi_I^g)f$ depends on the relationship between $(1-\beta)\pi_I^g$ and X_I which is similar, but not identical, to the relation between π_I^g and A_I. If $(1-\beta)\pi_I^g \gtrless X_I$, $F_I \lessgtr 0$. In the case where $\pi_I^g > 0$, a one unit increase in I can increase expected gross profit flows by either more or less than the increase in current autonomy payments and thus either decrease or increase the probability of short-term financial strife.

As noted above, $D'=D-\hat{D}$ where $\hat{D}$ is the product of $\hat{L}$ and $(P^K K(I;K^0))$, $\hat{L}$ is the maximum leverage ratio considered safe or prudent by management, and $P^K K\ (I;K^0)$ represents the value of the firm's assets. Since $\hat{L}=\frac{\hat{D}}{P^K K}$, it can be thought of as the maximum acceptable debt to debt-plus-equity ratio. As noted (fn. 12), an increase in I^g will always raise D by more than the marginal increase in the "admissible" level of long-term debt emanating from the increase in the firm's K. Thus, <u>investment always initially increases the firm's long-term debt dependency and therefore reduces security</u>.

In order to simplify notation, we respectively define a series of vectors that contain the relevant exogenous parameters for the R, X, a, D and $\hat{D}$ functions: $\bar{R}, \bar{X}, \bar{a}, \bar{D}$ and $\bar{\hat{D}}$. These vectors are fully specified in Appendix A and include the dividend payout rate, the interest rate, the price of a unit of capital, the depreciation rate, the maximum acceptable debt to debt-plus-equity ratio, and the initial values of the stocks of capital and debt, and last period's profit flow.

Assuming that there are no costs of adjustment, the specification of the firm's investment decision is given by:

Maximize

$$0 = O[G(R(I;\bar{R}), K(I;K^0)),\ S(\int_{a(I;\bar{a})}^{X(I;\bar{X})} f(\pi;\pi^g(K(I;K^0)),\sigma^2)\,d\pi,\ D(I;\bar{D}) - \hat{D}(I;\bar{\hat{D}}))] \qquad (3a)$$

Subject to

$$I \geq 0 \qquad (3b)$$

Thus, an investment-induced G-S tradeoff is the essence of the firm's optimization problem. The G-S tradeoff is best understood by examining $G_I = G_R R_I + G_K$ and $S_I = S_F F_I + S_{D'} D'_I$ where $G_R > 0$, $G_K > 0$, $S_F < 0$ and $S_{D'} < 0$ are preference weights for the R, K, F and D' subobjectives.

Given our above discussion of the sign indeterminacy of F_I

and R_I and thus S_I and G_I, the nature of the tradeoff is undetermined. However, it is shown in Appendix C that the first order conditions for (3) and the assumptions that $P_Q < 0$ and $P_{QQ} \leq 0$, restrict $S_I < 0$ and $G_I > 0$ in the neighborhood of equilibrium: <u>an investment-induced G-S tradeoff is operational</u>. The mechanics of the tradeoff are straightforward. A one unit increase in I increases G, and thus utility, either through a simultaneous increase in both firm size and net revenues or an increase in firm size that outweighs, in utility units, a decline in net revenues. At the same time, marginal I decreases S either through a simultaneous increase in the probability of short-term financial strife and long-term debt dependency or an increase in long-term debt dependency that outweighs, in utility units, the decline in F.

The dependence of 0 in equation (3a) on multiple objectives (G and S) and subobjectives (R,K,F, and D') requires that management's relative subjective ranking of these objectives and subobjectives be made explicit. For simplicity, it is assumed that S and G are linear in their arguments: $S_{FF} = S_{D'D'} = S_{FD'} = G_{RR} = G_{KK} = G_{KR} = 0$. In contrast, the relative preference ordering for G and S is variable and endogenous. It is assumed that $O_{GG} = O_{GS} = 0$, while $O_{SS} < 0$: the firm's imperative to grow is a constant unyielding commitment that is independent of the size of the firm, while the firm's response to financial security and uncertainty is variable. In particular, at lower levels of financial security management responds to the threat of encroachment on its decision-making autonomy and the possible threat to the firm's immediate and long-run survival by choosing an investment/debt strategy which focuses on restoring financial security even at the expense of maintaining or promoting the firm's growth objective. A financially fragile firm will sacrafice potential growth to lower the probability of crisis. As in the credit rationing literature, financial structure influences investment. I is inversely related to the debt-equity ratio. However, in this Keynesian model credit affects I through the demand side. Note that the intensity of the G-S tradeoff is variable. Ceteris paribus, at higher levels of I (and thus higher levels of G and lower levels of S) the relative preference for security increases.

III. The Optimal Investment Stratesv

The first order condition for an interior solution to maximization problem (3) is $O_G G_I = -O_S S_I$ or alternatively

$$O_G\,[G_R R_I + G_K K_I] + O_S\,[S_F F_I + S_D D_1'] = 0$$

The firm invests to the point where the marginal utility gains (losses) from growth are exactly offset by the marginal utility losses (gains) from financial security/autonomy. In equilibrium $sgn(-S_I) = sgn(G_I)$ -- the firm faces a G-S tradeoff. The exact nature of the tradeoff is discussed below.

The second order condition for a maximum requires that:

$$O_G G_{II} + G_I O_{GI} + O_S S_{II} + S_I O_{SI} < 0$$

Recognizing that $O_{SI} = O_{SS} S_I$ and that $O_{GI} = O_{GG} G_I = 0$, the second order condition can be stated as:

$$O_G G_{II} + O_S S_{II} + S_I^2 O_{SS} < 0 \qquad (4)$$

This condition is met if

$$G_{II} = G_R (1-\beta)\pi^g_{II} < 0 \text{ and } S_{II} = S_F [\pi^g_{II} f (\beta-1)] < 0 \text{ or}$$

alternatively if $\pi^g_{II} = Q_I^2 [QP_{QQ} + 2P_Q] < 0$ where β is the dividend payout rate.[14] Thus under our assumptions that $P_Q < 0$ and $P_{QQ} \leq 0$ the second order condition holds.

In Appendix C, we show that a strong condition for $S_I < 0$ and $G_I > 0$ in the neighborhood of equilibrium is a minimal preference for the firm's size subobjective. Assuming that this preference exists, the nature of the G-S tradeoff is such that management must sacrifice financial security to obtain growth and vice-versa.

The managerial firm's optimal I decision is summarized by Figure I. In finding the I^* that ensures $G_I(I) = -\frac{O_S}{O_G} S_I(I)$ management must resolve the G-S tradeoff. At levels of $I < I_1$ marginal increases in I increase gross profits by enough to (1) offset the marginal increments in the costs of autonomy and thus ensure that G rises $(G_I > 0)$, and (2) ensure that F declines by enough, despite the increase in financial obligations (X), to offset the increase in D', thus S increases and (3) increase the relative preference weight, $\frac{O_G}{O_S}$ assigned to the G objective as safety increases and thus O_S declines. Thus for $I < I_1, O_I > 0$: total utility increases with I. For $I_1 < I < I_2$ marginal increases in investment result in smaller increases in gross profits as the firm's profit per unit decreases at higher levels of output. As a result, π^g_I offsets the marginal increments in

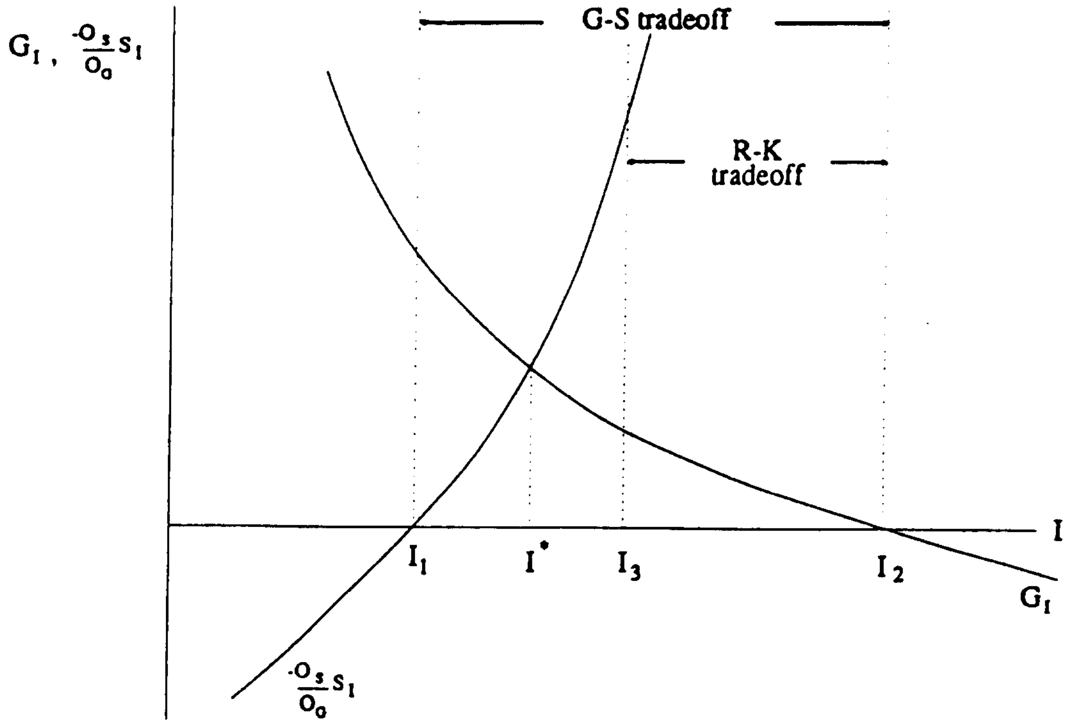

Figure I: the optimal solution and G-S and R-K tradeoffs

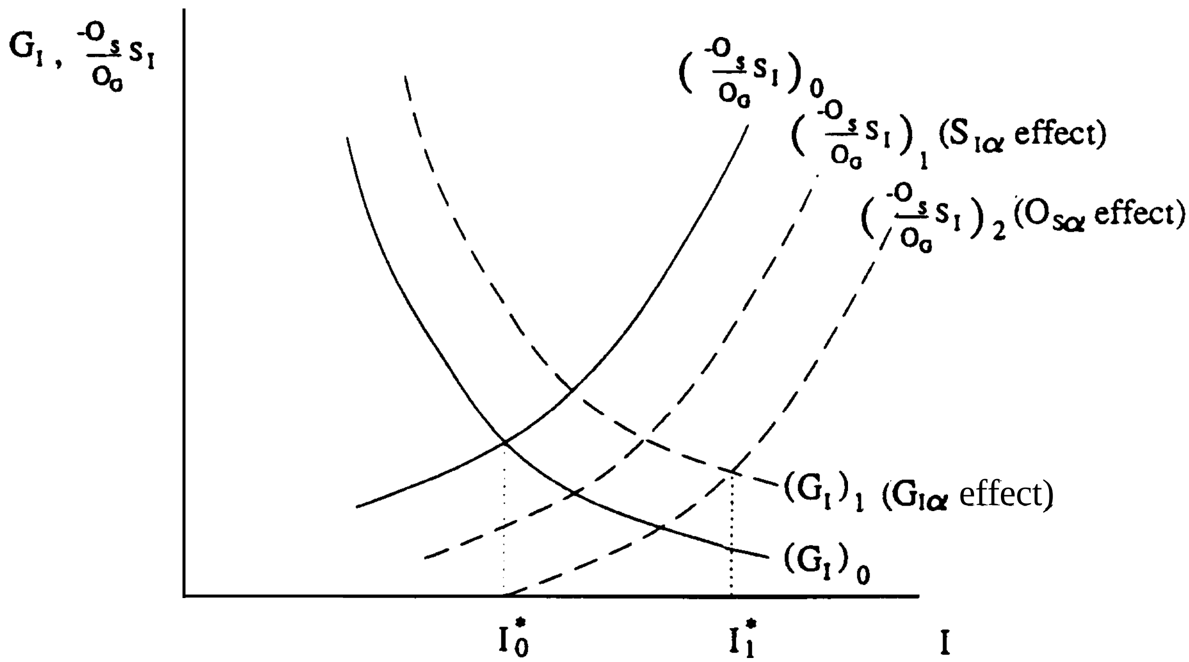

Figure II: the effect of $d\alpha$ on optimal I the costs of autonomy by less -- G_I declines -- and is no longer capable of reducing F by enough to offset the rise in D' -- S_I becomes negative. In addition, $\frac{O_G}{O_S}$ declines. Thus G continues to rise but at the expense of a decline in S ($S_I < 0$): the G-S tradeoff is operable. As long as $G_I > -\frac{O_S}{O_G} S_I$, marginal increments in I will increase 0. But, given that $\pi^g_{II} < 0$ (which ensures that $S_{II} < 0$), $G_{II} < 0$, and $O_{SS} < 0$, beyond I' marginal increments to I will no longer generate enough profits to ensure that the appropriately weighted increase in G offsets the increasingly more heavily weighted declines in S.

Depending on the specific nature of the $-\frac{O_S}{O_G} S_I$ and G_I functions, the firm may also sacrifice net revenue in order to increase the size of the firm. While there are many different sets of circumstances under which this tradeoff may be operable, the most obvious one is the case of a financially robust firm. In this situation, O_S is small even at higher levels of I, thus the relative preference weight assigned to the negative values of S_I is small and marginal increments to I beyond the point where $R_I = 0$ are likely to be utility enhancing. In the extreme case where $O_S = 0$ for all I, I^* satisfies $G_I = 0$. Thus $I^* = I_2 > I_3$.

Thus an important connection between the two tradeoffs facing the firm becomes apparent: when the growth objective dominates the security objective, the firm is more likely to pursue growth to the point where net revenues are traded off for size -- $R_I < 0$. In particular, as the intensity of the G-S tradeoff declines beyond a particular point the intensity of the

R - K tradeoff increases. For the financially fragile firm, where security objectives dominate growth objectives, restrictions on I make it likely that $R_I > 0$. In this situation our model reproduces the NPV > 0 result of the irreversible investment literature (surveyed in Pindyck(l991)). The possibility that either one or both tradeoffs are operable and thus $R_I \gtrless 0$ or, $(1-p)\ \pi_I^g \gtrless A$, further distinguishes our model from the neoclassical theory of I -- the optimal level of I can be greater or less than in neoclassical theory. It includes as a special but extremely relevant case the solution associated with irreversible investment models.

IV. Comparative Statics

In this section, the comparative static effects of $\alpha, \hat{L}, D^0, \sigma^2, r, P^K, \delta$ and K^0 are discussed. Detailed derivations are contained in Appendix D.

In general the effect on I^* and K^* of a one unit change in any parameter, p, (with the exception of K^0, discussed below) can be expressed as

$$\frac{dI}{dp} = -\frac{(O_G G_{Ip} + O_s S_{Ip} + S_I S_p O_{SS})}{|H|} \qquad (5)$$

where $O_{GG}=0$ is invoked, $|H|$ is the second order condition in equation (4), and O_{Sp} is written as $S_p O_{SS}$. Given that $|H|<0$, the sign of $\frac{dI}{dp}$ depends on the sign of three separate effects: $O_G G_{Ip}$, $O_S S_{Ip}$, and $S_I S_p O_{SS}$. These effects respectively represent: (1) the change in investment-induced increases in growth objectives evaluated in utility terms by O_G: (2) the change in investment-induced reductions in financial security evaluated in utility terms by O_S: and (3) the change in the evaluation of the investment-induced reduction in financial security (S,) as a result of changes in the preference weight $O_{Sp}=(S_p O_{SS})$ that occur as S changes. Given that $O_G>0$, $O_S>0$, $O_{SS}<0$, and $S_I<0$ in the neighborhood of equilibrium, the sign of $\frac{dI}{dp}$ depends on the signs of G_{Ip}, S_{Ip}, and S_p.

The comparative static results are best understood by recognizing that each of these three effects alters the intensity of the G-S tradeoff. Unambiguous increases in the intensity of the tradeoff (any combination of $G_{Ip}<0$ or $S_{Ip}<0$ or $S_p<0$) will result in less I and conversely.

To show how the model works, we consider in detail a change in α, the firm's profit markup. Changes in α are the primary channel through which real sector developments directly affect the pace of accumulation of the model. $dI/d\alpha$ can be expresses as

$$\frac{dI}{da} = -\frac{[O_G(G_R(1-\beta)\pi^g_{I\alpha}) + O_S(S_F f(\beta-1)\pi^g_{I\alpha}) + S_I O_{SS}(S_F\ (P-1)\ Qf)\ 1}{|H|} \qquad (6)$$

where $\pi^g_{I\alpha}=Q_I(1+QP_{Q\alpha})$. If demand increases such that $P_{Q\alpha}\geq 0$ and $\pi^g_{I\alpha}>0$, then $G_{I\alpha}>0$, $S_{I\alpha}>0$ and $S_\alpha>0$ implying that $\frac{dI}{da}>0$.

A rise in **a** stimulates I three ways. First, it increases the marginal return to growth -- marginal gross profits are increased because the additional output is sold at a higher *a,* marginal costs remain the same, and the marginal decline in price when Q grows is either unaffected or reduced. Second, it reduces the marginal decline in safety because F is reduced. Third, it increases the level of S through higher gross profits that reduce F, and thereby lower the weight on the investment-induced decline in S. All three effects reduce the intensity of the G-S tradeoff and result in optimal trades of investment-induced reductions in S in favor of investment-induced increases in G. Thus, I increases. As can be seen in Figure II, the first (or demand) effect shifts the G_I curve to the right while the latter effects ($S_{I\alpha} > 0$ and $S_\alpha > 0$) both shift the $-\frac{O_S}{O_G}S_I$ curve to the right.

This result is important on both the micro and macro levels. On the micro level, it shows how shifts in demand and cost functions change I demand. On the macro level, it provides a feedback mechanism through which macroeconomic variables shift the firm's demand and cost functions and thus influence microeconomic profitability and I.

We next consider the effect of D^0, the initial level of debt, on I'. Changes in D^0 affect I in our model because they change S. Since S_{D^0} (the only shift parameter (in Figure II) that is operable in this case) is negative, $dI/dD^0 < 0$. An increase in D^0 raises D and X and thus reduces the level of financial security by increasing both F and D'. As a result the preference weight assigned to the investment-induced reduction in S is increased.

The unknowability of the future is reflected in σ^2, a measure of short-term uncertainty, and $\hat{\pounds}$, an index of perceived long-term uncertainty.

$\frac{dI}{d\hat{\pounds}} > 0$. $S_{\hat{\pounds}}$ is positive because a rise in $\hat{\pounds}$ (and thus in

$\hat{D}$) means that management feels more secure at any given debt level. Moreover, because S rises with $\hat{L}$, investment-induced reductions in safety cause smaller declines in utility. Finally, I causes a smaller reduction in S at higher $\hat{L}$ levels because the "acceptable" debt to debt-plus-equity ratio has risen.

It is shown in Appendix D that, in general, $\frac{dI}{d\sigma^2} < 0$. There are two exceptions, however. If a firm's financial condition is either extremely robust (with S so high that $(1-\beta)\pi_I^g < A$,, or the growth-profits tradeoff is operable) or is extremely fragile (where debt is so high that $X > \pi^g$, or an autonomy crisis is likely), then $\frac{dI}{d\sigma^2} > 0$. For the extremely robust firm, a rise in σ^2 which reduces the probability of all undesirable profit outcomes (or all $\pi < X$), lowers the probability of the net additional undesirable profit outcomes associated with $F_I > 0$. Thus $F_{I\sigma^2} < 0$, $S_{I\sigma^2} > 0$ and the firm's utility maximizing opportunities are enhanced as uncertainty rises. For the very fragile firm, $S_{\sigma^2} > 0$ -- security actually rises as the level of uncertainty increases. In this case the reduction in the probability of existing undesirable outcomes outweighs the addition of new undesirable outcomes and $F_{\sigma^2} > 0$: the only hope for a firm that faces relatively certain bankruptcy lies in the additional desirable outcomes associated with an increase in σ^2 which provide a chance, however slim, for survival. Put differently, a firm on the verge of bankruptcy will take an investment gamble that an ordinary firm would not consider. Ceteris paribus, this increase in security induces an increase in I.

Given that σ^2, $\hat{\pounds}$ and π^g or α are all conditional on the existing information set, additional adverse information generated by the passage of time might (depending on the exact nature of expectation formation) reduce α and $\hat{\pounds}$ and increase σ^2, thus causing a reduction in I. In addition, the sequential nature of decision making in our model implies that a reversal in π in subsequent periods would result in a revival of investment. Thus our model can reproduce the '*waiting to invest" result of the irreversible investment literature.

We now consider the effect of a change in K^0, the initial stock of illiquid capital, on I^* and K^*. (Note that $\frac{dI}{dK^0} = \frac{dK}{dK^0} - 1$.) In neoclassical models with reversible investment $\frac{dK^*}{dK^0} = 0$ because the same user cost is applied to old and new capital. Under the assumption of illiquid capital, however, the use of K^0 is "free" in the sense that the costs of autonomy associated with K^0 are fixed or sunk costs. Marginal profit per unit of K^0 is larger than that associated with I because I has variable costs of autonomy. The greater the proportion of K^* represented by K^0 as opposed to I, the lower D (given D^0) and the higher S. Thus, $dK^*/dK^0 > 0$. The effect of K^0 on I' cannot be determined a priori because $\frac{dK^*}{dK^0}$ may be $\gtrless 1$.

Another noteworthy characteristic of the model is that cash flow is positively related to $I: \frac{dI}{d\pi^0} > 0$. (If the model was extended to include a stock of liquid assets, this stock would also be positively related to I.) Given the relation between D and π^0 discussed in Appendix A, this result is qualitatively the same as the $\frac{dI}{dD^0}$ result discussed above. Thus, our model generates the major theoretical relation empirically tested and confirmed by Fazzari, Hubbard and Peterson (1988). While Fazzari et. al. assume that the cash flow (or debt-equity) effect on I is generated on the supply side of financial markets, the relations they estimate are reduced form equations which cannot distinguish between the demand side effect of our model and the supply side influences of theirs.

These comparative static results can serve as a microfoundation for Minsky's financial theory of investment instability. In particular, the results associated with the expectational (π^g or α), attitudinal ($\hat{\pounds}$ and σ^2), and financial variables (r, D^0 and π^0) can be used to model the main characteristics of post-war business cycles in a manner consistent with Minsky's work. In our concluding section, we outline how this can be accomplished.

Finally, it can be shown under reasonable assumptions that changes in the interest rate, the purchase price of a unit of capital, or the rate of depreciation have the expected effect on I'.[15]

V. Conclusion

The comparative static properties of the model suggest that it is a sensible or reasonable formulation of the problem. They also demonstrate that this model of the firm can be used to underpin Minsky's theory of cyclical investment instability.

In the model, investment is affected by three distinct clusters of variables: (1) variables that reflect subjective manaserial attitudes -- O_s/O_G, $\pounds$, $\pi^g(K)$ (which reflects management's optimism about future market growth), and σ^2 (which reflects management's confidence in its ability to forecast meaningfully); (2) variables that describe the financial status of the firm -- D^0, the initial stock of debt, Y, the firm's liquid assets, and r, the interest rate; and (3) CC, the profit markup determined in the real sector of the economy. An advantage of our model as a microfoundation for Minsky's financial theory of the investment cycle, then, is its rich menu of subjective and financial variables. Because of space constraints, we limit our discussion of the role of the model in a Minsky cycle to the behavior of investment at the cycle's end-of-expansion, onset-of-crisis stage.

In the mid-expansion phase of a Minsky cycle investment spending rises rapidly because boom euphoria raises managerial optimism and confidence (π^g rises due to revised estimates of future aggregate demand while σ^2 falls). Boom euphoria may also dampen management's concern with safety (O_G/O_s may rise). $\pounds$, the maximum acceptable leverage ratio, either creeps up (if the boom is modest) or leaps up (if the boom is expected to be long and vigorous): the firm is now willing to use debt-finance more aggressively. Working against these developments, we may see a modest rise in r and in D^0 (as borrowing outstrips cash-flow and the financing gap widens). However, according to Minsky and

Keynes (1936; CH 22), changes in the subjective variables will dominate any modest objective deterioration in the firm's financial status in the heat of the boom and investment spending will accelerate.

Both Keynes and Minsky blame the onset of recession on some combination of rising interest rates and the inevitable disappointment of the euphoric profit expectations of the mid-expansion. Bouyant profit expectations (reflected in π^g) confidently held (i.e., with low σ) may have outpaced π, realized profits. (Keep in mind that α is assumed constant and therefore cannot rise in Minsky's model, while it may fall in Keynes's.) Therefore, the firm must now make downward revisions in its profit expectations just as interest payments rise unexpectedly. If its forecasts errors are large enough, management might lose confidence in its ability to make meaningful forecasts (so σ^2 might explode). Meanwhile, it will have to adjust downward its estimate of $\pounds$ and will see its liquid assets erode (or Y fall).

As these developments depress I*, desired investment, actual investment expenditures will be sustained for some time by the need to complete unfinished projects. Thus, the need to borrow, even at high interest rates will continue right into the recession. I* will now be ready to collapse, leading the economy into a downward spiral of uncertain, historically contingent dimensions. For example, if π is low enough and D^o is high enough, or the typical firm is financially "fragile," a financial panic might take place.

The model developed here can also reproduce the other phases of Minsky's business cycle. And its set of investment determinants is rich enough to enable it to "explain" business cycles with distinct patterns. Of course, we have paid a price for this richness. The theory is too complex to find incorporation in a formal, mathematical business cycle model and the problem as we have posed it seems to elide any simple, mathematically elegant formulation. Nevertheless, we believe that the benefits outweigh the costs. And we hope that our efforts will stimulate others to develop more attractive models of the enterprise investment decision in a Keynes-Minsky setting.

Appendix A

In this appendix we formally specify the financing and debt structures described in the body of the paper. The representative firm is modelled as a net debtor with no liquid assets and with only two sources of investment finance: Y, the internal funds or cash-flow carried over from last period ($Y=\pi^{o}-X^{o}$) and if $Y<P^{K}I^{g}$, new debt **(AD).** If $Y>P^{K}I^{g}$, the residual goes to debt reduction. All debt is assumed to be variable rate consols.

The firm's level of debt is thus given by

$$D = D^{o}+(P^{K}I+\delta P^{K}K(I;K^{o}))-Y \tag{A1}$$

where δ is the rate of depreciation and $(P^{K}I+\delta P^{K}K)=P^{K}I^{g}$. The interest cost, direct and/or opportunity, associated with $P^{K}I^{g}$ is $rP^{K}I^{g}=rP^{K}I=r\delta P^{K}K$ which is equivalent to X, the current cost of autonomy in a world without dividends.

The application of present valuation rules to $rP^{K}I^{g}$ given the independence of K' in successive periods leads to the particular form of A in equation *(2).* Abstracting for the moment from dividend payments,

$$A=rP^{K}I+\delta P^{K}K=r(D^{o}-Y)$$

While the independence of succesive X's implies the independence of replacement investment (δK), it also establishes a dependence between net investment (I) in successive periods -- a one unit increase in current period I must reduce I in the next period by an equivalent amount in order to preserve the independence of the $K's$. Thus, the current financing of δK generates an infinite stream of debt payments ($r\delta P^K K$) with a present value of $\delta P^K K$, while the net effect of financing I is limited to a one time, current period, debt payment $rP^K I$ -- the future stream of debt payments associated with I is offset by the equivalent reduction in the debt payments associated with I in the next period. Therefore, when dividends are zero the present discounted value of : (1) debt payments associated with I^g; and (2) total debt payments or total current costs of autonomy, X, reduces respectively to $rP^K I+\delta P^K K$ and $rP^k I+\delta P^K K+r(D^0-Y)$ where $r(D^0-Y)$ can be interpreted as a onetime fixed or sunk cost of K^0 generated from the carry-over debt (D^0-Y) associated with K^0.

Finally, to preserve the consistency of the static problem in equation (1) with the dynamic problem under our simplifying assumption (in Appendix B) requires that expected dividend payments be defined as a percent of gross profits minus ~~last period's~~ interest payments rather than current interest payments: $\beta(\pi^g-rD^0)$ where $0<\beta<1$. We model dividends in this manner to avoid the reintroduction of minor future effects -- R' would depend on both the D and K trajectories -- that could disrupt the independence of K in successive periods. Including expected dividend payments as a current cost of autonomy implies that

$$X=rP^K I+r\delta P^K K(I;K^0)+r(D^0-Y)+\beta(\pi^g-rD^0)$$

and that $Y=\pi^0-X^0=\pi^0-rD^0-\beta(\pi^0-rD^0)$. Substituting for Y in equation (Al) results in a fuller specification of current debt:

$$D=(1+(1-\beta)r)D^0+P^K I+\delta P^K K(I;K^0)-(1-\beta)\pi^0$$

Incorporating dividend payments and the cash-flow relation, Y, expected net revenues -- expected gross profit flows minus the expected present value of the costs of autonomy associated with I^g -- can now be fully specified as

$$R=(1-\beta)\pi^g(K(I;K^0))-[rP^K I+\delta P^K K(I;K^0)+(1-\beta)(r(D^0(1+r)-\pi^0))]$$

Given the complete specification of X, A, and D, the series of parameter vectors referred to in the text can be written as:

$$\bar{R}=(\beta,r,P^K,D^0,\pi^0,K^0),\bar{X}=(r,P^K,\bar{\delta},D^0,\pi^0,\beta,K^0),\bar{a}=(\sigma^2,K^0),$$

$\bar{D}=(\beta,r,D^0,P^K,\delta,K^0,\pi^0)$ and $\hat{D}=(\hat{L},P^K,K^0)$.

The firm's full optimization problem becomes equation *(3)* subject to the specifications of R, A, X and D in this appendix.

Appendix B

In this Appendix we develope the relationship between the full dynamic control problem suggested in Section I, the equivalent dynamic model under the three simplifying assumptions made in Section I, and the equivalent static model examined in the body of the paper.

The full dynamic model can be expressed as

Maximize

$$O[G(R'(\bar{K},\bar{I}),\ K'(\bar{K})),\ S(F(K_t,I_t)),\ S(F(\bar{K}_{t+1},\ \bar{I}_{t+1}))$$

$$\ldots\ S(F(\bar{K}_T,\ \bar{I}_T))] \qquad \text{(B1a)}$$

Subject to

$$I_t = K_{t+1}\ K_t \text{ for } t = 1 \ldots T \qquad \text{(B1b)}$$

$$K_0 = K^0 \qquad \text{(B1c)}$$

$$X_0 = X^0 \qquad \text{(B1d)}$$

where R' is the present discounted value of the net revenue stream over the planning horizon (t = 1 ... T), K' is a time-weighted average of the firm's real capital stock, $\bar{K}$ and $\bar{I}$ are the capital stock and net investment trajectories from t = 1 ...T. $\bar{K}_{t+1}$ and $\bar{I}_{t+1}$ are the K and I trajectories from t = 1 ...t+ i, the sequence of S functions represent S in sucessive time periods, and all subscripts refer to time periods.

In this specification. all future effects of current I decisions, on G and S are included.

Under simplifying assumption (2) and the respecification of S, elaborated in Section I, which preserves the dynamic nature cf the firm's S objectives in a world in which future effects are unknowable, problem (B1) reduces to

Maximize

$$O[G(R'(\bar{K},\bar{I}),\ K'(\bar{K})),\ S(F(K_t,I_t),\ D'(K_t,I_t))] \qquad (B2)$$

subject to (B1b), (B1c) and (B1d).

In this specification, the firm chooses K_t for $t = 1\ldots T$ to optimize (B2). K_1 is chosen to satisfy the G and S objectives of the firm, while $K_2 \ldots K_T$ are chosen to satisfy only the firm's G objectives. Assuming that the respecification of S adequately captures the firm's inherently dynamic S objectives under assumption (2), as is argued in the text, problem (B2) can be considered to be equivalent under our core and subsidiary assumptions to problem (B1).

Under assumption (3) in Section I, K_t^* and K_{t+i}^* are independent for all i. In particular $K_t^* \neq K_{t+i}^*$ for all $i > 0$. Under assumption (1). only the K_1^* element of the K^* trajectory is implemented implying that the maximization of (B2) with respect to the firm's growth objectives is equivalent to the maximization of the firm's current growth objectives which in turn are a function of current expected net revenues (R) and the current size (K) of the firm. Treating the relation between K and I implicitly. problem (B2) is equivalent to problem (1) or problem (3) in the text.

Accepting the specification of dividends as $\beta(\Pi^g - RD^0)$ in order to avoid the introduction of minor future effects, letting $T = \infty$, and recognizing differences in the specification of R_t in problem (B2) and R in problem (3) that result respectively from the explicit and implicit treatment of the relation between K and I and thus the interdependence of I_t and I_{t+i} (implied by the independence of K_t and K_{t+i}) in each problem, it can readily be shown that the first order conditions for K_1 in problem (B2) are equivalent to the first order conditions for I in (3).

An extended appendix • vJith this proof is available upon request from the suthsrs. For the interested reader the differences in R_t in (B2) and R in (3), necessary to set up this proof, follow.

$$R_t = (1-\beta)\Pi^{\mathfrak{T}}(K_t) - [rP^K I + r\delta P^K K + (1-\beta)(r(D^0(1+r-\Pi^0))] \quad (B3)$$

$$R = (1-\beta)\Pi^{\mathfrak{T}}(K(I;K^0)) - [rP^K I + \delta P^K K(I;K^0) + (1-\beta)(r(D^0(1+r-\Pi^0))] \quad (B4)$$

The similar, although different after discounting, treatment of the interest payments associated with net investment, $(rP^K I)$, results from the treatment of the interdependence of I_t and I_{t+i} in problem (B2) via the first order conditions, while the difference in the treatment of the interest pavments associated with replacement investment $((r\delta P^K K)$ vs. $(\delta P^K K))$ results from the independence of K_t and K_{t+i} and the use of discounting in problem (B2). The proof 'follows from the application of the first order conditions associated with the theory of optimal control of dynamic systems to problem (B2) where (1) the interdependency between I_t and I_{t+i} and the independence of K_t and K_{t+i} and (2) the specific functional forms for I?+ in (B3),F and D' in the text and $K' = \sum_{t-1}^{T} \gamma^{t-1} K_t$ with $0 < \gamma < 1$ are incorporated. The first order conditions for K_1 in problem (B2) are equivalent to the first order conditions for problem (3) presented in Section Iii.

Appendix C

In this Appendix, we consider a proposition that establishes the signs of S_I and G_I in equilibrium and thus the exact nature of the tradeoff between G and S. Two cases are considered: (1) $\delta = 0$; and (2) $\delta > 0$.

<u>Proposition</u>:

Define I_1, I_2, I_3, and I^+ as the I levels at which $S_I(I_1) = 0$, $G_I(I_2) = 0$, $F_I(I_3) = 0$, and $G_I(I^+) = -\frac{\theta_S}{\theta_G} S_I(I^+)$. Define condition (1) as $S_{II} < 0$ and $G_{II} < 0$ for $I_1 \leq I \leq I_2$, and condition (2) as $\frac{G_K/P^K}{G_P} > \delta(1-r)$. If $\delta = 0$ and condition (1) holds, $G_I(I^+) > 0$ and $S_I(I^+) < 0$. If $\delta > 0$ and conditions (1) and (2) hold, a strong set of conditions is met for $G_I(I^+) > 0$ and $S_I(I^+) < 0$.

<u>Proof</u>:

Case I

Under the assumptions on the demand curve ($P_2 < 0$ and $P_{22} \leq 0$), $S_{II} < 0$ and $G_{II} < 0$. Thus condition (1) holds. At I_2, $G_P[(1-\beta)\frac{\pi_I^g}{P^K} P^K - rP^K] + G_K = 0$ or the marginal profit rate on investment $\frac{\pi_I^g}{P^K} \equiv \pi^r = \frac{r}{(1-\beta)} - \frac{G_K}{G_P P^K (1-\beta)}$. Thus $\pi^r(I_2) < \pi^r(I_3) = \frac{r}{1-\beta}$. Given that $\pi_{II} < 0$ for all I, there exists a level of investment, $I_3 < I_2$, where $\pi^r(I) = \frac{r}{1-\beta}$. At I_3, $S_I = S_F[(\pi^r P^K(\beta-1) + rP^K)f] + S_D[1-L^+] P^K$ reduces to $S_D[1-\hat{L}]P^K < 0$ and $G_I = G_K > 0$. Since $S_I(I)$ is a continuous function, $S_I(I_3) < 0$ and $S_{II} < 0$ then $I_1 < I_3 < I_2$. Given that $S_I(I) < 0$ for $I > I_1$, and $G_I(I) > 0$ for $I < I_2$, $S_I(I) > 0$ for $I < I_1$ and $G_I(I) < 0$ for $I > I_2$, the requirement that I^+ satisfy sgn $(-S_I(I^+))$ = sgn $G_I(I^+)$ implies $I_1 < I^+ < I_2$ and that $G_I(I^+) > 0$ and $S_I(I^+) < 0$.

Case II

Condition (1) holds as in Case I. If the firm has a minimal concern (preference) for the size of the firm subobjective, then condition (2) holds. Assuming this preference exists, $\Pi^r(I_2) = (r+\delta)/(1-\beta) - G_K/G_P P^K(1-\beta) < \Pi^r(I_3) = r(1+\delta)/(1-\beta)$. Thus as in Case I, $I_1 < I_3 < I_2$ and $G_I(I^*) > 0$ and $S_I(I^*) < 0$.

It must be emphasized that the minimal preference for the size objective is a strong condition. As $\delta \rightarrow 1$ the movement of the equilibrium point towards the fourth quadrant (due to $G_{I\delta} < 0$) where $G_I < 0$ and $S_I > 0$ is offset not only by a minimal preference for size that increases $G_I(I)$ for all I but by the shift in $S_I(S_{I\delta} < 0)$. Given the difficulty in comparing $|G_{I\delta}|$ and $|S_{I\delta}|$, the stronger condition is invoked.

Using realistic values for δ and $r-\delta=.08$ and $r = .1$ -- condition (2) requires that the response of G for a one dollar increase in $P^K K$ is 1/14 of the response in G for a one dollar increase in F!. Thus, a mild preference for the size subobjective is sufficient to satisfy this strong condition. In comparison to $G_K = 0$, the existence of a preference for size implies that I_2 is greater -- on the basis of G objectives alone, the firm will invest more. Thus, $\Pi_I^g(I_2)$ is smaller and is more likely to be less than $\Pi_I^g(I_3)$, and I_2 is more likely to be less than I_3.

Appendix D

In this appendix the comparative static results and conditions discussed in Section IV are formally derived. Total differentiation of the first order condition for the model in (3) yields

$$|H|dI = Ad\alpha + BdD^0 + Cd\hat{L} + Ed\sigma^2 + GdK^0 + Jdr + LdP^K + Md\Pi^0 + Nd\delta$$

where

$$A = -[O_G(G_R(1-\beta)\Pi^{\sigma}_{I\alpha}) + O_S(S_F f(\beta-1)\Pi^{\sigma}_{I\alpha}) + S_I O_{SS}(S_F(\beta-1)Qf] < 0$$

$$B = -[S_I O_{SS}(S_F(rf(1+r)(1-\beta)) + S_{D'}(1+(1-\beta)r))] > 0$$

$$C = -[O_S(-S_{D'}P^K) + S_I O_{SS}(-S_{D'}P^K K)] < 0$$

$$E = -[O_S f_{\sigma^2}((\beta-1)\Pi^{\sigma}_I + rP^K) + S_I O_{SS}(S_F(\frac{\beta}{b-a}f + \int_a^{\bar{\Pi}} f_{\sigma^2} d\Pi))] \gtrless 0$$

$$G = -[S_I O_{SS}(-S_F f r P^K - S_{D'}P^K)] - |H| \gtrless 0$$

$$J = -[O_G(-G_R P^K) + O_S(S_F P^K(1+\delta)f) + S_I O_{SS}(S_F f(D^0(r(1-\beta)-\beta)$$
$$+ S_{D'}(1-\beta)D^0)] \gtrless 0$$

$$L = -[O_G(-G_R(r+\delta)) + O_S(S_F fr(1+\delta) + S_{D'}(1+\delta-\hat{L}))$$
$$+ S_I O_{SS}(S_F r(I+\delta K)f + S_{D'}(((I+\delta K)-\hat{L}K))] \gtrless 0$$

$$M = -[S_I O_{SS}(S_F r(\beta-1)f + S_{D'}(\beta-1))] < 0$$

$$N = -[O_G(-G_R P^K) + O_S(S_F r P^K f + S_{D'}P^K) + S_I O_{SS}(S_F r P^K f + S_{D'}P^K K)] > 0$$

$|H|$ = LHE equation (4) < 0

Setting the appropriate terms equal to zero and solving for the comparative static results discussed in the text yields

$\frac{dI}{d\alpha} = \frac{A}{|H|}$ = RHS of equation (6), $\frac{dI}{dD^0} = \frac{B}{|H|}$, $\frac{dI}{d\hat{L}} = \frac{C}{|H|}$, $\frac{dI}{d\sigma^2} = \frac{E}{|H|}$, $\frac{dI}{dK^0} = \frac{G}{|H|}$, $\frac{dI}{d\Pi^0} = \frac{M}{|H|}$, $\frac{dI}{d\delta} = \frac{N}{|H|}$ and $\frac{dI}{dP^K} =$

$$\frac{-[O_G(-G_R(r+\delta))+O_S(S_F f r(1+\delta)+S_D(1+\delta-\hat{L}))+S_I O_{SS}(S_F r(I+\delta K)f+S_D((I+\delta K)-\dot{L}K))]}{|H|}$$

$$\frac{dI}{dr} = \frac{-[O_G(-G_R P^K)+O_S(S_F P^K(1+\delta)f+S_I O_{SS}(S_F f(D^0(r(1-\beta)-\beta)+D)+S_D(1-\beta)D^0)]}{|H|}$$

$$\frac{dI}{d\sigma^2} = \frac{-[O_S(S_F(f_{\sigma^2}((\beta-1)\Pi^\sigma_I+rP^K(1+\delta))))+S_I O_{SS}(S_F(\frac{2}{(b-a)}f+\int_a^X f_{\sigma^2}d\Pi))]}{|H|}$$

We **now** consider $\frac{dI}{d\sigma^2}$. Given that $f_{\sigma^2} = \frac{-6}{(b-a)^2}$ and that $\int_a^X f_{\sigma^2} d\Pi = (X-a)\frac{-6}{(b-a)^3}$, the necessary and sufficient condition for $S_{I\sigma^2} < 0$ is $\Pi^\sigma_I < \frac{rP^K(1+\delta)}{1-\beta}$ and the necessary and sufficient condition for $S_{\sigma^2} < 0$ is $\frac{2}{(b-a)^2} + (X-a)\frac{-6}{(b-a)^3} < 0$ or $X \geq \frac{b+a}{2} = \Pi^\sigma$. Thus a strong set of conditions for $\frac{dI}{d\sigma^2} < 0$ is (1) $\Pi^\sigma_I < \frac{rP^K(1+\delta)}{1-\beta}$ and (2) $X \geq \Pi^\sigma$. Recognizing that it is possible for $X \geq \Pi^\sigma$ at low levels of I (high values of Π^σ_I), conditions (1) and (2) can be combined by defining I_c as the level of I < I_2 where $X = \Pi^\sigma$. Thus the strong condition for $\frac{dI}{d\sigma^2} < 0$ requires that $I_c < I < I_2$ or that the firm's financial position is neither extremely robust (such that $S_{I\sigma^2} > 0$) nor extremely fragile (such that $S_{\sigma^2} > 0$). Assuming that the respresentative firm's financial position is captured by this description, then $\frac{dI}{d\sigma^2} < 0$. If it is assumed that the direct effect of σ^2 on S_I ($S_{I\sigma^2}$) dominates its affect on O_S ($O_{SS}S_{\sigma^2}$) then the strong condition for $\frac{dI}{d\sigma^2} < 0$ reduces to condition (2).

A strong condition for $\frac{dI}{dr} < 0$ is $r(1-\beta)D^0 - \beta D^0 + D > 0$. Recognizing that $D \equiv \frac{dD}{dt} + D^0$, a stronger condition is $\frac{dD}{dt} > (\beta-1)D^0$ -- the change in debt is greater than a $|\beta-1|$ (50% if $\beta = .5$) reduction in the stock of debt. This condition is extremely likely to hold for the representative firm. Thus, $\frac{dI}{dr} < 0$ requires no restricitive assumptions.

Finally, the large increase in admissable debt when P^K increases by a unit ($\hat{L}K$) implies that S_{P^K} is sign indeterminant. A strong condition for $\frac{dI}{dP^K} < 0$ is that $|O_S S_{IP^K}| > |S_I O_{SS} S_{P^K}|$ -- the direct effect of P^K on S_I dominates the preference weight change effect.

Footnotes

[1]The mainstream literature on "irreversible nvestment" that has arisen in the past decade is an excellent example of this problem. These models of enterprise decision making must rely on every conceivable, empirically repugnant, neoclassical, efficient-markets assumption to be able to derive an analytical solution to the investment problem they pose. See the literature on irreversible investment in Pinkyck (1991).

[2]See Crotty (1990) for an analysis of the contrasting views of Keynes and Minsky on this issue.

[3]The market value of the firm, he told us, "is the outcome of the mass psychology of a large number of ignorant individuals" (1936, p. 154).

[4]By avoiding the conflation of ownership and management we also allow for the theoretical possibility of partially autonomous and non-synchronic developments in the real and financial sectors of the economy.

[5]See Cyert and Hendrick (1972), Marris and Meuller (1980) and Williamson(1981) for surveys of managerial and behavioral theories of the firm.

In recent gears neoclassical financial economists concerned with the agency costs associated with conflicts between owners and managers have adopted a traditional "managerial" view of the objective function of the enterprise. For example. Jensen (1988) has argued that "managers have incentives to expand their firms beyond the size that maximizes shareholder wealth. Growth increases managers' power by increasing the resouces under their control, and changes in management compensation are positively related to growth. Moreover. the tendency of firms to reward middle managers through promotion rather than through year-to-year bonuses also creates an organizational bias toward growth to supply the new positions that such promotion-based reward systems require" (1388, p. 28). For a general discussion of the effects of owner-manager conflict on the investment decision of the firm, see Crotty (1990).

Lazonick's (1991) recent history of the evolution of the organizational structure of the business enterprise argues that US business has been "managerial" rather than neoclassical in form and behavior since the 1920s. Indeed, he refers to the US economic system from the 1920s to the present as "managerial capitalism."

[6]The ioint maximization of R' and K' best represents management's growth objective because the maximization of l?' with minimum size or the accumulation of a large productive capacity with poor profit prospects do no guarantee long-run survival in a dynamic competitive environment. Larger firms have easier access to financial markets, and economies of scale in research and development and in marketing. Finally, control of a large firm confers income, status and power on

management. See Donaldson and Lorsch(1983)for case-study evidence that supports our specification of the firm's <growth objective.

[7]It is common in the neoclassical investment literature to reduce a dynamic problem to a static one. See, for example, Jorgenson's rental price model. Such models are static because the assumptions of perfect certainty and/or investment reversibility eliminate any intertempcral profit tradeoff. Our reduction on the other hand, results from the combined assumptions of illiquid capital and Keynesian uncertainty.

[8]We specify the firm's short-term expectations of profitability in this way solely in order to make the analytics of our model tractable. Under Keynesian uncertainty rational agents could never formulate such a complete distribution and, even if they could, they would never have complete confidence that it represented the whole truth about likely future state of the world. Since all such forecast3 are built on hopes, fears and social conventions of various kinds, they can never attain even the subjective status of knowledge.

'Even so, the model as is, constitutes a more Keynesian than neoclassical formulation of the problem because the illiquidity of capital causes the investment decision to be constrained by the existing financial structure of the firm.

[10]Note that this formulation of the index of long-term vulnerability stresses the threat to autonomy from creditors and excludes the threat posed by shareholders.

[11]Note that while D chanqres slowly over time, $\hat{D}$ is subject to dramatic shifts at times of unforeseen change, 3uch a3 the onset of an unexpected recession or a financial market panic, when events reveal that the firm's expectations were in serious error. Investment demand itself would then experience dramatic shifts as well.

[12]Suppose the firm sets $\hat{D}$ by choosing a target ratio of debt to debt-plus-equity. A dollar used-to buy capital goods will raise D by one dollar because it was financed either by borrowing or by the use of internal fund3 that could have been used to retire debt instead. Eut D will rise by a fraction equal to the target debt to debt-plus-equity ration. Since this fraction must be less than one, D' will rise.

[13]If II is distributed normally, qualitatively similar results are found. See n. 15 for a comparison of comparative static results under these two alternative assumptions on f.

[14]While thi3 derivation of π^g_{II} invokes the production function assumption that $Q_{II} = 0$, all derivations and proofs that follow hold for the more general case where $Q_{II} \leq 0$.

[15]Qualitatively similar results to those derived in this section hold for the case where Π is distributed normally. In particular, the comparative static results for $\hat{L}, r,$ and P^K hold under the same conditions, while the remaining results require only marginally more restrictive conditions: $dI/d\alpha > 0$ requires as a strong condition $\Pi_I^g > 0$; $dI/dD^0 < 0$ requires as a strong condition $\Pi_I^g > 0$ and $X < \Pi^g$. With the exception of $dI/d\sigma^2$, the remaining comparative static results are qualitatively similar if $|\Pi_{IP}^g f(\Pi=X)| > | \Pi_I^g f_{\Pi^g}(\Pi=X)[X_P - \Pi_P^g] + X_I \Pi_P^g f_{\Pi^g}(\Pi=X)|$ where p is any relevant parameter. This condition is shown to hold in all cases for realistic parameter values. In addition $dI/d\sigma^2 < 0$ requires as a strong condition that $\Pi^g > rP^K/1-\beta$ and $X < \Pi^g - \sigma$. A technical appendix that derives these results is available upon request from the authors.

References

Crotty, J. R., "Owner-Manager Conflict and Financial Theories of Investment: A Critical Evaluation of Keynes, Tobin and Minsky," Journal of Post Keynesian Economics, Summer, 12:4, 1990, 519-42.

Cyert, R. M. and Hendrick, C. L., "Theory of the Firm: Fast, Present, and Future; An Interpretation," Journal of Economic Literature, 10:2, 1972.

Delli Gatti, F. and Galligati, M., "Financial Institutions, Income Distribution, and the Stock Market," Journal ofPost Kevnesian Economics, 1990, 356-74.

Donaldson, G. and Lorsch, J., Decision Makins at the Too: The Shaping of Strategic Direction, New York: Basic Books Inc., 1983.

Fazzari, S. M., Hubbard, R. G., and Petersen, B. C., "Financing Constraints and Corporate Investment," Brookings Papers on Economic Activity, 1988:1, 141-95.

Jarsulic, M., "Endogenous Credit and Endogenous Business Cycles," Journal of Post Keynesian Economics, 1989, 35-48.

Jensen, M., "Takeovers: Their Causes and Consequences," Journal of Economic Perspectives, Winter 1988, 2(1), 21-48.

Keynes, J. M., The General Theory of Income, Interest and Money, Cambridge: Cambridge University Press, 1972 [1936].

Lazonick, W., Business Orqanization and the Myth of the Market Economy, Cambridge: Cambridge University Press, 1991.

Marris, R. and Mueller, D. C., "The Corporation, Competition and the Invisible Hand," Journal of Economic Literature, 18:1, 1980.

Minsky, H., Stabilizins an Unstable Economy , New Haven: Yale University Press, 1986.

Minsky, H., Can "It" Happen Aqain?, M.E. Sharpe: Armonk, N.Y., 1982.

Minsky, H., John Maynard Keynes, New York: Columbia University Press, 1975.

Pindyck, R., "Irreversibility, Uncertainty and Investment," Journal of Economic Literature, September 1991, 1110-1148.

Semmler, W. and Franke, R., "Debt Financing of Firms, Stability, and Cycles in a Dynamic Macroeconomic Growth Model." In E. Nell and W. Semmler, eds, Nicholas Kaldor and Mainstream Economics, Macmillan, 1991.

Semmler, W., "A Macroeconomic Limit Cycle with Financial Perturbation," Journal of Behavior and Oraanization,

Skott, P., "On the Modeling of Systematic Financial Fragility," mimeo, University of Mass., 1991.

Taylor, L. and O'Connell, S., "A Minsky Crisis," Quarterly Journal of Economics, 1985, 871-875.

Vickers, D. Money Capital in the Theory of the Firm, Cambridge: Cambridge University Press, 1987.

Williamson, 0. E., "The Modern Corporation: Origins, Evolution, Attributes," Journal of Economic Literature, 19:4, 1981.