August, 1996

Dynamic Traffic Assignment for Automated Highway Systems: A Two-Lane Highway with Speed Constancy

H.-S. Jacob Tsao, *University of California - Berkeley*

CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
UNIVERSITY OF CALIFORNIA, BERKELEY

Dynamic Traffic Assignment for Automated Highway Systems: A Two-lane Highway with Speed Constancy

H.-S. Jacob Tsao

California PATH Working Paper
UCB-ITS-PWP-96-12

This work was performed as **part** of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department Transportation, Federal Highway Administration.

August 1996

ISSN 1055-1417

DYNAMIC TRAFFIC ASSIGNMENT FOR AUTOMATED HIGHWAY SYSTEMS: A TWO-LANE HIGHWAY WITH SPEED CONSTANCY

H.-S. Jacob Tsao

Institute of Transportation Studies, PATH

University of California, Berkeley

ACKNOWLEDGEMENT

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration. The author would like to thank Professor Bin Ran of University of Wisconsin, Madison for his valuable comments on an earlier version of this paper. He also likes to thank Bruce Hongola of PATH for helpful comments on an earlier version and for implementing the mathematical models with the CPLEX LP Solver.

DYNAMIC TRAFFIC ASSIGNMENT FOR AUTOMATED HIGHWAY SYSTEMS: A TWO-LANE HIGHWAY WITH SPEED CONSTANCY

ABSTRACT

Dynamic traffic assignment through analytical modeling and optimization has been widely accepted by the IVHS R&D community as a promising traffic control tool for understanding and relieving traffic congestion on conventional highways and city streets. Due to the completely controlled nature of AHS traffic, dynamic assignment of AHS traffic is even more promising. One added dimension of complexity associated with AHS dynamic traffic assignment is lane assignment. Lane changes, for fully utilizing AHS capacity or for exiting, incur disturbances to and hence reduction of longitudinal flow. Intelligent lane assignment is necessary to ensure a high rate of successful exiting, to another highway or to city streets, while minimizing the disturbances to the longitudinal flow. Merging maneuvers occurring at merge points such as on-ramps, highway-to-highway interchanges and locations where one lane is being dropped may also introduce disturbances to the longitudinal traffic flow on AHS. Intelligent metering and flow control at on-ramps and other merge locations is necessary to maximize the capacity while balancing the need to serve local demands with the need to optimize system throughput.

This paper identifies a general class of equations/inequalities to represent the impact of lane changes and merges on AHS longitudinal flow and develop mathematical models for system-optimal dynamic traffic assignment on a two-lane (one direction) automated highway. The amount of impedance to the longitudinal flow depends on the operating scenario and traffic conditions and can be highly stochastic. This paper focuses on deterministic impedance. This is a necessary step towards a complete study of stochastic impedance and the experience gained with the proposed focus is indispensable for studying the stochastic impedance.

Key Words: Automated Highway Systems (AHS), Dynamic Traffic Assignment, Lane Assignment, Flow Optimization, Linear Program

DYNAMIC TRAFFIC ASSIGNMENT FOR AUTOMATED HIGHWAY SYSTEMS: A TWO-LANE HIGHWAY WITH SPEED CONSTANCY

EXECUTIVE SUMMARY

AHS capacity prediction via microscopic simulation of vehicle movement for particular AHS operating scenarios has been progressing for several years. Some macroscopic simulation of aggregate traffic flow for smoothing traffic via local regulation of lane change activities started more recently. Based on the workload concept and using time-space consumption as a measure of workload and highway resources, models for static AHS traffic assignment and optimization have been developed. This paper focuses on **dynamic** AHS traffic assignment and optimization.

Dynamic traffic assignment through analytical modeling and optimization has been widely accepted by the IVHS R&D community as a promising traffic control tool for understanding and relieving traffic congestion on conventional highways and city streets. Due to the completely controlled nature of AHS traffic, dynamic assignment of AHS traffic is even more promising. Therefore, we adopt this approach. One added dimension of complexity associated with AHS dynamic traffic assignment is lane assignment.

Lane changes, for fully utilizing AHS capacity or for exiting, incur disturbances to and hence reduction of longitudinal flow. The amount of such disturbances depends on the operating scenario. If a lane change incurs significant amount of such disturbances **or** a large amount of lane changing is required, then traffic assignment at the additional detail level of lane assignment becomes necessary. Although trip lengths may be long, trip lengths on a particular highway could be significantly shorter. Although missing the desired exit and exiting at the next exit once in a while may be acceptable, missing a highway-to-highway connector ramp may incur unacceptable inconvenience. Therefore, intelligent lane assignment is necessary to ensure a high rate of successful exiting, to another highway or to city streets, while minimizing the resulting disturbances to and reduction of longitudinal flow. Merging maneuvers occurring at merge points such as on-ramps, highway-to-highway interchanges and locations where one lane is being dropped may also introduce disturbances to the longitudinal traffic flow on

AHS. Intelligent metering and flow control at on-ramps and other merge locations is necessary **to** maximize the capacity while balancing the need to serve local demands with the need to optimize system throughput.

This paper identifies a general class of equations/inequalities to represent the impact of lane changes and merges on **AHS** longitudinal flow and develop mathematical models for system-optimal **AHS** dynamic traffic assignment on a two-lane (one direction) automated highway. The amount **of** impedance to the longitudinal **flow** depends on the operating scenario and traffic conditions and can be highly stochastic. This paper focuses on deterministic impedance. This is a necessary step towards a complete study of stochastic impedance and the experience gained with the proposed focus is indispensable for studying the stochastic impedance. **A** computer program will be developed for numerical solution. The findings can be used in longer-term **AHS** capacity studies, especially the prediction of **AHS** network capacity, and be extended to analyze a comprehensive transportation network, including **AHS,** conventional freeways and arterials.

(1) INTRODUCTION

Shladover [9,10] initiated the research into AHS capacity prediction and estimated the achievable lane capacity. Predicting AHS in which lane changes are required is more complicated. The achievable capacity of such systems hinges upon how the traffic is organized, planned and controlled. Based on the concept of platooning, Varaiya and Shladover [15] proposed a five-layered system control architecture for a set of major system control tasks. Much research along this particular system design and operating scenario ensued [e.g. 8,161. Despite recent changes in direction, these efforts can be characterized as oriented towards simulating a ***particular*** AHS operating scenario, particularly ***microscopic*** vehicle movement, and ***local,*** as opposed to system, flow control.

Stevens [12] and Whitney [17] started two different but coordinated software requirements processes for AHS simulation tools under the sponsorship of FHWA. Their efforts can be characterized as geared towards simulating ***all*** major performance measures for ***various*** AHS operating scenarios.

A fundamental characteristic of AHS capacity is the conflicting relationship between the longitudinal flow and the lateral capacity. Tsao et al. [14] developed analytical models for vehicle/gap distribution and studied the probabilistic lane change completion time, a surrogate for lateral capacity, under different operating scenarios. Hall [3] used time-space product as the basic unit of both workload (incurred by lane changes and vehicle following) and capacity resource and studied the conflicting relationship between the longitudinal capacity and the lateral flow requirement (incurred by lane changes, but not merges) along a single "idealized AHS" where vehicles can enter and exit the highway anywhere along the highway (i.e., there are no designated locations as entrances or exits). Brouke and Varaiya [2] extended the workload concept to accommodate various maneuvers, including platoon splits, platoon joins, lane changes and merges. In both efforts, traffic assignment is not dynamic in the sense that either (i) time-varying demand and the resulting time-varying traffic flow are not considered or (ii) traffic state evolution through time is obtained through simulation rather than analytical means. These efforts can be characterized as oriented towards the ***analytical*** modeling, as opposed to simulation, of ***various*** major AHS operating scenarios, as opposed to any particular one.

AHS operation requires not only dynamic route assignment, among other things, but also lane assignment. Despite the wide acceptance of dynamic traffic assignment techniques as a promising approach to help solve the traffic congestion problem on conventional highways and city streets [e.g. 1,4,5], research into such techniques for **AHS** system capacity optimization is ***non-existent.***

In the area of traffic assignment for optimizing flow on conventional highways, research efforts on simulation and analytical modeling have progressed in parallel, with much synergy fostered and reaped between them. We propose to optimize system throughput via ***analytical*** dynamic traffic assignment. Our analytical approach would definitely complement the simulation efforts mentioned earlier. We can use simulation results [e.g. 8,6] to obtain the macroscopic (flow) representation of microscopic vehicle movement. Our results can be used to guide detailed simulation.

(2) PROBLEM DEFINITION AND BACKGROUND

This paper focuses on **AHS** traffic assignment and seeks to develop an analytical model for system flow optimization subject to the exiting requirements. **A** fundamental characteristic of an **AHS** is the conflicting relationship between the longitudinal flow and the lateral capacity. The robustness of the optimizer is achieved by employing an abstract representation of this relationship that is general enough for all major operating scenarios.

Specifically, we will investigate dynamic traffic assignment for capacity optimization for one automated highway (instead of a network) where

(A1) only one vehicle type (passenger vehicle) is accommodated,

(A2) automated traffic is isolated from the manual traffic,

(A3) the roadway consists of two automated lanes (one direction),

(A4) automated vehicles enter the automated traffic directly from dedicated on-ramps (i.e., dedicated to the exclusive use by automation-equipped vehicles) and leave the automated traffic directly into dedicated off-ramps; in other words, there is no transition lane,

(A5) On-ramps and off-ramps can be on-either side of the highway.

(A6) at all highway-to-street and highway-to-highway interchanges, the off-ramp precedes the on-ramp.

We vary the demand, i.e., dynamic O-D matrix, the locations of entrances and exits and length of automated highway. We seek to optimize the system throughput, subject to the exiting constraint. The "shadow prices" will be used to gauge the impact of the flow-hindering constraints. We consider not just trip length distribution but also the realistic need to change highway during a trip. Therefore, a trip length on a particular highway is bound to be shorter than its counterpart when multiple highways are considered. Furthermore, both inflows and outflows on highway-to-highway interchanges are likely much larger than those on regular exits. Their effects will be studied.

The amount of impedance to the longitudinal flow incurred by the lane changes and merges depends on the operating scenario and traffic conditions and can be highly stochastic. This paper focuses on deterministic impedance. This is a necessary step towards a complete study of stochastic impedance and the experience gained with the proposed focus is indispensable for studying the stochastic impedance.

We briefly describe what has been done in related areas and point out the distinction of the proposed work and how it complements the existing work. **Rao** and Varaiya [7] proposed a simple "link" controller that seeks to maximize throughput and maintain smooth traffic flow. Note that the objective is not to ensure a high rate of exiting success. Their aim is to determine the lane change to be advised at that link, not for a complete path to the exit to be decided." One of our goals, on the contrary, is to advise a complete path for vehicles with the common entry and exit points ***so*** that the system throughput is optimized.

For normal driving conditions, one of their objectives is to maintain a constant speed and, to achieve it, the amount of lane changes is constrained. Therefore, vehicles in unacceptable number missing their exits may result. Following the simulation nature **of** that research as well as the local (link) nature of the "pathing" (routing plus lane assignment) algorithm and assuming that speed constancy could be sacrificed for successful exiting, one way to find out how to exit all or most vehicles successfully is to adjust decision variables manually through some calibration process. Note that there are many decision variables and that this could be very computation-intensive as well as labor intensive. Our approach seeks to obtain the optimal decisions numerically using the techniques of mathematical programming.

Rao and Varaiya also suggested in the conclusion [7] that their lane-change policies needed to be augmented to prevent vehicles from missing exits **as** this is the main priority. Our analytical approach not only certainly complements their simulation approach; it also seeks the analytical solution of this problem of paramount importance.

Note that their work is geared towards the ***particular*** **AHS** operating scenario mentioned earlier. However, ours is oriented towards more ***abstraction*** so that the capacity achievable by specific **AHS** operating scenarios can be obtained by "***plugging in***" scenario specifics, particularly the macroscopic representations of the microscopic lane-change and merge behaviors as flow-hindering constraints.

Our ultimate goal of this line of research is to dynamically assign traffic to individual lanes on **AHS.** One approach is to directly formulate this problem as a capacitated dynamic multi-commodity flow problem. In this approach, the flow is defined at the detail level of both the origin and destination. In other words, the flow decision variables are origin-destination specific. However, it is well known that such formulation tend to involve such a large number of variables that efficient solution is difficult. To reduce the number **of** variables, we propose a two-stage approach that decomposes the assignment problem into ***optimal capacity allocation*** and ***path assignment.*** The basic principle behind the two-stage approach is **as** follows. Once a vehicle has entered the highway (i.e., moved beyond an on-ramp and merged into the existing traffic stream on the highway), it becomes part of the traffic currently present on that section. Particularly, it becomes part of the traffic present in that section that is destined for the off-ramp desired by the vehicle. Since AHS treat all vehicles destined for that section equally and does not grant any special treatment to that particular vehicle, it does not need to know the origin of that vehicle once it has entered the highway. ***Suppressing*** the reference to trip origin in mainline flow decision variables is the key to the proposed approach. This eliminates a full dimension for variable designation and helps greatly reduce the number of variables. During the first stage of optimal capacity allocation, highway capacity will be optimized and reserved for all the vehicle trips. Once sufficient capacity has been optimized and reserved, actual lane assignment for the vehicles with a common set of origin, destination and entry time can be made in the second stage. This paper focuses on the first stage - optimal capacity allocation.

We will formulate the problem as a linear program. The automated highway is divided into a number of sections and time is divided into multiple time periods. Each lane has a constant speed and lane-changing and merging maneuvers are not allowed to disturb the speed of either lane. The time-dependent traffic demand is then assigned to the combination of lane/section/time. The objective is to maximize the total longitudinal flow at selected points on the AHS subject to the constraint that all vehicles depart at the desired exits. Safety requirements, including safety spacing and speed limit, will be considered implicitly through problem input. The "shadow prices" and other "dual" information will be used to conduct sensitivity analysis. Assuming that all highway-to-highway interchanges are equipped with one-lane connector ramps, they are, from the view-point of the automated highway under consideration, effectively regular highway-to-street interchange (except the possibly high volume of traffic). Therefore, this model is actually general enough for highway-to-highway interchanges.

Clearly, if lane changes and merges do not affect the longitudinal flow at all, then the capacity of a two-lane automated highway is simply twice that of a single-lane automated highway and the problem is easy. In fact, the difficulty of the problem comes mainly from the the *interaction* between lane changeslmerges and longitudinal flow. We will model this interaction in two different ways. A ***key*** attribute of any such interaction is whether the speed of traffic is affected. On one extreme, no speed change is made to accommodate lane changes and successful exiting at the desired exit is made possible primarily by starting the lane change attempts early enough; merging of a vehicle into the existing traffic stream at on-ramps is to be performed by, among other things, having the vehicle wait for an opportunity, e.g., waiting for a sufficiently large gap. Note that the lane-changinglmerging vehicle as well as the traffic on the destination lane are not allowed to slow down. On the other extreme is the policy that, to ensure a **100%** exiting success rate even though the lane-changing vehicles started the lane-changing attempt late (e.g., when it is very near the desired off-ramp), the lane-changing vehicles as well as the traffic on the destination lane must slow down, safety permitting, to accommodate the lane changes. Apparently, there is a continuous spectrum of intermediate policies. This papers deals with the former extreme. We will identify a class of equations and/or inequalities to represent the microscopic lane-change behaviors as macroscopic flow-hindering constraints. Another classes of equation and inequalities will be identified for the merging behaviors. The insight gained in the process can be used in future attempts to study the spectrum **of** lane-changing and merging policies.

(3) OPTIMAL CAPACITY ALLOCATION

The objective of the stage of optimal capacity allocation is to set aside sufficient yet optimal capacity for the next stage of actual path/lane assignment in response to time-varying OD traffic. When approximation is needed for ease of formulation or algorithmic efficiency, we make sure to allocate sufficient capacity for the assignment stage. This paper deals with an idealized situation where the time-varying **OD** matrix is deterministic and known. It considers only one vehicle type on a fully automated two-lane highway that is dedicated to the use by automation-equipped vehicles. This paper studies an important operating strategy where (a) the speed of traffic on each of the two lanes is maintained at a given constant and (b) there exists a non-zero speed differential between the two lanes. The speed non-zero differential is assumed to facilitate lane-changing.

(3.1) Two Primary Classes of Decision Variables

In this operating scenario, the most important feature is the speed constancy. Each of the two lanes maintains a constant but different speed V_l on lane l, I $=$ **1,** 2, with the right lane (I $=$ 1) being the slower lane and the left lane (I $=2$) being the faster lane. **As** a result, inflow from on-ramps cannot interrupt the speed of the existing traffic and the traffic is not allowed to back up onto the automated highway from the off-ramp. Lane changes are not allowed to interrupt the traffic speed on either lane. Therefore, controlling the inflow (i.e. merging a maximum amount of incoming vehicles from the on-ramps without disturbing the speed of existing traffic on the highway) and controlling lateral flow (i.e. scheduling/locating lane changes ***so*** as not to disturb the traffic speed) are the two most important classes of decisions. (We assume sufficiently large buffer sizes at the off-ramps ***so*** that the outgoing traffic will not back up onto the AHS.)

(3.2) Discrete Formulation: Characteristics and Assumptions

In this paper, we adopt a discrete formulation approach to the dynamic traffic assignment problem in which the continuous time is divided into intervals of equal length δT. We discuss in this section some basic characteristics and assumptions about the discrete approximation to **AHS** traffic.

(3.2.1) Highway Representation:

The highway is divided into N sections. Figure **1** illustrates the highway under consideration. Conforming to the notational convention established in the discipline of dynamic traffic assignment, we use the subscript a to represent the section. There is at most one off-ramp in each section but it can be on either side **of** the highway. But, there can be one on-ramp on both sides of the highway. In practice, there is very little justification for constructing on-ramps on both sides for the same highway-to-street interchange. Accommodating such a possibility is motivated by the ease of formulation. When there is only one on-ramp in the section, the demand as well as the capacity of the ramp on the other side are set to zero so as to effectively reflect the absence of ramp. The on-ramp, if existing, is at the beginning of the section while the off-ramp, if existing, is at the end of the section. The lane in a section from which the off-ramp forks off from will be referred to as the exit lane while the other lane will be referred to as the non-exit lane.

(3.2.2) Section Length and Speed:

The section length L_a, length of time interval T and traffic speeds V_l are such that a vehicle cannot physically traverse longer than the length **of** any whole section in one time period. **As** a result, a vehicle cannot physically appear in more than 2 sections in a time period. In Figure 1, the shaded areas indicate the distances traveled by a vehicle in the respective lane during a time period.

(3.2.3) Unidirectional Lane Changing in Section:

Only one direction of lane changing is allowed in a section, i.e. either from the faster lane to the slower lane **or** the reverse direction, but not both. This is to simplify the task of quantifying the lateral capacity. With both directions of lane changing allowed, defining the concept of lateral capacity is more complicated and formulating the lateral capacity constraints becomes more difficult. Rao and

Varaiya made the same restriction [8]. In Figure **1,** the arrows across the lane line indicate direction **of** lane changing.

(3.2.4) Destination-Specific Flow:

Due to the need to ensure successful exiting for all vehicles, traffic movement between sections and lanes is identified to the detail of traffic destination, but not the origin and departure time, which are considered only in the traffic assignment stage.

(3.2.5) Uniformly Distributed Traffic in Section:

Several variables represent the traffic condition on a section. An important class **of** traffic variables is the amount of traffic that travels from one section to the next on a given lane. This amount depends on the speed of traffic on that lane as well as where the vehicles are. Suppose that all the vehicles that are present on a section/lane at the beginning of a time period are concentrated in the very beginning of the section/lane. The time period could be ***so*** short that none of the vehicles can reach the next section at the end of the time period. On the other hand, all the vehicles may instead be concentrated on the end **of** the section/lane. In this case, all these vehicles may be able to reach the next section by the end of the time period. The point is that, to properly model the progression of traffic, one needs to know where the vehicles are along the section/lane. To simplify the modeling effort, we make the following assumption.

Assumption I: At the beginning of each time period, the vehicles in a section/lane that are headed for any destination down the highway is uniformly distributed along that section. Similarly, at the beginning of each time period, the vehicles in a sectiodane destined for any section that desire to change lanes and are able to change lanes during the time period are also uniformly distributed along that section.

As will be seen later in this paper, this assumption simplifies the calculation of the longitudinal as well as lateral traffic flow. Although Assumption **1** will simplify the modeling effort, it alone may introduce unacceptable modeling errors. We first point out a possible error and then propose a way to amend the shortcoming.

Consider two consecutive sections where the upstream section is very short but the downstream section is very long. Concentrate on any of the two lanes. Assume that the upstream section is packed with vehicles while the downstream section has no vehicles at all. Also Assume that the length of time period is exactly the time required to travel the length of the upstream section. Therefore, all the vehicles in the upstream section can reach the downstream section in the next time period. By Assumption 1, all the vehicles will be uniformly distributed along the downstream section. Since the down stream section is very long, by the uniform distribution, some of the vehicles would be positioned at the end of the downstream section. However, this implies that some of the vehicles must have traveled at a very high speed ***so*** that they could reach those positions. By making the downstream section arbitrarily long, the implied speed of some of the vehicles could be made arbitrarily high. This is certainly unacceptable.

This shortcoming can be largely overcome by partitioning the highway into equi-length sections or sections of approximately equal length and setting the length of time period to the time required for the traffic on the faster lane to travel the whole length of the equi-length sections. This ensures that, in the absence of lane changes, all the vehicles on the faster lane in any section will reach the next section and hence the relative positions of the vehicles remain intact. In fact, the positions of vehicles become irrelevant for the faster lane. Since there exists a positive speed differential between the traffic on the two lanes, this arrangement does not render modeling the vehicle positions on the slower lane completely unnecessary. In fact, it is impossible to avoid the need to model the position of the vehicles on both lanes. However, when the speed differential is small, this arrangement should suffice even for the slower lane.

In this arrangement, the positions of interchanges may not be at or near the section boundaries. To ensure successful exiting, instead of placing an off-ramp at its actual position in the model, one can place it at the nearest section boundary upstream from the actual position creating a virtual off-ramp. Since the virtual off-ramp **is** before the actual one, ensuring successful exiting at the virtual exit guarantees successful exiting at the actual off-ramp. The section length should be set ***so*** that the need to have equi-length sections is balanced with the need to model the configuration accurately. For generality, we do not assume equi-length sections but call for careful partitioning of the highway during modeling.

(3.3) Model Building:

This subsection discusses some basic characteristics of the optimal capacity allocation problem. There are four basic different types of variables: metered inflows, lateral flows, outflows and longitudinal flows. Actually the only decision variables are the metered flows into the highway and the lateral flows. The outflow from a section can be derived and is simply the total number **of** vehicles that are (a) destined for that section, (b) present in the destination section at the beginning of the time period, and (c) are able to reach the off-ramp (at the end of the section) during the time period. This includes such vehicles that are either on the exit lane already or on the other lane (the non-exit lane). Those on the other lane have to make a lane change in that section, if the lane-changing direction is allowed. (Note that this amount of outflow may exceed the off-ramp capacity, in which case the capacity assignment plan is infeasible.) Given the metered inflow, outflow and lateral flow, longitudinal flow can be derived, as will be explained later in this subsection. We first summarize the notations defined ***so*** far and then define the input parameters, decision variables and derived variables as follows. We will discuss the decision variables and how the rest of the variables can be derived from the decision variables afterwards.

(3.3.1) Basic Notation:

In defining the notation, we adopt the following convention. All indices are lower case. Upper-case quantities are problem parameters, whose values will be given as input to the model. Lower-case quantities represent variables.

AT	the length of one time period
M	the total number of time periods
t	the time index, $t = 1, 2, \ldots, M$
N	the total number of sections on the highway
a	the section index, a $= 1, 2, \ldots, N$

L_a the length of section a

l the lane index, $l = 1, 2$

V_l the constant velocity of traffic on lane l

$\bar{V}$ the average of the two constant lane speeds

For notational convenience, we extend the range of the indices t and a. Any quantity associated with t=O is defined as that quantity already existing at the beginning of time period 1, e.g. existing traffic at the beginning of time period 1. Any quantity associated with $a=0$ should be interpreted as that quantity present at the very beginning of section 1, e.g. upstream demand. However, unless stated otherwise, the value of t ranges from 1 through M and that of a ranges from 1 through N.

(3.3.2) Input Parameters:

The demand parameters are as follows. We first define the traffic already existing on the highway at the beginning of the study horizon and then define the demand arriving at the sections during the succeeding time periods $t=1,2,...,M$.

X_{al}^s: the number of vehicles destined for section s that are traversing lane l of section a at the beginning of time period 1. This is simply the existing traffic at the beginning of the study horizon. It has an alternative notation $x_{al}^s(1)$, as will be defined later.

$F_{al}^s(0)$: When $a=0$, this denotes the number of vehicles already waiting in the queue at the beginning of the study horizon at the beginning of lane l of section 1 that are destined for section s. In this case, this is simply the initial upstream queue. When $a=1,2,...,N$, this denotes the number of vehicles already waiting in the queue at the beginning of time period 1 at the on-ramp connected to lane l of section a that are destined for section s. In other words, this is simply the existing demand at the beginning of the study horizon. $F_{al}^s(0)=0$ if lane l is not equipped with an on-ramp.

$F_{al}^s(t)$: When $a=0$, this denotes the number of vehicles arriving at the beginning of lane l of section 1 during time period t that are destined for section s. In this case, this is simply the upstream demand. When $a=1,2,...,N$, this denotes the number of vehicles arriving, during time period

t, at the on-ramp connected to lane l of section a that are destined for section s. In other words, this is simply the on-ramp demand. $F^s_{al}(t)=0$ if lane l is not equipped with an on-ramp, i.e. if there is no on-ramp directly connected to this lane.

Note that, with proper interpretation, the two quantities defined above can be combined into one, $F^s_{al}(t)$, where $a=0,1,2,...,N$ and $t=0,1,2,...,M$. With this convention, we define the following derived parameters:

$PF^s_{al}(t)$: $F^s_{al}(t)/\sum_{i=a}^{N} F^i_{al}(t)$, if $\sum_{i=a}^{N} F^i_{al}(t) \neq 0$,

0 otherwise, where $a=1,2,...,N$ and $t=0,1,2,...M$.

Often in the following discussion, we need the aggregate amount of traffic without regard to the destination, e.g. the total number of vehicles that are arriving, during time period t, at the on-ramp connected to lane l of section a. This aggregate amount is simply $\sum_{s=a}^{N} F^s_{al}(t)$. For convenience, we will denote this amount by $F^{\cdot}_{al}(t)$ with the symbol "." indicating the aggregation or sum. This symbol should be interpreted similarly for all other parameters and variables.

The supply parameters are:

$CN_{al}(t)$: When $a=0$, this denotes the maximum number of vehicles that can enter lane l of section 1 during time period t from upstream. In this case, this is simply the capacity of upstream section/lane. When $a=1,2,...,N$, this denotes maximum number of vehicles that can enter the existing traffic stream during time period t through the on-ramp connected to lane l of section a. In other words, this is the on-ramp capacity. $CN_{al}(t) = 0$ if lane l is not equipped with an on-ramp.

$CF_{al}(t)$: maximum number of vehicles that can enter the off-ramp connected to lane l of section a during time period t. This is the off-ramp capacity. This quantity applies only to the exit lanes.

$CL_{al}(t)$: the maximum number of vehicles that can be safely accommodated in a unit length of lane l of section a at the beginning of time period t. This is the per-unit-length lane capacity and can be a function of the constant traffic speed of the lane.

(a_i, b_i): $a_i + b_i d$, where $a_i \geq 0$ and $b_i \leq 0$, $i=1,2$, is an upper bound for the maximum possible number of lane changes allowed per unit-length and per unit-time without slowing down the traffic in either lane, where d **is** the traffic density on the destination lane, i.e. the number of vehicles per unit-length. These are the two pieces of the piecewise linear function approximating the antagonistic relationship between the lateral capacity and the longitudinal flow. Note that a_i and b_i may depend on the lane speeds, vehicle organization and lane-change rules. **Also** note that it is possible that $a_2 = a_1$ and $b_2 = b_1$.

(a_i^0, b_i^0): $a_i^0 + b_i^0 d$, where $a_i^0 \geq 0$ and $b_i^0 \leq 0$, $i=1,2$, is an upper bound for the maximum possible number of vehicles entering the receiving lane from the on-ramp per unit-time without slowing down the traffic in the receiving lane, where d is the traffic density on the receiving lane, i.e. the number of vehicles per unit-length. These are the two pieces of the piecewise linear function approximating the antagonistic relationship between the on-ramp inflow and the longitudinal flow. Note that a_i^0 and b_i^0 may depend on the lane speeds, vehicle organization and lane-change rules. **Also** note that it is possible that $a_2^0 = a_1^0$ and $b_2^0 = b_1^0$.

(3.3.3) Decision Variables (Control Variables):

There are only two types of metered inflow variables, metered inflow from on-ramps and inflow metered from upstream.

$u_{al}(t)$: When $a=0$, this denotes the number of vehicles entering lane l of section 1 during time period t from upstream. When $a=1,2,\ldots,N$, this denotes the total number of vehicles entering lane l of section a, through metering (without disturbing the speed of the existing traffic) from the on-ramp during time period t. If lane 1 is not equipped with an on-ramp, then $u_{al}(t)$, or equivalently $CN_{al}(t)$, should be set **to** 0.

$v_{al}^s(t)$: the number of vehicles that are traversing lane 1 **of** section a at the beginning of time period t with destination s and change lanes from lane 1 to lane $\bar{l}$ (the complement of lane 1 or the lane other than lane 1) during time period t. ($\bar{1}=2$ and $\bar{2}=1$.) If lane changing from lane 1 to lane $\bar{l}$ is disallowed, this variable can be set to 0 explicitly in the formulation. To achieve the same effect, the upper bound can be set to 0, i.e. $a_1=a_2=b_1=b_2=0$.

(3.3.4) Derived Variables:

This subsection defines variables that can be derived from the decision variables. Their definition is motivated by notational convenience only. How they are derived will become clear later when linear constraints are defined for the linear programming formulation.

Metered inflow from an on-ramp destined for section s down the highway can be obtained by multiplying the total metered inflow by the corresponding time-varying OD proportion:

$u_{al}^{s}(t)$: When $a=0$, this denotes the number of vehicles entering lane l of section 1 during time period t from upstream with destination s. When $a=1,2,...,N$, this denotes the total number of vehicles entering lane I of section a, through metering from the on-ramp during time period t that are destined for section **s.**

Two types of lateral flow can be derived - lateral flow that also reaches the next section by the end of the time period and lateral flow that cannot reach the next section by the end of the time period.

$v_{al}^{s1}(t)$: the number **of** vehicles traversing lane I of section a at the beginning of time period t with destination s that change lanes from lane I to lane $\bar{l}$ and move from section a to section a+1 during time period t.

$v_{al}^{s0}(t)$: $v_{al}^{s}(t) - v_{al}^{s1}(t)$, i.e. the number of vehicles traversing lane I of section a at the beginning of time period t with destination s that change lanes from lane l to lane $\bar{l}$ during time period t but cannot reach section $a+1$ during time period t. These two types of lateral flow are referred to as cross-section lateral flow and within-section lateral flow respectively. Note that the superscripts 1 and 0 signify cross-section and within-section flow respectively.

There are three other major types of derived variables, the outflow, longitudinal flow and existing traffic. There are two types of longitudinal flow: cross-section and within-section.

$w_{al}^{s1}(t)$: the number of vehicles with destination s that are traversing lane l of section a at the beginning of time period t and are able to to move from section a to section $a+1$ on lane I during time period t.

$w_{al}^{s0}(t)$: the number of vehicles with destination s that are traversing lane 1 of section a at the beginning of time period t and moved forward on lane l of section a without being able to reach section $a+1$ during time period t.

$x_{al}^{s}(t)$: the number of vehicles that are destined for section s and are traversing lane 1 of section a at the beginning of time period t.

$y_{al}(t)$: the number of vehicles leaving the highway from lane 1 (an exit lane) of section a during time period t. **This** variable applies only when lane l is an exit lane.

(3.3.5) Lateral flow:

Lateral flow variables and the metered inflow variables are the only two classes of decision variables. For simplicity of formulation and reduction of decision variables, we make the following assumption:

Assumption 2: Vehicles that enter a section through an on-ramp are not allowed to change lanes in that section.

By this assumption, a lateral flow from one lane to another refers to the movement of only those vehicles that are already traversing the origin lane of that section at the beginning of the time period and the lateral flow does not include any vehicles that have just entered the section through an on-ramp.

The maximum number of vehicles that are present in a section at the beginning of the time period and successfully change lanes into the other lane (lateral capacity) during the time period is limited by the longitudinal flow of the destination lane, the section length and the length of time period. The higher the longitudinal flow, the lower the maximum number. The longer the section, the larger the maximum number. Similarly, the longer the time period, the larger the maximum number. Since the speed is kept as a constant, the only determinants are the density of traffic on the destination lane, the section length and the length of time period.

We use a piecewise linear function to represent the per-section-length and per-time-length maximum number, or simply referred to as the lateral capacity, as a function of the traffic density of the destination lane. Note that the function could depend on the speed. **Also** note that different operating scenario may impose different limit on lateral capacity and hence the piecewise-linear function may vary with respect to the operating scenario. For convenience, we make the following assumption.

Assumption 3: The speed differential between the two lanes is small enough so that a single piecewise-linear function is sufficient to represent the lateral capacity as a function of the longitudinal capacity for both lane-changing directions.

Note that the lateral capacity and all other capacity constraints limit the total number of lane changes made by vehicles aggregated over all destinations.

Finally, we discuss the speed aspect of the lateral flow. There are two issues, mixed speeds and cross-section lateral flow. The speed of a vehicle that successfully changes lanes changes from the speed of the origin lane to that of the destination lane. To simplify the formulation, we choose the average speed, i.e. the average of the two lane speeds, as the effective constant speed for these vehicles, instead of considering detailed speed profile during the lane change. Note that such a choice implies that all vehicles that successfully change lanes have a common effective constant speed, regardless of the direction of lane change.

By the speed limitation, some of these vehicles, i.e. only a portion of these vehicles, can reach the next section by the end of the time period while the other cannot. Denote the portion by P_{al}^{1}, to be referred to as lateral progression factor in the rest of this paper, By Assumption 1, it is simply the proportion, in terms of length and with respect to the whole section, of the latter part of the section from which vehicles can reach the next section by the end of the time period. Therefore,

$$P_{al}^{1} \equiv \bar{V}\Delta T / L_a \ .$$

Note that $p_{a1}^{1} = p_{a2}^{1}$. Given the decision variable $v_{al}^{s}(t)$, the cross-section and within-section respectively lateral flow are simply, by Assumption 2,

$$v_{al}^{s1}(t) = v_{al}^{s}(t) P_{al}^{1} \quad \& \quad v_{al}^{s0}(t) = v_{al}^{s}(t)(\ 1 - P_{al}^{1})$$

(3.3.6) Inflow:

Inflow amounts are decision variables. However, inflow amounts are further limited by the longitudinal flow due to the need to maintain constant lane speeds. The way they are limited by the longitudinal flow is similar to the way that the lateral flow is limited by the longitudinal flow. However, we use a different set of piecewise-linear function to represent the limit on inflow. **As** we commented for the lateral flow, different operating scenarios may impose different capacity limit on the inflows and hence the piecewise linear function may vary with respect to the operating scenario.

We make two assumptions regarding the time-varying O-D demand and its influence on the actual metered inflow.

Assumption 4: For each ramp and each time period, the total demand (aggregated over all destination) is known. Also known is the proportion of demand trips that are destined for each section down the highway. The metering process does not discriminate against trip's exit. **As** a result, the actual metered inflow consists of traffic destined for all destinations down the highway, with their proportions identical to the proportions of demand at the on-ramp.

(3.3.7) outflow:

Outflow amounts are determined as follows. Let us concentrate on the lane of a section that is equipped with an off-ramp (i.e. the exit lane). (We refer to a lane of a section as an entry lane if there is an on-ramp in that section which directly feeds traffic into it.) Only vehicles with the section as the destination may exit from the off-ramp. The outflow of such vehicles from lane 1 of section a is the sum of (i) the number of vehicles that are destined for the off-ramp, are present in the section at the beginning of the time period and can reach the end of the section (where the off-ramp is located) and (ii) the number of vehicles that are destined for the off-ramp, present in the other lane at the beginning of the time period, can change lanes in the section and can reach the end of the section. The number defined in (i) can be easily obtained by multiplying, x_{al}^{a}, i.e. the number of vehicles with the off-ramp at the end of section a as the destination that are traversing section a at the beginning of time period t, by the following quantity, to be referred to as the longitudinal progression factor:

$$P_{al}^{0} = V_l \Delta T / L_a \ .$$

Note that the longitudinal progression factor is lane-specific. The number defined in (ii) can be obtained similarly as $v_{a\bar{l}}^{a}(t) P_{al}^{1}$. Therefore,

$$y_{al}(t) = x_{al}^{a}(t)\ P_{al}^{0} + v_{a\bar{l}}^{a}(t)\ P_{a\bar{l}}^{1}\ .$$

The total amount of outflow ***so*** defined and obtained will then be subject to the off-ramp capacity.

(3.3.8) Longitudinal Flow:

First, note that, by definition, the longitudinal flow in a lane of a section refers to the movement of only those vehicles that are already traversing that lane of that section at the beginning of the time period. Particularly, it does not include the movement of the inflow and the movement of the lateral flow from the neighboring lane. The longitudinal flow can be derived from the lateral flow out of the lane and the outflow from that lane to the off-ramp in that section. We explain as follows. First, concentrate **on** a particular section and consider the longitudinal flow of traffic destined for a section different from the current section down the highway. Now, imagine that there are longitudinal flows but no lateral flows and outflows. In this case, the longitudinal flow in a time period is determined completely by the amount of vehicles present in the lane at the beginning of the time period, the length of the section and the speed of the traffic in the lane. Particularly, we have

$$w_{al}^{s1}(t) = x_{al}^{s}(t)\ P_{al}^{0}\ ;$$

$$w_{al}^{s0}(t) = x_{al}^{s}(t)\ (1 - P_{al}^{0}\ )\ .$$

Note that this way of calculating longitudinal flows ensures that traffic with different destinations advances simultaneously and proportionally. This avoids a potential problem of favoring traffic with certain destinations over that with other destinations. Such a potential problem is considered a possible violation of the natural phenomenon of first-in-first-out (FIFO). Other possible violations of FIFO are also avoided by the technique of two-stage decomposition.

Now, consider the possibility of vehicles changing lanes out into the neighboring lane. But, there are still no outflow to off-ramps. Then, the cross-section longitudinal flow becomes

$$w_{al}^{s1}(t) = (\ x_{al}^{s}(t) - v_{al}^{s}(t)\ )\ P_{al}^{0}$$

$$w_{al}^{s0}(t) = (\ x_{al}^{s}(t) - v_{al}^{s}(t)\ )\ (\ 1 - P_{al}^{0}\ )\ .$$

Now, consider the effect of outflow on the cross-section longitudinal flow. Since there is no outflow from the section (the destination is not the current section), the formula obtained above remains valid.

Finally, consider the longitudinal flow of the traffic destined for the section under consideration. We deal with the exit lane first. Since the traffic will leave the section and will not reach the next section, the cross-section longitudinal flow is simply zero, i.e.

$$w_{al}^{a1}(t) = 0\ .$$

However, assuming that no vehicles on the exit lane with current section as the destination will be allowed to change lanes into the non-exit lane (the other lane), since not all of $x_{al}^{a}(t)$ can reach the off-ramp, the within-section longitudinal flow is

$$w_{al}^{a0}(t) = x_{al}^{a}(t)\ P_{al}^{0}$$

Note that so far in this subsection we have been dealing with the exit lane. We now deal with the non-exit lane. Similar to its exit-lane counterpart, the cross-section longitudinal flow is simply zero, i.e.

$$w_{al}^{a1}(t) = 0\ .$$

Note that the subscript l above refers to the non-exit lane.

Assumption ***5:*** All the vehicles destined for the section under consideration (i.e. current section) that are traversing the non-exit lane of the current section at the beginning **of** the time period make a lane change into the exit lane in that time period.

In our notation, this requirement simply means

$$v_{al}^{a}(t) = x_{al}^{a}(t)$$

and hence

$$w_{al}^{a0}(t) = w_{al}^{a1}(t) = 0 \ .$$

Although some vehicles will not have reached the end of the section by the end of the time period, i.e. have not passed the off-ramp by the end of the time period, and may still have sufficient time to make the lane change during the next time period, we make such an requirement for ease of formulation. Such a requirement is a mild one when all sections have an equal length and the speed differential between the two lanes is small. In any event, this requirement leaves some slack in favor of successful exiting.

(3.3.9) Single-directional lane changing in each section:

To ensure single-directional lane changing but also to simplify the formulation, disallow lane changing in the direction not permitted by explicitly setting the lateral flow variable to zero in the formulation or setting the lateral capacity to zero.

(3.4) Linear Programming Formulation

We formulate the problem such that the longitudinal flow is maximized subject to constraints of successful exiting and capacity. We first state the objective function and then the constraints.

(3.4.1) The objective function:

The objective is to maximize the total cross-section flow, i.e.

$$\max \sum_{t=1}^{M}\sum_{a=1}^{N} \left[w_{a1}^{\cdot 1}(t) + w_{a2}^{\cdot 1}(t) + v_{a1}^{\cdot 1}(t) + v_{a2}^{\cdot 1}(t) \right]$$

(3.4.2) The on-ramp capacity constraints:

The total metered inflow from an on-ramp is limited by its capacity as follows:

$$u_{al}(t) \le CN_{al}(t) .$$

Note that this constraint applies only if lane l is an entry lane of section a

(3.4.3) The on-ramp merging constraint:

The total metered inflow from an on-ramp has to merge into the existing traffic on the receiving lane at the merge point. Since the speed of the existing traffic is not to be interrupted, the inflow is further limited by the existing flow on the receiving lane.

$$u_{al}(t) \le \left[a_1^0 + b_1^0 \frac{\left\{x_{al}(t)\right\}}{L_a} \right] \Delta T \ ;$$

$$u_{al}(t) \le \left[a_2^0 + b_2^0 \frac{\left\{x_{al}(t)\right\}}{L_a} \right] \Delta T \ .$$

These constraints apply only if lane 1 is an entry lane of of section a.

(3.4.4) The demand constraint on on-ramp:

The total on-ramp metered inflow cannot exceed the demand at the ramp:

$$u_{al}(t) \le \sum_{i=0}^{t} F_{al}(i) - \sum_{i=1}^{t-1} u_{al}(i)$$

This constraint applies only if lane l is an entry lane of section a. Note that spillback to city streets is not considered in this formulation.

(3.4.5) Capacity of upstream section/lane:

The upstream metered inflow is limited by the capacity of the upstream section/lane as follows:

$$u_{0l}(t) \le CN_{0l}(t) ,$$

where $1 = 1, 2$, and $t = 1, 2, ..., M$.

(3.4.6) The upstream demand constraints:

The upstream metered inflow cannot exceed the demand.

$$u_{0l}(t) \leq \sum_{i=0}^{t} F_{0l}(i) - \sum_{i=1}^{t-1} u_{0l}(i) \ ,$$

where 1 = 1, 2.

(3.4.7) The demand distribution:

The total on-ramp metered inflow as well as the total upstream metered inflow are segregated into that for each of the downstream sections according to the time-varying OD proportion matrix.

$$u_{al}^{s}(t) = u_{al}(t)\ PF_{al}^{s}(t) \ .$$

$$u_{0l}^{s}(t) = u_{0l}(t)\ PF_{0l}^{s}(t) \ .$$

These constraints apply only when lane *1* is an entry lane of section a.

(3.4.8) Outflow:

For the exit lane only, we have the following constraint that ensures that all the traffic destined for the section exit the highway from the off-ramp.

$$y_{al}(t) = x_{al}^{a}(t)\ P_{al}^{0} + v_{a\bar{l}}^{a}(t)\ P_{a\bar{l}}^{1} \ .$$

This constraint applies only if lane l is an exit lane of section a.

Also, make all the vehicles that are destined for the section and are traversing the non-exit lane of the section change lanes to the exit lane for exiting as follows, if such lane-changing is allowed in that section:

$$v_{al}^{a}(t) = x_{al}^{a}(t) \ .$$

This constraint applies only if lane *1* is the non-exit lane of section a and if such lane-changing is allowed. If such lane-changing is disallowed, then the following constraint should be imposed:

$$x_{al}^{a}=0.$$

Also, allow no lane changes from the exit lane to the non-exit lane for those vehicles that are destined for the section and are traversing the exit lane of the section. In other words, set

$$v_{al}^{a}(t) = 0\ ,$$

where *1* is the exit lane of section **a** . This constraint applies only if lane *1* is the exit lane of section **a** .

These equations, together with the flow conservation equations to be provided later, ensures that all traffic destined for a section cannot travel beyond it.

(3.4.9) Off-ramp capacity constraints:

Although the outflow comprises all the traffic destined for that section, it may exceed the off-ramp capacity. To prevent this, we need the off-ramp capacity constraints:

$$y_{al}(t) \leq CF_{al}(t)\ ,$$

where *1* is the exit lane of section **a** . This constraint applies only if lane *1* is the exit lane of section **a** . Note that spillback onto AHS is not allowed. In other words, any such spillback will cause infeasibility if the linear program.

(3.4.10) Flow conservation and propagation constraints:

The following constraints relate the existing traffic on lane l of section **a** at the beginning of time period **t+l** to its counterpart at the beginning of time period t. We first deal with $a \neq 1$. There are four different cases:

Case **1:** Section **a**$\neq$**1** is not the destination, i.e. $s \neq a$.

$$x_{al}^{s}(t+1) = x_{al}^{s}(t) + u_{al}^{s}(t) + w_{a-1,l}^{s1}(t) + v_{a-1,\bar{l}}^{s1}(t) + v_{\bar{l}}^{s0}(t) - w_{al}^{s1}(t) - v_{al}^{s}(t)$$

By allowing on-ramps on both sides of the highway, one equation suffices for describing the flow conservation and propagation constraint for Case **1.** If only one on-ramp is allowed in one section and if lane 1 happens not to be an entry lane of section a, a separate equation, with the term $u^s_{al}(t)$ removed from the above equation, would have been required.

Case 2: Section $a \neq 1$ is the destination, i.e. $s = a$, and 1 is the exit lane.

$$x^a_{al}(t+1) = x^a_{al}(t) + u^a_{al}(t) + w^{a\,1}_{a-1,l}(t) + v^{a\,1}_{a-1,\bar{l}}(t) + v^a_{a\bar{l}}(t) - x^a_{al}(t)\, P^0_{al} - v^a_{a\bar{l}}(t) P^1_{al}\ .$$

Case 3: Section $a \neq 1$ is the destination, i.e. $s = a$, and l is not the exit lane.

$$x^a_{al}(t+1) = x^a_{al}(t) + w^{a\,1}_{a-1,l}(t) + v^{a\,1}_{a-1,\bar{l}}(t) \ - v^a_{al}(t)$$

$$= w^{a\,1}_{a-1,l}(t) + v^{a\,1}_{a-1,\bar{l}}(t)\ .$$

The conditions under which these constraints apply are specified in the Case descriptions. Note that traffic entering the AHS and exiting it in the same section but on the opposite side is not allowed because no lane changing is allowed in the section in which the vehicle enters the highway. Also note that, on an actual highway, this is translated into entering the highway system and exiting at the off-ramp on the opposite side at the next interchange is not allowed. This can be relaxed, if desirable, though by (a) relaxing Assumption 2 and (b) requiring that all such entering flow during the time period be moved into the exit lane during the same time period.

Case 4: $a=1$. The conservation equations for all three cases above remain valid except that the two terms $w^{a\,1}_{a-1,l}(t) + v^{a\,1}_{a-1,\bar{l}}(t)$ should be replaced by $u^s_{0l}(t)$.

(3.4.11) Longitudinal capacity:

The number of vehicles on lane 1 of section a is limited by its capacity:

$$x^{\cdot}_{al}(t) \leq L_a CL_{al}(t)\ ,$$

where a = 1, 2, ..., N, and l = 1, 2. (This constraint applies to both lanes of all sections.)

(3.4.12) Lateral/longitudinal flow interaction:

Since traffic speed is not to be disrupted, the lateral capacity is limited by the longitudinal flow:

$$v_{al}^{\cdot}(t) \leq \left[a_1 + b_1 \left(\frac{x_{a\bar{l}}^{\cdot}(t)}{L_a} \right) \right] L_a \; AT \;;$$

$$v_{al}^{\cdot}(t) \leq \left[a_2 + b_2 \left(\frac{x_{a\bar{l}}^{\cdot}(t)}{L_a} \right) \right] L_a \; AT$$

These constraints apply only if lane changing **from** lane l to lane $\bar{l}$ is allowed in section a. **If** such lane changing is not allowed, then set

$$v_{al}^{s}(t) = 0 \; .$$

Therefore, $v_{al}^{\cdot}=0$ and such constraints, if included, will automatically be satisfied.

(3.4.13) Definitional Constraints:

$$v_{al}^{s1}=v_{al}^{s}(t)P_{al}^{1}$$

$$v_{al}^{s0}=v_{al}^{s}(t)(1-P_{al}^{1})$$

$$w_{al}^{s1}=(x_{al}^{s}(t)-v_{al}^{s}(t))P_{al}^{0}$$

$$w_{al}^{s0}=(x_{al}^{s}(t)-v_{al}^{s}(t))(1-P_{al}^{0})$$

$$v_{al}^{\cdot 1} = \sum_{i=a}^{N} v_{al}^{i1}(t)$$

$$v_{al}^{\cdot}(t) = \sum_{i=a}^{N} v_{al}^{i}(t) \; .$$

$$w_{al}^{s}(t) = w_{al}^{s0}(t) + w_{al}^{s1}(t) \; .$$

$$w_{al}^{\cdot 1}(t) = \sum_{i=a}^{N} w_{al}^{i1}(t) \; .$$

$$w_{al}^{\cdot}(t) = \sum_{i=a}^{N} w_{al}^{i}(t)$$

$$x_{al}(t) = \sum_{i=a}^{N} x_{al}^{i}(t)$$

(3.4.14) Non-negativity Constraints:

$$u_{al}(t) \geq 0 \, .$$

$$u_{0l}(t) \geq 0 \, .$$

$$v_{al}^{s}(t) \geq 0 \, .$$

(4) FUTURE WORK PLAN

A computer program will be developed **to** generate electronically a linear program based on input specifications. CPLEX, a linear programming software package, will be used to solve the linear program. Debugging the LP-generation program and refining the model will continue. **A** set **of** numerical experiments will be developed. **A** final report will summarize the theory as well as the numerical results.

REFERENCES

[1} Boyce D.E., Ran B. and LeBlanc L.J. 1991. Solving an Instantaneous Dynamic User-Optimal Traffic Assignment Model, to appear in Transportation Science.

[2] Brouke, M. and Varaiya, P., "A Theory of Traffic Flow in Automated Highway Systems", presented at the 75th Annual Transportation Research Board Annual Meeting, Paper No. 960430, Washington, D.C., 1996.

[3] Hall, R.W., "Longitudinal and Lateral Throughput on an Idealized Highway", Transportation Science, Vol. 29, No.2, pp. 118-127, 1995.

[4] Ran, B. and Boyce D.E. Dynamic Urban Transportation Network Models: Theory and Implications for IVHS, in preparation for Springer-Verlag, 1994.

[5] Ran, B., Boyce D.E. and LeBlanc L.J. 1993. A New Class **of** Instantaneous Dynamic User-Optimal Traffic Assignment Models. Operations Research, 41, 1, 192-202.

[6] Rao, B.S.Y. and Varaiya, **P.,** "Flow Benefits of Autonomous Intelligent Cruise Control in Mixed Manual and Automated Traffic", Transportation Research Record No. 930803, 1993.

[7] Rao, B.S.Y. and Varaiya, P., "Roadside Intelligence for Flow Control in an IVHS", to appear in Transportation Research **Part** C, 1993.

[8] Rao, B.S.Y., Varaiya, P. and Eskafi, **F.,** "Investigations into Achievable Capacity and Stream Stability with Coordinated Intelligent Vehicles", Transportation Research Record No. 930803, 1993.

[9] Shladover, S, "Operation **of** Automated Guideway Transit Vehicles in Dynamically Reconfigured Trains and Platoons," (Extended Summary, Vol. I & 11), UMTA-MA-06-0085-79-1, UMTA-MA-06-0085-79-2 and UMTA-MA-06-0085-79-3, U.S. Department **of** Transportation, Urban Mass Transportation Administration, Washington, D.C., July, 1979.

[IO] Shladover, S., "Potential Freeway Capacity Effects of Advanced Vehicle Control Systems," Second International Conference on Applications of Advanced Technologies in Transportation Engineering, Minneapolis, Minnesota, August 18-21, 1991.

[11] Stevens, W.B., "The Automated Highway System (AHS) Concept Analysis", MITRE Research Report MTR-93W0000123 (draft, Aug. 1993), McLean, Virginia.

[12] Stevens, W.B., MITRE, McLean, Virginia, private discussion, Aug. 1993.

[13] Tsao, H.-S.J., "A Functional Architecture for Automated Highway Traffic Planning", to appear in Proceedings of IVHS America Fourth Annual Meeting, Atlanta, GA, April, 1994.

[14] Tsao, H.-S.J., Hall, R.W. and Chatterjee, I., "Analytical Models for Vehicle/Gap Distribution on Automated Highway Systems", to appear in Transportation Science.

[15] Varaiya, P. and Shladover S.E., "Sketch **of** an IVHS System Architecture", PATH Research Report UCB-ITS-PRR-91-3, University of California, Institute of Transportation Studies, Berkeley, California, USA, Feb. 1992.

[16] Varaiya, P., "Smart Cars on Smart Roads: Problems **of** Control", IEEE Transaction on Automatic Control, Vol. 38 (1993), 195-207.

[17] Whitney, D., TASC Corporation, Boston, MA, private discussion, Sept. 1993.