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A PROBABILISTIC MODEL FOR AVCS LONGITUDINAL COLLISION/ SAFETY ANALYSIS

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This paper develops a probabilistic model for analyzing longitudinal collision/safety between an abruptly decelerating vehicle and its immediate follower. The input parameters are the length of the gap between the two vehicles, their common speed prior to the failure, the reaction delay of the following vehicle and a bivariate distribution for the two deceleration rates. The output includes the probability of a collision and the probability distribution of the relative speed at collision time.

We use this model to compare the safety consequences associated with the platooning and "free-agent" longitudinal-separation rules. We also demonstrate that the free-agent rule implemented with a potential technology of fast and accurate emergency deceleration, under some reasonable conditions, can avoid collisions while offering a high freeway capacity previously thought possible only under the platooning rule. This model has many other applications.

Key words: AVCS, collision speed distribution, platooning, free-agent.

An Advanced Vehicle Control System (AVCS) consists of two major components: vehicle automation technology and freeway operating strategy. A full-automation technology integrates the communication technology between vehicles and between vehicles and roadside, sensing technology and sophisticated automatic vehicle control. An operating strategy is a collection of operating rules that govern the movement of automated vehicles based on their capability and reliability.

Two primary objectives of Advanced Vehicle Control System (AVCS) are enhancements of highway capacity and safety. Capacity gain is achieved by reducing the average spacing, longitudinal and lateral, between vehicles. Safety improvement comes from the removal of human errors, which currently account for more than 90% of roadway accidents. However, automation may introduce new kinds of safety hazards through possible failures of additional equipment, the roadside control system and the communication system. Any of these failures may lead to collisions of a vehicle with other vehicles or with objects on the roadway.

For a given automation technology, different operating strategies for AVCS will result in different degrees of capacity and safety enhancements. Five major categories of operating rules are lane flow, lane change, lane selection, automated access and automated egress. The longitudinal-separation rule in the lane-flow category has been the focal point of recent studies because of its direct impact on both the capacity and safety enhancements. Central to any longitudinal-separation rule is the longitudinal spacing, i.e. the length of the gap between two adjacent vehicles. This paper concentrates on the failures that cause a sudden deceleration, and the resulting safety hazards as a function of the longitudinal spacing. A vehicle failure under different longitudinal-separation rules will result in collisions of different severity with different probability. We will use *collision speed*, i.e. relative speed between two colliding vehicles at the time of collision, as a *surrogate* for collision severity. We limit our consideration to only the initial collision after a failure. For ease of discussion, the deceleration of a failed vehicle and that of its immediate follower will be referred to as *failure deceleration* and *emergency deceleration* respectively.

A Probabilistic Model

We develop a probabilistic model for obtaining the probability of collision and the distribution of collision speed. This model can help determine safe distances between two vehicles, the target emergency deceleration, the specification for the accuracy and the response time of the braking system, the specification for the response time of the communication system and other quantities of great importance.

The input parameters considered in our probabilistic model are:

(I1) common speed prior to the failure deceleration,
(I2) spacing between the two vehicles,
(I3) reaction delay of the rear vehicle,
(I4) correlated bivariate distribution of the two deceleration rates.

The bivariate deceleration distribution is needed to allow possible correlation between the two random deceleration rates due to common driving conditions, e.g. slippery road conditions on a rainy day. The bivariate deceleration distribution can be *any* discrete probability distribution over *any* possible finite state space. The output will be the probability of a collision and also the probability distribution of the collision speed, denoted by Δv. Parametric study can be conducted by varying the input parameters and examining the resulting collision probability and Δv distribution.

The most complicated input to the model is the bivariate distribution. To justify particular selections for it in the absence of data on the future technology, or simply to facilitate the complex task of its determination, we will use the *Principle of Maximum Entropy* to derive a discrete bivariate distribution that satisfies any given marginal expectations, marginal standard deviations and coefficient of correlation. This distribution can be determined by solving a convex mathematical programming problem with linear equality constraints. The theoretical justification of this principle actually translates into the conservativeness appropriately required in a safety study like this. The adoption of this principle together with the discrete representation of the joint distribution of the two deceleration rates enable realistic and efficient parametric probabilistic studies of AVCS longitudinal safety.

Two Basic Vehicle Following Rules

Two basic longitudinal-separation rules are the *platooning rule* and the *free-agent rule*. The platooning rule was first proposed and studied by Shladover in the late 70's [Shladover, 1979] and has received renewed attention in the last few years. Under this rule, two adjacent vehicles in the same lane are kept either very close to, or very far from, each other. As a result, vehicles are organized in a clustered formation. Each cluster of vehicles is called a *platoon*. This rule fully utilizes the fact that, after a failure, the Δv, if any, is small if they are either very close to each other or very far apart. Shladover [Shladover, 1979] showed that the capacity increases significantly with platoon size. Under the free-agent rule, vehicles move without any clustered formation and the minimum longitudinal spacing is significantly longer than typical intra-platoon spacings, but significantly shorter than typical inter-platoon spacings.

The validity of the platooning concept hinges upon the crucial assumption that a failure would lead to, at the most, low-relative-speed collisions between vehicles in one lane. If this assumption proves to be true, then the platooning rule should be safer than the free-agent rule. However, so far very little is known about what other collisions may occur after the initial low-relative-speed collision. Could this initial collision lead to vehicles' skidding, spinning or swaying into other lanes? Could it cause some of the sensors or other on-board automation devices to malfunction and render vehicles out-of-control? Tongue [Tongue, 1993] is investigating the consequences of such low-relative-speed collisions using the technique of computer simulation. The major weakness of the free-agent rule is that in the event of a collision, the Δv tends to be more severe compared to the platooning rule. Other advantages of the free-agent rule include simplified control protocols and perhaps more stable traffic flow.

The above uncertainties suggest that we should not rule out the free-agent rule. In addition, the possibility of fast emergency deceleration, which has the potential of avoiding collisions even with short spacing, has not been fully explored in the literature.

A Probabilistic Comparison Between the Platooning and Free-Agent Rules

We will use the probabilistic model to compare these two basic longitudinal-separation rules. We will further demonstrate that, with *fast and accurate emergency braking* and under some other assumptions about the automation technology of the future, the free-agent rule might guarantee no collision after a failure while offering the high capacity thought possible with platooning.

Organization of the Paper

This paper is organized as follows: Section 2 explains our probabilistic approach. Section 3 contains the solution to this general problem. Section 4 briefly discusses the concept of maximum entropy and its role in our approach. Section 5 is devoted to the comparison between the two basic longitudinal-separation rules. Section 6 concludes this paper.

A PROBABILISTIC APPROACH

The goal is to provide the collision probability and Δv distribution for any given combination of the four input quantities, (I1) through (I4). To simplify the analysis, we use a finite number of discrete values as the domain, denoted by D, of the deceleration rates. In this way, an input distribution can be any possible discrete bivariate distribution over $D \times D$. Note that discretization is a powerful tool because it can be used to approximate any probability distribution to any desired accuracy.

The assumptions of our model are:

(A1) Prior to the failure, the two vehicles are moving on a straight lane at a common speed V with a spacing (the distance between the rear end of the front vehicle and the front end of the rear vehicle) of S.

(A2) The failed vehicle decelerates at a constant but random rate D_f.

(A3) The following vehicle decelerates at a constant but random rate D_r after a reaction delay T (if it has not already collided with the failed vehicle).

(A4) The two rates are possibly correlated.

We use a two-dimensional coordinate system to represent the position of the two vehicles as a function of time. The horizontal axis represents the time and the time of failure is the origin, i.e. the deceleration of the front vehicle occurs at time zero. The vertical axis represents vehicle position, with the origin set at the position of the rear end of the front vehicle at the time when the failure deceleration begins. We now introduce more notation, which is depicted in Figure 1:

$p(d_f,d_r) \equiv$ the probability of $D_f = d_f$ and $D_r = d_r$.

$\Delta v(d_f, d_r, S, T, V) \equiv$ the speed difference at collision time given d_f, d_r, S, T and V. For ease of notation, this will simply be abbreviated as Δv.

$t \equiv$ the elapsed time after the start of the front vehicle's deceleration.

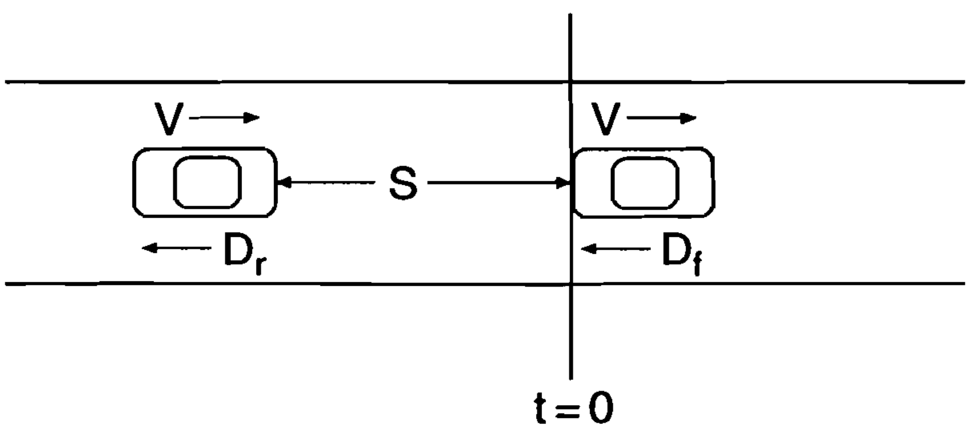

FIGURE 1 Initial condition at t=0.

$x_f(t) \equiv$ the position of the rear end of the front vehicle at time t, in absence of collision. In particular, $x_f(0)=0$.

$x_r(t) \equiv$ the position of the front end of the rear vehicle at time t, in absence of collision.

To find the probability distribution of Δv, we first determine, given a particular pair of deceleration rates $D_f=d_f$ and $D_r=d_r$, if the two vehicles would collide at all and, if so, when they do. We can then determine their respective speeds and the difference. Finally, adding up the probabilities associated with the pairs (d_f,d_r) that lead to the same collision speed produces the Δv distribution.

To determine if the two deceleration rates d_f and d_r would lead to a collision, we use the following approach. Since a collision can only take place while the rear vehicle is moving, and the rear vehicle stops at $t=T+V/d_r$ in absence of collision, we need only pay attention to the time period $(0,T+V/d_r)$. We will refer to this period as *the relevant interval.* It is obvious that *the two vehicles would collide if and only if the two curves defined by* $x_f(t)$ *and* $x_r(t)$ *intersect in the relevant interval.* If they intersect multiple times, the earliest crossing time is the collision time.

In absence of collision, the trajectory for the front vehicle is:

$$x_f(t) = Vt - d_f t^2/2 \text{ if } t \in [0, V/d_f];$$
$$V^2/2d_f \textit{ otherwise.}$$

In the absence of collision, the trajectory for the rear vehicle is:

$$x_r(t) = Vt - S \text{ if } t \in [0, T]$$
$$Vt - d_r(t-T)^2/2 - S \text{ if } t \in [T, T+V/d_r]$$
$$VT + V^2/2d_r - S \text{ if } t \in [T+V/d_r, \infty).$$

For convenience of discussion, the curve $x_f(t)$ will also be referred to as the front trajectory while $x_r(t)$ the rear trajectory. An example $x_r(t)$ is shown in Figure 2. Note that these two trajectories can intersect more than once. Figure 3 shows an example in which they intersect only once. They cross twice in Figure 4.

In terms of timing, there are only *four possible ways for the collision to occur:*

(C1) During the reaction period but before the front vehicle has stopped;
(C2) During the reaction period but after the front vehicle has stopped;
(C3) When both vehicles are decelerating;
(C4) After the front vehicle has stopped and while the rear vehicle is decelerating.

PROBLEM SOLUTION

We now summarize the derivation of the collision probability and Δv given any specific pair of deceleration rates $D_f=d_f$ and $D_r=d_r$. Let t^* denote a crossing time.

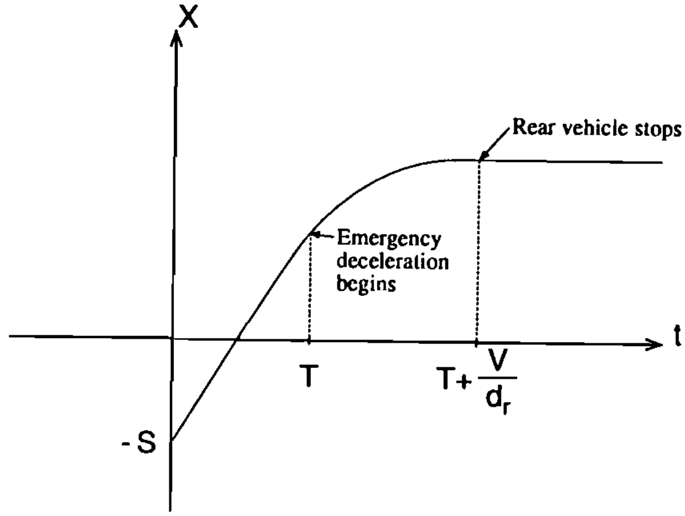

FIGURE 2 Trajectory $x_r(t)$ of the rear vehicle.

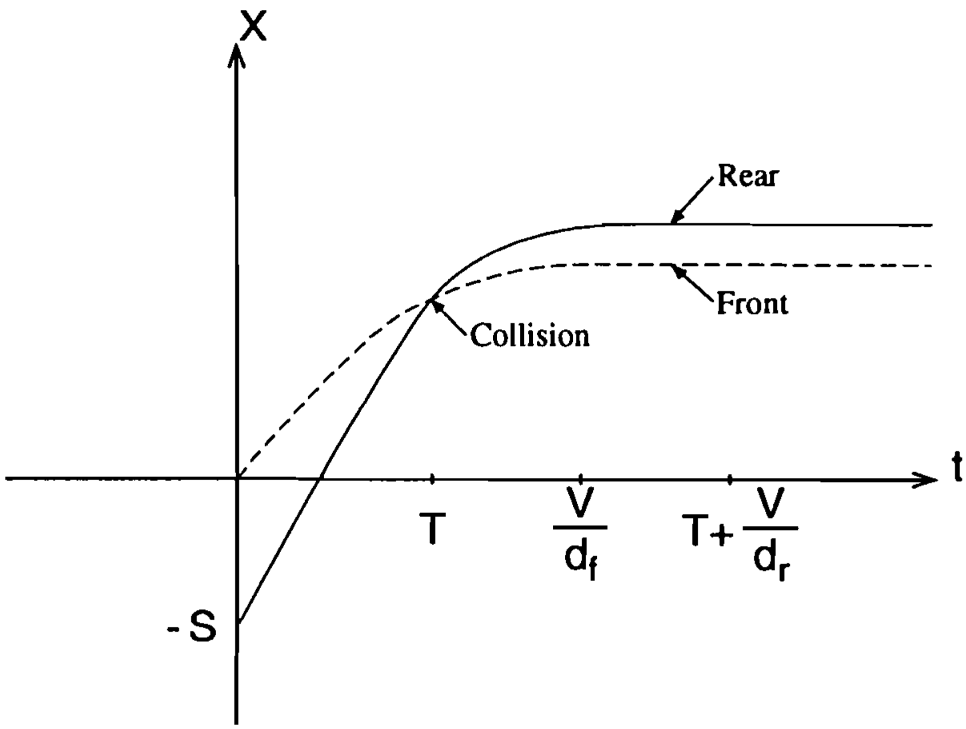

FIGURE 3 Two trajectories crossing only once.

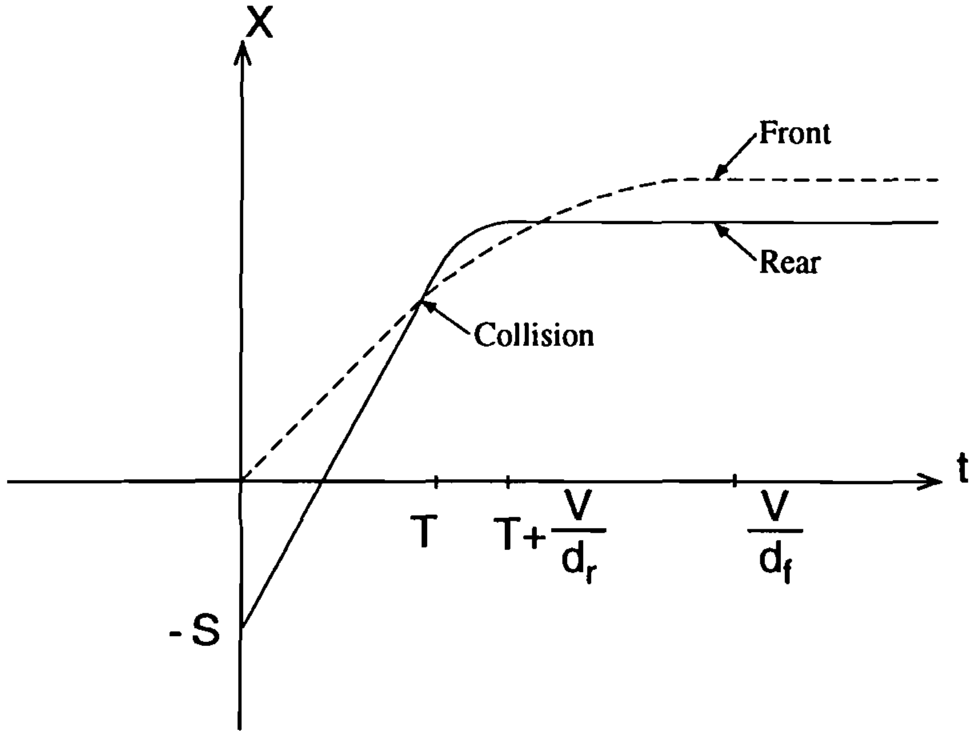

FIGURE 4 Two trajectories crossing twice.

For (C1) to occur, t^* must be on the first piece of the front trajectory and also on the first piece of the rear trajectory. Therefore, the prerequisites are $t^* \in [0, V/d_f]$ and $t^* \in [0, T]$. To determine the possible crossing times, solve:

$$Vt^* - d_f t^{*2}/2 = Vt^* - S.$$

The solutions are:

$$t^* = -(2S/d_f)^{1/2} \text{ and } (2S/d_f)^{1/2}.$$

Clearly, the first crossing time is not acceptable because it does not meet the prerequisites. The speed difference, if t^* indeed satisfies all prerequisites, will be $\Delta v = t^* d_f$.

In order for (C2) to occur, a prerequisite is $V/d_f \leq T$. Also required are $t^* \geq V/d_f$ and $t^* \in [0, T]$. For the possible crossing times, solve:

$$V^2/2d_f = Vt^* - S.$$

The solution is:

$$t^* = (V^2/2d_f + S)/V.$$

If t^* satisfies the prerequisites, then the speed difference would simply be $\Delta v = V$.

For (C3) to occur, a prerequisite is $T \leq V/d_f$. Also required are $t^* \in [0, V/d_f]$ and $t^* \in [T, T+V/d_r]$. To obtain the crossing times, we solve the following equation:

$$Vt^* - d_f t^{*2}/2 = Vt^* - d_r(t^* - T)^2/2 - S.$$

The solutions are:

$$t^* = \frac{d_r T + [d_r^2 T^2 - 2(d_r - d_f)(d_r T^2/2 + S)]^{1/2}}{d_r - d_f} \text{ and } \frac{d_r T - [d_r^2 T^2 - 2(d_r - d_f)(d_r T^2/2 + S)]^{1/2}}{d_r - d_f},$$

if $d_r^2 T^2 - 2(d_r - d_f)(d_r T^2/2 + S) \geq 0$ and $d_r \neq d_f$. If $d_r = d_f$,

$$t^* = (d_r T^2/2 + S)/d_r T.$$

The speed difference, if t^* satisfies the prerequisites, is $\Delta v = d_f t^* - d_r t^* + d_r T$.

Finally, in order for (C4) to occur, the prerequisites are $T + V/d_r \geq V/d_f$, $t^* \geq V/d_f$ and $t^* \varepsilon [T, T+V/d_r]$. To obtain the crossing times, solve:

$$V^2/2d_f = Vt^* - d_r(t^* - T)^2/2 - S.$$

The solutions are:

$$t^* = \frac{(d_r T + V) - [(d_r T + V)^2 - d_r(d_r T^2 + 2S + V^2/d_f)]^{1/2}}{d_r} \text{ and } \frac{(d_r T + V) + [(d_r T + V)^2 - d_r(d_r T^2 + 2S + V^2/d_f)]^{1/2}}{d_r},$$

if $(d_r T + V)^2 - d_r(d_r T^2 + 2S + V^2/d_f) \geq 0$. The speed difference, if the interval requirements are satisfied, will be $\Delta v = V - d_r(t^* - T)$.

Adding up the probabilities associated with the pairs (d_f, d_r) that lead to the same collision speed produces the Δv distribution.

MAXIMUM ENTROPY MODEL

The MAXENT technique can determine a unique distribution, univariate or multivariate (with correlation), discrete or continuous, that satisfies any "linear equality constraints" on the probability distribution. Such linear constraints can be used to express almost all common constraints on distributions,.e.g. expected value, percentage quantile, the variance and correlation when the expected value is given, etc. The "right-hand-side" of any such constraint is a parameter value of the MAXENT distribution.

Entropy of a probability distribution on a finite domain, p_i, i=1,2,...,n, is defined by $-\sum_{i=1}^{n} p_i \ln p_i$. It can be interpreted as a measure of uncertainty and its negation can be interpreted as a measure of information. The maximum-entropy distribution contains the least "information" out of all the distributions that satisfy the linear constraints. In other words, it picks the one that is "maximally non-committal." For example, the maximum-entropy distribution on any finite state space without any constraints is the uniform distribution. For an analysis like ours where information about the exact distribution is limited, the selected distribution should be as non-committal as possible. Therefore, adoption of this principle is especially appropriate. One final note about the maximum-entropy approach is that there exist very robust and efficient computational algorithms. For references on the subject of maximum entropy and details of an efficient algorithm, see [Fang, 1992].

A COMPARISON BETWEEN PLATOONING AND FREE-AGENT RULES

We first state the assumptions of comparison and then use the model and a software tool to produce the collision probability and Δv distribution for a set of failure/reaction scenarios. Note that we are not attempting a complete comparison, which involves, among many other things, the failure probability, traffic disruption due to collisions, complexity of vehicle control algorithm and protocol, complexity of operating strategy, and stability of traffic flow. For convenience, *meter* and *second* will be abbreviated as m and s respectively.

To set the stage for the comparison, we itemize the additional assumptions as follows:

(A5) The randomness of the failure deceleration rate is due to chance. A target constant emergency deceleration rate has been *preset* for responding to vehicle failures; but, due to inaccuracy of the braking system, the actual rate is random, but constant.

(A6) The distributions of these two rates are statistically independent.

(A7) We set the common speed prior to the failure at $25m/s$, which is approximately 55 miles/hour.

(A8) The reaction delay, including the communication delay and the brake actuation delay, is set at 100 milliseconds ($0.1s$). This choice of reaction delay is consistent with the current automatic control technology.

(A9) The possible rates, for both decelerations, are $i \times 0.5m/s^2$, i=1,2,...,20. The failure deceleration rate follows the maximum-entropy distribution with expected value $5.0m/s^2$ and standard deviation $1.0m/s^2$.

The spacing and the emergency deceleration distribution will be varied. The spacings for the two rules are chosen so that the two resulting capacities are identical. We consider two different platooning scenarios: (i) 20-vehicle platoon with $1m$ intra-platoon spacing and $61m$ inter-platoon spacing, and (ii) 5-vehicle platoon with $1m$ intra-platoon spacing and $31m$ inter-platoon spacing. With the vehicle length set at $5m$, their free-agent and identical-flow counterparts would have a common inter-vehicle spacing of $4m$ and $7m$ respectively. With 20% capacity reserved for lane-change maneuvers, the two capacities are 8,000 and 6,000 vehicles per lane per hour respectively.

For emergency deceleration, we consider many more maximum-entropy distributions with specified values of expected value and standard deviation, with the expected values ranging from $3.0m/s^2$ to $8.0m/s^2$ and the standard deviation ranging from $0.1m/s^2$ to $1.0m/s^2$.

Figure 5 shows five Maximum Entropy distributions with different expected values and standard deviations. For a clearer comparison, we connect, for each distribution, the neighboring points $(d_i, prob(d_i))$, where d_i is a possible deceleration rate and $prob(d_i)$ is the associated probability.

The result of our probabilistic comparison is tabulated in 2 tables. Table I contrasts the difference between the two rules for the case of a 20-vehicle platoon. Table II contains the same contrast as in Table I except that the platoon size is 5. A common measure for the severity of longitudinal collisions is the instantaneous change of speed experienced by the vehicle during the collision, which depends on the elasticity of the collision and the vehicle weight. Hitchcock [Hitchcock, 1992] estimated that a

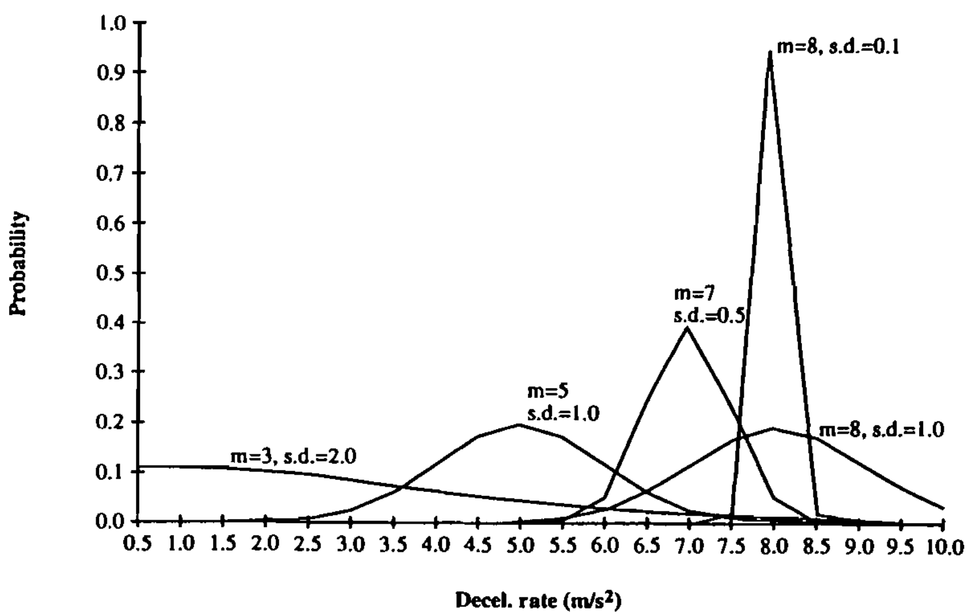

FIGURE 5 Five maximum entropy distributions.

change in velocity of 5 miles/hour will not result in major injuries in most cases and that of 10 miles/hour is dangerous (i.e. can cause serious injury or death). The consequence of velocity changes between these values is difficult to judge. Given the typical mix of automobile masses, this range roughly translates into collision speeds between 8 miles/hour (3.55*m/s*) and 16 miles/hour (7.1*m/s*). Therefore, we choose to display the probabilities of collision speed greater than 0*m/s*, 3.5*m/s* and 7.0*m/s* in the two tables. Note that under the platooning rule, the failed vehicle may be at the very end of a platoon, in which case the collision probability should be minute but the Δv, given the occurrence of a collision, may be high.

We will use the concept of *stochastic larger (smaller)* for comparing two random variables. However, since we are not interested in any exact ordering of the input or output distributions, we only use them in an approximate sense and, for ease of discussion, use the terms *larger* and *smaller* for abbreviation. Although we cannot rigorously compare the safety of the two basic rules, we will nevertheless, for convenience of discussion, use the term *safer* to loosely express our intuition.

It is apparent from the tables that when the mean emergency deceleration rate is

Table I. 20-Vehicle Platooning

Rear Decel. (m/s^2)		Rule	P(Col. Spd>0m/s)	P(Col. Spd>3.5m/s)	P(Col. Spd>7.0m/s)
mean	s.d.				
3	0.5	Platooning Free Agent	.9407 .9428	.0104 .5897	.0054 .0001
4	0.5	Platooning Free Agent	.8270 .7506	.0002 .2823	.0001 .0000
5	0.5	Platooning Free Agent	.5597 .4108	.0000 .1194	.0000 .0000
6	0.5	Platooning Free Agent	.2369 .1298	.0000 .0212	.0000 .0000
7	0.5	Platooning Free Agent	.0544 .0212	.0000 .0017	.0000 .0000
8	0.5	Platooning Free Agent	.0062 .0017	.0000 .0001	.0000 .0000
8	0.1	Platooning Free Agent	.0027 .0005	.0000 .0000	.0000 .0000
8	1	Platooning Free Agent	.0255 .0114	.0000 .0015	.0000 .0000

Front Decel. Mean = 5 meters/sec^2
s.d. = 1 meter/sec^2

Table II. 5-Vehicle Platooning

Rear Decel. (m/s^2) mean	s.d.	Rule	P(Col. Spd>0m/s)	P(Col. Spd>3.5m/s)	P(Col. Spd>7.0m/s)
3	0.5	Platooning	.9236	.1406	.1138
		Free Agent	.9428	.8702	.1298
4	0.5	Platooning	.7332	.0370	.0191
		Free Agent	.7506	.5892	.0212
5	0.5	Platooning	.4730	.0016	.0003
		Free Agent	.4072	.2494	.0017
6	0.5	Platooning	.1995	.0000	.0000
		Free Agent	.0969	.0572	.0001
7	0.5	Platooning	.0458	.0000	.0000
		Free Agent	.0071	.0065	.0000
8	0.5	Platooning	.0053	.0000	.0000
		Free Agent	.0003	.0002	.0000
8	0.1	Platooning	.0023	.0000	.0000
		Free Agent	.0000	.0000	.0000
8	1	Platooning	.0215	.0000	.0000
		Free Agent	.0062	.0043	.0000

Front Decel. Mean = 5 meters/sec²
s.d. = 1 meter/sec²

smaller than the mean failure deceleration rate, platooning is safer because its collision probability is not much different from its free-agent counterpart while its collision speed is smaller. Also, when the two rates are comparable, platooning seems safer for the same reason. However, when the mean emergency deceleration rate is significantly larger than the mean failure deceleration, the free-agent rule seems safer because its collision probability is significantly smaller while its collision speed distribution is not significantly larger. When the mean emergency deceleration rate is much larger than the mean failure deceleration rate and its accuracy is high, the collision probability can even be eliminated for very small longitudinal spacings under the free-agent rule. For example, (i) a longitudinal spacing of $7m$ and (ii) the MAXENT emergency deceleration rate with an expected value of $8m/s^2$ and standard deviation of $0.1m/s^2$ would virtually guarantee no collision after the failure. (See Table II). Note that the qualifier *virtually* is used because of potential numerical inaccuracy or possible insufficiency of the discrete approximation of a continuous distribution. In this particular example, the collision probability after a vehicle failure is 0.00001864, a very small probability that is less than 1% of its platooning counterpart.

Regarding the validity of these assumptions, Hedrick [Hedrick, 1992] is optimistic that a braking system capable of 0.8g (approximately $8m/s^2$) or higher deceleration under normal driving conditions can be successfully developed in the future. An apparent AVCS design objective is to lower the failure deceleration rate as much as the cost considerations allow. Although there is no concrete data to support the validity of the selected failure deceleration rate in this example, it seems quite conservative. (See Figure 5.)

CONCLUSION

We have proposed a model for calculating the probability of a two-vehicle collision and the resulting collision speed distribution after the front vehicle abruptly decelerates. Robust probabilistic modeling is possible by using the proposed discrete representation of the joint deceleration distribution. The adoption of the maximum entropy principle made possible the task of determining the input distribution conservatively and efficiently. The availability of the software tool enabled efficient parametric studies of the safety consequences of a vehicle failure under various longitudinal-separation rules.

Our comparison suggests that, in many cases, a vehicle failure would cause far more initial collision under platooning. If a small fraction of these low-relative-speed collisions lead to major collisions, then the platooning rule would actually be less safe. We also demonstrated that the free-agent longitudinal-separation rule implemented with a potential technology of fast and accurate emergency deceleration, under some plausible conditions, might avoid any collisions after a vehicle failure while offering the high freeway capacity thought possible with platooning.

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