Using GARCH Model in the Analysis of Trade Liberalization and Poverty in Developing Countries

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The current paper reviews impacts of trade liberalization on developing countries and levels of poverty. The expected impacts of multilateral trade liberalization on wage levels and subsequent poverty are implored. Empirical Auto-regression models are visualized to develop a different set of strategies and programs to provide real benefits to the poor with real benefits. It is concluded that GARCH updating formula takes the weighted average of the unconditional variance, the squared residual for the first observation and the starting variance and estimates the variance of the second observation. This input into the forecast of the third variance and so forth. Eventually, an entire time series of variance forecasts is constructed. Ideally, this series is large when the residuals are large and small when they are small. The likelihood function provides a systematic way to adjust the parameters $\alpha, \beta$ to give the best fit. It is possible that the true variance process can differ from the one specified by econometricians. In order to detect this, a variety of diagnostic tests are available. Various tests such as tests for autocorrelation in the squares are able to detect model failures.
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Contents
1. Abstract ........................................................................................................................................... 44
2. Introduction ....................................................................................................................................... 44
3. Modelling Trade Liberalization And Poverty ............................................................................. 45
4. Cross-Country Regression ................................................................................................................. 47
5. Partial-Equilibrium/Cost-Of-Living Analysis .............................................................................. 48
6. General-Equilibrium Simulation ....................................................................................................... 50
7. Micro-Macro Synthesis ...................................................................................................................... 53
8. Generalized Autoregressive Conditional Heteroskedasticity ....................................................... 55
9. Linear Regression and Autoregressive Models ............................................................................... 56
10. Conclusions ..................................................................................................................................... 63
11. References ....................................................................................................................................... 63

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1. Abstract
The current paper reviews impacts of trade liberalization on developing countries and levels of poverty. The expected impacts of multilateral trade liberalization on wage levels and subsequent poverty are implored. Empirical Auto-regression models are visualized to develop different set of strategies and programs to provide real benefits to the poor with real benefits. It is concluded that GARCH updating formula takes the weighted average of the unconditional variance, the squared residual for the first observation and the starting variance and estimates the variance of the second observation. This input into the forecast of the third variance and so forth. Eventually, an entire time series of variance forecasts is constructed. Ideally, this series is large when the residuals are large and small when they are small. The likelihood function provides systematic way to adjust the parameters \( \alpha, \beta \) to give the best fit. It is possible that the true variance process can differ from the one specified by econometricians. In order to detect this, a variety of diagnostic tests are available. Various tests such as tests for autocorrelation in the squares are able to detect model failures.

2. Introduction
The linkages between trade liberalization and poverty have been receiving a great deal of attention in recent years. At the 1999 Geneva conference on the WTO and the developing countries, Joseph Stiglitz, then Vice President of the World Bank, proposed that the next round of WTO negotiations be labeled the Development Round and that it incorporate an explicit emphasis on poverty reduction. This direction was reinforced by Mike Moore, who was Director General of the WTO at the time. The current WTO Round was subsequently named the Doha Development Round (3). In the light of this growing interest in the topic, the Swedish Parliament, in cooperation with the Trade Research Division of the World Bank, sponsored a conference in Stockholm in October of 2000 aimed at a quantitative assessment of the links between trade liberalization and poverty. It is fair to say that researchers are still struggling to reach a consensus on the best approach of analyzing the impacts of multilateral trade liberalization on poverty, let alone agreeing on the answer. Historically, most poverty research has focused on consumption side of the question since that is easier to measure, more reliable, and less volatile than income. Meanwhile many social, political, and economic factors contribute to poverty, the evidence shows that unregulated capital and trade flows contribute to rising inequality and impede progress in poverty reduction.

Trade liberalization leads to more import competition and to a growing use of the threat to move production to lower-wage locales, which in term depressing wages. Deregulated international capital flows have led to rapid increases in short-term capital flows and more frequent economic crises, while simultaneously limiting the ability of governments to cope with crises. Economic upheavals disproportionately harm the poor, and thus contribute to the lack of success in poverty reduction and to rising income inequality. The world’s poor may stand to gain from global integration, but not under the unregulated version currently promoted by the World Bank and others. The lesson of the past 20 years is clear: it is time for a different approach to global integration, whereby living standards of the world’s poor are raised rather than jeopardized. Criticism of the unregulated globalization agenda has been met with policymakers’ renewed adherence to the doctrine that greater global deregulation of trade and capital flows helps to raise equality within countries, and accelerate poverty reduction.

But income distribution between countries worsened in the 1980s, and its apparent improvement (or leveling off) in the 1990s is the result solely of rising per capita income in China, where the enormous population tends to distort world averages. Income
inequality is also growing and is a widespread trend in countries with both advanced and developing economies. Thus, success in reducing poverty has been limited. The number of poor people has increased, especially in Eastern Europe and Central Asia. At the same time the share of poor people in national income remained at 40-50% in Latin America, sub-Saharan Africa, and South Asia. The promises for more equal income distribution and reduced poverty around the globe have failed to materialize under the current form of unregulated globalization. Thus, it is time for multinational institutions and other international policy makers to develop a different set of strategies and programs to provide real benefits to the poor with real benefits.

3. Modelling Trade Liberalization and Poverty

There is wide variation in methods used for modelling the relationship between trade liberalization and poverty over the world, but it can be classified to two main categories the first of two key models based on the analysis of economic relations and the second method based on statistical technique. We will preview various models used in modelling for this relation.

Before describing each of the methodological approaches, it is useful to consider the linkages that exist between trade, trade policy, and poverty. L. Alan Winters (2000) identifies several key linkages, which are reiterated in large part by Bannister and Thugge (2001). Potential links include changes in:

(a) The price and availability of goods;
(b) Factor prices, income, and employment;
(c) Government transfers influenced by changes in revenue from trade taxes;
(d) The incentives for investment and innovation, which affect long-run economic growth;
(e) External shocks, in particular, changes in the terms of trade; side of the trade-poverty linkage (a). Linkages (b) tend to be less frequently considered. A study by Levin (2000) focuses on transfers, link (c). A number of economy-wide analyses account for terms of trade effects, link (e). Each study typically abstracts from at least two of the linkages in order to keep the model tractable, and because the necessary data may not have been available. When reading a paper one should keep in mind which linkages are excluded from the analysis, and how this may influence the results.

For instance Feenstra (1994) and Broda and Weinstein (2006) linkage (a) and (b) and charted new ground in the study of trade gains by examining expanded product varieties made available through imports. Broda and Weinstein concluded that conventional import price indices, which do not take into account new import varieties; overstate U.S. import inflation by 28%.

Using product level data on the fraction of imported goods in total consumption, Broda and Weinstein (2006) calculate that the 28 percent increase in consumer purchasing power is equivalent to a gain of 2.8 percent of 2001 U.S. GDP from greater import variety. Feenstra (2006) not only concludes that this estimate can be safely extended to other developed countries, but he also indicates that the likely gain is probably larger for countries with a larger trade exposure (e.g., South Korea). However, applying the 2.8 percent figure to Korean purchasing power suggests a gain of $375 per capita in 2005 GDP (measured in 2000 dollars), or a total of roughly $18 billion (2000 dollars) for the overall economy. Broda, Greenfield, and Weinstein (2006) they extend their

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5 Gary Hufbauer and Agustín Carnejo, “THE PAYOFF TO SOUTH KOREA FROM GLOBALIZATION”, Korea Economic Institute. 2007, p.7
6 The previous source, p.8
methodology to a number of countries, including South Korea, and find that increased import variety accounted for an average of 5 percent of total factor productivity (TFP) during 1994–2003 in the typical developed country, and that the South Korean experience was very close to this average. Import variety contributes to TFP because, with a wider range of choices, industrial firms can right size their inputs to the exact specifications of their own products. Making a bold assumption that increased import variety accounts for 5 percent of South Korea’s total factor productivity growth over the entire post war period (1959–2003), we can roughly calculate that this single channel contributed about $550 per capita (2000 dollars) to the total gains.

As suggested in the introduction, the factor price, income, and employment link (b) may have the greatest relative importance of all the links between trade and poverty. Household survey data used in the Hertel, Preckel, Cranfield, and Ivanic study as well as casual observation suggests that people tend to be much more heterogeneous with respect to income than with respect to consumption. In other words, two households may have identical commodity budget shares, and the same level of income, but entirely different sources of income; e.g., one derives all income from agricultural labour, while the other relies on transfers from a relative who works abroad. This point is underscored by the fact that opposition to free trade initiatives often arises from groups with highly specialized income, such as steel workers and sugar farmers in the U.S., to name just two examples.

Within the world of classical trade theory, income effects are key to the famous Stolper-Samuelson theorem, which relates international trade to the domestic distribution of income (Dixit and Norman). By the Heckscher-Ohlin theorem, a country has a comparative advantage in the good that intensively uses the country’s relatively abundant factor. Free trade will increase the relative price of that good and so, by the Stolper-Samuelson theorem; increase the real return of the relatively abundant factor by an even larger percentage. At the same time, trade will reduce the return to the relatively scarce factor, though to a smaller degree. As a result, it can be said that changes in commodity prices due to trade liberalization magnify the resulting changes in factor prices. The presence of this Magnification Effect (due to Jones, 1965) in theoretical trade models is one reason why trade economists tend to focus on factor market effects when analyzing trade liberalization and poverty. Some (e.g. Winters, 2000) have argued that the practical relevance of the Stolper-Samuelson/Magnification result is negligible, since it rests on so many restrictive assumptions as to be a special case. Nevertheless, this theoretical insight underscores the importance of considering factor earnings effects when examining the relationship between trade liberalization and poverty. Three empirical studies reinforce this view. A general equilibrium analysis of technical change in the Philippines by Coxhead and Warr (1995) found earnings effects to be substantially more important than consumption effects. In particular, income effects accounted for two-thirds of poverty alleviation when there was a rise in agricultural productivity. While this is not a trade liberalization study, the nature of the shock is not dissimilar since the adjustments are transmitted through commodity and factor markets. Harrison, Rutherford, and Tarr (2000) find that factor price changes

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drive the incidence of trade liberalization in Turkey. They demonstrate this by employing three counterfactuals in which the 40 representative households in the analysis (differentiated by rural/urban orientation and by income level) have (i) identical consumption shares, (ii) identical factor income shares, and then (iii) identical consumption and factor income shares. Since counterfactual (i) provide nearly identical results to those generated when the heterogeneity of the 40 households is left intact, the authors conclude that clearly, for the poor it is the source of income, not the pattern of expenditure that is driving the adverse impact relative to the average household.

A general equilibrium analysis by Warr (2001)\(^\text{13}\) of Thailand’s proposed rice export tax also suggests that factor earnings effects are the driving force behind welfare and distributional effects. Although an export tax generates government revenue and lowers the price of rice for consumers, it also lowers the return to unskilled labour, which is used intensively in the Thai rice industry. Because both the rural and urban poor derive more than 40 percent of their income from unskilled labour (according to the Thai survey upon which the stylized households are based), the negative income effect ends up outweighing the consumption benefit, such that both the rural and urban poor are harmed by the export tax.

Despite the apparent importance of factor earnings effects, they are often not accounted for in studies that quantify the effects of external shocks on the poor in developing countries. This is particularly the case for analyses based on detailed household surveys, at least historically. Because abstracting from this particular linkage may be quite misleading, this survey will pay particular attention to how each analysis deals with the income side of the story. At the same time, the issue of whether a focus on factor markets is the same as a focus on income is not explored in depth here. It can be argued that many of the poor are subsistence farmers and largely disconnected from markets because of trade liberalization, or that their well being can be largely determined by their net trade position in a staple commodity such as rice. Studies that take this latter perspective include Ravallion (1990)\(^\text{14}\), Deaton (1989), and Ravallion and van de Walle (1991)\(^\text{15}\). As to the importance of thinking about a household’s income in terms of commodities versus factors, Hertel, Preckel, Cranfield\(^\text{16}\), and Ivanic (2001) provide interesting survey evidence on this issue for seven developing countries.

4. Cross-Country Regression

Four general methodologies are in current use for estimating the poverty impacts of trade liberalization. The first approach considered in this survey is cross-country regression, as exemplified in a recent paper by Dollar and Kraay (2001)\(^\text{17}\). These authors first categorize developing countries as either globalizers or non-globalizers based on changes in trade volumes and tariff rates since 1980, then carry out case study as well as statistical analysis. Looking at anecdotal evidence on poverty, including time-series Gini coefficients and income growth rates for average households versus the poorest quintile, they find no general trend in inequality among countries classified as globalizers. However, that term tend to have higher rates of growth than non-globalizers. This leads to the conclusion that globalization tends to be associated with a decline in absolute poverty. Verifying these findings in a more rigorous manner, the authors undertook cross-country regression analysis, and determined that no systematic relationship exists between changes in trade volumes and changes in the

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income share of the poorest. Additionally, no statistical relationship between changes in trade volumes and changes in income inequality could be found. Rodrik (2000) offers a cogent critique of Dollar and Kraay’s study. In general his remarks relate to issues with the data, to the difficulty of distinguishing between correlation and causation in cross-country regression analysis, and to the challenge of obtaining results that are robust to specification changes. Estimating the relationships that exist between trade policy, growth, and poverty depends critically on finding appropriate measures of these variables, and carefully sorting out omitted variable and endogenously problems, all of which are quite challenging given the very limited data available. The fact that Dollar and Kraay include results obtained using Instrumental Variables provides some reassurance against Rodrik’s critique.

Most trade and poverty researchers forego cross-country regression analysis and instead carry out some form of simulation. The hallmark of simulation analysis is the use of a counterfactual, which literally means contrary to the facts and enables investigation of what might have been, had a certain shock taken place. The great advantage of counterfactuals is that the effects of a specific shock can be isolated from the effects of all other events occurring during the period of interest. Counterfactual analysis, therefore, provides an elegant means of avoiding the identification problems inherent to cross-country regression, while allowing the researcher to pose specific policy questions once the appropriate simulation model has been operationalized.

The cross-country regression approach nevertheless has a number of advantages for understanding the links between trade and poverty. First of all, it enables the use of traditional statistical tools for testing results and hypotheses, as opposed to only making predictions. Secondly, cross-country regression results are typically much more general than the country-specific results of many applied simulation models. Thirdly, cross-country regression may be able to account for some of the dynamic aspects of trade reform that are missed by static simulation models. Given the differing advantages and disadvantages associated with the cross-country regression and simulation approaches, they should probably be viewed as complementary forms of analysis as opposed to substitutes.

5. Partial-Equilibrium/Cost-of-Living Analysis

The second general methodology identified as a means of estimating the poverty impacts of trade liberalization is partial-equilibrium/cost-of-living analysis. The awkwardness of this characterization reflects the fact that more than one type of study is included in this category. In general, however, all of the studies in this category are partial equilibrium in nature, since they focus on one or a limited number of markets in an economy. Additionally, most can be considered cost-of-living studies since they tend to focus on household expenditure as a measure of poverty. The majority of studies in this category can also be regarded as micro simulation models. Micro simulation is distinguished by a focus on behaviour at the individual or household level, as opposed to using any sort of representative household. As such, individual or household survey data are key to applications of the micro simulation approach. It should be noted that micro simulation is sometimes associated with general equilibrium contexts as well (see, for example, the studies by Cogneau and Robilliard, and Cockburn).

A great many papers fit into the partial-equilibrium/cost-of-living analysis category. One fairly representative approach is by Levinsohn, Berry, and Friedman (1999), who

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examine how the Indonesian economic crisis affected poor households in that country. The author combined 1993 consumption data for 58,100 households from the survey, along with price changes due to the 1997-1998 crisis to compute household changes in the cost-of-living. The salient findings were that very low income households were not insulated from the international shocks, and in fact tended to be hurt the most. Regardless of being urban or rural, households at lower expenditure levels experienced larger cost-of-living increases, a relationship that is monotonic. Additionally, the consumer price impacts of the crisis were greater for urban than for rural areas, and greatest overall for the urban poor. From a methodological perspective, the Levinsohn, Berry, and Friedman analysis has two principal drawbacks. That is by focusing only on the consumption side of the crisis link that requires precluded calculation of its real effects. This may not have been so critical for this particular application, since increases in nominal wages were overshadowed by increases in general commodity prices (an average of 26.0% versus 92.5% according to Levinsohn, Berry, and Friedman). However, studies focusing on trade liberalization generally find factor market effects to be at least as important as commodity market effects. Secondly, the Levinsohn, Berry, and Friedman analysis did not allow the effects of the crisis to be isolated from other phenomena, including the El Nino drought and widespread forest fires that occurred in the same period as the crisis. This drawback could have been avoided in a model enabling the specification of counterfactual simulations.

Another methodological limitation of significance to this survey was that household expenditure shares in the Levinsohn, Berry, and Friedman study were assumed to stay fixed throughout the crisis. Changes in demand due to changes in income or the prices of other goods were ignored. In terms of a household’s demand schedule for a given good, movements along as well as shifts in the demand curve were precluded. Estimation of a demand system, particularly one that is non-homothetic, would have avoided this issue. Another limitation is that the expenditure shares were outdated, since Indonesia in 1997 had changed substantially from where it had been in 1993. The authors point out, however, that the consumption baskets of poor households relative to rich ones, and rural households relative to urban ones, likely did not vary much over this period. This is relevant because the authors were primarily interested in assessing the relative impact of the crisis across income levels, and between rural and urban areas.

Another approach to trade, price changes, and poverty is provided by Case (1998), in a paper for the October 2000 Conference on Poverty and the International Economy. She quantifies the extent that trade reform in South Africa will affect households as consumers, using household budget shares and estimates from a Linear Expenditure System estimated separately for Africans and Whites. Budget shares and the demand system estimates were calculated using the nationally representative 1993 South African Living Standards Survey, which covers 43,794 individuals in 8,848 households drawn from 360 clusters. Using outside estimates of the price changes following tariff reform, it is found that the cost of reaching the household’s initial level of utility falls by roughly 2 percent for African households and by 1 percent for White households. As with the Levinsohn, Berry, and Friedman study, potential factor earnings effects do not enter into Case’s analysis, despite the availability of employment and income information in the household survey.

A third example of how partial equilibrium models are being used to address trade and poverty issues is Minot and Goletti (2000), who offer an extensive examination of how rice market liberalization in Viet Nam may affect income and poverty in that country. They employ a variety of methods to reach their research objective, including

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descriptive analysis based on surveys of rice producers, traders, and other market participants; time-series analysis of rice prices and production; and estimation of household demand behaviour based on the nationally representative 1992-93 Vietnam Living Standards Survey of 4,800 households. Households in poverty are defined as those below the 25th percentile in terms of per capita expenditure, and results are provided in terms of the Foster-Greer-Thorbecke poverty index as well.

The centerpiece of Minot and Goletti's analysis is a multitasked spatial equilibrium model that is used to conduct a series of policy experiments, including (i) removing the rice export quota, (ii) changing the quota level, (iii) replacing the quota with a tax, and (iv) removing restrictions on the internal movement of food. The distributional consequences of these counterfactuals are determined by way of the net rice sales position of different household classes. It is found that export liberalization raises rice prices within the country, particularly in the country's rice exporting areas. The higher prices have a positive effect on rural incomes, and are generally favourable with regard to the number of people in poverty. Relaxing the restrictions on the internal movement of rice from south to north generates net benefits for the country, without increasing most measures of poverty.

Since rice production is quite labour intensive in Viet Nam, a rise in rice prices should increase demand for agricultural labour, and consequently the agricultural wage rate. Higher rice prices would then lead to a greater decrease in poverty, particularly in households that derive a share of their income from agricultural labour. Unfortunately, Minot and Goletti's counterfactual analysis assumes that labour demand and wage rates remain constant. While they point out that landlessness and the use of hired labour are not widespread in Viet Nam, inclusion of a factor earnings link (b) would have quantified this perception.

6. General-Equilibrium Simulation

If a researcher is interested in how trade liberalization will affect only a limited number of an economy's markets, needs to incorporate a great amount of sectoral detail, or has limited time available, then partial-equilibrium/cost-of-living analyses are logical approaches. They also have the advantage of being easier to understand than general equilibrium modelling. When examining the question of poverty, however, partial-equilibrium/cost-of-living analysis usually requires a researcher to abstract from the income side of the issue, or limit the analysis to consideration of a single factor (typically labour). The focus on commodity markets is due in part to a traditional lack of good data on household earnings, since in household surveys income information tends to be less complete and less reliable than expenditure information (Cockburn; Hertel et al.)

However, several recent empirical studies provide evidence that – regardless of the data limitations. This abstraction is not innocuous.

General equilibrium analysis of poverty and distribution issues in a developing country context has its origins in work by Adelman and Robinson for Korea (1978), along with Lysy and Taylor for Brazil (1980). General equilibrium models are now widely used to assess the impact of economic shocks that reverberate across sectors and, in some cases, regions of a country or even the world. They are capable of producing disaggregated results at the microeconomic level, while providing a consistency check on macroeconomic accounts. A general equilibrium model is generally calibrated to a Social Accounting Matrix, which is a complete, consistent, and disaggregated data system. The salient feature of Social Accounting Matrices is that they quantify – at a single point in

time – the interdependence of sectors and regions in an economy. General equilibrium models are typically based on neoclassical theories of firm and household behaviour, and have a time frame long enough to achieve equilibrium in markets. While most are comparative static in nature, dynamic versions have also been developed to address certain types of issues.

A study by Löfgren (1999)\textsuperscript{27} is representative of how applied general equilibrium models are currently being used to analyze trade and poverty issues. Löfgren investigates how reduced agricultural and industrial protection will affect representative Moroccan households in the short run. The general equilibrium model is multi-sector, single-region, static, and calibrated to a 1994 Social Accounting Matrix\textsuperscript{6}, which captures the pronounced rural/urban disparity in economic structure, wages, and education that is characteristic of Morocco. Four household groups are distinguished according to whether they are rural or urban, poor or non-poor. Unlike two studies examined below in this section, the distribution of income within the groups is not modelled, as the study does not seek to make statements about the total income distribution. Based on information in the Social Accounting Matrix, Löfgren divides factor markets into four types of agricultural resources, five types of capital, and a variety of labour types, differentiated by skill, urban/rural orientation, and use in agriculture. Production is specified as a Leontief function of aggregate value-added and an aggregate intermediate input, which are a constant elasticity of substitution (CES) function of primary factors, and a Leontief function of intermediate inputs, respectively. Consumer demand is represented by the Linear Expenditure System. The model relies on standard neoclassical assumptions and is set up in real terms, such that there are no asset markets, money is neutral, and all agents make decisions as a function of relative prices. Löfgren’s simulations assess the impact of removing border protection under different assumptions about labour market rigidity. The essential results are that trade liberalization in agriculture will result in gains for the country as a whole, while the rural poor loses out. Compensation in the form of government transfers as well as education and infrastructure investments for rural areas would likely be needed if liberalization were to be pursued.

On the methodological side, Löfgren finds that the results are strongly influenced by the commodity, factor, foreign exchange, and government budget links between agriculture and the rest of the economy, which correspond to links (a), (b), (c), and (e) listed in section II of this survey. Of all the potential linkages identified by Winters (2000), Löfgren’s analysis excludes only the investment and innovation link (d), and risk and adjustment cost link (f). Ignoring these two effects would likely result in systematic underestimation of the long-run benefits and short-run costs of trade liberalization, respectively. Determining the ultimate importance of these linkages would require specification of a dynamic model. Löfgren’s general approach is more or less representative of a large number of trade and poverty studies carried out over the past decade. One variant of this basic paradigm is to address in greater detail how external shocks affect the total income distribution of a country. For this purpose it is necessary to postulate a distribution of income for each representative household type (as in Adelman and Robinson, 1978)\textsuperscript{28} or to work at the level of actual households (as in Cogneau and Robilliard, and Cockburn)\textsuperscript{29}. If a distribution is assumed \textit{a priori}, it can then be used in conjunction with the general equilibrium model to assess the impact of exogenous shocks on the income distribution of a country, as well as poverty. In this framework, it is typically the case that the mean and total income levels for a household group are endogenous while the higher moments of the distribution are fixed.


\textsuperscript{29} Cogneau, Denis, and Anne Sophie Robilliard, previous source.
In an interesting paper, Decaluwé, Patry, Savard, and Thorbecke (1999) consider this basic approach and provide some refinements to it. They model an archetype African economy with two agricultural activities, four non-agricultural activities, and six representative household groups. One of the innovations is the use of a flexible Beta functional form to model the income distribution within household groups, instead of the more common - and restrictive - lognormal or Pareto distributions. The parameters of the Beta distribution are specified to conform to observed socio-economic characteristics of each household type, and it is shown that the shape of the distribution may indeed vary markedly across them. Another of the model’s refinements is the specification of a poverty line in the LES demand system based on a unique and fixed bundle of basic-needs commodities. Because commodity prices are endogenously determined, the poverty line is as well. Although no empirical results are presented, Decaluwé, Patry, Savard, and Thorbecke suggest that their innovations will help shed more light on the black box pertaining to the behaviour of poverty following a shock.

The authors also emphasize in the first part of their paper that Social Accounting Matrixes can be used on their own to analyze issues related to income distribution, and to a lesser extent, poverty. This involves the use of accounting multipliers in conjunction with information on the factor income of disaggregated household types.

Another approach to trade, poverty, and income distribution modelling is offered by Cogneau and Robilliard. In many ways their general equilibrium model is fundamentally different from the general equilibrium models described above. Their aim is to assess the impact of different growth strategies on welfare and poverty in Madagascar. To meet this goal they embed an econometrically estimated labour allocation model based on 4,508 households within a general equilibrium framework. The combination of a micro simulation and general equilibrium model facilitates the modelling of a country’s overall income distribution, since it is no longer necessary to a priori assume an income distribution for each household type. The combination of approaches also allows endogenous variables to be determined at the level of individual households, thereby eliminating the representative household assumption (for the most part) and its associated theoretical shortcomings.

Three aggregate sectors of the Madagascar economy are modelled: a formal sector that produces a tradable commodity, an informal sector that produces a non-tradable, and an agricultural sector that produces both a tradable and non-tradable. Productive factors include labour, agricultural capital, and formal sector capital. Agricultural and informal activity is endogenous and determined at the household level, as is agricultural labour demand. Informal labour demand is determined at the aggregate level based on demand for the informal good and agricultural labour. The supply of labour to the agricultural and informal sectors is endogenous and determined at the individual level using the labour allocation model. Consumer demand is modelled with the Linear Expenditure System. Macroeconomic data were from a 1995 Social Accounting Matrix constructed by Razafindrakoto and Roubaud. Microeconomic data, covering 4,508 households, are from the 1993 EPM (Enquête Permanente auprès des Ménages) survey carried out by INSTAT (the Institute National de la Statistique) on behalf of the Malagasy government. Although endogenous variables are based on individual household behaviour via the micro simulation model, results of the simulations are presented in terms of 14 representative households.

Four of the representative households are urban, and differentiated according to educational attainment and gender. Eight households are rural, agricultural, and differentiated according to region and farm size. The remaining two types of households are rural, non-agricultural, and distinguished according to wealth. Although it is not

31 Cogneau, Denis, and Anne Sophie Robilliard, previous source.
clear from the paper, these typologies appear to be based on the 1993 EPM survey data set and the 1995 Social Accounting Matrix. These same data sources also provide information for the disaggregating of household income. Earnings are based on receipts from agricultural labour, informal labour, formal sector labour, capital dividends, sharecropping income, and transfers from other households or the government.

Cogneau and Robilliard consider six counterfactuals, including (i) an increase in formal sector labour demand, (ii) an increase in formal sector wages, (iii) an increase in agricultural productivity, (iv) an increase in food crop productivity, (v) an increase in cash crop productivity, and (vi) an increase in the world cash crop prices. While relative income and price changes are significant in most simulations, the effect of shocks on poverty and inequality are small. The authors identify several reasons for this finding, including the unequal distribution of productive factors across households, and the ability of households to diversify their income sources through reallocation of productive activity.

Cogneau and Robilliard’s analysis is a unique melding of micro simulation and general equilibrium modeling. Basing the analysis on actual households facilitates the study of income distribution, since restrictive assumptions about within-group distributions and certain other aggregation issues can be avoided. Working with actual households also lends an air of realism, and allows for the possibility that there is considerable heterogeneity across households. Meanwhile, incorporation of general equilibrium mechanisms captures the redistributive effects of shocks on both sectors and households. These accomplishments entail higher data requirements and computational costs, however. Working with 4,508 agents requires other model dimensions to be scaled back, since, for example, the income of each agent needs to be tied to each commodity represented. As a result, the sectors and commodities of Cogneau and Robilliard’s model are highly aggregated, and a number of critical macroeconomic features are ignored. Another consideration is that it is not practical to inspect the impact of a simulation on each of several thousand households. Accordingly, results must be aggregated and analyzed for a limited number of representative households, just as in conventional general equilibrium models.

7. Micro-Macro Synthesis

While the approach of Cogneau and Robilliard is innovative, there are other ways to capitalize on the detail of household survey data while availing the ability of general equilibrium models to capture the numerous links between trade and poverty. A somewhat simpler, more pragmatic means to the same end is offered by the studies in this fourth and final category of the survey, which, for lack of a better label, is entitled micro-macro synthesis. An alternative description might be general equilibrium simulation with post-simulation analysis. This approach is best characterized by its sequential, two-step nature. In general, a general equilibrium model is first shocked to get commodity and factor price changes. These are then fed into or calibrated to a post-simulation framework that calculates the effects on actual or highly disaggregated representative households. Various poverty measures can then be applied to assess the distributional effects of the shocks.

This two-step approach is similar to that employed in some partial-equilibrium/cost-of-living analyses, except that in those studies the price changes are typically for consumer goods only, and are purely hypothetical or based on real-world observations – in other words, not from a counterfactual simulation. A limitation of post-simulation analysis, at least in the view of general equilibrium practitioners, is that the reactions of households to commodity and factor price changes in the post-simulation analysis are not transmitted back to the general equilibrium model. Although this absence of feedback is not satisfactory from a theoretical point of view, the resulting error is likely to be small.
Robilliard, Bourguignon, and Robinson (2001) are one of a growing number of micro-macro studies to recently emerge. As in the Levinsohn, Berry, and Friedman paper, the authors study the effects of the 1997 Indonesian crisis on poor households. The general equilibrium model is based on a single-region Social Accounting Matrix that captures macroeconomic constraints along with inter-sectoral flows for 38 sectors and 15 factors of production. The post-simulation analysis is a micro simulation model based on the Susenas (1996) survey, with 33,000 individuals in 9,800 households. Conducting the analysis with actual households facilitates calculation of changes in the income distribution, since one can avoid strong assumptions about intra-group distributions and certain other aggregation issues. Robilliard, Bourguignon, and Robinson’s micro simulation model represents the way in which households generate their income, by focusing on how earnings are determined and how occupational choices are made. Workers are divided into eight groups according to skill, gender, and area of residence. Functions corresponding to wage worker earnings, farm and non-farm worker profits, and occupational choices are estimated. Labour supply is modelled as a discrete choice between inactivity and full time work.

The general equilibrium model relies on standard neoclassical assumptions and is set up in real terms, with no asset markets, neutral money, and decisions based on relative prices. The model is dualistic in that it distinguishes between formal and informal activities in each sector, both of which produce the same good. Eight labour categories, six types of capital, and 10 household types are distinguished, along with macro accounts for enterprises, government, the rest of the world, and for savings-investment. The real wage is assumed to be fixed in formal-sector labour markets, while informal sector labour markets absorb any labour displaced from the formal sectors.

The general equilibrium model is linked to the micro simulation model through (i) the wage level in each wage labour market, (ii) the income level for the informal self-employed sector, (iii) the number of wage workers and self-employed by labour market segment, and (iv) consumption prices. The micro simulation model is solved so that it generates equilibrium values and changes that are consistent with the results from the general equilibrium model.

Simulations can be carried out to (a) decompose and reproduce the crisis impact, (b) examine how the Indonesian economy would have fared with the same adjustment in trade balance but no credit crisis, and (c) examine different policy options, including a food price subsidy, a public work program for unskilled workers, and transfers to target groups. It is found that poverty increases over the 1997-98 period were due in equal measure to the El Nino drought and to the financial crisis (a very different perspective from that of the Levinsohn, Berry, and Friedman paper). The second set of experiments suggests that some of the available policy options would have resulted in a smaller increase in poverty.

On the methodological side, the Robilliard, Bourguignon, and Robinson approach is somewhat costly since the unit of analysis is an actual household, and a great deal of estimation work is required. To assess the benefits of this approach, they carried out the analysis using representative households for comparison. They determine that a representative household assumption biases most experiments and leads to incorrect results in the case of targeted policies. The representative household approach appears to systematically underestimate the effect of shocks on income inequality and poverty.

Another interesting approach to trade and poverty issues is offered by Hertel, Preckel, Cranfield, and Ivanic (2001). They examine how global trade liberalization affects poverty in each of seven different developing countries. While they center their analysis on factor market effects, they also allow for commodity market and terms of trade effects (altogether incorporating links (a), (b), and (e) described in section II). The first
The step of the authors’ analysis involves conducting a policy experiment in the Global Trade Analysis Project (GTAP) model of trade (Hertel, 1997) to generate a vector of factor and commodity price changes for 17 regions of the world. Since the GTAP database is designed for broad country coverage, it is limited to one representative household per region – clearly not adequate for an investigation of poverty. The price changes are therefore fed into a post-simulation framework that characterizes households according to factor income and consumption profiles, which are based on International Comparison Project data, and household surveys for seven countries, respectively. One of the authors’ most striking findings is the extent to which households in each of the seven countries are specialized in terms of factor earning profiles. To capture the consequent vulnerability to trade liberalization, households are categorized into five strata, including those getting at least 95% of income from (i) transfers, (ii) agriculture, (iii) non-agricultural business, (iv) wages and then (v) a stratum for households that have diversified income sources. Within each stratum, the differences across income levels are preserved. Changes in real household incomes are calculated, and demand response is simulated by feeding commodity price changes into an estimated global AIDADS demand system. The demand system is used to calculate the poverty level of utility for each region. Equivalent variation (EV) and a first-order compensating variation (CV) measure are then calculated at both the per capita and poverty line levels. Since the CV approximation proves to be quite accurate compared to the exactly computed EV, it is used to decompose the results into underlying commodity and factor market adjustments. The Foster-Greer-Thorbecke measure of poverty is used to calculate the total transfer required to lift all households above the poverty level of utility, as a proportion of the poverty level of income. Hertel, Preckel, Cranfield, and Ivanic’s findings suggest that multi-lateral trade liberalization will reduce overall poverty in Indonesia, Philippines, Uganda, and Zambia, but increase overall poverty in Brazil, Chile, and Thailand. Within regions, the results vary considerably by household group. The largest poverty reduction occurs among agriculture-specialized households in Brazil, while the largest increase occurs among non-agricultural, self-employed, and wage-labour households in Brazil, Chile, and Thailand.

From the previous preview for the various models and methodologies we can conclude that quantifying the poverty impacts of trade liberalization and related external shocks is currently an area of intense research, and a variety of methodologies are being employed to address the issues involved. This survey provides a review of methods in current use, and classifies them into four broad categories, namely (i) cross-country regression analysis, (ii) partial-equilibrium/cost-of-living analysis, (iii) general-equilibrium simulation, and (iv) micro-macro synthesis (also referred to as general equilibrium simulation with post-simulation analysis). These four groups encompass both the bottom up and top-down traditions that are associated with poverty and trade specialists, respectively. The former approach builds on detailed survey information, and emphasizes the heterogeneity of individual households as well as commodity market linkages between trade and poverty. The latter approach begins with the representative household assumption from microeconomic theory, and generally incorporates additional linkages between trade and poverty such as factor earnings and terms of trade effects.

8. Generalized Autoregressive Conditional heteroskedasticity

We discussed the modeling of the time behaviour of the uncertainty related to many econometric models when applied to economical data. There are periods when unpredictable poverty fluctuations are larger and periods when they are smaller. This behavior, known as heteroskedasticity, refers to the fact that the size of volatility tends
to cluster in periods of high volatility and periods of low volatility. The discovery that it is possible to formalize and generalize this observation was a major breakthrough in econometrics. In fact, we can describe many economic and financial data with models that predict, simultaneously, the economic variables and the average magnitude of the squared prediction error.

We show here how the average error size can be modeled as an autoregressive process. Given their autoregressive nature, these models are called autoregressive conditional heteroskedasticity (ARCH) or generalized autoregressive conditional heteroskedasticity (GARCH). This discovery is particularly important in financial econometrics, where the error size is, in itself, a variable of great interest.

9. Linear Regression and Autoregressive Models
Let's first discuss two examples of basic econometric models, the linear regression model and the autoregressive model, and illustrate the meaning of homoskedasticity or heteroskedasticity in each case.

The linear regression model is the workhorse of economic modeling. A univariate linear regression represents a proportionality relationship between two variables:

\[ Y = \alpha + \beta x + \varepsilon \]

The preceding linear regression model states that the expectation of the variable \( y \) is \( \beta \) times the expectation of the variable \( x \) plus a constant \( \alpha \). The proportionality relationship between \( y \) and \( x \) is not exact but subject to an error \( \varepsilon \). In standard regression theory, the error \( \varepsilon \) is assumed to have a zero mean and a constant standard deviation \( \sigma \).

The standard deviation is the square root of the variance, which is the expectation of the squared error:

\[ \delta^2 = E(\varepsilon^2) \]

It is a positive number that measures the size of the error.

We presume that homoskedasticity the assumption that the expected size of the error is constant that does not depend on the size of the variable \( x \). We call heteroskedasticity the assumption that the expected size of the error term is not constant.

The assumption of homoskedasticity is convenient from a mathematical point of view and is standard in regression theory. However, it is an assumption that must be verified empirically. In many cases, especially if the range of variables is large, the assumption of homoskedasticity might be unreasonable. For example, assuming a linear relationship between consumption and household income, we can expect that the size of the error depends on the size of household income. But, in fact, high-income households have more freedom in the allocation of their income.

In the preceding household-income example, the linear regression represents a cross-sectional model without any time dimension. However, in finance and economics in general, we deal primarily with time series, that is, sequences of observations at different moments of time. Let's call \( X_t \) the value of an economic time series at time \( t \). Economic time series are considered to be realizations of stochastic processes. That is, each point of an economic time series is considered to be an observation of a random variable.

We can look at a stochastic process as a sequence of variables characterized by joint-probability distributions for every finite set of different time points. In particular, we can consider the distribution \( f_t \) of each variable \( X_t \) at each moment. Intuitively, we can visualize a stochastic process as a very large (infinite) number of paths. A process is called weakly stationary if all of its second moments are constant. In particular this means that the mean and variance are constants \( \delta^2_t = \delta^2 \) that do not depend on the time \( t \). A process is called strictly stationary if none of its finite distributions depends on time. A strictly stationary process is not necessarily weakly stationary as its finite
distributions, though time-independent, might have infinite moments. The terms $\mu_t$ and $\delta_t^2$ are the unconditional mean and variance of a process. In finance and economics, however, we are typically interested in making forecasts based on past and present information. Therefore, we consider the distribution $f_{t_2}(x/I_{t_1})$ of the variable $X_{t_2}$ at time $t_2$ conditional on the information $I_{t_1}$ known at time $t_1$, based on information available at time $t-1, I_{t-1}$ we can also define the conditional mean and the conditional variance $(\mu_t \mid I_{t-1})$, $(\delta_t^2 / I_{t-1})$.

A process can be weakly stationary but have time varying conditional variance. If the conditional mean is constant, then the unconditional variance is the unconditional expectation of the conditional variance. If the conditional mean is not constant, the unconditional variance is not equal to the unconditional expectation of the conditional variance; this is due to the dynamics of the conditional mean.

In describing ARCH/GARCH behavior, we focus on the error process. In particular, we assume that the errors are an innovation process, that is, we assume that the conditional mean of the errors is zero. We write the error process as: $\epsilon_t = \delta_t Z_t$ where $\delta_t$ is the conditional standard deviation and the $Z$ terms are a sequence of independent, zero-mean, unit-variance, normally distributed variables.

Under this assumption, the unconditional variance of the error process is the unconditional mean of the conditional variance. Note, however, that the unconditional variance of the process variable does not, in general, coincide with the unconditional variance of the error terms. In financial and economic models, conditioning is often stated as regressions of the future values of the variables on the present and past values of the same variable. For example, if we assume that time is discrete, we can express conditioning as an autoregressive model:

$$X_{t-1} = \alpha_0 + \beta_1 X_t + \ldots + \beta_n X_{t-n} + \epsilon_{t+1} + \ldots$$

The error term $\epsilon_{t+1}$ is conditional on the information $I_t$ that, in this example, is represented by the present and the past $n$ values of the variable $X$. The simplest autoregressive model is the random walk model of the logarithms of prices $P_t$:

$$P_{t+1} = \mu_t + P_t + \epsilon_t$$

In terms of returns, the random walk model is simply:

$$r_t = \Delta P_t = \mu + \epsilon_t$$

A major breakthrough in econometric modeling was the discovery that, for many families of econometric models, linear and nonlinear alike, it is possible to specify a stochastic process for the error terms and predict the average size of the error terms when models are fitted to empirical data. Two observations are in order. First, we have introduced two different types of heteroskedasticity. In the first example, regression errors are heteroskedastic because they depend on the value of the independent variables: The average error is larger when the independent variable is larger. In the second example, however, error terms are conditionally heteroskedastic because they vary with time and do not necessarily depend on the value of the process variables. Later in this chapter we will describe a variant of the ARCH model where the size of volatility is correlated with the level of the variable. However, in the basic specification of ARCH models, the level of the variables and the size of volatility are independent.
Second, let's observe that the volatility (or the variance) of the error term is a hidden, no observable variable. Later in this chapter, we will describe realized volatility models that treat volatility as an observed variable. Theoretically, however, time-varying volatility can be only inferred, not observed. As a consequence, the error term cannot be separated from the rest of the model. This occurs both because we have only one realization of the relevant time series and because the volatility term depends on the model used to forecast expected returns. The ARCH/GARCH behavior of the error term depends on the model chosen to represent the data. We might use different models to represent data with different levels of accuracy. Each model will be characterized by a different specification of heteroskedasticity.

Consider, for example, the following model for returns:

\[ r_m^t = m + \varepsilon_t \]

Lent to the clustering of the squared returns (minus their constant mean). Now suppose that we discover that returns are predictable through a regression on some predictor \( f \):

\[ r_m^t = m + f_{t-1}^t + \varepsilon_t \]

As a result of our discovery, we can expect that the model will be more accurate, the size of the errors will decrease, and the heteroskedastic behavior will change.

Note that in the model \( r_m^t = m + \varepsilon_t \), the errors coincide with the fluctuations of returns around their unconditional mean. If errors are an innovation process, that is, if the conditional mean of the errors is zero, then the variance of poverty coincides with the variance of errors, and ARCH behavior describes the fluctuations of returns. However, if we were able to make conditional forecasts of returns, then the ARCH model describes the behavior of the errors and it is no longer true that the unconditional variance of errors coincides with the unconditional variance of returns. Thus, the statement that ARCH models describe the time evolution of the variance of returns is true only if returns have a constant expectation.

ARCH/GARCH effects are important because they are very general. It has been found empirically that most model families presently in use in econometrics and financial econometrics exhibit conditionally heteroskedastic errors when applied to empirical economic and financial data. The heteroskedasticity of errors has not disappeared with the adoption of more sophisticated models of financial variables. The ARCH/GARCH specification of errors allows one to estimate models more accurately and to forecast volatility.

**ARCH Models**

ARCH models attempt to explain variance clustering in the residuals and imply nonlinear dependence among the squared errors of the first moment model. Engle (1982) relaxes the constant conditional variance assumption in traditional Box-Jenkins ARIMA models and allows it to follow a process as below

\[ a_t = \sigma_t \varepsilon_t \]

And

\[ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m} \]

Where

\[ \sigma_t = E(a_t^2/F_{t-1}) = \text{var}^\dagger(a_t^2/F_{t-1}) = \text{var}^\ddagger(Y/F_{t-1}) \]

Letting

\[ e_t = a_t^2 - E(a_t^2/F_{t-1}) \]
This is same as,
\[ e_t = \sigma_t^2 - \sigma_i^2 \]
AR(m) model also can be written as below
\[ a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots + \alpha_m a_{t-m}^2 - e_t \]
where \( e_t \) is a white noise process. A model with \( \sigma_t^2 \) is referred to as an autoregressive conditional heteroskedastic (ARCH) model, or ARCH(m) model. For such models, it is required that \( \alpha_0 > 0 \) and \( \alpha_i \geq 0 \) for \( i > 0 \).

The log likelihood function of an ARCH model, with the assumption that \( e_t \) follows a Normal distribution is
\[
\ell(\alpha) = \sum_{i=m+1}^{n} \left( -0.5 \ln(\sigma_i^2) - 0.5 \left( \frac{e_t}{\sigma_i^2} \right) \right)
\]

In practice, there is substantial evidence showing that the Normality assumption may not always be satisfactory. Non-normal distributions, such as the Student-t distribution (Bollerslev, 1987)\(^{35}\). In econometrics an autoregressive conditional heteroskedasticity (ARCH) model considers the variance of the current error term to be function of the variance of the previous time period’s error term ARCH relates the error variance to the square of a previous period’s error it’s employed commonly in modelling economic time series that exhibit time varying volatility clustering. Since the introduction of the autoregressive conditional heteroskedasticity ARCH\(^{36}\) model over many years ago, the variations, extensions, and applications have become breathtaking and intimidating.

**Problems with ARCH (q) Models**

1. The required value of \( q \) might be very large.
2. Non-negativity constraints might be violated.

When we estimate an ARCH model, we require \( ai > 0 \) \( i = 1, 2, \ldots, q \) (since variance cannot be negative). A natural extension of an ARCH(q) model which gets around some of these problems is a GARCH model. If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a Generalized Autoregressive Conditional heteroskedasticity (GARCH, Bollerslev(1986))\(^{37}\) model.

**GARCH Model**

Although the ARCH model is simple, it restricts the model for the conditional variance \( \sigma_t^2 \) (or equivalently \( h_t \)) to follow a pure AR process and hence it may require more parameters to adequately represent the conditional variance process in comparison with other more generalized models. Bollerslev (1986) extends Engle’s original work by allowing the conditional variance to follow an ARMA process. This model is known as a generalized ARCH model, or GARCH model.

Even a casual look through the econometric literature of the last two decades reveals a drastic change in the conceptual treatment of economic time series. The modelling of such time series moved from a static set-up to one that recognizes the importance of fitting the time-varying features of macro-economic and financial data.

In particular, it is now widely accepted that the covariance structure of returns (referred often as volatility) changes through time. A large part of the modern econometric literature frames modelling of time-varying volatility in the autoregressive conditional heteroskedasticity (ARCH) framework, a stationary, parametric, conditional approach that postulates that the main time-varying feature of returns is the conditional covariance

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structure3 while assuming in the same time that the unconditional covariance remains constant through time (for example, the survey Bollerslev)\textsuperscript{38}. While the autoregressive conditional heteroskedastic approach to modeling time-varying volatility is currently prevalent, alternative methodologies for volatility modeling exists in the econometric literature. In particular, the non-stationary framework that assumes the unconditional variance to be the main time varying feature of returns has a long tradition that can be traced back to Officer\textsuperscript{39}, Hsu, Miller and Wichern\textsuperscript{40}, Merton\textsuperscript{41}, French\textsuperscript{42}. ARCH and GARCH models have become important tools in the analysis of time series data, particularly in financial applications. These models are especially useful when the goal of the study is to analyze and forecast volatility. This paper gives the motivation behind the simplest GARCH model and illustrates its usefulness in examining portfolio risk. Extensions are briefly discussed. The great workhorse of applied econometrics is the least squares model. This is natural because applied econometricians are typically called upon to determine how much one variable will change in response to a change in some other variable. Increasingly however, econometricians are being asked to forecast and analyze the size of the errors of the model. In this case the questions are about volatility and the standard tools have become the ARCH/GARCH models.

The basic version of the least squares model assumes that, the expected value of all error terms when squared, is the same at any given point. This assumption is called homoskedasticity and it is this assumption that is the focus of ARCH/GARCH models. Data in which the variances of the error terms are not equal, in which the error terms may reasonably be expected to be larger for some points or ranges of the data than for others, are said to suffer from heteroskedasticity. The standard warning is that in the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH and GARCH models treat heteroskedasticity as a variance to be modelled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance of each error term. This turns out often to be of interest particularly in finance.

The warnings about heteroskedasticity have usually been applied only to cross sectional models, not to time series models. For example, if one looked at the cross-section relationship between income and consumption in household data, one might expect to find that the consumption of low-income households is more closely tied to income than that of high-income households, because the dollars of savings or deficit by poor households are likely to be much smaller in absolute value than high income households. In a cross-section regression of household consumption on income, the error terms seem likely to be systematically larger in absolute value for high income households than for low-income households, and the assumption of homoskedasticity seems implausible. In contrast, if one looked at an aggregate time series consumption function, comparing national income to consumption, it seems more plausible to assume that the variance of the an error term doesn’t change much over time.

A recent development in estimation of standard errors, known as robust standard errors, has also reduced the concern over heteroskedasticity. If the sample size is large, then robust standard errors give quite a good estimate of standard errors even with

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heteroskedasticity. If the sample is small, the need for a heteroskedasticity correction that doesn’t affect the coefficients, and only asymptotically corrects the standard errors, can be debated. However, sometimes the natural question facing the applied econometrician is the accuracy of the predictions of his model. Thus the key issue is the variance of the error terms and what makes them large. This question often arises in financial applications where the dependent variable is the return on an asset or portfolio and the variance of the return represents the risk level of those returns. These are time series applications, but it is nonetheless likely that heteroskedasticity is an issue. Even a cursory look at financial data suggests that some time periods are riskier than others; that is, the expected value of the magnitude of error terms at some times is greater than at others. Moreover, these risky times are not scattered randomly across city, are designed to deal with just this set of issues. They have quarterly or annual data. Instead, there is a degree of autocorrelation in the riskiness of financial returns. Financial analysts, looking at plots of daily returns.

The ARCH and GARCH models stand for autoregressive conditional heteroskedasticity and generalized autoregressive conditional heteroskedastic become widespread tools for dealing with time series heteroskedastic models. The goal of such models is to provide a volatility measure – like a standard deviation - that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing. The extension of the ARCH model to the GARCH model is similar to the extension of the standard time series AR model to the general ARMA model. The great workhorse of applied econometrics is the least squares model. The basic version of the model assumes that, the expected value of all error terms, in absolute value, is the same at any given point. Thus, the expected value of any given error term, squared, is equal to the variance of all the error terms taken together. This assumption is called homoskedasticity. Conversely, data in which the expected value of the error terms is not equal, in which the error terms may reasonably be expected to be larger for some points or ranges of the data than for others, is said to suffer from heteroskedasticity.

It has long been recognized that heteroskedasticity can pose problems in ordinary least squares analysis. The standard warning is that in the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression is still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. However, the warnings about heteroskedasticity have usually been applied only to cross sectional models, not to time series models. For example, if one looked at the cross-section relationship between income and consumption in household data, one might expect to find that the consumption of low-income households is more closely tied to income than that of high-income households, because poor households are more likely to consume all of their income and to be liquidity-constrained. In a cross-section regression of household consumption on income, the error terms seem likely to be systematically larger for high-income than for low-income households, and the assumption of homoskedasticity seems implausible. In contrast, if one looked at an aggregate time series consumption function, comparing national income to consumption, it seems more plausible to assume that the variance of the error terms doesn’t change much over time.

In that case, the GARCH(p, q) model (where p is the order of the GARCH terms $\sigma^2$ and q is the order of the ARCH terms $\epsilon^2$) is given by

$$\sigma^2_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \ldots + \alpha_q \epsilon^2_{t-q} + \beta_1 \sigma^2_{t-1} + \ldots + \beta_p \sigma^2_{t-p} = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon^2_{t-i} + \sum_{i=1}^{p} \beta_i \sigma^2_{t-i}$$

The asymptotic, that is for large samples, standard deviation of (i) is $T^{-\frac{1}{2}}$. Individual values that are larger than this indicate GARCH errors. To estimate the total number of

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lags, use the Ljung-Box test until the value of the these are less than, say, 10% significant. The Ljung/Box Q-statistic follows χ2 distribution with n degrees of freedom if the squared residuals are uncorrelated. The null hypothesis states that there are ARCH or GARCH errors. Rejecting the null thus means that there are no such errors in the conditional variance.

After we select our mode we will test model under Akaike information criterion developed by Hirotsgu Akaike under the name of an information criterion (AIC) in 1971 and proposed in Akaike (1974), is a measure of the goodness of fit of an estimated statistical model. It is grounded in the concept of entropy, in effect offering a relative measure of the information lost when a given model is used to describe reality and can be said to describe the tradeoff between bias and variance in model construction, or loosely speaking that of precision and complexity of the model.

The AIC is not a test on the model in the sense of hypothesis testing, rather it is a tool for Model selection. Given a data set, several competing models may be ranked according to their AIC, with the one having the lowest AIC being the best. From the AIC value one may infer that e.g the top three models are in a tie and the rest are far worse, but one should not assign a value above which a given model is 'rejected'.

In the general case, the AIC is

\[ \text{AIC} = 2K - 2\ln(L) \]

where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model. Over the remainder of this entry, it will be assumed that the model errors are normally and independently distributed. Let n be the number of observations and RSS be

\[ RSS = \sum_{i=1}^{n} \hat{\beta}_i \]

the residual sum of squares. Then AIC becomes

\[ \text{AIC} = 2k + n \left[ \ln(2\pi RSS/n) + 1 \right] \]

the residual sum of squares. Then AIC becomes increasing the number of free parameters to be estimated improves the goodness of fit, regardless of the number of free parameters in the data generating process. Hence AIC not only rewards goodness of fit, but also includes a penalty that is an increasing function of the number of estimated parameters. This penalty discourages overfitting. The preferred model is the one with the lowest AIC value. The AIC methodology attempts to find the model that best explains the data with a minimum of free parameters. By contrast, more traditional approaches to modeling start from a null hypothesis. The AIC penalizes free parameters less strongly than does the Schwarz criterion. AIC judges a model by how close its fitted values tend to be to the true values, in terms of a certain expected value. In order to ensure our model we will use additional test denotes as Schwarz\textsuperscript{44} Bayesian criterion (BIC), Schwarz Bayesian criterion (BIC) In order to describe a particular dataset, one can use non-parametric methods or parametric methods. In parametric methods, there might be various candidate models with different number of parameters to represent a dataset. The number of parameters in a model plays an important role. The likelihood of the training data is increased when the number of parameters in the model is increased but it might result in overtraining problem if the number of parameters is too large. In order to overcome this problem one can use Bayesian Information Criterion (parametric method) which is a statistical criterion for model selection.

The BIC is sometimes also named the Schwarz Criterion, or Schwarz Information Criterion (SIC). It is so named because Gideon E. Schwarz (1978) gave a Bayesian argument for adopting it. The BIC is an asymptotic result derived under the assumptions that the data distribution is in the exponential family. Let,

\( n \) = the number of observations, or equivalently, the sample size;
\( k \) = the number of free parameters to be estimated. If the estimated model is a linear regression, \( k \) is the number of regressors, including the constant;
\( L \) = the maximized value of the likelihood function for the estimated model.
The formula for the BIC is:
\[
\text{BIC} = -2 \cdot \ln L + k \ln(n).
\]
Under the assumption that the model errors or disturbances are normally distributed, this becomes:
\[
\text{BIC} = n \ln \left( \frac{\text{RSS}}{n} \right) + K \ln(n).
\]
where RSS is the residual sum of squares from the estimated model.

Given any two estimated models, the model with the lower value of BIC is the one to be preferred. The BIC is an increasing function of RSS and an increasing function of \( k \). That is, unexplained variation in the dependent variable and the number of explanatory variables increase the value of BIC. Hence, lower BIC implies either fewer explanatory variables, better fit, or both. The BIC penalizes free parameters more strongly than does the Akaike information criterion.

10. Conclusions
The BIC can be used to compare estimated models only when the numerical values of the dependent variable are identical for all estimates being compared. The models being compared need not be nested, unlike the case when models are being compared using an F or likelihood ratio test. The GARCH model that has been described is typically called the GARCH (1, 1) model. In parentheses it is a standard notation in which the first number refers to how many autoregressive lags or ARCH terms appear in the equation, while the second number refers to how many moving average lags are specified which is often called the number of GARCH terms. Sometimes models with more than one lag are needed to find good variance forecasts.

Although this model is directly set up to forecast for just one period, it turns out that based on the one period forecast a two period forecast can be made. Ultimately by repeating this step, long horizon forecasts can be constructed. For the GARCH model the two step forecast is a little closer to the long run average variance than the one step forecast and ultimately, the distant horizon forecast is the same for all time periods. This is just the unconditional variance. Thus the GARCH models are mean reverting and conditionally heteroskedasticity but have a constant unconditional variance.
The GARCH updating formula takes the weighted average of the unconditional variance, the squared residual for the first observation and the starting variance and estimates the variance of the second observation. This is input into the forecast of the third variance and so forth. Eventually, an entire time series of variance forecasts is constructed.
Ideally, this series is large when the residuals are large and small when they are small. The likelihood function provides a systematic way to adjust the parameters \( \alpha, \beta \) to give the best fit. Of course, it is entirely possible that the true variance process is different from the one specified by the econometrician. In order to detect this, a variety of diagnostic tests is available. The simplest is to construct the series of \( \varepsilon_t \), which are supposed to have constant mean and variance if the model is correctly specified. Various tests such as tests for autocorrelation in the squares are able to detect model failures.

11. References

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