Summer 2023

Who Fares Better in Teamwork?

Huseyin Yildirim, *Duke University*

Who Fares Better in Teamwork?*

Huseyin Yildirim
Department of Economics
Duke University
Box 90097
Durham, NC 27708
E-mail: yildirh@econ.duke.edu

July 8, 2022

Abstract

This article establishes a tenuous link between ability and relative well-being in teamwork. It shows that higher-ability or lower-cost members can easily fare worse than their lower-ability counterparts due to free-riding. The extent of free-riding hinges crucially on log-concavity of effort cost, which its convexity restricts little. The article further shows how to compose teams that allocate effort efficiently and equalize payoffs in equilibrium. Efficient teams must have sufficiently diverse abilities and sizes at most the number of cost log-inflections plus one. These findings can explain the evidence of significant dislike for teamwork in the workplace and classroom.

Keywords: Teamwork, Payoff Ordering, Easy vs. Challenging Project, Transparency
JEL Classification: C73, D63, H41, L23

*I thank David Martimort (Co-Editor) and the referees for their valuable comments. I also thank Arjada Bardhi, Silvana Krasteva, Tracy Lewis, Alvaro Name-Correa, Philipp Sadowski, and the 2021 European Econometric Society Meeting participants for suggestions. Jiaqi Huang and Zhichao Wang provided able research assistance. All errors are mine.

1 Introduction

In a knowledge-driven world with increasingly complex problems, teamwork has become the norm across various settings. For instance, organizations of all sizes frequently rely on problem-solving teams for their critical decisions (Postrel, 2002). Educators encourage group assignments in classrooms (Thom, 2020). And, researchers collaborate on projects more than ever (Jones, 2021).

Despite its prevalence, however, there is compelling evidence that teamwork is disliked by many of its participants. For example, a Dropbox survey of 2,000 workers reveals that less than half enjoy working in a team, reporting the main difficulty to be "freeloaders" not pulling their weight.[1] Similarly, a University of Phoenix survey of 1,000 U.S. residents shows that 95 percent of those who have worked on a team believe teamwork is crucial in the workplace. Yet more than three-quarters of employees would prefer to work alone.[2] In education, Thom (2020) reviews the literature on groupwork efficacy and concludes that "high-achieving students generally dislike group assignments, while low-achieving students embrace them... Free riders – i.e., individuals who contribute little to the group but often incur just as little, if any, real consequence – are perhaps their most common grievance...Distaste among high-achieving students may stem from the likelihood that group assignments tend to redistribute performance, lowering their grades while raising them for low-achieving students and free riders."

This evidence suggests that individuals dislike teams primarily because they worry about an unfair workload allocation and, in turn, their well-being relative to others – a comparison that seems especially pertinent in teamwork (Gill and Stone, 2015). Although how individuals determine their relative well-being remains unsettled in the behavioral literature (Camerer et al., 2004), it seems plain that better performers will at least expect to fare no worse than poor performers in a team. In particular, endowed with superior skills, more able employees, students, and researchers will probably anticipate pulling more weight in a group project but resent doing so to the point of feeling "exploited" by their less able counterparts. Such resentment may then turn into a reluctance to participate in future teams.

In this article, I provide a first systematic analysis of relative payoffs in teams. I will, how-

[1] https://www.hrmagazine.co.uk/content/news/half-hate-teamwork.

[2] https://www.prnewswire.com/news-releases/university-of-phoenix-survey-reveals-nearly-seven-in-ten-workers-have-been-part-of-dysfunctional-teams-187090161.html.
For additional evidence, see also https://www.prweb.com/releases/2017/06/prweb14452679.htm.

ever, do so without directly introducing social preferences to identify the source of inequity concerns and bridge the gap with the extant literature on teamwork (reviewed below).[3]

My model of problem-solving teams is a simple variation of Lee and Wilde's (1980) R&D race. Rather than competing, a group of agents is asked to work together for a breakthrough, e.g., a new product design. Time is continuous, and there is no deadline. At each instant without success, agents privately choose their efforts. Agents do not accumulate knowledge, so their optimal efforts remain stationary, but they may learn from each other (research spillover), depending on transparency in the working relationship. The team dissolves at the first successful attempt, upon which each member obtains a fixed and equal reward (pure teamwork). The only heterogeneity among agents is their publicly known abilities, where higher ability means a lower marginal cost of effort.[4]

Under the standard cost assumptions, i.e., increasing and convex, there is a unique (Nash) equilibrium in this model. Although, as expected, higher-ability agents work harder in equilibrium, they do not always fare better. I show that regardless of the rest of the team, any two agents' equilibrium payoffs are in *reverse* order to their abilities when the cost function is (locally) log-concave between their equilibrium efforts and in the same order to their abilities when the cost function is (locally) log-convex. For expositional convenience, I say that an agent views the team project as "easy" if his cost function is log-concave at his equilibrium effort and as "challenging" if it is log-convex. In other words, an agent views the project as easy if a marginal increase in his equilibrium effort would increase his marginal cost more than his total cost, and vice versa for a challenging project.[5] With this language, any two agents' equilibrium payoffs are reverse-ordered to their abilities if both perceive the project easy in-between their efforts and same-ordered if they perceive it challenging. Intuitively, unable to observe others' efforts, higher-ability agents cannot commit to not overworking for an easy project.[6] The same is not true for a challenging project because the steep marginal cost endows higher-ability agents with some commitment to shifting the workload to others in equilibrium.

[3]Gill and Stone (2015) incorporate such preferences into teamwork with homogenous agents.

[4]The exogenous reward can be a bonus promised to team employees, the grade assigned to group students, or the equal credit attributed to collaborating researchers. I establish the robustness of my findings to heterogenous rewards and multiple breakthroughs in Appendix B.

[5]In a sense, the agent's perception of the project reflects whether he views its success as a downhill or an uphill battle at his equilibrium effort. This project classification is different from Ozerturk and Yildirim's (2021), which I discuss in the analysis.

[6]Formally, a higher-ability agent ends up with a greater marginal cost of effort in equilibrium.

Note that under a globally log-concave cost such as the iso-elastic specification often used in the teamwork literature, the above result implies that equilibrium payoffs would be reverse-ordered to abilities in the entire team. Such a payoff reversal under a globally log-concave cost was first observed in a two-agent setting by Bowen et al. (2019), which I further discuss below. While not my main focus here, I add to their result that the payoff reversal also holds under a general cost function if agents expect to exert minimal effort and thus view the project as easy. And this will be the case if agents are sufficiently patient or low ability, or attach a low enough value to success.

However, the team's payoff order is surprisingly rich when the cost function is not globally log-concave, though still increasing and convex, and agents' efforts are significant.[7] In particular, as mentioned above, the payoffs would be same-ordered as abilities if the cost function is (locally) log-convex such that all agents view the project as challenging. Perhaps more interesting is that if the cost function has a *single* log-inflection point (log-concave turning log-convex), there are 2^{n-1} possible strict equilibrium payoff orders for heterogeneous n-member teams, the best-off member being either the most or the least able in each.[8] If the cost function has multiple log-inflections, payoff orders in which medium-ability agents receive the highest payoff are also possible.

Such ambiguity in relative payoffs in teams makes this article's main point: there is a weak relationship, if any, between the team's ability and payoff rankings. Put differently, the cost specification plays an essential role in:

- team incentives of heterogeneous agents (perhaps, more than one would think),
- identifying potential equity and fairness concerns in the team, and
- determining agents who are more eager to team up.[9]

These observations raise two questions: how should agents of heterogeneous abilities be matched into teams to maximize their average payoff? Would inequity be necessarily concerning in such a team?

[7] As shown in the analysis, standard cost assumptions, i.e., increasing and convex, put little structure on its log properties. For instance, straightforward extensions of the iso-elastic cost can readily admit multiple log-inflections because the sum of two log-concave functions need not be log-concave.

[8] More precisely, Proposition 2 establishes that a payoff order can emerge as equilibrium for some ability profile if and only if the best-off member in any subteam is the least or the most able. Note that despite their richness, $n! - 2^{n-1}$ payoff orders are ruled out.

[9] These issues are much less relevant in homogenous-ability teams, especially when the focus is on symmetric equilibrium.

In a second-best world with unobservable efforts, the designer, e.g., an employer or a teacher, would group individuals to equalize their equilibrium marginal costs. Interestingly, such an allocatively efficient matchup would also equalize individual payoffs in my model. Thus, efficiency need not create inequity in carefully composed teams. However, because they are designed to control the free-rider problem, second-best teams must have the "right" size and members with sufficiently diverse abilities. Specifically, the maximum team size is the number of log-inflections plus one. The role of log-inflections in team design further implies that second-best teams will often have members with differing perceptions of the joint project, some on the log-concave and others on the log-convex part of their cost in equilibrium. For instance, with a unique log-inflection cost, the second-best team will have exactly *two* heterogeneous members – one viewing the project as easy and the other as challenging. On the other hand, there is no second-best team with heterogeneous members for a globally log-concave cost.

Finally, I investigate how team members respond to transparency in the workplace, facilitating research spillover. I find that despite the free-riding, transparency creates an endogenous effort complementarity in the team and is desired by members of all abilities. But, much like the payoff order, whether higher- or lower-ability agents are motivated more by transparency is ambiguous, depending now on the log-concavity of *marginal* cost.

Related Literature. This article is closest to those that model teamwork as a dynamic public good provision with complete information. These articles characterize non-stationary team dynamics when agents accumulate effort/contribution toward a pre-specified project size, e.g., Admati and Perry (1991), Marx and Matthews (2000), Compte and Jehiel (2003), Yildirim (2006), Kessing (2007), Georgiadis (2015), Bhattacharya et al. (2017), and Bowen et al. (2019).[10] However, with few exceptions, their models assume ex ante homogenous agents, so payoff ordering is a nonissue. As Bowen et al. nicely put it: "...little is known about this [free-riding] problem when agents are heterogeneous. We begin by studying a simple two-agent model."

Bowen et al.'s two-agent model exhibits effort accumulation, creating intertemporal strategic complementarity. They assume a globally log-concave cost and, though not their primary focus, find in their Propositions 2 and 9 a similar payoff reversal to one here: the low-cost

[10] I briefly consider non-stationary dynamics with multiple breakthroughs in Appendix B. Non-stationary team dynamics also arise in models with incomplete information when agents learn about the project's or teammates' potentials over time, e.g., Bonatti and Horner (2011), Guo and Roesler (2018), and Cetemen et al. (2020). They, too, mostly assume ex-ante homogenous agents.

agent obtains a lower discounted payoff. In comparison, I assume away effort accumulation for tractability and study the payoff ranking and optimal composition in n-agent teams under general cost. I show, among others, that the opposite payoff ranking obtains in the presence of (local) log-convexity, and the exact ranking can be highly non-monotone in ability, depending crucially on the number of log-inflections in the cost function. As such, medium-ability agents can be the best-off in some teams – a case that would not emerge with only two agents. Equally importantly, I further explore efficient team composition and the role of transparency in team incentives.[11]

Employing a similar model to one here, Ozerturk and Yildirim (2021) also uncover a payoff reversal in heterogeneous teams. But the driving force in their case is the *endogenous* reward in the form of credit attribution by the public. For instance, they obtain no payoff reversal under the iso-elastic cost when it is sufficiently convex, e.g., the cubic cost. Here, keeping up with the extant literature, I assume agents receive exogenous rewards from success.

In terms of team design, this article complements those that show the optimality of asymmetric incentive pays for symmetric team members when production technology exhibits complementaries (e.g., Segal, 2003; Winter, 2004; Bose et al., 2010). In the absence of incentive pays, I find that asymmetric team members receive symmetric payoffs in allocatively efficient teams when their efforts are substitutes. In terms of team composition, this article complements Franco et al. (2011), Kaya and Vereshchagina (2014), Bel et al. (2015), Bonatti and Rantakari (2016), and Glover and Kim (2021), among others, in arguing that the free-riding problem can be less severe in heterogeneous teams.

In particular, within a Lee-Wilde type model, Bonatti and Rantakari (2016) explore agents' incentives to develop projects (or solutions) for the organization when they have conflicting preferences on which one to adopt. In their benchmark analysis, which is closest to my teamwork setting, the authors allow two symmetric agents to choose the projects collectively and adopt the first developed. They show that agents would agree to pursue different projects to alleviate the free-rider problem in developing them. In my model, agents are exogenously assigned to the same project, and the free-rider problem is mitigated by having those with sufficiently diverse abilities work on it.

Finally, this article contributes to the recent debate on the value of peer transparency or

[11] The payoffs of heterogeneous agents have also been studied in static models of public good provision. In their seminal paper, Bergstrom et al. (1986) show that when agents differ only in income, they receive the same payoff in the unique interior equilibrium. In particular, higher-income contributors are never worse off than lower-income individuals.

close working relationship in teamwork. Several studies, including Winter (2010), Bose et al. (2010), and Bag and Pepito (2012), have argued that transparency is valuable to an organization only if agents' efforts are complementary. With substitute efforts, transparency exacerbates free-riding and is undesirable. Although efforts are substitutes in my model, transparency is preferred by heterogeneous agents. Transparency encourages agents of all abilities to work harder despite their free-riding incentives by facilitating research spillover and speeding up the project's completion.

The remainder of the article is organized as follows. Section 2 lays out the model. Section 3 establishes the unique equilibrium and shows the weak relationship between the team's ability and payoff rankings. Section 4 explores the optimal team design. Section 5 determines the incentive effect of transparency that facilitates research spillover. Finally, Section 6 concludes. The Appendix contains the proofs omitted from the main text as well as the extensions to heterogeneous rewards and multiple breakthroughs.

2 Model

My formal framework builds on Lee and Wilde's (1980) R&D race. Instead of racing, a group of $n > 1$ risk-neutral agents is asked to work together on a joint project toward a breakthrough.[12] They continuously choose their effort levels over an infinite horizon. Effort is costly and unobservable to others. Let $x_i(t)$ and $c_i(x_i(t))$ represent agent i's nonnegative flow effort and its cost at time t, respectively. For ease of comparative statics, I posit a separable cost function:[13]

$$c_i(x) = \frac{c(x)}{a_i}, \tag{1}$$

where

$$c' > 0, c'' > 0, \text{ and } c''' \geq 0, \text{ with } c(0) = c'(0) = 0 \text{ and } c'(\infty) = \infty. \tag{A1}$$

As usual, the cost of effort is increasing and convex, with the cost of negligible effort being negligible. (A1) will guarantee a unique and interior equilibrium, and is satisfied by many cost functions, including $c(x) = x^k$ for $k \geq 2$, $c(x) = x^k + x^m$ for $k, m \geq 2$, and $c(x) = x^k e^{x^m}$ for $k \geq 2$ and $m \geq 0$. I will call the parameter $a_i > 0$ agent i's "ability" because higher ability implies a uniformly lower marginal cost: $c_i'(x) < c_j'(x)$ for $a_i > a_j$ and all $x > 0$. Agents'

[12]In Appendix B, I show the robustness of my results to joint projects that require two successive breakthroughs, perhaps building on each other, for completion.

[13]The separable cost simplifies the exposition but is not essential for my results. As will be seen below, the model is rich enough even with separability.

abilities are publicly known, perhaps, because of prior interactions and achievements, ruling out any reputational concerns.

Similar to Lee and Wilde, I assume no knowledge accumulation for agents, which allows me to drop the time index and focus on their stationary strategies, x_i. Agents may, however, learn from each other. Following Kamien et al. (1992) on research joint ventures, agent i's rate of discovery is given by

$$y_i = x_i + \beta \sum_{j \neq i} x_j, \tag{2}$$

where $\beta \in [0, 1]$ is the rate of research spillover. Later, I will interpret β as the level of transparency within the team: with probability β, agent i may get inspired by teammates' attempts at discovery.[14,15]

Given the stationary strategies, agent i's random time for the breakthrough is exponential with rate y_i. Consequently, the team's random time for the breakthrough is also exponential with the aggregate rate:

$$\sum_i y_i = \alpha X,$$

where

$$\alpha = 1 + (n-1)\beta \ \text{ and } \ X = \sum_i x_i.$$

Here, X is the total effort, and $\alpha \in [1, n]$ is the aggregate impact of one's effort on the team's discovery rate. The project is completed, and the team dissolves as soon as the breakthrough is made. Upon completion, each agent receives an exogenous reward $v > 0$ and zero, otherwise. Thus, the only source of heterogeneity for agents is the marginal cost of effort, though I also show the robustness of my results to heterogeneous rewards in Appendix B. I will sometimes refer to the team as a partnership in which agents equally share a fixed total prize V, i.e., $v = V/n$.[16] Agents discount the future benefits and costs at a common rate $r > 0$.

[14] The fact that the discovery rate, y_i, is linear in efforts and not directly dependent on abilities is without loss. I could let $y_i = R_i(x_i) + \beta \sum_{j \neq i} R_j(x_j)$ for some strictly increasing and concave function R_i; e.g., $R_i(x_i) = a_i x_i$. Then, by a change of variables: $x_i := R_i^{-1}(x_i)$, any nonlinearity and ability-dependence in the rate would be absorbed by the cost of effort, c_i.

[15] The assumption of no knowledge accumulation is obviously unrealistic. Yet, it streamlines the analysis and seems reasonable for a highly innovative project. The project is known to be feasible, with an uncertain completion time. Nevertheless, Bonatti and Horner (2011) imply that my results would be robust to a small perturbation in the project's feasibility. This is because their Extension A to heterogenous cost agents would reduce to the present setup if the project were almost surely feasible ($\overline{p} \to 1$), and the cost of effort were linear. As mentioned above, the nonlinear cost assumption (A1) ensures interior equilibrium and, in turn, a more meaningful payoff comparison, which is the main focus here.

[16] Gill and Stone (2015) cite strong evidence about equal-sharing rules in team settings.

To determine agent i's expected discounted payoff, note that given the exponential arrival time, the probability of no breakthrough until time t is $e^{-\alpha X t}$. In the next instant dt, agent i incurs his flow cost $c_i(x_i)dt$ and receives his reward v if the team succeeds with probability $\alpha X dt$. If the team fails, the game is reset to $t = 0$. As a result, agent i's expected discounted payoff at any point in time without the breakthrough becomes

$$\begin{aligned} u_i &= \int_0^\infty e^{-rt} e^{-\alpha X t} \left(\alpha X v - c_i(x_i) \right) dt \\ &= \frac{\alpha X}{r + \alpha X} v - \frac{c_i(x_i)}{r + \alpha X}, \end{aligned} \tag{3}$$

where the first term is his expected benefit and the second term is his expected cost.

3 Analysis

I begin my investigation by establishing a unique equilibrium and then show that higher-ability agents work harder in equilibrium but need not be better off. In subsection 3.2, I consider large teams with a sufficiently dispersed ability profile to make the relationship between the ability and payoff rankings precise.

Equilibrium and payoff ordering. In (Nash) equilibrium, agent i chooses his effort that best responds to his teammates'. Namely, given $X_{-i} = X - x_i$, agent i solves[17]

$$\max_{x_i \geq 0} u_i.$$

The first-order condition requires[18]

$$c_i'(x_i) \left(\frac{r}{\alpha} + X \right) - c_i(x_i) = rv. \tag{4}$$

The left-hand side of (4) is a *dynamic* marginal cost. Re-arranging it as

$$\frac{c_i'(x_i)}{\alpha} r + [c_i'(x_i) x_i - c_i(x_i)] + c_i'(x_i) X_{-i},$$

notice that the first term, $\frac{c_i'(x_i)}{\alpha} r$, refers to the marginal cost of increasing effort now rather than in the next instant. The marginal cost is scaled down by the aggregate spillover factor,

[17]Since efforts are unobservable, agents cannot commit to their effort strategies in my model. Commitment, however, would have no value due to exponential (or memoryless) discovery rates induced by no knowledge accumulation; see Reinganum (1982) for a similar observation.

[18]The second-order condition is easily verified: $\left.\frac{\partial^2 u_i}{\partial x_i^2}\right|_{\frac{\partial u_i}{\partial x_i} = 0} = -c_i''(x_i)(\frac{r}{\alpha} + X) < 0$.

α, which amplifies the impact of one's effort The second term, $[c_i'(x_i)x_i - c_i(x_i)]$, is the net increase in the flow cost of exerting effort x_i, and the last term, $c_i'(x_i)X_{-i}$, is the marginal opportunity cost of effort in case teammates make the breakthrough. The agent trades off the dynamic marginal cost against the dynamic marginal benefit rv on the right-hand side of (4), which is the time value of receiving the project prize, v.

It is worth observing from (4) that agent i's dynamic marginal cost is increasing in his effort, x_i, (because $c_i'' > 0$) and in the others', X_{-i}, implying $\partial x_i / \partial X_{-i} < 0$. The downward-sloping reaction function points to the classical free-riding incentive: project completion being a public good, each agent would find it costlier to put in the same effort if he knew his teammates were working harder.

An effort profile $\mathbf{x}^*$ constitutes an equilibrium if it solves (4) for all i. Proposition 1 establishes the equilibrium and some of its intuitive properties.

Proposition 1 *(**existence**) There is a unique and interior equilibrium, i.e., $x_i^* > 0$ for all i. Conversely, given a positive effort profile, $\mathbf{x}$, there is a unique ability profile that engenders $\mathbf{x}$ as the equilibrium. Moreover, in equilibrium,*

- **(a)** *agent i's effort always lies in a well-defined interval $(\underline{x}_i, \overline{x}_i)$,*
- **(b)** *a higher-ability agent exerts greater effort: $a_i > a_j$ implies $x_i^* > x_j^*$, and*
- **(c)** *every agent increases his effort with the rate of research spillover, β.*

The equilibrium is interior because exerting little effort is assumed to cost little, i.e., $c_i(0) = c_i'(0) = 0$. Its uniqueness obtains because, as argued above, agents' efforts are strategic substitutes, i.e., $\partial x_i / \partial X_{-i} < 0$. For the converse of the equilibrium construction, I fix an effort profile $\mathbf{x}$ and simply solve (4) for the ability a_i:

$$a_i = \frac{c'(x_i)\left(\frac{r}{\alpha} + X\right) - c(x_i)}{rv}, \tag{5}$$

which will be useful in Section 4 when determining the optimal team composition.

The rest of Proposition 1 lays out some key properties of the equilibrium. Part (a) offers exogenous bounds for an agent's equilibrium effort. Roughly, the upper bound $\overline{x}_i$ corresponds to agent i's "stand-alone" effort while the lower bound $\underline{x}_i$ corresponds to his best response to the others' stand-alone efforts. Though loose, these bounds may be easier to check for some results than the equilibrium itself. Part (b) confirms our intuition: a higher-ability agent, having a lower marginal cost, works harder for the project. Perhaps the least obvious observation in Proposition 1 is part (c). It says that a higher rate of research spillover, β, or, equivalently, a higher aggregate rate α, motivates every team member regardless of their abilities. Note from (4) that the direct effect of spillover is positive: fixing teammates' efforts, X_{-i}, a higher α would reduce agent i's dynamic marginal cost (the left-hand side of (4)), encouraging him to work harder. Put differently, all else equal, an increase in research spillover would encourage agent i by speeding up the discovery and bringing forward the returns. However, the indirect effect of spillover is negative due to free-riding: expecting others to work harder, agent i would also have an incentive to slack. Part (c) shows that this free-riding effect is partial in that $\partial x_i/\partial X_{-i} \in (-1, 0)$. To understand, suppose, to the contrary, that $\partial x_i/\partial X_{-i} \leq -1$ so that agent i's action crowded out an increase in the others' and (weakly) lowered the team's total. Then, the agent's dynamic marginal cost would fall below the marginal benefit rv in (4), leading him to exert more effort – not less. In a sense, it pays an agent to free ride on others but not to the extent of stalling or slowing down the rate of discovery. In Section 5, I will re-interpret the spillover parameter β as the level of transparency within the team, and further show that all team members prefer more transparency.

Armed with Proposition 1, I now turn my attention to the main question of this article: who fares better in teamwork? It is tempting to answer that a higher-ability agent would never be worse off than his lower-ability teammates because he could easily mimic their lower efforts. However, this answer will generally be incorrect as it ignores the commitment issue with unobservable efforts.

Inserting (4) into (3), note that agent i's expected equilibrium payoff reduces to

$$u_i^* = v - \frac{c_i'(x_i^*)}{\alpha}, \tag{6}$$

which is the difference between the fixed prize, v, he receives upon the project's completion and his marginal cost, discounted by the research spillover factor, α. Hence, comparing agents' equilibrium payoffs amounts to comparing their equilibrium marginal costs, $c_i'(x_i^*)$, in my model. To this end, recall that $c_i(x) = \frac{c(x)}{a_i}$ and factor out (4) as:

$$c_i'(x_i^*) \left[\frac{r}{\alpha} + X^* - \frac{c(x_i^*)}{c'(x_i^*)} \right] = rv. \tag{7}$$

Given that the right-hand side of (7) is fixed, agent i's marginal cost $c_i'(x_i^*)$ and the "hazard rate" term $\frac{c'(x_i^*)}{c(x_i^*)}$ are inversely related.[19] For notational brevity, let

$$h(x) \equiv \frac{c'(x)}{c(x)},$$

denote the hazard rate so that $h'(x) = (\ln c(x))''$. Then, (6) and (7) readily imply that agents' equilibrium payoffs are ordered the same as their hazard rates:

$$sgn\left(u_i^* - u_j^*\right) = sgn\left(h(x_i^*) - h(x_j^*)\right). \tag{8}$$

Because equilibrium efforts are monotone in abilities by Proposition 1, the slope of hazard rate function, $h(x)$, or, equivalently, the log-concavity of the cost function, $c(x)$, will prove crucial for the payoff ordering. Given this, though not needed for the subsequent results, I find the following project classification insightful.

Definition 1 *(**easy vs. challenging project**) In equilibrium, agent i is said to view the project as*

$$\begin{cases} \textit{easy} & \textit{if} \quad h'(x_i^*) < 0 \\ \textit{challenging} & \textit{if} \quad h'(x_i^*) > 0 \\ \textit{neither} & \textit{if} \quad h'(x_i^*) = 0. \end{cases}$$

That is, an agent perceives the project as easy (resp. challenging) if his marginal cost increases slower (resp. faster) than his cost at the equilibrium effort. As such, besides the cost function's shape, an agent's perception of the project is likely to vary with the team's composition. And team members may have heterogeneous perceptions of the project, depending on their anticipation of teammates' effort.[20]

[19] I borrow the term "hazard rate" for the log derivative from the procurement design literature (e.g., Laffont and Tirole, 1993) in which the ratio F'/F, F being the cumulative density of marginal production cost, frequently appears and is referred to as such.

[20] To elaborate on the project classification, consider two agents $i = 1, 2$ such that $u_1^* = u_2^*$. Then, $c_1'(x_1^*) = c_2'(x_2^*)$ by (6) and thus, $h(x_1^*) = h(x_2^*)$ and $c_1(x_1^*) = c_2(x_2^*)$ by (7).

Suppose agent i considers marginally increasing his effort x_i^* (off-equilibrium). Clearly, by (6), his payoff would decrease. To relate this change to his hazard rate, note that (6) can be re-written: $u_i^* = v - \frac{c_i(x_i^*)h(x_i^*)}{\alpha}$. Thus, his payoff would decrease by:

$$\frac{\partial u_i^*}{\partial x_i^*} = \frac{-c_i'(x_i^*)h(x_i^*) - c_i(x_i^*)h'(x_i^*)}{\alpha}.$$

Given the above equalities, if $h'(x_1^*) < 0 < h'(x_2^*)$, then $\left|\frac{\partial u_1^*}{\partial x_1^*}\right| < \left|\frac{\partial u_2^*}{\partial x_2^*}\right|$: agent 1, who views the project as easy, would lose less utility than agent 2, who views the project as challenging, by marginally increasing his equilibrium effort.

Before proceeding, it is worth noting that the project classification here is different from Ozerturk and Yildirim's (2021). Because, in their model, each agent's reward depends on the market's equilibrium belief about his contribution to the project's success, these authors call a project "easy" if marginal cost is concave and "difficult" if sufficiently convex. For instance, a cubic cost of effort would correspond to a difficult project in Ozerturk and Yildirim but an easy project here. The main reason is that I assume exogenous rewards in this investigation, in line with the extant literature.

The next result, which is immediate from (8), shows a reversal in the equilibrium payoff order based on agents' perceptions of the project.

Proposition 2 *(**payoff reversal**) In equilibrium, a higher-ability agent fares worse [resp. better] than a lower-ability if both view the project as easy [resp. challenging] in-between their efforts. Formally,* $a_i > a_j$ *implies*

$$\begin{cases} u_i^* < u_j^* \quad \textit{if} \quad h'(x) < 0 \textit{ or } c(x) \textit{ is log-concave on } [x_j^*, x_i^*] \subset (\underline{x}_j, \overline{x}_i), \\ u_i^* > u_j^* \quad \textit{if} \quad h'(x) > 0 \textit{ or } c(x) \textit{ is log-convex on } [x_j^*, x_i^*] \subset (\underline{x}_j, \overline{x}_i). \end{cases}$$

Proof. By Proposition 1(b), $a_i > a_j$ implies $x_i^* > x_j^*$. Futhermore, $\underline{x}_j < x_j^*$ and $x_i^* < \overline{x}_i$ by Proposition 1(a). Thus, $[x_j^*, x_i^*] \subset (\underline{x}_j, \overline{x}_i)$. The payoff comparison is immediate from (8) and the fact that $h'(x) = (\ln c(x))''$. ■

The intuition behind Proposition 2 is that with unobservable efforts, the higher-ability agent cannot commit to not overworking for an easy project to the point of being worse off. Formally, by (6), the higher-ability agent i ends up with a higher marginal cost: $c_i'(x_i^*) > c_j'(x_j^*)$. Thus, if side payments were feasible, the two agents would be better off by marginally shifting the workload between themselves. The payoff reversal does not occur when the two agents perceive the project as challenging in-between their equilibrium efforts: the steep marginal cost of effort endows the higher-ability agent with some commitment not to overwork. In fact, in this case, it is the lower-ability agent who overworks.

Proposition 2 is, however, not a full characterization: it cannot rank two equilibrium payoffs unless the agents' views of the project's difficulty match in-between their efforts; i.e., the cost of effort is locally log-concave or log-convex. As we will see below, this feature will lead to a surprisingly weak relationship between agents' abilities and payoffs. Nevertheless, many familiar cost functions that satisfy the assumption (A1) are globally log-concave. One prominent example is the iso-elastic cost $c(x) = x^k$, $k \geq 2$, often used in the teamwork literature.

Thus, from Proposition 2, the entire team's equilibrium payoffs are in reverse order to abilities under a globally log-concave cost. As discussed in the introduction, this finding is consistent with Bowen et al.'s (2019) two-agent setting.

In general, (A1) does not impose much structure on the log properties of a cost function, but the following lemma indicates it must be log-concave for sufficiently low effort levels.

Lemma 1 *$c(x)$ is log-concave on $(0, \underline{x})$ for some $\underline{x} > 0$.*

Proof. Because $c(0) = 0$ and $c'' > 0$, we have $\frac{c(x)}{x} < c'(x)$, or equivalently, $\frac{1}{x} < \frac{c'(x)}{c(x)}$ for all $x > 0$. Hence, $\lim_{x \to 0} \frac{c'(x)}{c(x)} = \infty$, which implies that $\frac{c'(x)}{c(x)}$ must be decreasing, or equivalently, c must be log-concave in some non-empty interval $(0, \underline{x})$. ■

Lemma 1 says that irrespective of his cost function, an agent who expects to exert minimal effort and thus bear a small cost will perceive the project as easy. And, in equilibrium, all agents will expend minimal effort if they are sufficiently patient so that they can afford to postpone the discovery by choosing little effort each time or if they have sufficiently low abilities or low project value. I collect these observations in:

Corollary 1 *Let $a_1 > a_2 > ... > a_n$. Then, $u_1^* < u_2^* < ... < u_n^*$ if*

(a) (Bowen et al., 2009) *the cost of effort is globally log-concave, e.g., $c(x) = x^k$ for $k \geq 2$,*

(b) *the discount rate, r, agents' abilities, or project value, v, are sufficiently low, or*

(c) *the team is a partnership of a sufficiently large size, i.e., $v = V/n$ and n is large.*

Lemma 1 also says that the cost of effort satisfying (A1) cannot be globally log-convex.[21] Thus, I next consider a family of cost functions with a unique log-inflection point:

$$\begin{cases} c \text{ is log-concave} & \text{if} \quad x < x_c \\ c \text{ is log-convex} & \text{if} \quad x > x_c \end{cases} \tag{9}$$

for some $0 < x_c < \infty$.

(9) describes a U-shaped hazard rate, $h = \frac{c'}{c}$, reaching its minimum at x_c. Because, by definition, $h'(x_c) = 0$, an agent who chooses the critical effort x_c would view the project

[21] Evident from the proof of Lemma 1, a globally log-convex cost would require a fixed cost of effort, i.e., $c(0) > 0$, under which an interior equilibrium would no longer be guaranteed for all parameter values. Extending the model in this direction would unduly complicate the analysis without adding new insights.

neither easy nor challenging. One example of (9) is a straightforward extension of the iso-elastic cost:[22] $c(x) = x^k e^{x^m}$ for $k \geq 2$ and $m > 1$, where $x_c = \left(\frac{k}{m(m-1)}\right)^{1/m}$. Figure 1 illustrates the case for $k = m = 2$.

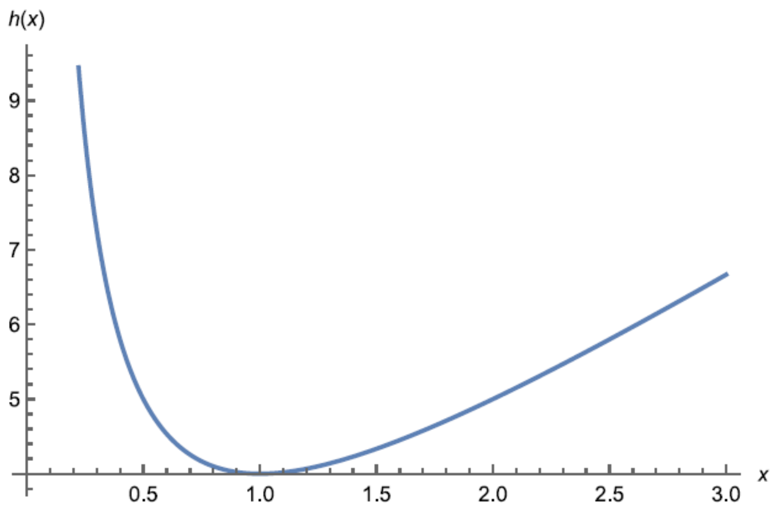

Figure 1. Unique Log-Inflection: $c(x) = x^2 e^{x^2}$ and $h(x) = \dfrac{2}{x} + 2x$

Before presenting Proposition 3, it is worth noting from (4) that a proportional increase in project value v is strategically equivalent to a proportional increase in an agent's ability because agent i cares about their product va_i in choosing his effort. For ease of reference, I, therefore, define an agent's "value-adjusted" ability:

$$A_i = va_i.$$

Proposition 3 *(**unique log-inflection**) Let $a_1 > a_2 > ... > a_n$ and consider the cost specification in (9). Then,*

(a) *there are value-adjusted abilities $0 < A_L < A_H < \infty$ such that*

$$\begin{cases} u_1^* < u_2^* < ... < u_n^* & \text{if} \quad va_1 < A_L \\ u_1^* > u_2^* > ... > u_n^* & \text{if} \quad va_n > A_H. \end{cases}$$

(b) *In any team, the highest equilibrium payoff belongs to its least or most able member.*

[22]Other examples with a unique log inflection point are: $c(x) = e^{x^k} - 1$ for $k > 1$, and $c(x) = \frac{e^{x ArcTan(x)}}{\sqrt{1+x^2}} - 1$.

(c) *A payoff order arises as equilibrium for some ability profile if and only if the highest payoff belongs to the least or most able member in any subset of the team.*

Part (a) follows because, as in Corollary 1, agents with sufficiently low abilities or project value all exert low efforts, $x_1^* < x_c$, and view the project as easy in equilibrium. By the same logic, team members with sufficiently high abilities or project value all put in the high effort, $x_n^* > x_c$, and, operating on the log-convex part of the cost, view the project as challenging. The payoff ordering is then obtained from Proposition 1.

Part (a), however, is mute when team members have more dispersed abilities or moderate project value. Indeed, part (b) indicates that without knowing the team's ability profile and project value, the most one can conclude under the cost structure (9) is that *the best-off member is of the lowest or the highest ability*. Part (c) refines this conclusion by showing the feasible equilibrium payoff orders. Specifically, a payoff order with the highest payoff belonging to its least or most able member in any subteam can emerge as equilibrium for some ability profile. As such, for an n-agent team with heterogeneous abilities, there are 2^{n-1} possible *strict* payoff orders in equilibrium.[23] For instance, for a three-agent team, all payoff orders but those with the medium-ability faring the best are possible in equilibrium. I numerically illustrate this point in the next example.[24]

Example 1. (payoff orders) *Consider a three-member team with* $(a_1, a_2, a_3) = (30, 20, 10)$ *and* $r = \alpha = 1$.

Project value	Payoff order
$0 < v < 1.57$	$u_1^* < u_2^* < u_3^*$
$1.57 < v < 2.51$	$u_2^* < u_1^* < u_3^*$
$2.51 < v < 3.09$	$u_2^* < u_3^* < u_1^*$
$3.09 < v$	$u_3^* < u_2^* < u_1^*$

Table 1. $c(x) = x^2 e^{x^2}$

Nevertheless, part (c) does rule out $n! - 2^{n-1}$ payoff orders beyond identifying the best-off agent. For instance, there is no four-member team with $a_1 > a_2 > a_3 > a_4$ such that in equilibrium, $u_4^* > u_2^* > u_1^* > u_3^*$ or $u_4^* > u_2^* > u_3^* > u_1^*$. The reason is that despite being the medium ability, agent 2 would claim the highest payoff in the subteam $\{1, 2, 3\}$, contradicting

[23] Two heterogeneous agents can obtain the same equilibrium payoff, which would further increase the number of possible payoff orders.

[24] In the example, I vary the project value, but alternatively, one can fix the project value and, using (5), construct an ability profile for each payoff order.

part (c). However, the following payoff rankings are possible: $u_4^* > u_1^* > u_2^* > u_3^*$ or $u_4^* > u_3^* > u_2^* > u_1^*$.

Proposition 3 raises an obvious question: can a medium-ability member ever be the best-off in a team? The answer is yes, but in light of Proposition 3, only if the cost of effort has at least two log-inflections. To this end, I next posit such a family of cost functions:

$$\begin{cases} c \textit{ is log-concave} & \textit{if} \quad x < x_{c_1} \\ c \textit{ is log-convex} & \textit{if} \quad x_{c_1} < x < x_{c_2} \\ c \textit{ is log-concave} & \textit{if} \quad x > x_{c_2}, \end{cases} \tag{10}$$

where $0 < x_{c_1} < x_{c_2} < \infty$.

An example for (10) is, again, a trivial extension of the iso-elastic cost:[25]

$$c(x) = x^k + x^m \text{ for } k \geq 2 \text{ and } \sqrt{m} > 1 + \sqrt{k}.$$

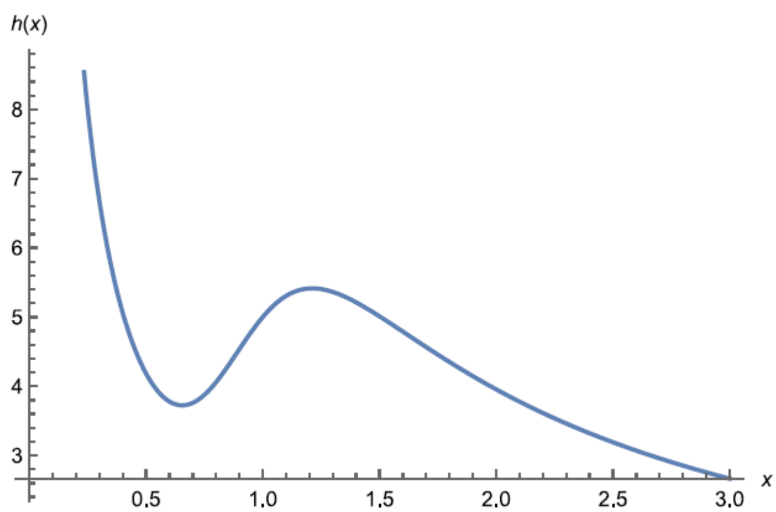

Figure 2. Two Log-Inflections: $c(x) = x^2 + x^8$ and $h(x) = \dfrac{2 + 8x^6}{x + x^7}$

Proposition 4 ***(two log-inflections)*** *Let $a_1 > a_2 > ... > a_n$ and consider the cost specification in (10). Then, there exist teams in which the highest equilibrium payoff belongs to a medium-ability*

[25] It is readily verified that $x_{c_i} = \left(\frac{(m-k)(m-k-1)-2k \pm (m-k)\sqrt{(m-k-1)^2-4k}}{2m} \right)^{\frac{1}{m-k}}$. The reader will recall that the sum of two log-concave functions, here x^k and x^m, need not be log-concave.

agent, i.e. $i \neq 1, n$. Moreover, there exist value-adjusted abilities $0 < A_L < A_{M_1}, A_{M_2} < A_H < \infty$ such that

$$\begin{cases} u_1^* < u_2^* < ... < u_n^* & \text{if} \quad va_1 < A_L, \\ u_1^* > u_2^* > ... > u_n^* & \text{if} \quad A_{M_1} \leq va_i \leq A_{M_2} \text{ for all } i, \\ u_1^* < u_2^* < ... < u_n^* & \text{if} \quad va_n > A_H. \end{cases}$$

A team in which a medium-ability agent receives the highest equilibrium payoff is easily constructed by an effort profile around the second log-inflection point, x_{c_2}. Consider, for instance, a three-member team with the effort profile: $x_1 = 2x_{c_2}$, $x_2 = x_{c_2}$, and $x_3 = x_{c_1}$. From (5), this effort profile is the unique equilibrium for some ability profile $a_1 > a_2 > a_3$. Because the hazard rate, $h(x)$, reaches its maximum at x_{c_2} for $x \geq x_{c_1}$, agent 2 would obtain the highest payoff in this team.

The second part of Proposition 4 indicates two equilibrium payoff reversals with two log-inflections: unlike in Proposition 3, agents' payoffs are also in reverse order to their abilities when they are of sufficiently high ability. Again, this is because unlike (9), the cost under (10) is log-concave for high enough effort levels. Interestingly, both very low and very high ability agents view the project as easy: the former expect to exert relatively low effort, whereas the latter expect to incur a relatively low cost. With two log-inflections, it is also possible that two agents who view the project as easy may have no payoff reversal if, as indicated in Proposition 2, the cost function is not log-concave between their efforts.[26] The following example demonstrates the additional richness of payoff orders with two log-inflections.

Example 1. (cont'd) *Consider a three-member team with $(a_1, a_2, a_3) = (30, 20, 10)$, and $r =$*

[26]For instance, inspecting Figure 2, a two-member team with equilibrium effort levels $x_1^* = 1.5$ and $x_2^* = 0.5$ for some abilities $a_1 > a_2$ would have $u_1^* > u_2^*$ since, clearly, $h(x_1^*) > h(x_2^*)$. No payoff reversal occurs even though both agents view the project easy: $h'(x_1^*) < 0$ and $h'(x_2^*) < 0$. The reason for no payoff reversal is that $h'(x) \nless 0$ for some $x \in [x_2^*, x_1^*]$, so Proposition 2 does not apply.

$\alpha = 1$.

Project value	*Payoff order*
$0 < v < 0.17$	$u_1^* < u_2^* < u_3^*$
$0.17 < v < 0.31$	$u_2^* < u_1^* < u_3^*$
$0.31 < v < 0.37$	$u_2^* < u_3^* < u_1^*$
$0.37 < v < 5.83$	$u_3^* < u_2^* < u_1^*$
$5.83 < v < 8.81$	$u_3^* < u_1^* < u_2^*$
$8.81 < v < 10.90$	$u_1^* < u_3^* < u_2^*$
$10.90 < v$	$u_1^* < u_2^* < u_3^*$

Table 2. $c(x) = x^2 + x^8$

Large teams. It is evident from the analysis so far that agents' equilibrium payoff order closely tracks log-concavity of the cost function. I now demonstrate that this relationship becomes exact in a large team approximated by a continuum of agents.[27] Consider such a team and index each agent by his effort $x \in [x_L, x_H]$, where $0 < x_L < x_H < \infty$ are fixed bounds. Then, the team's total effort is:

$$X = \int_{x_L}^{x_H} x dx = \frac{x_H^2 - x_L^2}{2}.$$

By (5), this effort profile is the unique equilibrium for a team with the ability profile:

$$a(x) = \frac{c'(x)\left(\frac{r}{\alpha} + X\right) - c(x)}{rv},$$

where $a'(x) > 0$ because $c''' \geq 0$, i.e., a higher-ability agent exerts greater effort, as before.

Suppose such a dense ability profile, $a(x) \in [a(x_L), a(x_H)]$, exists in the population. Using (6) and (7), agent x's equilibrium payoff is found to be

$$u(x) = v - \frac{\frac{r}{\alpha}v}{\frac{r}{\alpha} + X - \frac{1}{h(x)}}, \tag{11}$$

which implies

$$sgn[u'(x)] = sgn[h'(x)]. \tag{12}$$

In words, in a large team with a sufficiently heterogeneous ability profile, i.e., a team member with ability $a(x)$ exists for every effort level x, equilibrium payoffs are reverse ordered with

[27] Strictly speaking, being measure zero, no agent would exert effort in a continuum team. The continuum team, however, is the limit of a large but finite team. To see this, simply take two agents with effort levels x and $x + \varepsilon$, and note that $u(x + \varepsilon) - u(x) \approx u'(x)\varepsilon$ in a sufficiently large team with $\varepsilon \approx 0$.

ability if the cost of effort is log-concave, and positively ordered if the cost is log-convex. As such, local minima and local maxima for payoffs are attained at exactly log-inflection points of the cost function. Figure 3 illustrates this relationship with the cost specification in Figure 2.

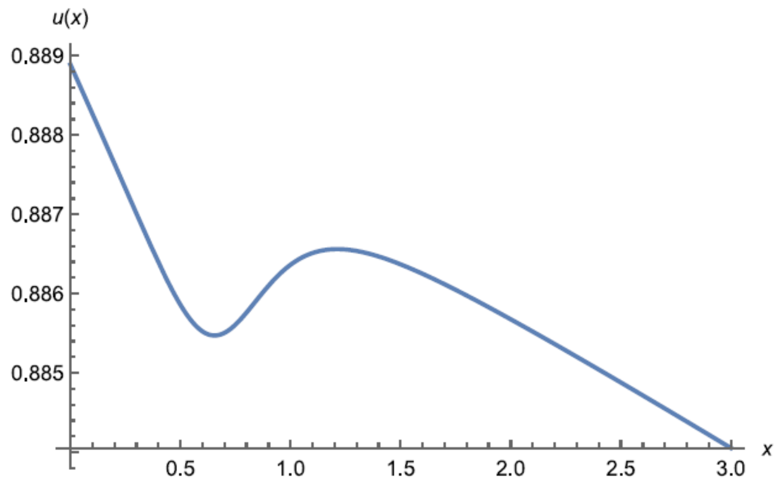

Figure 3. Payoff Non-monotonicity

$c(x) = x^2 + x^8$, $v = r = \alpha = 1$, $x_L = 0$, and $x_H = 3$

4 Second-best teams

Up to now, my analysis has demonstrated that despite working harder, higher-ability team members can easily be worse off than their lower-ability counterparts in the unique equilibrium with standard (increasing and convex) cost functions. Accordingly, agents who worry about such inequitable workload will be reluctant to participate in teamwork.

This observation begs the following question: can teams be designed to have both allocatively efficient and equitable workloads? The answer is yes if the designer, e.g., an employer or a teacher, has access to a sufficiently rich pool of agents, and the cost function has at least one log-inflection.

Note that unable to observe efforts, the designer cannot dictate a workload allocation as would be in a first-best world. But if they are available in the pool, she can match agents i and j into a second-best team where their equilibrium efforts are allocatively efficient: $c_i'(x_i^*) = c_j'(x_j^*)$. Interestingly, such efficiency also ensures equal payoffs by (6) and, in turn, equal costs, $c_i(x_i^*) = c_j(x_j^*)$, by (3). Consequently, the agents in a second-best team must have equal hazard rates – a condition that turns out to be sufficient, too, as formalized in Lemma 2.

Lemma 2 *A set, S, of agents forms a second-best team if and only if they have the same equilibrium hazard rate, i.e., $h(x_i^*) = h_0$ for all $i \in S$ and some $h_0 > 0$.*

Proof. The necessity is immediate from the preceding argument. The sufficiency follows from (11): two agents with equal hazard rates in equilibrium must receive the same payoff and, in turn, have the same marginal cost by (6). ■

Lemma 2 tells us exactly how to compose second-best teams, which I summarize in the following steps.

- Fix $h_0 > 0$ and solve $h(x_i) = h_0$ for x_i.
- Pick a subset, S, of these solutions and find their sum: $X_S = \sum_{i \in S} x_i$.
- Using (5), let agent i expected to exert effort x_i in S have the ability:

$$a_{i,S} = c'(x_i) \left(\frac{\frac{r}{\alpha} + X_S - \frac{1}{h_0}}{rv} \right). \tag{13}$$

Notice that the second-best effort levels do not depend on the model parameters r, α, and v, but the ability levels that engender them as the unique equilibrium do. Notice also that the set S need not correspond to all the second-best efforts that solve $h(x_i) = h_0$. As such, there can be multiple small second-best teams matching different effort levels with the hazard rate h_0. If it contains all the effort levels at h_0, I will call that team *full-size*. Proposition 5 reports some general properties of second-best teams, followed by a numerical example of team design.

Proposition 5 *(**second-best teams**) An equilibrium effort profile is allocatively efficient if and only if all team members obtain the same payoff. In addition,*

(a) *if the cost of effort has $I \in \{0, 1, 2, ...\}$ log-inflections, then a second-best team can have at most $I + 1$ heterogeneous members, and the members of a full-size team cannot uniformly view the project as easy or challenging.*

(b) *In a second-best team, the members' ability levels and their ability gaps decrease with the rate of research spillover, discount rate, and project value. Formally, both $a_{i,S}$ and $|a_{i,S} - a_{j,S}|$ decrease with α, r, and v.*

(c) *If a new agent k is added to a second-best team S, then the ability levels and the ability gaps in S must increase for the expanded team to remain second best. Formally,*

$$a_{i,S} < a_{i,S\cup\{k\}} \text{ and } |a_{i,S} - a_{j,S}| < |a_{i,S\cup\{k\}} - a_{j,S\cup\{k\}}| \text{ for } i,j \in S.$$

Thus, a carefully designed team with standard preferences can be both efficient and equitable. The equal treatment of heterogeneous team members in my model nicely complements Winter (2004) and Bose et al. (2010). These authors find that the least costly way of motivating ex ante homogenous team members with complementary efforts is to offer them heterogeneous payoffs for success. This is done to solve the coordination problem in the team. In my setup, efforts are substitutes, and ex ante heterogeneous agents are treated equally in an optimal team.

Part (a) of Proposition 5 is a direct implication of Lemma 2. Because two agents on a second-best team must have the same hazard rate in equilibrium, i.e., $h(x_i^*) = h(x_j^*)$, there must be at least one log-inflection between their efforts (because $h'(x) = 0$ for some $x \in (x_i^*, x_j^*)$ by the mean-value theorem). Thus, the number of heterogeneous agents in a second-best team is bounded by the number of log-inflections. The role of log-inflections in the team design further implies that all members of a full-size team cannot view the project as easy or challenging. If, for instance, they all regarded the project as easy, the highest ability member would be working disproportionately hard to the point of being the worst off. Indeed, the designer cannot compose a heterogeneous second-best team under the iso-elastic cost $c(x) = x^k$, $k \geq 2$, which is globally log-concave and thus has no log-inflection point.

Parts (b) follows from (13). Evidently, the ability level of an agent who is expected to exert the second-best effort x_i decreases with the rate of research spillover, α, discount rate, r, (or the discovery's "urgency"), and the project value, v. The reason is that an increase in each parameter would motivate the agent, reducing the need for him to be high ability to achieve the effort x_i. Perhaps more interestingly, the ability gap in the second-best team also decreases with these parameters. Although more diverse agents, being cognizant of their cost differences, are less prone to free-riding, this incentive is less pronounced when agents are motivated. The intuition behind part (c) is similar: raising the team's total effort (X_S in (13)), a new member would exacerbate the free-riding incentive among the original members. To counter, the designer would replace them with more able and more diverse agents for the expanded team to stay second best. I illustrate the construction of second-best teams next.

Example 2. (team design) *Suppose $c(x) = x^2 e^{x^2}$, with hazard rate $h(x) = \frac{2}{x} + 2x$. Refer to*

Figure 4. For $h_0 = 6$ and 5, the agents $S_A = \{A_1, A_2\}$ and $S_B = \{B_1, B_2\}$ form second-best teams, with respective effort pairs: $(0.38, 2.62)$ and $(.5, 2)$. For $r = \alpha = v = 1$, (13) implies the following ability profiles in these teams: $(3.85, 151267)$ for S_A, and $(5.30, 3603.48)$ for S_B. Furthermore, (3) implies that each agent would receive the payoff $u_A = 0.74$ in S_A, and $u_B = 0.70$ in S_B. Finally, it is immediate from Figure 4 that both teams are full size, and that whereas agents A_1 and B_1 view the project easy, i.e., $h' < 0$, their teammates view it challenging, i.e., $h' > 0$.

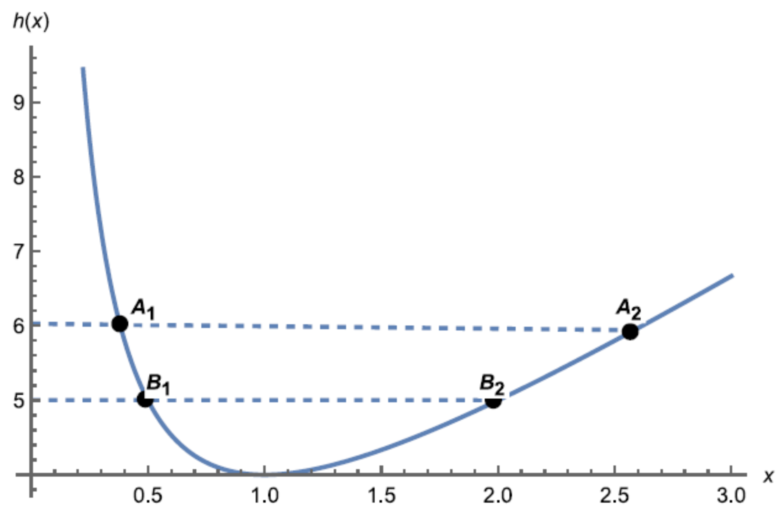

Figure 4. Second-Best Teams

5 Transparency and research spillover

In the base model, perhaps because of the project's highly innovative nature, agents are assumed not to accumulate knowledge, but they may learn from each other. Following the literature on research joint ventures (e.g., Kamien et al. 1992), I have captured the rate of such research spillover in (2) by the parameter $\beta \in [0, 1]$. As discussed in the model setup, this parameter can be interpreted as the level of transparency within the team: with probability β, agent i may get inspired by teammates' attempts at discovery. The amount of transparency is likely to depend on the close working environment that the team or its organization chooses at the outset. In Proposition 1, I have shown that transparency motivates *all* team members despite the free-rider problem, i.e., $\frac{\partial x_i^*}{\partial \beta} > 0$ for all i. In Proposition 6, I explore how transparency affects their payoffs and marginal incentives.

Proposition 6 ***(preference for transparency)*** *In equilibrium, every agent prefers more transparency:* $\frac{\partial u_i^*}{\partial \beta} > 0$ *for all* i*. Moreover,* $sgn\left(\frac{\partial x_i^*}{\partial \beta} - \frac{\partial x_j^*}{\partial \beta}\right) = sgn\left(\overline{h}(x_j^*) - \overline{h}(x_i^*)\right)$*, where* $\overline{h} = \frac{c''}{c'}$ *is the "hazard rate" of marginal cost so that* $\overline{h}' = (\ln c')''$*.*

Recall from part (c) of Proposition 1 that increased transparency encourages agents to exert more effort by speeding up the discovery and bringing forward the returns. Although the free-riding incentive partially mitigates such encouragement, the overall effect of transparency on individual effort is positive. In other words, transparency creates endogenous complementarity in agents' efforts. Proposition 6 reveals that such complementarity outweighs the additional cost of effort, leading each agent to opt for more transparency, regardless of their abilities. One implication of this result is that if they could costlessly choose it at the outset, agents would agree on the full transparency, $\beta = 1$; in turn, by (2), each agent would be *equally likely* to make the breakthrough, i.e., $y_i^* = nX^*$ for all i, regardless of his ability.

Proposition 6 further reveals that which agent is marginally more motivated by transparency depends on the log-concavity of *marginal* cost. In particular, regardless of the rest of the team, the higher ability of two agents is motivated more [resp. less] if the marginal cost is log-concave [resp. log-convex] in-between their equilibrium efforts. It can be verified that marginal cost has as many log-inflections as the cost itself.[28] Thus, team members' responsiveness to transparency can be as heterogeneous as their payoff orders. For instance, marginal cost is globally log-concave for the iso-elastic cost, which means higher-ability team members would be more responsive to transparency than the lower-ability under this cost specification. On the other hand, if marginal cost, and thus the cost itself, has a unique log-inflection, then the most we can say for an arbitrary team is that the most responsive team member is of the lowest or the highest ability.

As discussed in the introduction, some recent studies, including Bose et al. (2010), Winter (2010), and Bag and Pepito (2012), have found that peer (or effort) transparency is valuable to an organization only if agents' efforts are complementary. With substitutes, transparency worsens free-riding and is undesirable. Although efforts are also substitutes in my model, transparency is preferred by all agents, as explained above.

[28]This follows from the fact that $\left(\frac{c'}{c}\right)' = \left(\frac{c''}{c'} - \frac{c'}{c}\right)\frac{c'}{c}$, which implies $sgn(h'(x)) = sgn\left(\overline{h}(x) - h(x)\right)$. Hence, hazard rates of cost and marginal cost have as many crossings as the log inflection points of the cost, i.e., the points at which $h'(x) = 0$.

6 Discussion and concluding remarks

Motivated by the evidence of significant dissatisfaction with teamwork in the workplace and classroom, this article has explored the relationship between the ability and relative well-being of team members. The relationship is surprisingly weak: the free-rider problem can be so severe that higher-ability agents fare worse than their lower-ability teammates (Propositions 2-4). I have shown that the extent of free-riding depends crucially on the team's ability profile and the log-concavity of the effort cost, which the (standard) convexity assumption imposes little structure. On the other hand, I have also shown that carefully matching agents into teams can ensure an efficient and equitable workload allocation, provided the team has the "right" size and ability diversity (Proposition 5). Consequently, I view a misalignment in either dimension as a major source of distaste for collaboration among employees and students alike.

Indeed, based on recent teamwork statistics, human resource professionals conclude that "[T]he most successful workplace teams ideally consist of between 4 and 9 members. Employers have a tendency of adding more members to an existing team to accelerate the work process. And when this backfires, which it often does, they are not only faced with reduced employee morale but also failure."[29] Similarly, in his review of teamwork efficacy in education, Thom (2020, p. 263) writes: " Group structure, including the number of individuals in each group and how they are selected, is perhaps the most crucial factor in performance. With regard to size, studies show that smaller groups – generally four or fewer members – are more effective than larger groups... Among other problems, larger groups tend to experience higher levels of free-riding, interpersonal conflict, and general dissatisfaction." These best practices in business and education resonate well with the message of Proposition 5. For instance, adding a new member to a carefully selected team will require re-grouping abilities for the entire team. Otherwise, per Propositions 2-4, some members are very likely to view the new workload allocation as inequitable, a potential source of "reduced employee morale" and "general dissatisfaction."

The teamwork statistics furthermore reveal the importance of effective communication among team members. They point to "a staggering 85% of employees saying they feel happier at work because they have access to collaborative management tools." Therefore, it is not sur-

[29]Refer to https://teamstage.io/teamwork-statistics and https://www.zippia.com/advice/workplace-collaboration-statistics.

prising that "[T]he collaboration tools and software market value has increased by more than $5 billion between 2015 and 2019," a 43.5 percent growth even in this pre-pandemic period. These figures support my findings on the positive role of research spillover and the desirability of transparency by *all* team members despite the free-riding issue. I conjecture one reason why employees prefer collaborative tools is that they can better see their co-workers' progress, which may inspire their own. Interestingly, according to Proposition 5, a disagreement on the project's difficulty need not mean ineffective communication among team members. It may be natural even in well-functioning teams.

While providing new insights into the extent of free-riding in heterogeneous teams, this article has only scratched the surface. Future research may fruitfully adopt the model and study team incentives for skill acquisition and transfer. Specifically, if agents could invest in their own and teammates' abilities before working on the project, how would they spend their resources, e.g., time? Future research may also study optimal project design: should group assignments be given in piecemeal or as a whole, where the reward v is spread across pieces?

My analysis of heterogeneous teams may, however, have implications beyond teamwork. For instance, Doraszelski (2008) examines a patent race a la Lee and Wilde (1980) between two homogenous firms where ex-post imitation is feasible. He observes that although perfect patent protection would lead to overinvestment by engendering a winner-take-all competition, no patent protection would lead to underinvestment by turning the race into teamwork, as in my base model. Therefore, he concludes that "the misallocation of resources in the non-cooperative game can be reduced by reducing the asymmetry in the rewards to winning and losing the R&D race. One way to accomplish this is to partially insure the participating firms against losing the R&D race, e.g., by making patent protection less than perfect. Another way is to "throw money" at all participating firms." My results reveal that when firms are heterogeneous in their R&D abilities, the shape of the cost function will also play a key role in the recommended policies. For example, if, as in Doraszelski's model, the cost function is iso-elastic and thus globally log-concave, the more efficient firm will likely require a more generous compensation than the less efficient. And the reverse is likely to hold if the cost is log-convex on the region of equilibrium investments.

Appendix A: omitted proofs

Proof of Proposition 1. As I demonstrate in Appendix B how my results extend to heterogeneous rewards, I offer a more general proof of equilibrium uniqueness here. To this end, let $v_i > 0$ be agent i's exogenous reward from the team's success.

Define

$$\Phi(x, X) = c'(x)\left(\frac{r}{\alpha} + X\right) - c(x) \tag{A-1}$$

so that the first-order condition (4) reads

$$\Phi(x_i, X) = rv_i a_i. \tag{A-2}$$

Φ has the following properties:

$$\Phi(0, X) = 0$$

(because $c'(0) = c(0) = 0$),

$$\Phi(\infty, X) = \infty$$

(because $c'' > 0$, $c'(\infty) = \infty$, and $\Phi(x, X) \geq c'(x)\frac{r}{\alpha}$), and

$$\Phi_X = c'(x) > 0 \text{ and } \Phi_x = c''(x)\left(\frac{r}{\alpha} + X\right) - c'(x) > 0 \text{ for } x > 0$$

(because $c''(x) > 0$, and $c''(x)x - c'(x) \geq 0$ by $c'''(x) \geq 0$).

Then, by the properties of Φ, (A-2) has a unique solution:

$$x_i = f_i(X). \tag{A-3}$$

Summing both sides of (A-3) for all i, the equilibrium X^* must solve the following equation:

$$g(X) \equiv \sum_i f_i(X) - X = 0. \tag{A-4}$$

Note that

$$f_i'(X) = -\frac{\Phi_X}{\Phi_x} < 0 \quad \text{and} \quad \lim_{X\to\infty} f_i(X) = 0, \tag{A-5}$$

where the limit follows because the left-hand side of (A-2) would diverge if $\lim_{X\to\infty} f_i(X) > 0$.

Hence,

$$g'(X) < 0 \text{ and } \lim_{X\to\infty} g(X) = -\infty. \tag{A-6}$$

The equilibrium is established if $g(X) > 0$ for some $X \geq 0$. To this end, I consider two cases:

Case 1. $c'(\frac{r}{\alpha})\frac{r}{\alpha} - c(\frac{r}{\alpha}) > r\max_i\{v_i a_i\}$.

Then, $f_i(0) > 0$ for all i by (A-2), and, in turn, $g(0) = \sum_i f_i(0) > 0$.

Case 2. $c'(\frac{r}{\alpha})\frac{r}{\alpha} - c(\frac{r}{\alpha}) \leq r\max_i\{v_i a_i\}$.

Let $i_{\max} = \arg\max_i\{v_i a_i\}$. Note that $(c'(x)x - c(x))' = c''(x)x > 0$ for $x > 0$. Thus, there is some $\widehat{X} \geq 0$ such that

$$c'(\frac{r}{\alpha} + \widehat{X})\left(\frac{r}{\alpha} + \widehat{X}\right) - c(\frac{r}{\alpha} + \widehat{X}) = rv_{i_{\max}}a_{i_{\max}},$$

which implies $f_{i_{\max}}(\widehat{X}) = \frac{r}{\alpha} + \widehat{X}$.

Next, consider $i \neq i_{\max}$. Then, there is some $\widehat{x}_i = f_i(\widehat{X}) > 0$ such that

$$c'(\widehat{x}_i)\left(\frac{r}{\alpha} + \widehat{X}\right) - c(\widehat{x}_i) = rv_i a_i.$$

From here, it follows that

$$\begin{aligned} g(\widehat{X}) &= \sum_i f_i(\widehat{X}) - \widehat{X} \\ &= \left(\frac{r}{\alpha} + \widehat{X}\right) + \sum_{i \neq 1} f_i(\widehat{X}) - \widehat{X} \\ &= \frac{r}{\alpha} + \sum_{i \neq 1} f_i(\widehat{X}) \\ &> 0. \end{aligned}$$

Given (A-6) and the two cases, there is a unique $X^* > 0$ that solves (A-4). Therefore, from (A-3) and the fact that $f_i' < 0$, there is a unique and interior equilibrium: $x_i^* = f_i(X^*) > 0$ for all i.

Conversely, let $x_1 \geq x_2 \geq ... \geq x_n \geq 0$ be an arbitrary effort profile. By (A-2), define $v_i a_i = \frac{\Phi(x_i, X)}{r} > 0$. Clearly, $v_i a_i \geq v_j a_j$ for $x_i \geq x_j$ (with a strict inequality for $x_i \neq x_j$) because $\Phi_x > 0$. From here, it also follows that $x_i^* > x_j^*$ for $v_i a_i > v_j a_j$. In Proposition 1(b), $v_i = v$ for all i, so its statement highlights only the heterogeneity in ability.

For part (a), define

$$\Phi(\overline{x}_i, \overline{x}_i) = rva_i \text{ and } \Phi(\underline{x}_i, \underline{x}_i + \sum_{j \neq i} \overline{x}_j) = rva_i.$$

Given the properties of Φ above, $\overline{x}_i$ and $\underline{x}_i$ uniquely exist, and $0 < \underline{x}_i < \overline{x}_i$. Next, observe that

$$\Phi(x_i^*, x_i^*) < \Phi(x_i^*, X^*) = rva_i = \Phi(\overline{x}_i, \overline{x}_i) \Longrightarrow x_i^* < \overline{x}_i.$$

Moreover,

$$\Phi(\underline{x}_i, \underline{x}_i + \sum_{j\neq i} \overline{x}_j) = rva_i = \Phi(x_i^*, X^*) < \Phi(x_i^*, x_i^* + \sum_{j\neq i} \overline{x}_j) \Longrightarrow \underline{x}_i < x_i^*.$$

Hence, $\underline{x}_i < x_i^* < \overline{x}_i$, as claimed.

I relegate the proof of part (c) to that of Proposition 6 below, as it relates to the parameter β. ■

Proof of Corollary 1. Given globally log-concave cost, part (a) is immediate from Proposition 2. For part (b), recall $c_i(x) = \frac{c(x)}{a_i}$ and re-arrange (4):

$$c'(x_i^*)\left(\frac{r}{\alpha} + X^*\right) - c(x_i^*) = rva_i.$$

Suppose $r \to 0$ but $x_i^* \nrightarrow 0$. Then, the left-hand side remains strictly positive as $c'(x_i^*)x_i^* - c(x_i^*) > 0$ by $c''(x) > 0$, whereas the right-hand side approaches zero, a contradiction. Hence, $x_i^* \to 0$ as $r \to 0$. The same limit argument also applies for $v \to 0$ or $a_i \to 0$. Thus, by continuity, x_i^* is sufficiently small for a sufficiently low r, a_i, or v. The result then follows from Lemma 1 and Proposition 2. Finally, for part (c), recall that agents equally share a fixed total prize in a partnership: $v = V/n$. Thus, v is sufficiently small for a sufficiently large n. The result then obtains from part (b). ■

Proof of Proposition 3. Let $a_1 > a_2 > ... > a_n$ and consider the cost specification in (9). From (A-2), in equilibrium, $\Phi(x_i^*, X^*) = rva_i$. Let $A_L \equiv \frac{\Phi(x_c, x_c)}{r} > 0$.

Suppose $va_1 \leq A_L$ but, to the contrary, $x_1^* > x_c$ at some equilibrium. Using (A-1), note that

$$\begin{aligned} \frac{d}{dx_i}\Phi(x_i, x_i + X_{-i}) &= c''(x_i)\left(\frac{r}{\alpha} + x_i + X_{-i}\right) > 0, \\ \frac{d}{dX_{-i}}\Phi(x_i, x_i + X_{-i}) &= c'(x_i) \geq 0. \end{aligned} \tag{A-7}$$

Thus,

$$rva_1 = \Phi(x_1^*, X^*) > \Phi(x_c, x_c + X_{-1}^*) \geq \Phi(x_c, x_c) = rA_L, \tag{A-8}$$

which implies $va_1 > A_L$, a contradiction.

Hence, $va_1 \leq A_L$ implies $x_1^* \leq x_c$ at any equilibrium. By Proposition 1, this means $x_n^* < ... < x_1^* \leq x_c$ and, by Proposition 2, $u_1^* < u_2^* < ... < u_n^*$.

Next, I define agent i's stand-alone effort $x_i^s > 0$, which uniquely solves

$$\Phi(x_i^s, x_i^s) = rva_i. \tag{A-9}$$

Clearly, $x_i^* < x_i^s$ in any equilibrium with $n > 1$ because $\frac{\partial x_i}{\partial X_{-i}} < 0$ by (A-5). Now I define

$$A_H = \frac{\Phi(x_c, x_c + \sum_{i \neq n} x_i^s)}{r}.$$

Suppose $va_n \geq A_H$ but, to the contrary, $x_n^* \leq x_c$ at some equilibrium. Then,

$$rva_n = \Phi(x_n^*, X^*) \leq \Phi(x_c, x_c + X_{-n}^*) < \Phi(x_c, x_c + \sum_{i \neq n} x_i^s) = rA_H, \tag{A-10}$$

a contradiction. Hence, $va_n \geq A_H$ implies $x_n^* > x_c$; in turn, $x_1^* > ... > x_n^* \geq x_c$ and $u_1^* > u_2^* > ... > u_n^*$ at any equilibrium.

For part (b), the conclusion is automatic for $n = 2$. Let $n \geq 3$, and recall that $h = \frac{c'}{c}$.

Suppose, to the contrary, that some agent $i \neq 1, n$ has the highest equilibrium payoff in the team. That is, $u_i^* \geq \max\{u_1^*, u_n^*\}$, which, by Proposition 1, is equivalent to: $h(x_i^*) \geq \max\{h(x_1^*), h(x_n^*)\}$. But, by (9) and the fact that $x_n^* < x_i^* < x_1^*$, either $h(x_i^*) < h(x_n^*)$ if $x_i^* \leq x_c$, or $h(x_i^*) < h(x_1^*)$ if $x_i^* \geq x_c$, yielding a contradiction. Hence, $u_i^* < \max\{u_1^*, u_n^*\}$ for all $i \neq 1, n$.

For part (c), I first prove the following claim:

Claim. if $u_{(1)}^* \geq ... \geq u_{(i)}^* \geq ... \geq u_{(n)}^*$ is an equilibrium payoff order, then the following effort monotonicity conditions must hold: (1) $x_{(i)}^* \leq x_{(i+1)}^*$ implies $x_{(i)}^* \leq x_{(j)}^*$ for all $j \geq i$, and (2) $x_{(i)}^* \geq x_{(i+1)}^*$ implies $x_{(i)}^* \geq x_{(j)}^*$ for all $j \geq i$.

Proof. Suppose $x_{(i)}^* \leq x_{(i+1)}^*$ for some i. Then, because $u_{(i)}^* \geq u_{(i+1)}^*$, it must be that $x_{(i)}^* \leq x_c$ (otherwise, $x_c < x_{(i)}^*$ would imply $x_c < x_{(i)}^* \leq x_{(i+1)}^*$ and, in turn, $h(x_{(i)}^*) < h(x_{(i+1)}^*)$ by (9), revealing $u_{(i)}^* < u_{(i+1)}^*$). Suppose, to the contrary, that $x_{(i)} > x_{(j)}$ for some $j > i$. Then, together, $x_c \geq x_{(i)}^* > x_{(j)}^*$, which implies $h(x_{(i)}^*) < h(x_{(j)}^*)$ and $u_{(i)}^* < u_{(i+1)}^*$, a contradiction. Hence, $x_{(i)}^* \leq x_{(j)}^*$ for all $j \geq i$, as claimed. Similarly, suppose $x_{(i)}^* \geq x_{(i+1)}^*$ but, to the contrary, $x_{(i)}^* < x_{(j)}^*$ for some i and j. Then, $x_c \leq x_{(i)}^* < x_{(j)}^*$ and, in turn, $h(x_{(i)}^*) < h(x_{(j)}^*)$ and $u_{(i)}^* < u_{(i+1)}^*$, a contradiction. Hence, $x_{(i)}^* \geq x_{(j)}^*$ for all $j \geq i$. □

($\Longrightarrow$) Suppose that $u_{(1)}^* \geq ... \geq u_{(i)}^* \geq ... \geq u_{(n)}^*$ is an equilibrium payoff order for some ability profile, but $u_{(i)}^*$ does not belong to the least or most able member of the subteam $\{(i), ...(n)\}$ for *some* i. Equivalently said, the effort $x_{(i)}^*$ is neither the highest nor the lowest in the subteam. But this would contradict the claim because $x_{(i)}^*$ must be one of the extreme efforts.

($\Longleftarrow$) Suppose $u_{(i)}^*$ belongs to the least or most able member of the subteam $\{(i), ...(n)\}$ for all i. Then, the two effort monotonicity conditions hold; in turn, an ability profile generates the proposed payoff order as equilibrium. ■

Proof of Proposition 4. Consider a three-agent team in which $x_1 = 2x_{c_2}$, $x_2 = x_{c_2}$, and $x_3 = x_{c_1}$. Using (5), this effort profile can be engendered as the unique equilibrium for some ability profile: $a_1 > a_2 > a_3$. By (10), $h(x_{c_2}) > \max\{h(x_{c_1}), h(2x_{c_2})\}$ and thus, $u_2 > \max\{u_3, u_1\}$.

Next, recall from (A-9) that x_i^s refers to i's stand-alone effort, and define

$$A_L = \frac{\Phi(x_{c_1}, x_{c_1})}{r},$$

$$A_{M_1} = \frac{\Phi(x_{c_1}, x_{c_1} + \sum_{i \neq n} x_i^s)}{r} \text{ and } A_{M_2} = \frac{\Phi(x_{c_2}, x_{c_2})}{r},$$

$$A_H = \frac{\Phi(x_{c_2}, x_{c_2} + \sum_{i \neq n} x_i^s)}{r}.$$

As in the proof of Proposition 3, it can be shown that in any equilibrium, $x_1^* \leq x_{c_1}$ if $va_1 < A_L$, and $x_n^* \geq x_{c_2}$ if $va_n > A_H$. Because c is log-concave for both $x < x_{c_1}$ and $x > x_{c_2}$ by (10), we have $u_1^* < u_2^* < ... < u_n^*$.

Next, suppose $A_{M_1} \leq va_i \leq A_{M_2}$ for all i, but, to the contrary, that $x_n^* \leq x_{c_1}$ in some equilibrium. Then, using the properties of Φ from (A-7) and recalling that $x_i^* < x_i^s$, we observe

$$rva_n = \Phi(x_n^*, X^*) \leq \Phi(x_{c_1}, x_{c_1} + X_{-n}^*) < \Phi(x_{c_1}, x_{c_1} + \sum_{i \neq n} x_i^s) = rA_{M_1},$$

which implies $va_n < A_{M_1}$, a contradiction. Hence, $x_n^* > x_{c_1}$ in any equilibrium.

Similarly, suppose, to the contrary, $x_1^* \geq x_{c_2}$. Then, because $X_{-1}^* > 0$ for $n > 1$,

$$rva_1 = \Phi(x_1^*, x_1^* + X_{-1}^*) \geq \Phi(x_{c_2}, x_{c_2} + X_{-1}^*) > \Phi(x_{c_2}, x_{c_2}) = rA_{M_2},$$

which implies $va_1 > A_{M_2}$, a contradiction. Hence, $x_1^* < x_{c_2}$.

Together with Proposition 1, this means $x_{c_1} \leq x_n^* < ... < x_1^* \leq x_{c_2}$. Because c is log-convex on $[x_{c_1}, x_{c_2}]$ by assumption, it follows that $u_1^* > u_2^* > ... > u_n^*$ by Proposition 2. ■

Proof of Proposition 5. By definition, an equilibrium effort profile is allocatively efficient if and only if $c_i'(x_i^*) = z^*$ for all i and some $z^* > 0$. Then, by (6),

$$u_i^* = v - \frac{c_i'(x_i^*)}{\alpha} = v - \frac{z^*}{\alpha} \text{ for all } i.$$

For part (a), let S be the set of agents in the second-best team. Suppose $\ln c(x)$ has $I \in \{0, 1, 2, ...\}$ inflection points, or equivalently, $h'(x) = 0$ has I distinct solutions, where $h =$

$\frac{c'}{c}$. Suppose, to the contrary, that S has $I+2$ heterogeneous members. Then, by definition, $c_i'(x_i^*) = z^*$ for all $i \in S$, where, without loss of generality, $x_1^* > x_2^* > ... > x_{I+1}^* > x_{I+2}^*$. Or equivalently, $h(x_i^*) = h_0$ for all $i \in S$ and some $h_0 > 0$ by Lemma 2. Then, by the mean-value theorem, there exists some $\overline{x}_i \in (x_i^*, x_{i+1}^*)$ such that $h'(\overline{x}_i) = 0$ for all i. This implies that $h'(x) = 0$ has at least $I+1$ solutions, yielding a contradiction. Hence, the second-best team has at most $I+1$ heterogeneous members.

Next, suppose that S is a full-size second-best team, and to the contrary, all its members view the project as easy, i.e., $h'(x_i^*) < 0$ for all $i \in S$. Consider two members such that $x_i^* > x_{i+1}^*$, and suppose that there is no other member with a second-best effort in-between. Then, $h(x_{i+1}^* + \varepsilon) < h_0 < h(x_i^* - \varepsilon)$ for some $\varepsilon \in (x_{i+1}^*, x_i^*)$. Because h is continuous, the intermediate value theorem implies that there is some $\overline{x}_i \in (x_{i+1}^* + \varepsilon, x_i^* - \varepsilon)$ such that $h(\overline{x}_i) = h_0$. But this contradicts that S has no other member between i and $i+1$. A similar argument also proves that $h'(x_i^*) > 0$ for all $i \in S$ or $h'(x_i^*) = 0$ for all $i \in S$ cannot arise, either.

For part (b), I use (13) and observe

$$\left|a_{i,S} - a_{j,S}\right| = \left|c'(x_i) - c'(x_j)\right| \left(\frac{\frac{r}{\alpha} + X_S - \frac{1}{h_0}}{rv} \right),$$

where x_i and x_j solve $h(x_i) = h(x_j) = h_0$ and thus, they are independent of the parameters α, r, and v. The comparative statics are then immediate.

Part (c) follows similarly because the new member k must have $h(x_k) = h_0$ by Lemma 2, and because $X_S < X_{S\cup\{k\}}$. ■

Proof of Proposition 6. From (A-3), define a new function F_i such that

$$f_i(X) = F_i(\frac{r}{\alpha} + X).$$

Then, X^* uniquely solves

$$\sum_i F_i(\frac{r}{\alpha} + X^*) - X^* = 0.$$

Differentiating with respect to α, we obtain

$$\frac{\partial X^*}{\partial \alpha} = \frac{\sum_i F_i'}{(\sum_i F_i' - 1)} \frac{r}{\alpha^2}.$$

Because $x_i^* = F_i(\frac{r}{\alpha} + X^*)$, further differentiation reveals

$$\begin{aligned} \frac{\partial x_i^*}{\partial \alpha} &= F_i' \times \left(-\frac{r}{\alpha^2} + \frac{\partial X^*}{\partial \alpha} \right) \\ &= F_i' \times \left(-\frac{r}{\alpha^2} + \frac{\sum_i F_i'}{\sum_i F_i' - 1} \frac{r}{\alpha^2} \right). \end{aligned}$$

Simplifying terms and noticing $F_i' = f_i' < 0$, we find

$$\frac{\partial x_i^*}{\partial \alpha} = \frac{F_i'}{\left(\sum_i F_i' - 1\right)} \frac{r}{\alpha^2} > 0 \Longrightarrow \frac{\partial x_i^*}{\partial \beta} > 0, \tag{A-11}$$

because $\alpha = 1 + (n-1)\beta$. This proves part (c) of Proposition 1.

To prove Proposition 6, recall from (6) that

$$u_i^* = v - \frac{c_i'(x_i^*)}{\alpha} = v - \frac{1}{a_i}\left(\frac{c'(x_i^*)}{\alpha}\right).$$

From here,

$$\frac{\partial}{\partial \alpha}\left(\frac{c'(x_i^*)}{\alpha}\right) = \frac{c''(x_i^*)\frac{\partial x_i^*}{\partial \alpha}\alpha - c'(x_i^*)}{\alpha^2}.$$

Using (A-11),

$$\begin{aligned}
\frac{\partial}{\partial \alpha}\left(\frac{c'(x_i^*)}{\alpha}\right) &= \frac{c''(x_i^*)\left(\frac{F_i'}{\sum_i F_i' - 1}\frac{r}{\alpha^2}\right)\alpha - c'(x_i^*)}{\alpha^2} \\
&= \frac{1}{\alpha^2}\left[c''(x_i^*)\frac{F_i'}{\left(\sum_i F_i' - 1\right)}\frac{r}{\alpha} - c'(x_i^*)\right].
\end{aligned}$$

Substituting for F_i',

$$\begin{aligned}
\frac{\partial}{\partial \alpha}\left(\frac{c'(x_i^*)}{\alpha}\right) &= -\frac{1}{\alpha^2}\left[c''(x_i^*)\frac{\frac{-c'(x_i^*)}{c''(x_i^*)\left(\frac{r}{\alpha}+X^*\right)-c'(x_i^*)}}{\sum_i F_i' - 1}\frac{r}{\alpha} - c'(x_i^*)\right] \\
&= -\frac{c'(x_i^*)}{\alpha^2\left(\sum_i F_i' - 1\right)}\left[\frac{c''(x_i^*)\frac{r}{\alpha}}{c''(x_i^*)\left(\frac{r}{\alpha} + X^*\right) - c'(x_i^*)} + \sum_i F_i' - 1\right] \\
&= -\frac{c'(x_i^*)}{\alpha^2\left(\sum_i F_i' - 1\right)}\left[-\frac{c''(x_i^*)X^* - c'(x_i^*)}{c''(x_i^*)\left(\frac{r}{\alpha} + X^*\right) - c'(x_i^*)} + \sum_i F_i'\right] \\
&< 0,
\end{aligned}$$

because $\sum_i F_i' < 0$, and $c''(x_i^*)X^* - c'(x_i^*) \geq 0$ given $c''' \geq 0$.

Hence,

$$\frac{\partial u_i^*}{\partial \alpha} = -\frac{1}{a_i}\frac{\partial}{\alpha}\left(\frac{c'(x_i^*)}{\alpha}\right) > 0 \implies \frac{\partial u_i^*}{\partial \beta} > 0.$$

Finally, observe that

$$\frac{\partial x_i^*}{\partial \alpha} - \frac{\partial x_j^*}{\partial \alpha} = \left(F_i' - F_j'\right)\left[\frac{r}{\alpha^2\left(\sum_i F_i' - 1\right)}\right].$$

Given that $\sum_i F_i' - 1 < 0$, this implies

$$sgn\left(\frac{\partial x_i^*}{\partial \alpha} - \frac{\partial x_j^*}{\partial \alpha}\right) = sgn\left(F_j' - F_i'\right).$$

Note from (A-5) that

$$\begin{aligned} F_i' = f_i' &= -\frac{c'(x_i^*)}{c''(x_i^*)\left(\frac{r}{\alpha} + X^*\right) - c'(x_i^*)} \\ &= -\frac{1}{\frac{c''(x_i^*)}{c'(x_i^*)}\left(\frac{r}{\alpha} + X^*\right) - 1}. \end{aligned} \tag{A-12}$$

Hence,

$$sgn\left(F_j' - F_i'\right) = sgn\left(\frac{c''(x_j^*)}{c'(x_j^*)} - \frac{c''(x_i^*)}{c'(x_i^*)}\right)$$

and, in turn, given that $\alpha = 1 + (n-1)\beta$,

$$sgn\left(\frac{\partial x_i^*}{\partial \beta} - \frac{\partial x_j^*}{\partial \beta}\right) = sgn\left(\overline{h}(x_j^*) - \overline{h}(x_i^*)\right)$$

where $\overline{h} = \frac{c''}{c'}$. ■

Appendix B: Extensions

Heterogeneous rewards. Let (a_i, v_i) be agent i's ability and project-value pair. In the proof of Proposition 1, I have shown that a unique and interior equilibrium with heterogeneous rewards exists. In equilibrium, i's first-order condition (4) is modified to be

$$c_i'(x_i^*)\left(\frac{r}{\alpha} + X^*\right) - c_i(x_i^*) = rv_i. \tag{B-1}$$

Inserting (B-1) into (3), (6) becomes

$$u_i^* = v_i - \frac{c_i'(x_i^*)}{\alpha},$$

which is equivalent to

$$\frac{u_i^*}{v_i} = 1 - \frac{1}{\alpha}\frac{c_i'(x_i^*)}{v_i}. \tag{B-2}$$

Factoring out (B-1), we find

$$\frac{c_i'(x_i^*)}{v_i}\left[\frac{r}{\alpha} + X^* - \frac{1}{h(x_i^*)}\right] = r. \tag{B-3}$$

where $h = \frac{c'}{c}$, as defined in the text.

Together, (B-2) and (B-3) reveal a slight modification of (8):

$$sgn\left(\frac{u_i^*}{v_i} - \frac{u_j^*}{v_j}\right) = sgn\left(h(x_i^*) - h(x_j^*)\right). \tag{B-4}$$

In light of (B-4), it is evident that all the results in the text about payoff ordering with equal rewards v would apply to the ordering of $\frac{u_i^*}{v_i}$ with heterogeneous rewards. Hence, when v_i and v_j are not too different, it would also imply the same ordering of u_i^*.

Two-stage projects. Here, I extend the base model to two-stage projects where the team needs to make two successive breakthroughs for completion. Perhaps, the project has two complementary parts building on each other. I show that the equilibrium characterization remains largely intact, but effort and payoff rankings may differ in the initial stage of the project, especially under log-concave cost. In what follows, I present the formal results in Lemma B1 and Proposition B1, and then provide intuition in three remarks.

Let $s = 1$ and $s = 2$ denote the project's initial and final stage, respectively. Team members receive the reward $v > 0$ if and only if both stages are complete. Because, as in the base model, there is no knowledge accumulation, optimal strategies remain stationary within each stage. Let $x_{i,s}^*$ and $u_{i,s}^*$ represent agent i's equilibrium effort and expected payoff in stage s, respectively, and $X_s^* = \sum_i x_{i,s}^*$ represent the total effort.

Lemma B1. *There is a unique and interior equilibrium. In equilibrium,*

- **(a)** *every agent works harder in the final stage: $x_{i,1}^* < x_{i,2}^*$ for all i,*
- **(b)** *every agent's payoff increases at an increasing rate as the project gets closer to completion:*

$$u_{i,1}^* < u_{i,2}^* < v \text{ and } u_{i,2}^* - u_{i,1}^* < v - u_{i,2}^* \text{ for all } i.$$

- **(c)** $sgn\left(u_{i,2}^* - u_{j,2}^*\right) = sgn\left(h(x_{i,2}^*) - h(x_{j,2}^*)\right)$ *and* $sgn\left(\frac{u_{i,1}^*}{u_{i,2}^*} - \frac{u_{j,1}^*}{u_{j,2}^*}\right) = sgn\left(h(x_{i,1}^*) - h(x_{j,1}^*)\right)$.

Proof. Working backward, note that the subgame in stage 2 is equivalent to the base model. Thus, there is a unique and interior equilibrium by Proposition 1. And, by (4), (6), and (8), I have the following:

$$c_i'(x_{i,2}^*)\left(\frac{r}{\alpha} + X_2^*\right) - c_i(x_{i,2}^*) = rv, \tag{B-5}$$

$$u_{i,2}^* = v - \frac{c_i'(x_{i,2}^*)}{\alpha}, \tag{B-6}$$

and

$$sgn\left(u_{i,2}^* - u_{j,2}^*\right) = sgn\left(h(x_{i,2}^*) - h(x_{j,2}^*)\right). \tag{B-7}$$

From (B-6), it is immediate that

$$v - u_{i,2}^* = \frac{c_i'(x_{i,2}^*)}{\alpha} > 0. \tag{B-8}$$

Next, consider stage 1. This subgame is isomorphic to stage 2 with heterogeneous rewards, $u_{i,2}^*$'s. Hence, the above arguments for heterogeneous rewards apply. In particular, the equilibrium in stage 1 is also unique and interior, and substituting $u_{i,2}^*$ for v_i in (B-4), I have the following:

$$c_i'(x_{i,1}^*)\left(\frac{r}{\alpha} + X_1^*\right) - c_i(x_{i,1}^*) = ru_{i,2}^*, \tag{B-9}$$

$$u_{i,1}^* = u_{i,2}^* - \frac{c_i'(x_{i,1}^*)}{\alpha}, \tag{B-10}$$

and

$$sgn\left(\frac{u_{i,1}^*}{u_{i,2}^*} - \frac{u_{j,1}^*}{u_{j,2}^*}\right) = sgn\left(h(x_{i,1}^*) - h(x_{j,1}^*)\right). \tag{B-11}$$

To prove part (a), I first establish that $X_1^* < X_2^*$. Suppose, to the contrary, that $X_1^* \geq X_2^*$. Then, $x_{i,1}^* \geq x_{i,2}^*$ for some i. Now observe that

$$\begin{aligned}
c_i'(x_{i,2}^*)\left(\frac{r}{\alpha} + X_1^*\right) - c_i(x_{i,2}^*) &\leq c_i'(x_{i,1}^*)\left(\frac{r}{\alpha} + X_1^*\right) - c_i(x_{i,1}^*) && \text{(because } c''' \geq 0\text{)} \quad \text{(B-12)}\\
&= ru_{i,2}^* && \text{(by (B-9))}\\
&< rv && \text{(because } u_{i,2}^* < v \text{ by (B-8))}\\
&= c_i'(x_{i,2}^*)\left(\frac{r}{\alpha} + X_2^*\right) - c_i(x_{i,2}^*) && \text{(by (B-5)),}
\end{aligned}$$

which implies $X_1^* < X_2^*$, a contradiction. Hence, $X_1^* < X_2^*$.

Next, suppose, to the contrary, $x_{i,1}^* \geq x_{i,2}^*$ for some i. Then, the same sequence of inequalities in (B-12) and the fact that $X_1^* < X_2^*$ yield a contradiction. Hence, $x_{i,1}^* < x_{i,2}^*$ for all i, proving part (a).

To prove part (b), note that

$$\begin{aligned}
v - u_{i,2}^* &= \frac{c_i'(x_{i,2}^*)}{\alpha} && \text{(by (B-6))}\\
&> \frac{c_i'(x_{i,1}^*)}{\alpha} && \text{(by part (a))}\\
&= u_{i,2}^* - u_{i,1}^* && \text{(by (B-10))}\\
&> 0. && \text{(because } c_i' > 0\text{).}
\end{aligned}$$

Finally, part (c) records (B-7) and (B-11). ■

Proposition B1. *Consider the unique log-inflection cost in (9), and the cutoffs A_L and A_H defined in Proposition 3.*

(a) *If $va_n > A_H$ so that all equilibrium efforts are on the log-convex part of the cost, then*

$$a_i > a_j \Longrightarrow u^*_{i,2} > u^*_{j,2} \text{ and } u^*_{i,1} > u^*_{j,1}.$$

(b) *Suppose $va_1 < A_L$ so that all equilibrium efforts are on the log-concave part of the cost. Then, $a_i > a_j$ implies $u^*_{i,2} < u^*_{j,2}$. Moreover,*

$$\begin{cases} u^*_{i,1} < u^*_{j,1} & \text{if} \quad a_i - a_j > \frac{c'(\overline{x}_i) - c'(\underline{x}_j)}{\alpha v} \\ \frac{u^*_{i,1}}{u^*_{j,1}} > \frac{u^*_{i,2}}{u^*_{j,2}} & \text{if} \quad a_i - a_j < \frac{c'(\underline{x}_i) - c'(\overline{x}_j)}{\alpha v} \text{ and } \underline{x}_i > \overline{x}_j, \end{cases}$$

where $\underline{x}_i$ and $\overline{x}_i$ are the bounds for i's equilibrium effort from Proposition 1(a).

Proof. Suppose $va_n > A_H$. Then, $a_i > a_j$ implies $u^*_{i,2} > u^*_{j,2}$ by Proposition 3, and, in turn, $a_i u^*_{i,2} > a_j u^*_{j,2}$. The proof of Proposition 3 would further imply $x^*_{i,1} > x_c$ for all i (because $u^*_{i,2} < v$ and (A-10) would continue to yield a contradiction). Hence, $x^*_{i,2} > x^*_{i,1} > x_c$ for all i by Lemma B1, i.e., the equilibrium efforts in both stages are on the log-convex part of the cost, i.e., $h' > 0$.

Next, because $a_i u^*_{i,2} > a_j u^*_{j,2}$, we have $x^*_{i,1} > x^*_{j,1}$ by (A-2). From here and the fact that $h' > 0$, part (c) of Lemma B1 implies that

$$\frac{u^*_{i,1}}{u^*_{i,2}} > \frac{u^*_{j,1}}{u^*_{j,2}},$$

which, given $u^*_{i,2} > u^*_{j,2}$, implies $u^*_{i,1} > u^*_{j,1}$, proving part (a).

To prove part (b), suppose $va_1 < A_L$. Then, $a_i > a_j$ implies $u^*_{i,2} < u^*_{j,2}$ by Proposition 3. Moreover, $x^*_{i,2} < x_c$ for all i by the proof of Proposition 3, and $x^*_{i,1} < x^*_{i,2} < x_c$ by Lemma B1, i.e., the equilibrium efforts in both stages are on the log-concave part of the cost, i.e., $h' < 0$. Unlike in part(a), however, we cannot rank $a_i u^*_{i,2}$ and $a_j u^*_{j,2}$, which is needed to compare the equilibrium efforts in stage 1. Using the bounds in Proposition 1(a) and (B-6), notice that if $a_i - a_j > \frac{c'(\overline{x}_i) - c'(\underline{x}_j)}{\alpha v}$ or, equivalently, $a_i \left(v - \frac{c'_i(\overline{x}_i)}{\alpha}\right) > a_j \left(v - \frac{c'_j(\underline{x}_j)}{\alpha}\right)$, then $a_i u^*_{i,2} > a_j u^*_{j,2}$ and, in turn, $x^*_{i,1} > x^*_{j,1}$ by (B-9). Lemma B1 then implies

$$\frac{u^*_{i,1}}{u^*_{i,2}} < \frac{u^*_{j,1}}{u^*_{j,2}},$$

and in turn, $u^*_{i,1} < u^*_{j,1}$.

Similarly, if $a_i - a_j < \frac{c'(\underline{x}_i) - c'(\overline{x}_j)}{\alpha v}$ and $\underline{x}_i > \overline{x}_j$, then $a_i u^*_{i,2} < a_j u^*_{j,2}$, and in turn, $x^*_{i,1} < x^*_{j,1}$. Hence, $\frac{u^*_{i,1}}{u^*_{i,2}} > \frac{u^*_{j,1}}{u^*_{j,2}}$ by Lemma B1, which is equivalent to

$$\frac{u^*_{i,1}}{u^*_{j,1}} > \frac{u^*_{i,2}}{u^*_{j,2}}.$$

■

Remark 1 *Together with Lemma B1, part (a) of Proposition B1 says that higher-ability agents work harder and are better off in both stages of the project. The intuition behind the final (second) stage is similar to the base model: higher-ability agents face a less severe free-rider problem under log-convex cost. Expecting to fare better in the final stage and having the cost advantage, higher-ability agents also act like "higher-ability" in the first stage. Thus, they continue to work harder and fare better in the first stage.*

Remark 2 *Part (b) of Proposition B1 says that as with the base model, higher-ability agents are worse off in the project's final stage under log-concave cost. This is because free-riding is more rampant in this case, as explained in Proposition 2. Anticipating to work disproportionately harder in the final stage, higher-ability agents also work harder in the initial stage if they are significantly more able or cost-effective than the rest. Otherwise, they will be less enthusiastic in the initial stage, as indicated in part (b). Therefore, whether or not higher-ability agents are better off than the lower-ability in the initial stage is ambiguous. Nevertheless, part (b) reveals that higher-ability agents will be relatively better off in the initial stage.*

Remark 3 *Although their model features continuous project progress as a function of the team effort, part (a) of Proposition B1 implies Bowen et al.'s (2009) payoff result is unlikely to hold under log-convex cost. Furthermore, part (b) reveals that even with a log-concave cost, the effort and payoff ordering can differ across project stages if effort accumulation is imperfect.*

References

Admati, A. R., and Perry, M. "Joint Projects Without Commitment." *Review of Economic Studies* Vol. 58 (1991), pp. 259-276.

Bag, P. K. and Pepito, N. "Peer Transparency in Teams: Does It Help or Hinder Incentives?" *International Economic Review* Vol. 53 (2012), pp. 1257-1286.

Bag, P. K. and Pepito, N. "Harmful Transparency in Teams." *Economics Letters* Vol. 144 (2016), pp. 88-91.

Bel, R., Smirnov, V., and Wait, A. "Team Composition, Worker Effort and Welfare." *International Journal of Industrial Organization* Vol.41 (2015), pp. 1-8.

Bergstrom, T., Blume, L., and Varian, H. "On the Private Provision of Public Goods." *Journal of Public Economics* Vol. 29 (1986), pp. 25-49.

Bhattacharya, S. K., Tokovenko, O., and Sardana, K. "Heterogeneity, Impatience, and Dynamic Private Provision of a Discrete Public Good." *Journal of Public Economic Theory* Vol. 19 (2017), pp. 1043-1054.

Bonatti, A., and Hörner, J. "Collaborating." *American Economic Review* Vol. 101 (2011), pp. 632-663.

Bonatti, A., and Rantakari, H. "The Politics of Compromise." *American Economic Review* Vol. 106 (2016), pp. 229-59.

Bose, A., Pal, D., and Sappington, D. E. M. "Asymmetric Treatment of Identical Agents in Teams." *European Economic Review* Vol. 54 (2010), pp. 947-961.

Bowen, T. R., Georgiadis, G., and Lambert, N. S. "Collective Choice in Dynamic Public Good Provision." *American Economic Journal: Microeconomics* Vol. 11 (2019), pp. 243-98.

Camerer, C. F., Loewenstein, G., and Rabin, M., eds. *Advances in Behavioral Economics*. Princeton University Press, 2004.

Cetemen, D., Hwang, I., and Kaya, A. "Uncertainty-Driven Cooperation." *Theoretical Economics* Vol. 15 (2020), pp. 1023-1058.

Compte, O., and Jehiel, P. "Gradualism in Bargaining and Contribution Games." *Review of Economic Studies* Vol. 71 (2004), pp. 975-1000.

Doraszelski, U. "Rent Dissipation in R&D Races." In R. Cellini, and L. Lambertini, eds., The Economics of Innovation, Emerald Group Publishing Ltd., 2008.

Fehr, E., and Schmidt, K. M. "A Theory of Fairness, Competition, and Cooperation." *Quarterly Journal of Economics* Vol. 114 (1999), pp. 817-868.

Franco, A., Mitchell, M., and Vereshchagina, G. "Incentives and the Structure of Teams." *Journal of Economic Theory* Vol. 146 (2011), pp. 2307-2332.

Georgiadis, G. "Projects and Team Dynamics." *Review of Economic Studies* Vol. 82 (2015), pp. 187-218.

Gill, D., and Stone, R. "Desert and Inequity Aversion in Teams." *Journal of Public Economics* Vol. 123, pp. 42-54.

Glover, J., and Kim, E. "Optimal Team Composition: Diversity to Foster Implicit Team Incentives." *Management Science* Vol. 67 (2021), pp. 5800-5820.

Guo, Y., and Roesler, A-K. "Private Learning and Exit Decisions in Collaboration." *Working Paper*, Northwestern University, 2018.

Jones, B. F. "The Rise of Research Teams: Benefits and Costs in Economics." *Journal of Economic Perspectives* Vol. 35 (2021), pp. 191-216.

Kamien, M. I., Muller, E., and Zang, I. "Research Joint Ventures and R&D Cartels." *American Economic Review* Vol. 82 (1992), pp. 1293-1306.

Kaya, A., and Vereshchagina, G. "Partnerships versus Corporations: Moral Hazard, Sorting, and Ownership Structure." *American Economic Review* Vol. 104 (2014), pp. 291-307.

Kessing, S. G. "Strategic Complementarity in the Dynamic Private Provision of a Discrete Public Good." *Journal of Public Economic Theory* Vol. 9 (2007), pp. 699-710.

Laffont, J. J., and Tirole, J. *A Theory of Incentives in Procurement and Regulation*. MIT press, 1993.

Lee, T., and Wilde, L. "Market Structure and Innovation: A Reformulation." *Quarterly Journal of Economics* Vol. 94 (1980), pp. 429-436.

Marx, L. M., and Matthews, S. A. "Dynamic Voluntary Contribution to a Public Project." *Review of Economic Studies* Vol. 67 (2000), pp. 327-358.

Ozerturk, S., and Yildirim, H.. "Credit Attribution and Collaborative Work." *Journal of Economic Theory* Vol. 195 (2021). Article 105264.

Postrel, S. "Islands of Shared Knowledge: Specialization and Mutual Understanding in Problem-Solving Teams." *Organization Science* Vol. 13 (2002), pp. 303-320.

Reinganum, J. F. "A Class of Differential Games for Which the Closed-Loop and Open-Loop Nash Equilibria Coincide." *Journal of Optimization Theory and Applications* Vol. 36 (1982), pp. 253-262.

Segal, I. "Coordination and Discrimination in Contracting with Externalities: Divide and Conquer?" *Journal of Economic Theory* Vol. 113 (2003), pp. 147-181.

Thom, M. “Are Group Assignments Effective Pedagogy or a Waste of Time? A review of the literature and implications for practice.” *Teaching Public Administration* Vol. 38 (2020), pp. 257-269.

Winter, E. “Incentives and Discrimination.” *American Economic Review* Vol. 94 (2004), pp. 764-773.

Winter, E. “Transparency and Incentives Among Peers.” *RAND Journal of Economics* Vol. 41 (2010), pp. 504-523.

Yildirim, H. “Getting the Ball Rolling: Voluntary Contributions to a Large-Scale Public Project.” *Journal of Public Economic Theory* Vol. 8 (2006), pp. 503-528.