On the Role of Confidentiality and Deadlines in Bilateral Negotiations

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Abstract

The preference between public and private negotiations for a buyer who sequentially visits two sellers is examined. It is shown that the buyer (weakly) prefers private negotiations so as to create strategic uncertainty about the trade history. With substitute goods, such uncertainty is valuable only when price offers have short deadlines that prevent a head-to-head competition. With complementary goods, strategic uncertainty is valuable to the extent that price coordination becomes a concern for sellers, which is likely to be the case when sellers possess high bargaining powers; their price offers have short deadlines; and/or goods are weak complements. Sellers’ strategic deadline choices as well as their incentives to disclose information about negotiations are also investigated.

JEL Classifications: C70, L23.

Keywords: public negotiations, private negotiations, exploding offers, open-ended offers, bargaining power.

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1 Introduction

When Walter E. Disney decided to purchase some 39 square mile land for his new theme park in Central Florida, he knew that any publicity would result in a burst of land prices. To avoid this, Disney kept his land deals confidential by creating dummy corporations at the onset of the negotiations (Mannheim 2002, ch.4). In contrast, James B. Duke’s project for a new university was well-known in his small community and led to substantially high land prices at the desired location. Instead of abandoning the project, Duke slightly changed the location and reverted to secrecy by acquiring property through real-estate agents (Durden 1993, pp. 30-32.).

From the Disney and Duke stories, it is clear that confidentiality can be an important strategic tool for a buyer who performs bilateral negotiations with several sellers. Other examples of such one-to-many negotiations include a firm trying to reach deals with multiple labor unions, a manufacturer bargaining with various sellers, and an academic department negotiating with multiple faculty candidates. In addition to the information flow between the sellers, the trade outcomes in these negotiations are likely to depend on sellers’ offer deadlines; in particular whether or not deadlines afford the buyer enough time to compare price offers before a purchasing decision. In fact, certain government policies such as the Federal Trade Commission’s (FTC) cooling-off rule effectively extend deadlines by enabling consumers to cancel a contract or return a purchase within a fixed time period.\(^1\)

In this paper, we aim to shed light on the buyer’s desire to keep negotiations confidential as well as on the sellers’ deadline choices. Specifically, we explore the buyer’s preference between public negotiations, in which she fully discloses the trade history, and private negotiations, in which she keeps this information confidential. In doing so, we also identify the social value of various disclosure laws such as sunshine laws\(^2\) that require government buyers to conduct public negotiations and the social value of deadline policies such as the FTC’s cooling-off rule mentioned above.

Our formal setting consists of three risk-neutral players – two sellers who each provide a differentiated good and a buyer with unit demands. The buyer sequentially negotiates with

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\(^1\)The cooling-off rule gives buyers three days to cancel a contract or return a purchase without any penalty, especially if the transaction amount is more than $25 and the agreement is made outside the vendor’s permanent place of business (see, [http://www.ftc.gov/bcp/edu/pubs/consumer/products/pro03.shtm](http://www.ftc.gov/bcp/edu/pubs/consumer/products/pro03.shtm)).

\(^2\)Over seventy countries, including the U.S. and Canada, have implemented some form of sunshine laws that set rules on public access to information about business dealings of government bodies, including participation in actual meetings. For an overview of the U.S. sunshine laws, see Berg et al. (2005).

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sellers in the order of her choosing. Under public negotiations, this order and the buyer’s purchasing history are disclosed to the sellers, whereas under private negotiations, both pieces of information remain confidential to the buyer. We model a buyer-seller negotiation as a random proposer bargaining in which each player makes a take-it-or-leave-it price offer with some pre-specified probability, but a seller’s price offer is also subject to a deadline. In particular, a price offer is said to be “exploding” if it obliges the buyer to decide before visiting the rival seller, and “open-ended” if it lets her decide after visiting both sellers. We characterize perfect Bayesian equilibria of public and private negotiation games.

Our analysis shows that the buyer (weakly) prefers to conduct private negotiations so as to create strategic uncertainty between the sellers. With substitute goods, such strategic uncertainty is valuable to the buyer only when offers are exploding. When offers are open-ended, the buyer is able to compare prices and engender a Bertrand competition, irrespective of the negotiation type. With complementary goods, strategic uncertainty benefits the buyer to the extent that this makes sellers concerned about price coordination. We discover that such a concern for sellers is most pronounced when they make exploding offers and when they are powerful bargainers. With exploding offers, since the buyer pays as she goes, sellers post moderate prices to entice the buyer to purchase in the first negotiation without worrying about the holdup problem in the second. When sellers are powerful, each anticipates the rival to make an aggressive price offer to the buyer, which, in turn, leads both to set relatively low prices.

While private negotiations are (weakly) preferred by the buyer, they may yield an inefficient trade unlike public negotiations. Thus, our results suggest that various mandatory disclosure laws imposed on government buyers are likely to be constraining, especially for the acquisition of complementary goods. Our results also suggest that the extended return and cancelation policies that effectively cause price offers to be open-ended are most useful for the purchases of substitute goods. In contrast, for the purchases of complementary goods, we show that such policies may actually hinder an efficient trade by giving sellers an incentive to price aggressively. In fact, with complements our analysis indicates that both the buyer and sellers could be better off under a policy of “all sales are final”.

We also investigate sellers’ strategic deadline choices as a first stage of an extended game, in which sellers decide on their deadlines simultaneously before the buyer’s choice of disclosure policy. We show that adopting a uniform deadline policy, as assumed in the basic model, is an equilibrium of this extended game. Finally, we explore sellers’ incentives
to disclose information about the buyer’s trade history, and demonstrate that they may conflict with the buyer’s desire for confidentiality.


Perhaps, the two papers that are closest to our investigation in this literature are by Daughety and Reinganum (1992), and Noe and Wang (2004). Daughety and Reinganum consider a costly search model with duopolists selling perfect substitutes and they offer an interesting strategic explanation for the firms’ choices of deadline policies. In contrast, our emphasis here is on the strategic impact of the information flow available to the sellers whose products can be imperfect substitutes or complements, although we also endogenize their deadline policies.

Noe and Wang (2004) provide an insightful analysis of the confidentiality issue similar to ours, but our model and focus differ from theirs in some significant ways too. First, Noe and Wang consider only exploding offers, whereas we consider both exploding and open-ended offers. Second, we characterize both efficient and inefficient equilibria under simple price offers while Noe and Wang restrict attention to efficient equilibria. As our analysis suggests, inefficient trade equilibria may be endemic (and unique) when offers are open-ended and goods are complements.

Our paper is also related to the literature on bilateral contracting with externalities, introduced by Segal (1999), and later generalized by Segal and Whinston (2003). With a random proposer bargaining, our model combines some elements of Segal and Whinston’s “offering game,” in which all offers are made by the buyer, and their “bidding game,” in which all offers are made by the sellers. Some recent papers, e.g., Genicot and Ray (2006), and Moller (2007), investigate sequential bilateral contracting with endogenous timing to determine when simultaneous contracting may or may not be optimal for the buyer, but they do so by assuming that contracts and agreements are public.

The question of equilibrium sequencing of sellers and its impact on efficiency in our
paper, though not our main focus, seems to parallel the question of issue sequencing in a multi-issue bargaining (e.g., Fershtman, 1990; Inderst, 2000; Winter, 1996). Finally, our paper complements the recent literature on consumer privacy in which it is the sellers, rather than the buyer, who initiate communication (e.g., Calzolari and Pavan, 2006; Taylor, 2004). We also briefly consider the possibility of sellers’ communication.

The rest of our paper is organized as follows. In the next section, we lay out the basic model and present two motivating examples with perfect complements and perfect substitutes. In section 3, we provide a general analysis of complements and substitutes, and determine the buyer’s preference between public and private negotiations. In Section 4, we extend the analysis to a setting in which the buyer cannot commit to holding public or private negotiations. In Section 5, we endogenize offer types by letting sellers strategically choose and commit to them, followed by an analysis of sellers’ incentives to share information about trade history in Section 6. Finally, we conclude in Section 7. The proofs of all formal results appear in the Appendix.

2 The Basic Model

There are three risk-neutral parties: one buyer, $b$, and two sellers, $s_i$, $i = 1, 2$. Each seller provides a (potentially) differentiated good at zero cost. The buyer has a unit demand for good $i$ and values it alone at $v_i \in [0, 1]$. Her joint valuation is normalized to 1. Letting $\Delta \equiv 1 - v_1 - v_2$, we say that goods are stronger complements as $\Delta$ increases from $-1$ (perfect substitutes) to $+1$ (perfect complements). More broadly, we say that goods are complements if $\Delta > 0$; substitutes if $\Delta < 0$; and independent if $\Delta = 0$. It is assumed that each party has a no-trade payoff of 0, and that the payoffs are common knowledge. Note that it is (socially) efficient for the buyer to purchase both goods.\(^3\)

The buyer negotiates with sellers sequentially and only once.\(^4\) We capture each negotiation as a one-shot random proposer bargaining: with probability $\alpha_i \in (0, 1)$, $s_i$ makes a simple (non-contingent and non-random) price offer for product $i$, and with probability

\[^3\]We do not consider here the possibility of some good $i$ being individually undesirable for the buyer, i.e., $v_i < 0$. Our results would, however, go through with a positive cost, $c_i$, of production as long as $v_i - c_i \geq 0$ for all $i$, and $1 - \sum c_i \geq \max_i \{v_i - c_i\}$.

\[^4\]Precluding renegotiation is clearly restrictive and may require some commitment power. Like Moller (2007), and Noe and Wang (2004), we make this assumption to better focus on other aspects of bargaining, but it would be satisfied if, for instance, the buyer has a relatively short time to purchase the goods. Sequentiality, however, should not be taken literally, because some sequential negotiations in our framework will be strategically equivalent to simultaneous ones.
$1 - \alpha_i$, $b$ does.\textsuperscript{5} It is assumed that $\alpha_i$ is common knowledge, but the identity of the proposer is realized at the beginning of negotiation $i$ and observed only by $s_i$ and $b$. Note that a greater $\alpha_i$ refers to an ex ante more powerful seller vis-à-vis the buyer, and that we eliminate the (uninteresting) cases in which one player has all the bargaining power, though one can take the limits.\textsuperscript{6} For simplicity, we ignore discounting between negotiations.

As alluded to in the introduction, price offers by sellers are associated with a deadline policy, $D$. In this respect, we consider both exploding offers, $D = X$, that compel the buyer to decide as soon as bargaining ends, and open-ended offers, $D = O$, that permit the buyer to decide after visiting both sellers. Initially, we take deadline policies to be exogenous – perhaps because they are the industry standards or because they are imposed by certain regulations such as the FTC’s cooling-off rule (see Footnote 1). We do, however, endogenize deadline policy choices in Section 5.

Being the central player, the buyer controls the information flow between the negotiations. In particular, she commits to perform either public negotiations, in which the full trade history is revealed to sellers, or private negotiations, in which she keeps this information confidential. In particular, under private negotiations, neither the order of negotiations nor the buyer’s purchasing history is disclosed to the sellers.\textsuperscript{7} The buyer’s ability to commit to a disclosure policy may simply be the result of her negotiation strategy through third parties as with Disney’s dummy corporations and Duke’s real estate agents, or the sellers’ legal requests for confidentiality in their business dealings. Nonetheless, in Section 4, we show that the buyer’s desired disclosure policy can be part of an equilibrium when she lacks commitment power, and in Section 6, we explore sellers’ incentives to share information about the trade history.

We denote by $P_D(\tilde{\rho}_i)$ and $P_{\overline{D}}(\tilde{\rho}_i)$ the price offer made by player $\tilde{\rho}_i \in \{s_i, b\}$ for product $i$ under the deadline policy $D \in \{X, O\}$, when negotiations are public and private, respectively. Also, we let $\phi_1 \in \{0, 1\}$ be the upstream trade outcome and $\rho_1 \in \{s_1, s_2, b\}$ be the upstream proposer.\textsuperscript{8} The extensive form of our basic negotiation game proceeds

\textsuperscript{5}This is a convenient way of parameterizing the distribution of bargaining power, and widely used in similar negotiation models, e.g., Marx and Shaffer (2007, 2010), and Raskovich (2007). We further discuss its relation to a two-stage bargaining protocol in the next section.

\textsuperscript{6}For instance, $\alpha_i$ may reflect a seller’s likelihood of having other customers or a landowner’s likelihood of having an alternative use for his land, which can be learned only if the two parties negotiate.

\textsuperscript{7}It is worth pointing out that private negotiations exist only if there are more than two contracting periods so that the suppliers cannot infer the order by simply observing the calendar time. Moreover, our assumption of no discounting eliminates the strategic effect of calendar time.

\textsuperscript{8}To simplify exposition, we abuse terminology by referring to the (endogenous) first and second nego-
in three stages. In stage 0, deadline policies are observed by all players. In stage 1, the buyer commits to engaging in public or private negotiations. In stage 2, she proceeds to negotiations in the order of her choosing. We characterize perfect Bayesian equilibria of this game. We say that trade is efficient if, in equilibrium, the buyer purchases both goods with probability 1 (as she should), and inefficient otherwise. In case of indifference, we assume that all players break ties in favor of Pareto efficient decisions, i.e., buying and selling more units.

**Remark.** It is worth noting that our setting exhibits the features of a common agency model because with probability $\alpha_i \alpha_j$ the sellers make offers to the same buyer. In contrast to the more sophisticated trade mechanisms allowed by much of the common agency literature (e.g., Epstein and Peters, 1999; Martimort and Stole, 2009; Pavan and Calzolari, 2009; Peters, 2003), we focus on simple (non-random and non-contingent) price offers. Under public negotiations, since the buyer’s valuations as well as the sellers’ mechanisms are publicly observable, Pavan and Calzolari (2009) imply that simple prices create no loss of generality in determining the equilibrium outcomes of our model. Under private negotiations, however, other trade mechanisms may also emerge in equilibrium. Characterizing the entire equilibrium set is beyond the scope of this paper.\(^9\)

2.1 Two Motivating Examples

To fix ideas, we present here two examples corresponding to perfect complements and perfect substitutes. Assume $\alpha_1 = \alpha_2 = \alpha$ and let $P_1$ and $P_2$ denote sellers’ prices. Since the buyer always makes a price offer equal to marginal cost, 0, we focus our arguments on sellers’ prices.

**Perfect Complements.** Suppose that the buyer has to acquire both units for her project, i.e., $v_1 = v_2 = 0$, and $\Delta = 1$. For the sake of brevity in this example, let $\alpha < \frac{1}{2}$, which makes the buyer the more powerful party and is likely to be the case in the Disney and Duke cases. Consider first exploding offers. If negotiations are public, then the second negotiation is valuable to the buyer only if the first one was successful. In this case, the

\(^7\)Characterizing the entire equilibrium set would require a sufficient enrichment of the communication space (Epstein and Peters, 1999), which is often very complex and impractical. While several recent papers, including Martimort and Stole (2009), Pavan and Calzolari (2009), and Peters (2003), have studied more structured settings for a less complex characterization, their results do not directly apply to our framework at hand.

\(^9\)Characterizing the entire equilibrium set is beyond the scope of this paper.
buyer’s expected payoff from the second negotiation is \((1 - \alpha)\) because with probability \(\alpha\), the second seller extracts all the surplus by charging \(P_2 = 1\) – an extreme form of hold-up. Therefore, seller 1 sets \(P_1 = 1 - \alpha\), resulting in an efficient trade. Accounting for the likelihood of all four realizations of proposers in the two negotiations, the buyer’s ex ante payoff is

\[
\pi^X(b) = (1 - \alpha)^2 + \alpha(1 - \alpha)[1 - (1 - \alpha + 0)] + (1 - \alpha)\alpha[1 - (0 + 1)] + \alpha^2[1 - (1 - \alpha + 1)] = (1 - \alpha)^2.
\]

If, on the other hand, negotiations are private, to alleviate the hold up problem, the buyer exercises her ability to act confidentially and strictly randomizes over the order of the negotiations. The following constitutes an efficient equilibrium: the buyer picks each seller to be the first with probability \(\frac{1}{2}\), and sellers propose (unique) prices \(P_1 = P_2 = \frac{1}{1+\alpha}\). Note that any unilateral attempt to charge a higher price, \(P' \in (\frac{1}{1+\alpha}, 1]\), by any seller would result in a sale only if approached second, and would yield a strictly lower payoff since \(\frac{1}{2}P' < \frac{1}{1+\alpha}\).

In the general analysis, we show that there is no inefficient equilibrium under exploding offers. The buyer’s ex ante payoff is \(\pi^X(b) = 1 - 2\alpha\frac{1}{1+\alpha} = \frac{1-\alpha}{1+\alpha} > (1-\alpha)^2 = \pi^X(b)\), revealing that the buyer strictly prefers private negotiations.

Next, consider open-ended offers. If negotiations are public, being a first-mover, seller 1 charges \(P_1 = 1\) since he knows that the rival will accommodate by setting \(P_2 = 0\). Of course, if seller 2 observes that the buyer has made the offer in the first negotiation, then he sets \(P_2 = 1\). The buyer’s ex ante payoff in this case is

\[
\pi^O(b) = (1 - \alpha)^2 + \alpha(1 - \alpha)[1 - (1 + 0)] + (1 - \alpha)\alpha[1 - (0 + 1)] + \alpha^2[1 - (1 + 0)] = (1 - \alpha)^2.
\]

Interestingly, \(\pi^O(b) = \pi^X(b)\). That is, under public negotiations, the buyer does not gain from the ability to compare prices and make an ex post purchasing decision. Compared to exploding offers, open-ended offers result in a less aggressive second seller but more aggressive first seller leading to the buyer’s indifference. If negotiations are private, then with open-ended offers, sellers effectively play a simultaneous pricing game. Clearly, \(P_1 + P_2 < 1\) cannot be part of an equilibrium in this game, since, with a guaranteed purchase by the buyer, one seller could slightly raise his price. Given \(\alpha < \frac{1}{2}\), a coordination equilibrium in which \(P_1 + P_2 = 1\) cannot result, either. To see this, note that given \(P_2\), seller 1 has two relevant choices: either set a coordinating price, \(P_1 = 1 - P_2\), and sell his unit with
probability 1, or set a noncoordinating price, $P_1 = 1$, and sell his unit with probability $1 - \alpha$ (only when the buyer is the proposer against the rival). For seller 1 to coordinate, it must be that $1 \times (1 - P_2) \geq (1 - \alpha) \times 1$, or equivalently $P_2 \leq \alpha$, i.e., the rival’s price must be low enough. But, for seller 2 to coordinate, $P_2$ must also be high enough, namely $P_2 \geq 1 - \alpha$, which, given $\alpha < \frac{1}{2}$, is not possible. Hence, there is no coordination equilibrium.

It is easy to verify that the unique noncoordination equilibrium is $P_1 = P_2 = 1$, since, given perfect complementarity, a unilateral deviation to a lower non-coordination price by any seller would not increase the probability of a sale. In our terminology, equilibrium trade in this case is inefficient because the buyer does not purchase any good if both sellers end up proposing. The buyer’s ex ante payoff is $\pi^O(b) = (1 - \alpha)^2$, implying that with open-ended offers, the buyer is indifferent between public and private negotiations.

To summarize, with perfect complements, if offers are exploding, the buyer strictly prefers private negotiations, whereas, if offers are open-ended, she is indifferent between public and private negotiations. Public negotiations always yield efficient trade, but private negotiations may yield inefficient trade when offers are open-ended.

**Perfect Substitutes.** Suppose now that the buyer needs at most one unit, i.e. $v_1 = v_2 = 1$, and $\Delta = -1$. If offers are open-ended, then the buyer is able to engender Bertrand competition between the two sellers resulting in $P_1 = P_2 = 0$, irrespective of whether the negotiations are public or private.

If offers are exploding, however, the buyer strictly prefers private negotiations. To see this, consider public negotiations and suppose, without loss of generality, that the buyer visits seller 1 first. In this case, $P_2$ is relevant only when the buyer has not already purchased the good from seller 1, in which case seller 2, observing the trade history, charges the monopoly price $P_2 = 1$. This implies that the buyer’s expected payoff from the second negotiation is $\alpha(1 - 1) + (1 - \alpha)(1 - 0) = 1 - \alpha$. Anticipating this, seller 1 sets $P_1$ such that $1 - P_1 = 1 - \alpha$, or equivalently $P_1 = \alpha$, guaranteeing a sale. In contrast to public negotiations, private negotiations lead to Bertrand pricing, $P_1 = P_2 = 0$, because, not observing the rival’s price or order, each seller tries to undercut the rival’s price. In the general analysis, we show that there may also exist a monopoly pricing equilibrium in which $P_1 = P_2 = 1$ and trade is inefficient, but with perfect substitutes Bertrand prices emerge as the unique equilibrium.

To summarize, with perfect substitutes, if offers are exploding, the buyer strictly prefers private negotiations, whereas, if offers are open-ended, she is indifferent between public and
private negotiations. Trade is efficient, regardless of offer or negotiation types.

Though insightful, the motivating examples are special in that they do not allow for heterogenous bargaining powers across sellers or intermediate degrees of substitutability between goods, both of which can affect efficiency of trade as well as the buyer’s preference for confidentiality, necessitating a general analysis.

3 General Analysis

Consistent with the motivating examples, we start the general analysis with complements and then consider substitutes. Since it is quite intuitive and also clear from our proofs that the buyer will make a price offer of 0 in every equilibrium, we only report sellers’ prices in our results below. Also, unless otherwise stated, the buyer is assumed to visit $s_i$ first under public negotiations.

3.1 Complements

Suppose that goods are complements, $\Delta > 0$, and that offers are exploding, $D = X$. Proposition 1 broadly confirms the insights from our motivating example with perfect complements.

**Proposition 1.** Consider complementary goods, $\Delta > 0$, and exploding simple price offers. Then,

(a) equilibrium is efficient under both public and private negotiations, and it has these pairs of seller prices, respectively:

$$P^X(s_i) = 1 - v_j - \alpha_j \Delta \text{ and } P^X(s_j|\phi_1 = 1) = 1 - v_i;$$

and

$$P^X(s_i) = v_i + \frac{1 - \alpha_j}{1/(\alpha_i + \alpha_j)} \Delta \text{ for } i = 1, 2.$$  

Under private negotiations, the buyer visits $s_i$ first with probability $\sigma^X_i \in [1 - \frac{P^X(s_i)}{1-v_j}, \frac{P^X(s_i)}{1-v_i}]$.

(b) The buyer strictly prefers private negotiations.

Under public negotiations, because trade history is perfectly observed, sellers charge prices that induce an efficient trade in equilibrium. To understand their prices in (1), note
that following the purchase of good $i$, the buyer is willing to pay $1 - v_i$ for good $j$, which is what seller $j$ proposes. Anticipating that the buyer’s reservation payoff from purchasing good $j$ alone is $(1 - \alpha_j)v_j$, and that upon acquiring good $i$, the buyer expects to pay $\alpha_j(1 - v_i)$ in the downstream negotiation, seller $i$ proposes a price, $P^X(s_i)$, that solves $1 - P^X(s_i) - \alpha_j(1 - v_i) = (1 - \alpha_j)v_j$.

A closer look at sellers’ prices in (1) reveals that the buyer is subject to a holdup problem in the second negotiation because seller $j$ ignores any previous payment by the buyer, and that seller $i$ partially covers the cost of this holdup to induce a sale in the first negotiation because $P^X(s_i) - (1 - v_j) = -\alpha_j \Delta < 0$. Under private negotiations, the buyer alleviates the holdup problem by strictly randomizing over the order of negotiations so that no seller could assume to be the second with certainty in equilibrium. Indeed, it is easily verified that $P^X(s_j) < 1 - v_i$ and $P^X(s_i) + P^X(s_j) < P^X(s_i) + P^X(s_j|\phi_1 = 1)$. These facts also explain why, despite equilibrium randomization over the negotiation sequence, trade is still efficient under private negotiations and why the buyer strictly prefers private negotiations as recorded in part (b) of Proposition 1.10 Note, however, that keeping negotiations private does not completely solve the holdup problem because $P^X(s_i) + P^X(s_j) > 1$. That is, the buyer may receive a negative ex post payoff if both sellers end up proposing.11

Our results in Proposition 1 largely parallel those of Noe and Wang (2004). These authors also address the issue of confidentiality in negotiations, but with symmetric sellers and symmetric goods, as well as with a restriction to efficient equilibrium and exploding offers. Proposition 1 shows that equilibrium efficiency is obtained without loss of generality. The main difference from their results is that for weak complements, they find that the buyer strictly prefers public negotiations. This difference can be attributed to the assumption regarding the bargaining protocols. In particular, our one-shot random-proposer bargaining within each negotiation would be payoff-equivalent to Noe and Wang’s two-stage alternating offer bargaining if the seller –not the buyer – made the first offer. Although, in any alternating-offer bargaining protocol, the assumption about the first proposer is not innocuous and could be context-dependent, in this case we have proven that the buyer would receive a higher payoff and thus never prefer public negotiations if the sellers made

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10Although our model is probably too stylized to fully explain Disney or Duke’s land acquisitions, it does appear that both of them made the payments as the deal for each parcel is struck, resembling our exploding offers. Thus, Proposition 1 is consistent with their strict incentive to keep negotiations private.

11Of course, the buyer’s ex ante payoff is still nonnegative.
the first offers.\textsuperscript{12}

While, under public negotiations, the buyer is completely indifferent to the negotiation sequence, some insight can be gained from the buyer’s equilibrium mixing under private negotiations. For instance, for perfect complements and a fixed $\alpha_j$, it follows that $\sigma_i^X \rightarrow 0$ as $\alpha_i \rightarrow 1$. That is, the buyer almost surely leaves the more powerful seller to the end. Although, by doing so, she may appear to subject herself to a more severe holdup problem, it is the weaker seller who is willing to give a larger discount in the first negotiation because as $\alpha_i \rightarrow 1$, we have $P^X(s_j) \rightarrow 0$.

Next, we turn to open-ended offers, $D = O$. Under public negotiations, this means that the buyer obtains a price quote from the upstream seller, and makes all her purchasing decisions after bargaining in the downstream. This also means that the downstream seller’s price can only depend on the identity of the upstream proposer, $\rho_1$, as opposed to the trade outcome, $\phi_1$. Under private negotiations, sellers essentially play a simultaneous pricing game, which, given complements, may or may not result in coordinated prices in equilibrium. The following proposition fully characterizes the equilibrium with open-ended offers (recalling, again, that the buyer always proposes a price of 0).

**Proposition 2.** Consider complementary goods, $\Delta > 0$, and open-ended simple price offers. Then,\

(a) equilibrium is efficient under public negotiations, and it has these seller prices:

$$P^O(s_i) = 1 - v_j \quad \text{and} \quad P^O(s_j|\rho_1) = \begin{cases} v_j & \text{if } \rho_1 = s_i, \\ 1 - v_i & \text{if } \rho_1 = b. \end{cases}$$

(b) Under private negotiations, equilibrium seller prices are given by:

$$\begin{align*}
(P^O(s_i), P^O(s_j)) = & \begin{cases}
(1 - v_j, 1 - v_i) & \text{if } \sum_k \alpha_k (1 - v_k) < \Delta, \\
(1 - v_j, 1 - v_i) & \text{if } \sum_k \alpha_k (1 - v_k) \geq \Delta \quad \text{and} \\
(1 - P^O(s_j), P^O(s_j)) & \text{for } P^O(s_j) \in \Omega_j' \quad \text{if } \alpha_k (1 - v_k) \leq \Delta \quad \text{for all } k, \\
(1 - P^O(s_j), P^O(s_j)) & \text{for } P^O(s_j) \in \Omega_j'' \quad \text{if } \alpha_k (1 - v_k) > \Delta \quad \text{for some } k,
\end{cases}
\end{align*}$$

where $\Omega_j' \equiv [v_j + \Delta - \alpha_i (1 - v_i), v_j + \alpha_j (1 - v_j)]$, $\Omega_j'' \equiv [v_j, v_j + \Delta]$, and $k = 1, 2$.

\textsuperscript{12}A proof of this claim is available upon request. The intuition is that with complements, upon a failure of the first negotiation, the buyer can realize a positive payoff from the second negotiation by rejecting the supplier’s offer. If, instead, the buyer is the first proposer, she would realize a positive payoff in the second negotiation only if she discloses the failure of the first negotiation.
(e) The buyer weakly prefers private negotiations if \( \sum_k \alpha_k (1 - v_k) \geq \Delta \), but she is indifferent between public and private negotiations if \( \sum_k \alpha_k (1 - v_k) < \Delta \).

Part (a) of Proposition 2 is best understood in conjunction with Proposition 1. Comparing (3) with (1), it is evident that seller \( j \) in the downstream prices less aggressively under open-ended offers to accommodate \( i \)'s offer; and anticipating this behavior, seller \( i \) demands a higher price than he does under exploding offers, i.e., \( P^O(s_i) > P^X(s_i) \).\(^{13}\) It is also worth remarking that unlike with exploding offers, the buyer never receives a negative ex post payoff with open-ended offers.

To understand part (b), note that under private negotiations each seller chooses between charging a (moderate) coordination price \( P^O(s_i) = 1 - \overline{P}^O(s_j) \), thereby guaranteeing a sale, and charging a (aggressive) noncoordination price \( P^O(s_i) = 1 - v_j \), thereby selling only if the buyer proposes in the other negotiation. Clearly, the latter is a more profitable option if the rival is less powerful against the buyer, and this is exactly what part (b) shows: for sufficiently small \( \alpha_k \)'s, only the noncoordination equilibrium exists, whereas for sufficiently large \( \alpha_k \)'s, only the coordination equilibria exist. For instance, in our motivating example with perfect complements, we had \( \alpha_1 = \alpha_2 < \frac{1}{2} \), and \( v_1 = v_2 = 0 \) (so \( \Delta = 1 \)), which, from (4), result in a unique inefficient equilibrium with \( P^O(s_1) = P^O(s_2) = 1 \). Note also that all else equal, a coordination equilibrium is less likely to occur as goods become stronger complements, i.e., a larger \( \Delta \). In particular, for perfect complements, there always exists a noncoordination equilibrium. The intuition is that strong complements require the buyer to purchase both goods, and knowing this, each seller targets the buyer’s joint valuation in pricing his product.

Further inspecting equilibrium prices in (4), it is true that trade is efficient at a coordination equilibrium and inefficient at a noncoordination equilibrium.\(^{14}\) It is also true, and proved in the appendix (Claim A1), that when both coordination and noncoordination equilibria exist for some (intermediate) parameter values, the buyer is weakly better off under coordination equilibrium, because the sellers’ prices are lower. In contrast to exploding offers, the order of negotiations is inconsequential when offers are open-ended. However, it is still the sellers’ strategic uncertainty under private negotiations that leads to low coordination prices. We show that the buyer’s ex ante payoff under public negotiations

\(^{13}\)Indeed, it is easy to verify that public negotiations lead to a first-mover advantage under open-ended offers, and a second-mover advantage under exploding offers.

\(^{14}\)Because, at a noncoordination equilibrium, both goods are purchased only if the buyer proposes in at least one negotiation.
turns out to be equal to her payoff under the inefficient noncoordination equilibrium. As a result, the buyer weakly prefers private negotiations whenever coordination equilibrium emerges; otherwise she is indifferent between public and private negotiations, as stated in part (c). This implies that with open-ended offers, the buyer is more likely to hold private negotiations as sellers become more powerful and/or goods become weaker complements.\textsuperscript{15}

3.2 Substitutes

Suppose that goods are substitutes, $\Delta < 0$. When offers are open-ended, it is intuitive that the buyer’s ability to compare prices will induce a Bertrand competition between sellers and result in an efficient trade, regardless of the negotiations being public or private. When offers are exploding, however, the upstream seller does not face a head-to-head competition by the downstream seller, and thus he is likely to raise his price above Bertrand. Similar to complements, private negotiations can foster a more fierce competition with exploding offers by creating strategic uncertainty about the buyer’s decision. This intuition for substitutes are formalized in,

**Proposition 3.** Consider substitute goods, $\Delta < 0$.

(a) If simple price offers are open-ended, then under both public and private negotiations, equilibrium is efficient and has Bertrand prices:

$$ P^O(s_i) = P^O(s_i) = 1 - v_j. $$

(b) If simple price offers are exploding, then

- under public negotiations, equilibrium is efficient and has these seller prices:

$$ P^X(s_i) = 1 - v_j - \alpha_j \Delta \text{ and } P^X(s_j|\phi_1 = 1) = 1 - v_i; $$

- under private negotiations, there always exists an efficient equilibrium with Bertrand prices as in part (a); but there may also exist an inefficient equilibrium with monopoly prices $P^X(s_i) = v_i$, where the buyer strictly randomizes over the order of negotiations;

\textsuperscript{15}A corollary to Proposition 2 is that all else equal, equilibrium is likely to be inefficient under private negotiations as the buyer gains bargaining power against sellers. This may partially explain why the European and the U.S. antitrust authorities are increasingly concerned about large industrial buyers such as supermarkets, and “buyer groups” in general (e.g., Inderst and Mazzarotto, 2008; Mathewson and Winter, 1997).
the buyer weakly prefers private negotiations, which becomes strict if no inefficient equilibrium exists.

Part (a) simply confirms that open-ended offers facilitate a Bertrand competition. Part (b) implies that when offers are exploding, public negotiations do not produce Bertrand competition. In particular, because the buyer cannot directly compare prices before purchasing, and because seller \( j \) can perfectly observe the trade history, seller \( i \) charges a price higher than Bertrand.\(^{16} \) The buyer can, however, restore Bertrand competition by holding private negotiations: conjecturing that his rival sets a Bertrand price and thus guarantees a sale, it is best for a seller to charge his Bertrand price as well. The potential cost of private negotiations to the buyer is that sellers’ posting their “monopoly” prices may form another equilibrium, resulting in an inefficient trade. Intuitively, if seller \( i \) expects the rival to set his monopoly price \( v_j \) and believes that his chances of being approached second are sufficiently high, then it is optimal for him to charge his monopoly price \( v_i \) too.\(^{17} \) Nevertheless, we show that the buyer’s payoff at this monopoly pricing equilibrium is the same as her payoff under public negotiations. Since the buyer’s payoff from Bertrand prices is strictly higher, she prefers private negotiations when offers are exploding. To glean some insight into the conditions under which the inefficient equilibrium exists, we consider two special cases.

**Corollary 1.** With identical substitutes, \( v_1 = v_2 > \frac{1}{2} \), and exploding simple price offers, there is no inefficient equilibrium under private negotiations.

Corollary 1 implies that with identical substitutes, the buyer strictly prefers private negotiations when offers are exploding. A special case of identical substitutes is the case of perfect substitutes as assumed in the motivating example. Corollary 1 also implies that for an inefficient equilibrium with proper mixing to exist, the buyer must possess different valuations for the goods. We next demonstrate this point.

**Corollary 2.** Let \( v_1 \in \left(\frac{1}{2}, 1\right) \), \( v_2 = 1 \), and \( 0 < \alpha_2 < 1 - v_1 < \tilde{\alpha}(v_1) < \alpha_1 < 1 \), where\(^{18} \)

\[
\tilde{\alpha}(v_1) = \frac{2v_1 - 1 + \sqrt{1 + 4v_1}}{2v_1(2 - v_1)} (1 - v_1).
\]

Consider private negotiations with exploding offers.

\(^{16}\)It is interesting to note that \( P_X(s_i) \) and seller \( i \)'s expected payoff are increasing in \( j \)'s bargaining power: as \( \alpha_j \) gets larger, the buyer’s reservation payoff from obtaining only good \( j \) diminishes, improving \( i \)'s bargaining position. A similar point has also been made by Marx and Shaffer (2010), who study sequential contracting with public and contingent payments.

\(^{17}\)As is evident from the proof, in such an equilibrium, the buyer always rejects the upstream supplier’s offer, but accepts the downstream’s whenever she has not acquired the upstream good by proposing a price of 0 herself.
Then, in addition to the Bertrand pricing equilibrium, there also exists a monopoly pricing equilibrium, in which the buyer first negotiates with seller 1 with probability 

\[ \sigma_1^X \in (\sigma_1^{-1}, 1) \] 

where 

\[ \sigma_1^X = \frac{1-(1-\alpha_1)v_1}{1-(1-\alpha_1)^2v_1}. \]

### 3.3 Discussion

From Propositions 1-3, we can say that the buyer is (weakly) better off conducting private negotiations. Since, unlike public negotiations, private negotiations may result in an inefficient trade, our results may provide an explanation for the presence of mandatory disclosure laws mentioned in the introduction, and why such laws are likely to be constraining for the buyer.\(^{18}\) When these laws do not apply to the buyer, it appears that the extended return and cancelation policies such as the FTC’s cooling-off rule that essentially require sellers’ price offers to be open-ended are most likely to achieve efficiency for substitute goods because open-ended offers engender Bertrand competition. For the purchases of complementary goods, however, such policies may actually interfere with an efficient trade by giving sellers an incentive to price more aggressively. In fact, in light of Propositions 1 and 2, it seems that an alternative policy of “all sales are final!” could be socially more desirable for complements (see also Proposition 6b on this point).

It is also worth noting that with substitutes, the buyer could eliminate the inefficient (monopoly pricing) equilibrium if she could credibly announce the order but not the outcome of negotiations.\(^{19}\) This way, she could prevent both sellers from simultaneously placing a positive probability on being the last in sequence. In our working paper, Krasteva and Yildirim (2010), we considered such “partially” private negotiations and found that while with substitutes the buyer (weakly) prefers partially private negotiations, with complements she prefers private negotiations.

\(^{18}\)This accords well with the Florida Statutes 373.139 stipulating that during real property acquisitions by the state’s water management districts “appraisal reports, offers, and counteroffers are confidential and exempt from the provisions of [sunshine law] until an option contract is executed...” (see, \url{http://www.leg.state.fl.us/Statutes}).

\(^{19}\)Such a negotiation scheme is not unrealistic. For instance, in July 2000, Motorola alleged that the state of Florida violated its own sunshine law by holding closed meetings with the rival Com-Net during a competitive procurement process for a public safety communications system. See, \url{http://urgentcomm.com/mag/radio_motorola_sues_florida/}

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4 Negotiating without Commitment to a Disclosure Policy

Up to this point, we have assumed that prior to visiting the sellers, the buyer can credibly commit to public or private negotiations. In this section we relax this assumption and let the buyer choose the amount of disclosure as she visits each seller. There are, however, a plethora of disclosure equilibria owing to different equilibrium beliefs that may be held by the sellers. To highlight the role of noncommitment, we show two such equilibria: one that generates private negotiations, coinciding with the commitment case, and the other that generates public negotiations.

Let $I_1 = \{s\}$ and $I_2 = \{s, \phi_1, P_1\}$ denote the buyer’s information sets prior to the upstream and downstream negotiations, respectively, where $s$ represents the negotiation sequence, and as before, $P_1$ denotes the upstream price and $\phi_1$ the purchasing decision. Clearly, the buyer can at most reveal the sequence, $s$, in the upstream whereas in the downstream, she can reveal either the full history (namely, $P_1$ if the offer is open-ended, and both $\phi_1$ and $P_1$ if it is exploding), only the sequence, or nothing. Let $d_1 \in 2^{I_1}$ and $d_2 \in 2^{I_2}$ be the buyer’s respective disclosure choices in each negotiation. Specifically, the extensive form game proceeds as follows. First, all players observe the deadline policies, $D$. Next, the buyer chooses the negotiation sequence. Then, she decides on her disclosure action $d_i$ in negotiation $i$, and the bargaining over the price begins. Proposition 4 is our main result in this section.

**Proposition 4.** Under non-commitment, there exist no-disclosure equilibria, $d_1^* = d_2^* = \{\}$, for both complements and substitutes. But, there also exist full-disclosure equilibria, $d_1^* = \{s\}$ and $d_2^* = I_2$, for both.

The intuition behind Proposition 4 is most transparent for exploding offers. With complements, recall from Proposition 1 that the buyer strictly prefers private negotiations. In an efficient equilibrium, any disclosure in the second negotiation subjects the buyer to a more severe holdup problem. In addition, a disclosure of sequence in the first negotiation does not change that seller’s price and so the buyer has no incentive to deviate from private negotiations. However, if each seller expects full disclosure in both negotiations, and a failure to do so by the buyer would lead each to believe that he is negotiated with second, then public negotiations also arise in equilibrium. A similar line of reasoning also holds for substitutes. In an efficient equilibrium, the buyer obtains the Bertrand prices under
private negotiations and thus cannot gain by disclosing the trade history. It is, however, also an equilibrium for the buyer to fully disclose in each negotiation if the sellers hold off-equilibrium beliefs to be the first in the negotiation sequence upon observing deviation from full disclosure by the buyer.

Proposition 4 is significant in two respects. First, since the buyer’s preferred mode of disclosure policy is sustained in equilibrium, the lack of commitment does not necessarily diminish her payoff. Second, since full disclosure is also sustained in equilibrium, mandatory disclosure laws may be less constraining for those buyers who lack credible means to commit to a disclosure policy.

5 Equilibrium Offer Deadlines

A major implication of our analysis so far is that the sellers’ deadline policies can have a significant impact on their equilibrium prices and payoffs. We have, however, assumed deadline policies to be exogenous and uniform across sellers. This assumption may be reasonable if these policies are industry standards or imposed by certain government regulations such as the FTC’s cooling-off rule alluded to above. In this section, we show that uniform deadline policies may emerge as sellers’ equilibrium choices in a “deadline game” that augments our basic negotiation game; in particular, before the buyer chooses between public and private negotiations, sellers simultaneously commit to making exploding or open-ended offers at the deadline stage, i.e., stage 0.

Our analysis in Section 3 has already produced sellers’ payoffs for the two subgames of this deadline game. For an equilibrium analysis, however, it is necessary to determine their equilibrium payoffs in the remaining two subgames with different deadlines. For substitutes, it is obvious that facing one open-ended and one exploding offer, the buyer will prefer to negotiate first with the seller whose offer is open-ended, allowing her to uniquely engender Bertrand prices in equilibrium. For complements, finding the equilibrium of the subgames with different deadlines, especially under private negotiations, is complicated and thus presented in Lemma A1 in the appendix. Armed with all the sellers’ payoffs at the deadline stage, we reach,

Proposition 5. In the deadline game, it is an equilibrium for both sellers to adopt the same deadline policy.

That is, both $D = X$ and $D = O$ can emerge as equilibrium deadlines. To gain
some insight, note that for substitutes, if the rival chooses open-ended offers, Bertrand competition ensues regardless of the buyer’s negotiation choice, and thus it is a best response for a seller to also choose open-ended offers. If, on the other hand, the rival makes exploding offers, a deviation to open-ended offers by a seller would again induce Bertrand, rendering such deviation unprofitable. For complements, given open-ended offers by the rival, a seller would not have a strict incentive to deviate to exploding offers: anticipating private negotiations, if he is approached first, then he has to reduce his price below what he would charge under an open-ended offer so as to alleviate the buyer’s hold up problem; and if he is approached second, then, given aggressive pricing by the rival under an open-ended offer, he can only accommodate and, once again, set a low price. A similar reasoning also explains the equilibrium with exploding offers.

Proposition 5 implies that the assumption of exploding offers maintained in the extant literature, e.g., Cai (2000), and Noe and Wang (2004) may arise as the equilibrium sellers’ choices, though open-ended offers may also arise in equilibrium.

It is also instructive to compare more explicitly the buyer’s and sellers’ deadline preferences across $D = X$ and $D = O$. For substitutes, the comparison is straightforward: under public negotiations, sellers prefer exploding offers to avoid Bertrand competition, but the buyer prefers open-ended offers to generate one; under private negotiations, both sellers and the buyer are indifferent between $D = X$ and $D = O$ because Bertrand competition results regardless. For complements, the comparison is somewhat more involved and thus recorded in,

**Proposition 6.** Consider complements, $\Delta > 0$. Then,

a) under public negotiations, the buyer is indifferent between exploding and open-ended offers. However, each seller $i$ strictly prefers open-ended to exploding offers if and only if $\sigma_i^O > 1 - \sigma_i^X$.

b) Under private negotiations, the buyer and each seller strictly prefer exploding offers if $\sum_k \alpha_k (1 - v_k) < \Delta$. If $\alpha_i (1 - v_i) > \Delta$ for some $i$, then the buyer strictly prefers open-ended offers while at least one seller strictly prefers exploding offers.

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To be more precise, this indifference is true if there is no inefficient monopoly equilibrium under private negotiations. Otherwise, if the inefficient monopoly pricing equilibrium is anticipated, then the sellers weakly prefer $D = X$ whereas the buyer strictly prefers $D = O$. 

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The reason behind the buyer’s indifference in part (a) is as explained in the motivating example: while with open-ended offers there is a first-mover advantage for sellers, with exploding offers there is a second-mover advantage. Thus, if seller $i$ is more likely to be the first under open-ended offers than to be the second under exploding offers, he prefers open-ended offers. Under private negotiations, recall from Proposition 2 that when offers are open-ended, there is a unique inefficient (non-coordination) equilibrium if $\sum_k \alpha_k (1 - v_k) < \Delta$. Thus, under this parameter condition, the buyer prefers exploding offers, yielding a unique efficient outcome. More interestingly, sellers also benefit from this increased efficiency since, while they have to lower prices when offers are exploding, they are more likely to sell their products. If $\alpha_i (1 - v_i) > \Delta$ for some $i$, then open-ended offers also lead to efficient equilibrium. But since open-ended offers, unlike exploding offers, never produce a negative ex post payoff, the buyer strictly prefers open-ended offers.

6 Sellers’ Incentives to Disclose Information

Being the common player in both negotiations, we have assumed that the buyer controls the information regarding the trade history. It is, however, conceivable that sellers may also have incentives to share this information if doing so is in their best interest and not illegal per se. Understanding these incentives is important because a buyer who hopes to keep negotiations confidential may simply fail to do so. Indeed, as alluded to in the introduction, there is now a rapidly growing literature on “consumer privacy” dealing with this very issue, albeit in a complementary context focusing on the informational link between consumer demands and assuming a fixed negotiation sequence. Here, we investigate how seller competition through disclosure policies can affect consumer privacy when the negotiation sequence is chosen by the consumer. To do so, we modify our basic setup in the following way: upon observing the offer deadlines, sellers simultaneously commit to their disclosure policies, i.e. whether to disclose ($\text{dis}$) or not to disclose ($\overline{\text{dis}}$) their dealings with the buyer. Once these policies are observed, the buyer negotiates with the sellers in her preferred order.

To succinctly illustrate our point here, we only consider exploding offers, $D = X$, and assume symmetry, i.e., $v_i = v$ and $\alpha_i = \alpha$ for $i = 1, 2$. From Proposition 1 and Corollary 1, it then follows that the buyer strictly prefers private negotiations for both complements and substitutes. To secure private negotiations in the presence of information sharing by

\footnote{Disclosure policies may simply be the privacy contracts between the seller and consumers.}
the sellers, however, the buyer needs to start the negotiations with the non-disclosing seller. When indifferent between a disclosing and nondisclosing seller, we thus assume that the buyer opts to begin with the latter to preserve her privacy. Moreover, when both sellers disclose, resulting in public negotiations, the buyer is equally likely to approach each seller first. The following proposition is our main finding in this section.

**Proposition 7.** Consider the disclosure game described in this section and focus on pure strategy equilibria. Then,

(a) with substitutes, there are two equilibria: \((\text{dis}, \text{dis})\) and \((\overline{\text{dis}}, \overline{\text{dis}})\).

(b) With complements, there is only one equilibrium: \((\text{dis}, \text{dis})\).

An important implication of Proposition 7 is that both sellers disclosing is an equilibrium for both substitutes and complements. Thus, the buyer may have to perform public negotiations even though she strictly prefers private negotiations. The intuition is that given the rival’s disclosure, the seller of substitutes also discloses to avoid Bertrand competition, whereas the seller of complements discloses to have a chance to be the last in the negotiation sequence. Another implication of Proposition 7 is that while with substitutes the buyer can still execute private negotiations in equilibrium, with complements she can only execute public negotiations. Overall, the message of Proposition 7 is that the sellers’ incentives to disclose information may intervene with the buyer’s desire for privacy.

### 7 Concluding Remarks

In closing, we mention two future research avenues. The first one is to draw more upon the literature on bilateral contracting with externalities. In particular, the externality between the sellers in our model comes only from the buyer’s demands, which is likely if one has in mind pieces of land, products, political endorsements, and workers in different production lines. However, if one has in mind two skilled workers, say two faculty members, who may enjoy some direct benefit from their interaction, then our analysis has to be modified to capture two-sided externalities. Some recent papers, e.g., Genicot and Ray (2006), and

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22Lemma A2 shows that this tie-breaking rule indeed guarantees efficient trade.

23All we need for the result is that each seller has a strictly positive probability to be the first, not necessarily \(\frac{1}{2}\).

24In fact, with complements, disclosure strictly dominates nondisclosure for each seller.
Moller (2007), endogenize the timing of the bilateral negotiations with such externalities, but all assume contracts and agreements to be public. The second research avenue is to allow sellers to use a richer offer space (e.g. stochastic offers) compared to a simple price assumed here, and explore the extent to which this can elicit the buyer’s information under private negotiations. We believe that the buyer would still prefer private negotiations for a wide range of parameters in order to earn information rents. A full analysis, however, awaits further investigation.

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Appendix

Proof of Proposition 1. Suppose $\Delta > 0$ and $D = X$. To prove part (a), consider public negotiations. Note that the buyer will propose a price of 0 in the second negotiation regardless of the trade history. Moreover, since her past payment is ignored by the seller in the second negotiation, she will also propose a price of 0 in the first negotiation. To determine seller prices, assume that $s_i$ is approached first. If $\phi_1 = 0$, then $P^X(s_j|\phi_1 = 0) = v_j$, yielding an expected payoff of $(1 - \alpha_j)v_j$ to the buyer. If, on the other hand, $\phi_1 = 1$, then $P^X(s_j|\phi_1 = 1) = 1 - v_i$, in which case she acquires both goods. Anticipating these two continuation outcomes, $s_i$ sets the highest price subject to $1 - P^X(s_i) - \alpha_j(1 - v_i) \geq (1 - \alpha_j)v_j$, whose unique solution is $P^X(s_i) = 1 - v_j - \alpha_j \Delta$. The buyer then acquires both goods with probability 1.

The buyer’s ex ante payoff from negotiating with seller $i$ first is

$$\pi^X(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i.$$  \hfill (A-1)

which, by re-labeling, is equal to her expected payoff from negotiating with seller $j$ first, leaving her completely indifferent to the order.

Next, consider private negotiations. It is again clear that $P^X(b) = 0$ in equilibrium. Moreover, $P^X(s_i) \in [v_i, 1 - v_j]$ for both sellers, because $s_i$ guarantees a sale for any price
less than or equal to $v_i$, while his probability of selling for any price strictly above $1 - v_j$ is 0. Note that when approached first, $s_i$ sells his good with certainty if and only if $1 - \pi^X(s_i) - \alpha_j \pi^X(s_j) \geq (1 - \alpha_j)v_j$, or equivalently $\pi^X(s_i) \leq 1 - v_j - \alpha_j[\pi^X(s_j) - v_j]$. Thus, for all price offers to be accepted irrespective of the order, we must have $\pi^X(s_i) \leq 1-v_j - \alpha_j[\pi^X(s_j) - v_j]$ for both sellers. Since an efficient trade is obtained in this region, each seller will choose a price that binds this inequality. Solving for prices, the unique equilibrium price candidates are $\pi^X(s_i) = v_i + \frac{1-\alpha_i}{1-\alpha_i \alpha_j} \Delta$ as stated in (2). It is straightforward to show that these prices are part of an equilibrium if and only if the buyer visits $s_i$ with probability $\sigma_i^X \in [1 - \frac{\pi^X(s_i)}{1-v_j}, \frac{\pi^X(s_i)}{1-v_i}]$. Otherwise, there is a profitable deviation to $1 - v_j$ for $s_i$ or $1 - v_i$ for $s_j$. Since $1 - \frac{\pi^X(s_i)}{1-v_j} \leq \frac{\pi^X(s_i)}{1-v_i}$ holds for all parameter values, an equilibrium with these prices always exists. To show the uniqueness of these equilibrium prices, suppose that $\pi^X(s_i) > 1 - v_j - \alpha_j[\pi^X(s_j) - v_j]$ for some $s_i$. Observe that the only other equilibrium candidate lies in the region where $\pi^X(s_i) > 1 - v_j - \alpha_j[\pi^X(s_j) - v_j]$ for both sellers. For such prices, each seller sells only if he is approached second and the buyer proposes in the previous negotiation, leading each seller to set $\pi^X(s_i) = 1 - v_j$. For these prices to be an equilibrium, $s_i$ should not have an incentive to deviate to a price $\pi^X(s_i) = (1 - \alpha_j)(1 - v_j) + \alpha_j v_i$, which is accepted with probability $\sigma_i + (1 - \sigma_i)(1 - \alpha_j)$. This deviation is unprofitable to $s_i$ if and only if $\sigma_i \leq \frac{\alpha_i(1-\alpha_j)\Delta}{1-v_j-\alpha_j^2 \Delta}$. Removing the deviation incentives from both sellers requires that $\sum_k \frac{\alpha_i (1-\alpha_k) \Delta}{1-v_k-\alpha_k^2 \Delta} \geq 1$. It is straightforward to check that this condition is violated for $v_1 = v_2 = 0$ and that the left-hand side of this inequality is decreasing in $v_1$ and $v_2$. Therefore, there is no $\sigma_i$ that can support $\pi^X(s_i) = 1 - v_j$ in equilibrium.

To prove part (b), note that given efficient trade, the buyer’s ex ante payoff under private negotiations is $\pi^X(b) = 1 - \alpha_i \pi^X(s_i) - \alpha_j \pi^X(s_j)$. Using seller prices in (2) and simplifying terms, we obtain

$$\pi^X(b) - \pi^X(b) = \frac{\alpha_i \alpha_j (1 - \alpha_i)(1 - \alpha_j)}{1 - \alpha_i \alpha_j} \Delta > 0.$$  

Proof of Proposition 2. Suppose $\Delta > 0$ and $D = O$. To prove part (a), consider public negotiations, and assume that $s_i$ is approached first. It is straightforward to show that $s_j$’s best response to an upstream price offer, $P_i$ is:

$$P^*(s_j|P_i) = \begin{cases} 1 - v_i & \text{if } P_i \leq v_i \\ 1 - P_i & \text{if } v_i < P_i \leq 1 - v_j \\ v_j & \text{if } P_i > 1 - v_j \end{cases}$$ (A-2)

Clearly, $P^O(b) = 0$ in any equilibrium. To determine equilibrium $P^O(s_i)$, note that if $s_i$
anticipates $j$ to propose in the downstream negotiation, then according to (A-2), $s_i$ earns a nonnegative payoff whenever $P(s_i) \leq 1 - v_j$, leading him to post $P(s_i) = 1 - v_j$. If, on the other hand, $s_i$ anticipates the buyer to propose in the downstream negotiation, then conjecturing $P(b) = 0$, he sets a price such that $1 - P(s_i) - 0 = v_j$, or $P(s_i) = 1 - v_j$. Hence, $P^O(s_i) = 1 - v_j$. If $s_i$ proposes in the first negotiation so that $P_i = 1 - v_j$, then $P^O(s_j|P_1 = s_i) = v_j$ by (A-2). If, on the other hand, the buyer proposes in the first negotiation so that $P_i = 0$, then $P^O(s_j|P_1 = b) = 1 - v_i$. 

Given these equilibrium prices, the buyer purchases both goods with probability 1, yielding an expected payoff to the buyer:

$$\pi^O(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i. \quad (A-3)$$

By re-labeling $i$ and $j$, (A-3) implies that the buyer is completely indifferent to the negotiation sequence.

Next, consider private negotiations, whereby sellers play a simultaneous pricing game. Hence, equilibrium supply prices lie at the intersection of their best response functions. Noting that $P^O(b) = 0$ in any equilibrium and exhausting several cases, $s_i$’s best response $P^*(s_i|P(s_j))$ to $j$’s price offer $P(s_j)$ is found to be:

$$P^*(s_i|P(s_j)) = \begin{cases} 1 - v_j & \text{if } P(s_j) \leq v_j \\ 1 - \overline{P}(s_j) & \text{if } v_j < P(s_j) \leq 1 - (1 - \alpha_j)(1 - v_j) \\ 1 - v_j & \text{if } 1 - (1 - \alpha_j)(1 - v_j) \leq P(s_j) \leq 1 - v_i \\ 1 - v_j & \text{if } P(s_j) > 1 - v_i \text{ and } \alpha_j(1 - v_j) \leq \Delta \\ v_i & \text{if } P(s_j) > 1 - v_i \text{ and } \alpha_j(1 - v_j) \geq \Delta. \end{cases} \quad (A-4)$$

Using (A-4), it is then easily verified that equilibrium seller prices, $P^O(s_i)$ are exactly as described in (4), proving part (b).

To prove part (c), we first prove the following claim.

**Claim A1.** If $\alpha_i(1 - v_i) \leq \Delta$ for all $i$, and $\sum_k \alpha_k(1 - v_k) \geq \Delta$, then the buyer is weakly better off in every efficient equilibrium.

**Proof.** Under the hypotheses of the Claim, seller prices at an efficient equilibrium satisfy: $P^O(s_i) + P^O(s_j) = 1$ and $P^O(s_j) \in [v_j + \Delta - \alpha_i(1 - v_i), v_j + \alpha_j(1 - v_j)]$. Together with $\alpha_i(1 - v_i) \leq \Delta$ and $\alpha_j(1 - v_j) \leq \Delta$ by hypothesis, it follows that $P^O(s_i) \geq v_i$ and $P^O(s_j) \geq v_j$. Next, note that the buyer’s expected payoffs at efficient and inefficient equilibria are
given, respectively, by: \( \pi^O(b|\text{eff.}) = (1-\alpha_i)(1-\alpha_j) + \alpha_i(1-\alpha_j)\tilde{P}^O(s_j) + \alpha_j(1-\alpha_i)\tilde{P}^O(s_i) \), and \( \pi^O(b|\text{ineff.}) = \pi^O(b|\text{eff.})|_{\tilde{P}^O(s_i)=v_i,\tilde{P}^O(s_j)=v_j} \). Hence, \( \pi^O(b|\text{eff.}) \geq \pi^O(b|\text{ineff.}) \). ■

To complete the proof of Proposition 2, note that for \( \sum_k (1-\alpha_k)v_k < \Delta \), only the inefficient equilibrium is possible under private negotiations, and it follows that \( \pi^O(b) = \pi^O(b|\text{ineff.}) \). For \( \sum_k \alpha_k(1-v_k) \geq \Delta \), both efficient and inefficient equilibria can arise under private negotiations, and by Claim A1, we have \( \pi^O(b|\text{eff.}) \geq \pi^O(b|\text{ineff.}) = \pi^O(b) \), proving part (c). ■

**Proof of Proposition 3.** Suppose \( \Delta < 0 \). When \( D = O \), Bertrand prices under which \( s_i \) charges \( 1-v_j \) form an equilibrium for both public and private negotiations: given that the rival sets his Bertrand price, a seller will realize a sale with a positive probability only if he also sets his Bertrand price. The uniqueness of these prices for public negotiations follows from a backwards induction similar to that in part (a) of Proposition 2. To see their uniqueness for private negotiations, suppose that in equilibrium, \( \overline{P}^O(s_i) > 1-v_j \) for both sellers. Since the buyer will purchase only one good, she must be indifferent between them. But then, each seller has a strict incentive to slightly undercut the rival.

To prove part (b), let \( D = X \). The derivation of prices under public negotiations is exactly the same as in part (a) of Proposition 1. Next, suppose that negotiations are private. We first argue that in equilibrium, either \( \overline{P}^X(s_i) = 1-v_j \) or \( \overline{P}^X(s_i) = v_i \) for all \( i \).

Note that in any equilibrium, \( \overline{P}^X(s_i) \leq v_i \). We exhaust several regions.

If \( \overline{P}^X(s_i) \leq 1-v_j \) for both sellers, then each sells with certainty, leaving \( \overline{P}^X(s_i) = 1-v_j \) the only price candidate in this region. Next, consider the region in which \( \overline{P}^X(s_i) \leq 1-v_j \) and \( \overline{P}_j(s_j) > 1-v_i \). Then, \( s_i \) sells with certainty independent of the order. As for \( s_j \), if he is approached second, his offer is rejected with certainty since upon purchasing good \( i \), the buyer’s marginal valuation for good \( j \) is \( 1-v_i \). If, on the other hand, \( j \) is approached first, his offer will be accepted only if \( \max\{v_j-\overline{P}^X(s_j), 1-\overline{P}^X(s_j)-\alpha_i\overline{P}^X(s_i)\} \geq v_i-\alpha_i\overline{P}^X(s_i) \). Since \( \overline{P}^X(s_j) > 1-v_i \) by hypothesis, this requires \( v_j - \overline{P}^X(s_j) \geq v_i - \alpha_i\overline{P}^X(s_i) \), or equivalently \( \overline{P}^X(s_j) \leq v_j - v_i + \alpha_i\overline{P}^X(s_i) \), which, by using \( \overline{P}^X(s_i) \leq 1-v_j \), implies \( \overline{P}^X(s_j) \leq 1-v_i \), contradicting \( \overline{P}^X(s_j) > 1-v_i \). This means that in this region, \( s_j \)’s offer is rejected with certainty, independent of the order, giving him a strict incentive to lower his price. Hence, prices such that \( \overline{P}^X(s_i) \leq 1-v_j \) and \( \overline{P}^X(s_j) > 1-v_i \) cannot be sustained in equilibrium.

Now, suppose that \( 1-v_j < \overline{P}^X(s_i) < v_i-v_j+\alpha_j\overline{P}^X(s_j) \) and \( \overline{P}^X(s_j) > 1-v_i \). We argue by the following two cases that no price pair in this region can be equilibrium candidates either, because the buyer always rejects \( s_j \)’s offer, giving \( j \) a strict incentive to lower his
Then, the buyer would purchase good $i$ if and only if $\max\{v_i - \overline{P}(s_i), 1 - \overline{P}(s_i) - \alpha_j \overline{P}(s_j)\} \geq v_j - \alpha_j \overline{P}(s_j)$, which, given the hypothesis, is equivalent to $v_i - \overline{P}(s_i) \geq v_j - \alpha_j \overline{P}(s_j)$ and thus satisfied. Since good $i$ is purchased with certainty, the buyer’s marginal valuation for good $j$ is $1 - v_i$, which is strictly less than $\overline{P}(s_j)$ by hypothesis. Hence, $s_j$’s offer will be rejected.

$s_i$ is approached second. In this case, $s_j$ sells his good in the first negotiation if and only if $\max\{v_j - \overline{P}(s_j), 1 - \overline{P}(s_j) - \alpha_i \overline{P}(s_i)\} \geq v_i - \alpha_i \overline{P}(s_i)$. Since $\overline{P}(s_j) > 1 - v_i$, we have $1 - \overline{P}(s_j) - \alpha_i \overline{P}(s_i) < v_i - \alpha_i \overline{P}(s_i)$. Therefore, in order for good $j$ to be purchased, it is necessary that $v_j - \overline{P}(s_j) \geq v_i - \alpha_i \overline{P}(s_i)$, or equivalently $\overline{P}(s_j) \geq \frac{v_i - v_j + \overline{P}(s_j)}{\alpha_i}$. Note, however, that because $v_i - v_j + \overline{P}(s_j) \geq 0$, we have $\overline{P}(s_j) \leq \frac{v_i - v_j + \overline{P}(s_j)}{\alpha_i}$, which, using the hypothesis, implies $\overline{P}(s_j) < \frac{v_i - v_j + \overline{P}(s_j)}{\alpha_i}$. Hence, $s_j$’s offer will be rejected.

Finally, let prices be such that $\overline{P}(s_i) > \max\{1 - v_j, v_i - v_j + \alpha_j \overline{P}(s_j)\}$ for both sellers. Consider $s_i$. Since $\overline{P}(s_i) > 1 - v_j$, an exact argument to that above reveals that if $s_i$ is approached first, the buyer will reject his offer with certainty. If, on the other hand, he is approached second, he successfully sells only if $j$ does not. (The latter happens whenever $s_j$ is the first to negotiate and makes an offer.) This leads $s_i$ to set the highest price, $\overline{P}(s_i) = v_i$.

Next, we find conditions under which the surviving price pairs $\{\overline{P}(s_i) = 1 - v_j\}$ and $\{\overline{P}(s_i) = v_i\}$ can each be sustained as an equilibrium. Since $s_i$ who charges $\overline{P}(s_i) = 1 - v_j$ is guaranteed to sell his good independent of the order, the best $s_j$ can do is to charge $\overline{P}(s_j) = 1 - v_i$. Hence, the price pair $\{\overline{P}(s_i) = 1 - v_j\}$ is always an equilibrium. As for the pair $\{\overline{P}(s_i) = v_i\}$, note that given $\overline{P}(s_j) = v_j$, $s_i$’s expected payoff from setting $\overline{P}(s_i) = v_i$ is $(1 - \alpha_j v_i)$, where $\alpha_j$ is the probability that the buyer visits $s_i$ first. But, for $\overline{P}(s_i) = v_i$ to be a best response for $s_i$, there are two possible deviations to be discouraged. The first is $\overline{P}(s_i) = 1 - v_j$, leading to a payoff of $1 - v_j$, and the second is $\overline{P}(s_i) = v_i - (1 - \alpha_j) v_j$, leading to a payoff of $[\sigma^X_i + (1 - \sigma^X_i) \alpha_j]_j$ [v_i - (1 - \alpha_j) v_j]. Hence, it must be that $(1 - \sigma^X_i) \alpha_j v_i \geq \max\{1 - v_j, [\sigma^X_i + (1 - \sigma^X_i) \alpha_j]_j [v_i - (1 - \alpha_j) v_j]\}$.

Letting $f^+ \equiv \max\{0, f\}$ and $f^+_{[k>0]} \equiv \{ f, \text{ if } k > 0 \}$, if $k \leq 0$, this inequality is satisfied if and only if $\sigma^X_i \in (\overline{\sigma}_i, 1)$, where $\overline{\sigma}_i \equiv 1 - \min\left\{ \left( \frac{\alpha_i (1 - \alpha_j) v_i}{v_j - (1 - \alpha_j) v_i} \right)_{[v_j - (1 - \alpha_i) v_i > 0]} (1 - \frac{1 - v_i}{\alpha_i v_j}) \right\}$ and
\[ \tilde{\sigma}_i^X \equiv \min \left\{ \left( \frac{\alpha_i(1-\alpha_j)v_j}{v_i-(1-\alpha_j)v_j} \right)_{|v_i-(1-\alpha_j)v_j|>0}, \left( 1 - \frac{1-v_j}{\alpha_jv_i} \right)^+ \right\}. \]

To complete the proof of part (b), under public negotiations, the buyer’s payoff is

\[ \pi^X(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i. \quad (A-5) \]

Under private negotiations, the buyer’s payoff at an efficient equilibrium (with Bertrand prices) is \( \pi^X(b_{\text{eff.}}) = \pi^X(b) + \alpha_i\alpha_j(-\Delta) > \pi^X(b) \). If the equilibrium under private negotiations is inefficient, then it is easily verified that \( \pi^X(b_{\text{ineff.}}) = \pi^X(b) \). The buyer thus weakly prefers private negotiations, which becomes strict if there is no inefficient equilibrium. ■

**Proof of Corollary 1.** Suppose that \( v_1 = v_2 > \frac{1}{2} \). Then, using the characterization in the proof of Proposition 3b and simplifying terms we obtain \( \tilde{\sigma}_i^X \equiv \min \left\{ \frac{1-\alpha_i}{2-\alpha_i}, \left( 1 - \frac{1-v}{\alpha_i v} \right)^+ \right\} \) and \( \tilde{\sigma}_i^X \equiv \min \left\{ \frac{1-\alpha_i}{2-\alpha_i}, \left( 1 - \frac{1-v}{\alpha_i v} \right)^+ \right\} \). Again, an inefficient equilibrium exists if and only if \( \tilde{\sigma}_i^X < \tilde{\sigma}_i^X \). Simple algebra shows that this condition is never satisfied if \( v_1 = v_2 > \frac{1}{2} \). ■

**Proof of Proposition 4.** Let \( B_i^j(d_i) \) denote \( s_i \)’s belief about the buyer’s disclosure to \( s_j \) given that he is disclosed \( d_i \). First we show that \( d_i^* = \{\} \) can be sustained as an equilibrium under substitutes. From Proposition 3 we know that Bertrand pricing is the unique equilibrium under \( E = O \). Since disclosure cannot result in lower prices, the buyer has no incentives to deviate in this case. Suppose now that \( E = X \). Similar to open-ended offers, if the buyer correctly conjectures the emergence of an efficient equilibrium with Bertrand prices, she will have no incentives to deviate from \( d_i^* = \{\} \). The inefficient equilibrium with \( d_i^* = \{\} \) and \( \bar{P}^X(s_i) = v_i \) can also be supported as a PBE. To see this, consider the buyer’s incentives to change her disclosure strategy. Suppose without loss of generality that \( s_j \) is approached second. Then, deviating to \( d_2 = \{\phi_1\} \) cannot be profitable for the buyer since \( s_j \) will optimally extract the remainder of the buyer’s surplus by setting \( P_j(s_j|\{\phi_1\}) = \left\{ \begin{array}{ll} 1-v_i & \text{if } \phi_1 = 1 \\ v_j & \text{if } \phi_1 = 0 \end{array} \right. \). Consider now a deviation to \( d_2 = \{P_i\} \). Let \( B_i^j(\{P_i\}) = \left\{ \begin{array}{ll} \{\} & \text{if } P_i \neq 1 - v_j - \alpha_j\Delta \\ s & \text{if } P_i = 1 - v_j - \alpha_j\Delta \end{array} \right. \), \( B_i^j(\{\}) = \{P_i\} \) and \( B_i^j(\{\}) = \{\} \).

Then, \( s_j \)’s equilibrium conjecture \( \phi_i^*(\{P_i\}) = \left\{ \begin{array}{ll} 1 & \text{if } P_i = \{0, 1 - v_j - \alpha_j\Delta\} \\ 0 & \text{otherwise} \end{array} \right. \) will result in \( P_j(s_j|\{P_i\}) = \left\{ \begin{array}{ll} 1-v_i & \text{if } P_i = \{0, 1 - v_j - \alpha_j\Delta\} \\ v_j & \text{if } P_i = \{0, 1 - v_j - \alpha_j\Delta\} \end{array} \right. \). Given this belief system, it is straightforward to verify that deviations \( d_2 = \{P_i\} \) and \( d_1 = \{s\} \) are not profitable since they leave the buyer’s expected payoff unchanged. Similarly, \( d_2 = \{s\} \) with \( B_i^j(\{s\}) = \{\} \) will result in \( \phi_i^*(\{s\}) = 1 - \alpha_i \). Then, from the proof of Proposition 3b) we know that
\( \sigma_j^X \) satisfies \((1 - \sigma_j^X) \alpha_i v_j \geq \max \{1 - v_i, [\sigma_j^X + (1 - \sigma_j^X) \alpha_i][v_j - (1 - \alpha_i)v_j]\}\) implying that \( P_j(s_j) = v_j \). This ensures that the buyer does not have incentives to deviate to \( d_2 = \{s\} \).

Therefore, we have shown that given substitute goods and \( E = X \), the inefficient equilibrium with \( d_i^* = \{\} \) and \( P_i(s_i) = v_i \) can be supported as a PBE.

Next we consider the case of complements. To show that \( d_i^* = \{\} \) for all \( i \) is part of an equilibrium with complements, consider first the case of \( E = X \). We can find PBE beliefs that eliminate the incentives for the buyer to deviate from \( d_2^* = \{\} \). \( d_2 = \{\phi_1\} \) in the downstream would result in the buyer losing all the surplus since \( P_j(s_j)\phi_1 = 1) = 1 - v_i \). An equilibrium conjecture \( \phi_i^*(\{s\}) = 1 \), resulting in \( P(s_j) = 1 - v_i \), eliminates the buyer’s incentives to deviate to \( d_2 = \{s\} \). Finally, an equilibrium conjecture \( B_i^j(\{P_i\}) = \{\}, B_i^j(\{s\}) = \{\} \) and \( \phi_i^*(\{P_i\}) = \begin{cases} 1 & \text{if } P_i \leq P_i^X(s_i) \\ 0 & \text{if } P_i > P_i^X(s_i) \end{cases} \) will result in

\[
P_j(s_j|\{P_i\}) = \begin{cases} 1 - v_i & \text{if } P_i \leq P_i^X(s_i) \\ v_j & \text{if } P_i > P_i^X(s_i) \end{cases}
\]

and \( P_i(s_i|\{s\}) = P_i^X(s_i) \) since \( 1 - P(s_i) - \alpha_j P_i^X(s_j) = (1 - \alpha_j)v_j \) yields exactly \( P_i^X(s_i) \) specified in Proposition 1a). Therefore, given this belief structure, the buyer has no incentive to deviate to \( d_2 = \{P_i\} \) or \( d_1 = \{s\} \).

Note that a deviation to \( d_2 = \{P_i\} \) would not make the buyer better off because then the downstream supplier would extract the remaining surplus. Next consider \( E = O \). Since in this case the order is inconsequential to the buyer’s purchasing decision, a deviation to \( d_2 = \{s\} \) and \( d_1 = \{s\} \) does not affect the sellers’ pricing and thus is not profitable. Finally, a deviation \( d_2 = \{P_i\} \) is unprofitable either since it results in \( P(s_j|\{P_i\}) = 1 - P_i \), which extracts all of the buyer’s surplus. Thus, we have shown that there exists a set of beliefs that support \( d_i^* = \{\} \) for all \( i \) as a PBE.

We next consider the equilibrium disclosure \( d_1^* = \{s\} \) and \( d_2^* = I_2 \), corresponding to public negotiations. Suppose \( E = X \). For \( B_i^j(d_i) = d_j^* \), deviating to \( d_2 = \{s\} \) would not change the equilibrium prices because, knowing the sequence, the downstream supplier would conjecture \( \phi_1 = 1 \). Next, define \( \hat{\sigma}_i(d_i) \) to be \( s_i \)'s belief about being in the upstream when he is disclosed \( d_i \); in particular \( \hat{\sigma}_i(\{\}) = 1 \) if \( \Delta < 0 \) and \( \hat{\sigma}_i(\{\}) = 0 \) if \( \Delta \geq 0 \). Then, by Proposition 1 and 3, \( P_i(s_i|\{\}) = \begin{cases} 1 - v_j - \alpha_j \Delta & \text{if } \Delta < 0 \\ 1 - v_j & \text{if } \Delta \geq 0 \end{cases} \) and the buyer has no incentive to deviate to \( d_i = \{\} \). Now, suppose \( E = O \). If \( \Delta < 0 \), the buyer cannot improve her payoff by deviating to \( d_i \neq d_i^* \) since \( d_i^* \) already results in Bertrand pricing. For \( \Delta \geq 0 \), \( d_i = \{\} \) is analogous to \( E = X \): if \( \hat{\sigma}_i(\{\}) = 1 \), then \( P_i(s_i|\{\}) = 1 - v_j \), leaving no incentive to the buyer to deviate. A deviation to \( d_2 = \{s\} \) would also prove unprofitable to the buyer.
as long as $s_i$ conjectures that the upstream offer was made by the buyer. More formally, define $\hat{\rho}_i(d_i)$ to be $s_i$’s belief about the identity of the proposer in negotiation $j$. Under $\hat{\rho}_i(\{s\}) = b$, the buyer would not deviate to $d_2 = \{s\}$ because $P_i(s_i|\{s\}) = 1 - v_j$. Therefore, we have shown that $d^*_1 = \{s\}$ and $d^*_2 = I_2$ can be supported as a BPE. \(\blacksquare\)

**Lemma A1.** Suppose that $\Delta > 0$ and negotiations are private. Moreover, sellers have hybrid deadlines $D = H_j$, in that while seller $i$’s offer is exploding, $j$’s offer is open-ended. Then,

- there exist efficient equilibria all with $P^{H_j}(s_i) + P^{H_j}(s_j) = 1$ if and only if $\sum_k \alpha_k(1 - v_k) \geq \Delta - \frac{\theta}{1 - \theta}v_i$, where $\theta = \frac{\sqrt{1 + 4\alpha_i^2} - 1}{\sqrt{1 + 4\alpha_i^2} + 1}$. In an efficient equilibrium, the buyer visits $s_i$ first with probability $\sigma^H_i \in [\sigma_i^H, \overline{\sigma}_i^H]$, where $\overline{\sigma}_i^H = \max\{0, \frac{\Delta - \alpha_i(1 - v_i) - \alpha_j(1 - v_j)}{1 - v_j - \alpha_i(1 - v_i) - \alpha_j(1 - v_j)}\}$ and $\sigma^H_i \in [\theta, 1]$;

- there also exists an inefficient equilibrium at which $P^{H_j}(s_i) = 1 - v_j$ for all $i$ if and only if $\alpha_i(1 - v_i) \leq \Delta$ for all $i$. In an inefficient equilibrium, the buyer visits $s_i$ first with probability $\sigma^H_i \in [0, \overline{\sigma}_i^H]$, where $\overline{\sigma}_i^H = \min\{\frac{\alpha_i(1 - \alpha_j)}{1 - v_j - \alpha_i(1 - v_i) - \alpha_j(1 - v_j)}, \frac{\Delta - \alpha_j(1 - v_j)}{\alpha_j(1 - v_j)}\}$.

**Proof.** It is easily verified that $P^{H_j}(b) = 0$, and $P^{H_j}(s_i) \in [v_i, 1 - v_j]$ for both sellers in equilibrium. Given this, we search for the sellers’ equilibrium prices. To simplify the notation, we suppress $H_j$ in the rest of the proof.

$P(s_i) + P(s_j) < 1$ : Such prices cannot be sustained in equilibrium because at least one seller would have a strict incentive to deviate to a different price without affecting the probability of a sale.

$P(s_i) + P(s_j) = 1$ : These prices can be sustained as an equilibrium. Note that under these prices, each seller realizes a sale with certainty, and the buyer is indifferent to the order of negotiations. $s_j$ has only one profitable deviation corresponding to $P(s_j) = 1 - v_i$. Thus, no deviation incentives by $s_j$ requires $P(s_j) \geq [\sigma_i + (1 - \sigma_i)(1 - \alpha_i)](1 - v_i)$. Seller $s_i$ has two profitable deviations. If he deviates to $P(s_i) = 1 - P(s_j) + (1 - \alpha_j)[P(s_j) - v_j]$, his probability of sale is $[\sigma_i + (1 - \sigma_i)(1 - \alpha_j)]$. If he deviates to $P(s_i) = (1 - v_j)$, his probability of sale is $(1 - \sigma_i)(1 - \alpha_j)$. Therefore, no deviation incentives by $s_i$ requires that $P(s_j) \leq 1 - \max\{1 - \sigma_i, \frac{\sigma_i + (1 - \sigma_i)(1 - \alpha_j)}{1 - \sigma_i + (1 - \sigma_i)(1 - \alpha_j)}\}(1 - \alpha_j)(1 - v_j)$. The right-hand side of this inequality is maximized for $\sigma_i = \theta = \frac{\sqrt{1 + 4\alpha_i^2} - 1}{\sqrt{1 + 4\alpha_i^2} + 1}$. Combining the conditions that render deviation unprofitable results in a necessary and sufficient condition for an efficient equilibrium $\alpha_i(1 - v_i) + \alpha_j(1 - v_j) \geq ||\alpha_i(1 - v_i)||^{\alpha_j(1 - v_j)}$. 

\[\sigma_i = \theta = \frac{\sqrt{1 + 4\alpha_i^2} - 1}{\sqrt{1 + 4\alpha_i^2} + 1} \]
$\Delta - \frac{\theta}{1-\theta} v_i$. In any efficient equilibrium, the buyer randomizes over the order by approaching $s_i$ first with probability $\sigma_i^{H_j} \in [\sigma_i^{H_j}, \sigma_j^{H_j}]$ where $\sigma_i^{H_j} = \max\{0, \frac{\Delta - \alpha_i(1-v_i) - \alpha_j(1-v_j)}{1-v_j - \alpha_i(1-v_i) - \alpha_j(1-v_j)}\}$ and $\sigma_j^{H_j} \in (0, 1)$.

$1 < P(s_i) + P(s_j) \leq 1 + (1 - \alpha_j)[P(s_j) - v_j]$: These prices cannot be part of an equilibrium: if $s_j$ is approached first, then $i$ sells only if the buyer proposes against $j$. If, on the other hand, $s_i$ is approached first, he sells with certainty, and so does $j$. Since the probability of a sale does not change up to the upper bound of $P(s_i)$, it must be that in equilibrium, $P(s_i) = 1 - P(s_j) + (1 - \alpha_j)[P(s_j) - v_j]$. Together with the fact that $P(s_i) + P(s_j) > 1$, it follows that the buyer would strictly prefer to first negotiate with $s_j$ whose offer is open-ended. But then, $s_i$ is strictly better off by increasing his price to $P(s_i) = 1 - v_j$, yielding a contradiction.

$P(s_i) + P(s_j) > 1 + (1 - \alpha_j)[P(s_j) - v_j]$: These prices can be part of an equilibrium. Under this price structure, if $s_j$ is approached first, each seller sells only if the buyer proposes against the rival. If $s_j$ is approached first, then $i$’s offer is rejected with probability 1 whereas $j$’s offer is accepted only if the buyer makes an offer in the previous negotiation. Since the probability of a sale does not change as prices increase in this region, the only equilibrium candidates are $P(s_i) = 1 - v_j$ and $P(s_j) = 1 - v_i$. Note that with these prices, the buyer is indifferent to the order of negotiation. A possible deviation for $s_j$ is to reduce his price to $v_j$, in which case he sells with certainty. Such deviation is unprofitable if and only if $v_j \leq (1 - \alpha_i)(1 - v_i)$, or equivalently $\alpha_i(1 - v_i) \leq \Delta$. $s_i$, however, has two possible deviations: (1) Reducing his price to $P(s_i) = (1 - \alpha_j)(1 - v_j) + \alpha_j v_i$, in which case he sells if approached first or if approached second and the buyer proposes in the previous negotiation. (2) Charging $v_i$, which guarantees a sale. To prevent these deviations for $i$, we thus require: $|\sigma_i + (1 - \sigma_i)(1 - \alpha_j)(1 - \alpha_j)(1 - v_j) + \alpha_j v_i| \leq (1 - \sigma_i)(1 - \alpha_j)(1 - v_j)$ and $v_i \leq (1 - \sigma_i)(1 - \alpha_j)(1 - v_j)$. Combining these two conditions, we conclude that $P(s_i) = 1 - v_j$ and $P(s_j) = 1 - v_i$ constitute an equilibrium if and only if $\alpha_i(1 - v_i) \leq \Delta$ for all $i$. In this equilibrium, $\sigma_i^{H_j} \in [0, \sigma_j^{H_j}]$, where $\sigma_j^{H_j} = \min\{\frac{\alpha_j(1-v_j)}{1-v_j - \alpha_j(1-v_j)}, \frac{\Delta - \alpha_j(1-v_j)}{1-\alpha_j(1-v_j)}\}$.  

**Proof of Proposition 5.** Consider first the case of complements. We know from Propositions 1 and 2 that the buyer prefers privacy for both $D = O$ and $D = X$. From Lemma A1, it can be established that her preference for privacy extends to $D = H_j$. Thus, we assume that the buyer chooses private negotiations independent of the deadline policies. Then, in order to show that $D = O$ can be supported as an equilibrium outcome, it suffices to prove that there exists an equilibrium of the negotiation subgame under $D = H_j$ that
makes a unilateral deviation to $D = H_j$ unprofitable. Note by Lemma A1 that $\sigma_i^{H_j} = 0$ is always supported as an equilibrium. In addition, the set of equilibria under $D = H_j$ and $\sigma_i^{H_j} = 0$ coincides with the set of equilibria under $D = O$. Thus, a belief by $s_i$ that $\sigma_i^{H_j} = 0$ will result in no deviation incentives from $D = O$. To establish that $D = X$ is always an equilibrium, recall from Proposition 1 that $D = X$ results in an efficient equilibrium with $\pi^X(s_i) = \alpha_i(v_i + \frac{1-\alpha_j}{1-\alpha_i\alpha_j}\Delta)$. Note from Lemma A1 that $\sigma_i^{I_1} = 1$ with $\pi^{I_1}(s_i) = \alpha_i\max\{v_i, (1-\alpha_j)(1-v_j)\}$ can always be supported as an equilibrium payoff of the negotiation subgame. It is easy to check that $\pi^X(s_i) > \pi^{I_1}(s_i)$ and thus $s_i$ will have no incentives to deviate to $D = O$.

Next consider the case of substitutes. To establish that $D = O$ and $D = X$ can each be supported as an equilibrium, note that a deviation to $D = H_j$ results in Bertrand pricing as long as the seller with an open-ended offer is approached first. Since the Bertrand equilibrium generates the lowest possible payoff for each seller, neither will have a unilateral incentive to deviate from $D = O$ or from $D = X$. ■

**Proof of Proposition 6.** Suppose $\Delta > 0$. Under public negotiations, it immediately follows from (A-3) and (A-5) that $\pi^O(b) = \pi^X(b)$, and in each case, the buyer is completely indifferent to the order. Recalling that $\sigma_i^D$ denote the probability that seller $i$ is visited first under $D \in \{O, X\}$, seller $i$’s expected payoffs are $\pi^O(s_i|\sigma_i^O) = \alpha_i[1-v_j-(1-\alpha_i^O)\alpha_j\Delta]$ and $\pi^X(s_i|\sigma_i^X) = \alpha_i[1-v_j-\sigma_i^X\alpha_j\Delta]$. Thus, $\pi^O(s_i|\sigma_i^O) > \pi^X(s_i|\sigma_i^X)$ if and only if $\sigma_i^O > 1-\sigma_i^X$, showing part (a).

To show part (b), consider private negotiations, and suppose $\sum_k \alpha_k(1-v_k) < \Delta$. By Proposition 1, $\pi^X(b) = 1 - \alpha_i\overline{\pi}^X(s_i) - \alpha_j\overline{\pi}^X(s_j)$ with unique prices in (2), and by Proposition 2, there is a unique efficient equilibrium with $\pi^O(b)$ as in (A-3). Simplifying terms, we have $\pi^X(b) - \pi^O(b) = \frac{\alpha_i\alpha_j(1-\alpha_i)(1-\alpha_j)\Delta}{1-\alpha_i\alpha_j} > 0$. As for seller $i$, it is easily obtained that $\pi^X(s_i) = \alpha_i\overline{\pi}^X(s_i) > \pi^O(s_i) = \alpha_i(1-\alpha_j)(1-v_j)$. Therefore, each player strictly prefers $D = X$ to $D = O$, if $\sum_k \alpha_k(1-v_k) < \Delta$.

Next, suppose that $\alpha_i(1-v_i) > \Delta$ for some $i$. Then, by Proposition 2, there exist only efficient equilibria under both $D = O$ and $D = X$, and $\pi^D(b) = 1 - \alpha_i\overline{\pi}^D(s_i) - \alpha_j\overline{\pi}^D(s_j)$. To show that $\pi^O(b) > \pi^X(b)$, it is thus sufficient to show that

$$\alpha_i\overline{\pi}^X(s_i) + \alpha_j\overline{\pi}^X(s_j) > \alpha_i\overline{\pi}^O(s_i) + \alpha_j\overline{\pi}^O(s_j). \quad \text{(A-6)}$$

Note that $\overline{\pi}^X(s_i) + \overline{\pi}^X(s_j) > \overline{\pi}^O(s_i) + \overline{\pi}^O(s_j) = 1$ implies $\overline{\pi}^X(s_i) > \overline{\pi}^O(s_i)$ for at least one seller. If $\overline{\pi}^X(s_i) > \overline{\pi}^O(s_i)$ for both sellers, then (A-6) trivially follows. Consider now
$P^X(s_i) > P^O(s_i)$ and $P^X(s_j) < P^O(s_j)$. Note that in equilibrium $P^O(s_i) + P^O(s_j) = 1$ and $P^X(s_j) = 1 - v_i - \alpha_i(P^X(s_i) - v_i)$. Substituting for $P^O(s_j)$ and $P^X(s_j)$ in (A-6), we obtain $\alpha_i(1 - \alpha_j)P^X(s_i) - \alpha_j(1 - \alpha_i)v_i > (\alpha_i - \alpha_j)P^O(s_i)$. If $\alpha_i \geq \alpha_j$, (A-6) follows by observing that the inequality is satisfied for $P^X(s_i) = P^O(s_i)$ and the left-hand side is increasing in $P^X(s_i)$. If $\alpha_i < \alpha_j$, then (A-6) can be rewritten as $\alpha_i(1 - \alpha_j)P^X(s_i) + (\alpha_j - \alpha_i)P^O(s_i) > \alpha_j(1 - \alpha_i)v_i$. Note that $P^X(s_i) > P^O(s_i)$ implies $\alpha_i(1 - \alpha_j)P^X(s_i) + (\alpha_j - \alpha_i)P^O(s_i) > \alpha_j(1 - \alpha_i)P^O(s_i) \geq \alpha_j(1 - \alpha_i)v_i$ since $P^O(s_i) \geq v_i$.

Finally, to prove that at least one seller prefers $E = X$, suppose without loss of generality that $P^X(s_i) > P^O(s_i)$. Note that $\pi^X(s_i|\sigma^X_i) = \alpha_iP^X(s_i)$ and $\pi^O(s_i|\sigma^O_i) = \alpha_iP^O(s_i)$. Hence, $\pi^X(s_i|\sigma^X_i) > \pi^O(s_i|\sigma^O_i)$.

**Lemma A2.** Suppose that $\Delta > 0$ and $D = X$, and that $s_i$ commits to disclosure, while $s_j$ commits to non-disclosure. Then, $P^X(s_j) = 1 - v_i - \alpha_i\Delta$, $P^X(s_i) = 1 - v_j$ and $\sigma^X_i = 0$ constitute an efficient equilibrium.

**Proof of Lemma A2.** Given the sellers’ disclosure strategies, $s_i$ has two pricing options: $P(s_i) = 1 - v_j - \alpha_j\Delta$ and $P(s_i) = 1 - v_j$. To see this, note that if $s_i$ believes that he is approached first, then he will anticipate public negotiations and thus charge the highest price acceptable by the buyer (i.e. $P(s_i) = 1 - v_j - \alpha_j\Delta$). If, on the other hand, $s_i$ believes that he is approached second, he has to choose between $v_i$, accepted with probability one, and $1 - v_j$ accepted only if the first negotiations was successful. It is straightforward to show that $v_i$ cannot be part of an equilibrium since $s_j$, upon observing to be the first to negotiate with the buyer, will adjust his price to guarantee a sale in the first negotiation, prompting a deviation to $1 - v_j$ by $s_i$. In addition, note that $P(s_i) = 1 - v_j - \alpha_j\Delta$, requiring a mixing $\sigma_i \geq \frac{\alpha_i\Delta}{1 - v_j}$ by the buyer, will cause $s_j$ to increase his own price when approached first to $1 - P(s_j) - \alpha_i(1 - v_j - \alpha_j\Delta) = (1 - \alpha_i)v_i \implies P(s_j) = 1 - v_i - \alpha_i(1 - \alpha_j)\Delta$. This results in $\pi(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i + \alpha_i\alpha_j(1 - \alpha_j)\Delta$, generating incentives for the buyer to deviate to $\sigma_i = 0$. Therefore, $\sigma_i \geq \frac{\alpha_i\Delta}{1 - v_j}$ cannot be supported as an equilibrium. For $\sigma_i < \frac{\alpha_i\Delta}{1 - v_j}$, resulting in $P^X(s_i) = 1 - v_j$, the buyer is indifferent in the order with an expected payoff of $\pi(b) = (1 - \alpha_i)(1 - \alpha_j) + \alpha_i(1 - \alpha_j)v_j + \alpha_j(1 - \alpha_i)v_i$.

However, the only efficient equilibrium involves $\sigma_i = 0$ since the buyer would accept $P^X(s_i)$ only in the second negotiation.

**Proof of Proposition 7.** Suppose that $D = X$, and that $v_i = v$ and $\alpha_i = \alpha$. Let us consider first the case of complementary goods. The payoff for each seller from $(\text{dis}, \text{dis})$ and $(\text{dis}, \text{dis})$ is easily derived from Proposition 1 and the payoff from $(\text{dis}, \text{dis})$ is derived
from Lemma A2. The matrix below depicts the payoffs for each seller.

<table>
<thead>
<tr>
<th></th>
<th>dis</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\pi(s_i) = \alpha[1 - v - \frac{1}{2}\alpha \Delta]$</td>
<td>$\pi(s_j) = \alpha[1 - v - \frac{1}{2}\alpha \Delta]$</td>
</tr>
<tr>
<td></td>
<td>$\pi(s_i) = \alpha(1 - v)$</td>
<td>$\pi(s_j) = \alpha(1 - v - \alpha \Delta)$</td>
</tr>
</tbody>
</table>

Note that in this case, disclosing is a strictly dominant strategy for each of the sellers and thus $(\text{dis, dis})$ is the unique equilibrium.

We next consider the case of substitutes. The matrix below depicts the payoffs for each pair of disclosure strategies.

<table>
<thead>
<tr>
<th></th>
<th>dis</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi(s_i) = \alpha[1 - v - \frac{1}{2}\alpha \Delta]$</td>
<td>$\pi(s_j) = \alpha[1 - v - \frac{1}{2}\alpha \Delta]$</td>
</tr>
<tr>
<td></td>
<td>$\pi(s_i) = \alpha(1 - v)$</td>
<td>$\pi(s_j) = \alpha(v + \frac{1}{1+\alpha} \Delta)$</td>
</tr>
<tr>
<td></td>
<td>$\pi(s_i) = \alpha(1 - v - \alpha \Delta)$</td>
<td>$\pi(s_j) = \alpha(1 - v)$</td>
</tr>
</tbody>
</table>

There are two equilibria: $(\text{dis, dis})$ and $(\overline{\text{dis}}, \overline{\text{dis}})$. ■

References


