Information, Competition, and the Quality of Charities

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Abstract

Drawing upon the all-pay auction literature, we propose a model of charity competition in which informed giving alone can account for the significant quality heterogeneity across similar charities. Our analysis identifies a negative effect of competition and a positive effect of informed giving on the equilibrium quality of charity. In particular, we show that as the number of charities grows, so does the percentage of charity scams, approaching one in the limit. In light of this and other results, we discuss the need for regulating nonprofit entry and conduct as well as promoting informed giving.

Keywords: informed giving, quality of charity, competition, all-pay auctions.

JEL Classification: H00, H30, H50

1 Introduction

Up 30 percent from a decade ago, the number of public charities in the United States exceeded one million in 2013 (Urban Institute 2014).¹ Charity Navigator, the leading charity evaluator, offers free ratings of the largest 8000 by identifying 37 causes (e.g., humanitarian relief). Its ratings reveal that the quality of the charities within each cause varies

¹We thank two anonymous referees, the Co-Editor, seminar participants at Carlos III, Duke, Texas A&M, USC (Columbia), Toronto, and UT (Austin), and conference participants at 2015 SPI Conference and Public Choice Conference for comments. All remaining errors are ours.

¹Public charities constitute about three-quarters of all registered nonprofits in the U.S., and unlike other nonprofits such as private foundations, they rely heavily on contributions from the general public, which consistently total about 1.5 percent of GDP in the U.S – $241.3 billion in 2013. In this paper, we mainly focus on public charities and use the term nonprofit interchangeably.
significantly with about one-third failing industry standards. The challenge for donors is therefore not finding a worthy cause to support but choosing the charity that is most deserving of their hard-earned money. Despite its importance, however, informed giving appears to be the exception rather than the rule. In this paper, we propose a model of charity competition in which informed giving alone can explain the quality heterogeneity across similar charities. Our analysis highlights an adverse effect of competition and a beneficial effect of informed giving on the quality of charity.

Our baseline model contains a fixed number of ex ante identical charities that fundraise for a given cause by declaring their specific missions (e.g., helping children, improving women’s health) and pre-investing in their program and service quality such as infrastructure, planning, and staff training. We assume that while some donors are purely mission-oriented, others care about quality. Among the latter, informed donors seek the best charity, perhaps using a rating agency, whereas the uninformed pick one at random. Note that the presence of mission-based and uninformed giving in the population invites charity scams: fundraising with no intention of providing the public good. Hence, in our model, charities invest in quality to attract informed donations. And their incentives to “win” a lump-sum revenue by setting a slightly higher quality than the rivals’ turn the charity competition into an “all-pay auction” and leads to mixing over quality choices in equilibrium.

In the unique symmetric equilibrium, we establish that each charity continuously mixes over a positive interval of quality and has a mass point at zero. Such a strategy readily rationalizes the quality heterogeneity mentioned above and predicts a significant probability of charity scams. We show that by raising the stakes for being the best charity, informed giving (stochastically) increases the equilibrium quality of charities, and individual informed and uninformed gifts in turn. Nevertheless, the total provision of the public good

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2 For details, visit [www.charitynavigator.org](http://www.charitynavigator.org). Significant quality variation is also reported by other major charity evaluators including BBB Wise Giving Alliance and GuideStar. Not surprisingly, it is such variation that has facilitated recent empirical investigations of charity ratings (e.g., Yoruk (2016)).

3 Based on a nationwide survey of 4000 Americans with incomes in the top 10 percent, the 2010 Hope Consulting Report found that 9 out of 10 donors indicate that they care about nonprofit performance, but only 3 out of 10 donors actually research nonprofits and only 3 out of 100 ensure giving money to highest-performing nonprofits; see [www.hopeconsulting.us/moneyforgood](http://www.hopeconsulting.us/moneyforgood). Experimental evidence parallels this finding: whereas Eckel and Grossman (1996) document that individuals give generously when they are paired with recipients of preferred characteristics, Fong and Oberholzer-Gee (2011) observe that only one third of subjects are willing to pay for information about recipients.

4 Given that charitable contributions are often nonrefundable, donors are unlikely to find any promised and/or uncertified quality credible.
is likely to be maximized when there is an optimal mix of the two donor types in the population: with too many informed donors, charities compete away donations in the “race to the top”, while with too many uninformed, they have little incentive to offer (costly) quality, which would discourage giving. This implies that the total provision improves with informed giving if the initial level of informed giving is low, which seems to be the case in reality (see Footnote 3).

The positive effect of informed giving is, however, countered by the negative effect of charity competition. Most starkly, we prove that as the number of charities grows unbounded, the fraction of scammers in the economy approaches one, owing to a negligible probability of receiving the informed donations. Our analysis, therefore, suggests regulating the market structure of the nonprofit industry. As discussed in Section 5, this can be achieved by setting higher entry barriers, such as a more onerous application procedure for tax-exempt status, or providing stronger incentives for nonprofit mergers, such as funding their due diligence. Our analysis also suggests regulating charity conduct and sanctioning poor performers, although in practice nonprofit enforcement is bound to be too weak to completely deter charity scams because of the legal and financial obstacles that regulators have to overcome.

The policy implications of our baseline model are reinforced in the long run market with an endogenous number of charities. In particular, introducing a costly entry stage to the model, we find that only the best charity provides the public good in the long run, with many unsuccessful and/or scam organizations present. The reason is that under endogenous entry, charities exhaust what they expect to receive from the unsuspecting – mission-oriented and uninformed – donors, leaving only the informed donors as their source of a positive net revenue. Similar to the baseline model, this means that the nonprofit market is likely to be highly concentrated in the provision of the public good, as evidenced by Seeman et al. (2014) and discussed in Section 5.

Related literature. Our theoretical framework draws upon two influential papers on all-pay auctions: Varian (1980) and Che and Gale (2003). Varian considers a price competition with informed and uninformed consumers in order to explain equilibrium price dispersion. Roughly speaking, a price reduction in Varian’s model plays a similar – investment – role to a quality increase in ours, although unlike quality, a price change does not affect consumers’ reservation utility in Varian. Che and Gale examine a research tourna-

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3In the limit economy, the expected number of nonscam charities is nonzero and finite.
ment where there is only one buyer, the procurer, whose value of innovation is endoge-
nous to the winner’s effort and who decides informed. Siegel (2010) ably generalizes such
all-pay auctions with endogenous valuations.

On the nonprofit side, our paper relates to the few studies on charity competition. Rose-Ackerman (1982) shows that competitive fundraising can be “excessive” despite donors’
aversion to it. Castaneda et al. (2008) argue that such inefficiency may be reduced by non-
profits’ ability to contract on the use of donations while Aldashev et al. (2014) observe that
it can be overcome by fundraising coordination, though such coordination is often diffi-
cult in this voluntary sector. In the same vein, Bilodeau and Slivinski (1997) find that rival
charities may specialize in the provision of one public good or service in order to attract
donations. With over one million registered charities, there are nevertheless many that
provide similar – if not identical – services yet differ significantly in their quality of pro-
vision. In this sense, Scharf (2014) is closer to our work. Assuming an exogenous quality
distribution, Scharf points out that competition can induce too much entry by low quality
charities. We let quality choice be part of the competition and derive an endogenous distri-
bution for it. Like her, we argue that increased entry into the charitable market decreases
the (average) quality of charity.

On the role of informed giving, our paper also relates to Vesterlund (2003), Andreoni
(2006) and Krasteva and Yildirim (2013). Vesterlund shows that a large leadership gift can
signal the (exogenous) quality of the charity. Andreoni extends this argument by demon-
strating that all else equal, it is the most wealthy who will lead. Krasteva and Yildirim
explore a private value setting in which donors are uncertain about their private valua-
tions of the charity and thus no signaling incentive exists. In all these papers, informed
giving raises more funds on average and is therefore uniformly encouraged. In contrast,
our analysis emphasizes that both informed and uninformed giving might be important
for the performance of the charitable market.

The rest of the paper is organized as follows. In the next section, we present the baseline
model with an exogenous number of charities and exogenous donor information. In
Section 3, we characterize the equilibrium and consider key comparative statics, followed
by several extensions of the model in Section 4. In the last two sections, we discuss the
empirical and policy relevance of our findings and then conclude with final remarks. The
proofs of all formal results are relegated to the appendix.

\footnote{For a recent overview of the literature on charitable giving, see Andreoni and Payne (2013).}
2 Baseline model

There are $n \geq 2$ ex ante identical charities raising funds for a given cause such as humanitarian relief. Before appealing to donors, charities publicly announce their specific missions (e.g., helping children, improving women’s health) and simultaneously invest in their program quality $q_i \in [0, \infty)$ that costs $C(q_i) = q_i$. Unlike their missions, the charities’ quality is observed by donors only upon inspection, possibly through a rating agency, and it may reflect the investment in infrastructure, planning, and staff training, for instance. Let $\mathbf{q} = (q_1, ..., q_n)$ and $G_i(\mathbf{q})$ denote the quality profile of charities and the total gift received by charity $i$, respectively. Conjecturing the rivals’ quality profile $\mathbf{q}_{-i}$, charity $i$ chooses $q_i$ to maximize its net revenue to be spent toward its program:

$$R_i(\mathbf{q}) = G_i(\mathbf{q}) - C(q_i).$$

If $R_i(\mathbf{q}) \leq 0$ or $q_i = 0$, we assume that the charity provides no public good: in the former, funds fall short of the cost to run the program, and in the latter, the charity is a “scam”. In order to identify the need for regulation, we assume for now no legal sanctions against charity scams.

On the demand side of the charitable market, there is a mass $M$ of potential donors, each with a (normalized) unit wealth. The mass $A$ of them are purely mission-oriented or “attached” in that each gives a fixed amount $g_A > 0$ to a specific charity regardless of its quality. For simplicity, attached donors are assumed to be uniformly distributed across $n$ charities, so each charity claims $\frac{A}{n}g_A$ of their gifts. The “unattached” donors, in contrast, care about the broad cause, such as humanitarian relief, and aim to support high quality programs without a specific charity in mind. Among the unattached, informed donors, whose measure is $I$, can observe the quality of the charities prior to giving, whereas uninformed donors, whose measure is $U$, give to one charity at random. We assume that an unattached donor $j$ possesses CES preferences:

$$u(x_j, g_j; q_i) = x_j^{\rho} + q_i g_j^{\rho},$$

As previously mentioned, most charitable contributions are nonrefundable; hence, fearing hold up, donors are unlikely to trust any promised and/or uncertified quality by charities. Our results are, however, robust to “partial” promises, whereby, as in Siegel (2010, Section III), the charity invests a fraction of $q_i$, with the commitment of investing the rest if enough funding is received.

Alternatively, if $R_i(\mathbf{q}) \leq 0$, charity $i$ uses its funds $G_i(\mathbf{q})$ for another cause that donors do not care about. Moreover, our distinction between a failed fundraising effort and a charity scam is consistent with the popular view; see http://www.consumer.ftc.gov/features/feature-0011-charity-scams.
where $0 < \rho < 1$, $x_j \geq 0$ denotes private consumption, and $g_j \geq 0$ denotes gift to the charity of quality $q_i$. In particular, higher quality increases the marginal rate of substitution between private consumption and giving.\textsuperscript{9}

Note that our fundraising game is one with complete information and simultaneous moves among charities and donors, so we solve for a Nash Equilibrium. We restrict attention to symmetric equilibrium, however, both to generate equilibrium heterogeneity across symmetric charities and to keep the analysis tractable since, as with the extant literature on all-pay auctions alluded to above, our fundraising game also exhibits mixing in quality choices. Nevertheless, we briefly discuss the robustness of our results to \textit{ex ante} heterogeneous charities in the Conclusion.

\subsection*{2.1 More on the setup}

Our model is designed to highlight donor information as the unique source of equilibrium quality dispersion. We therefore assume that all charities and (strategic) unattached donors are \textit{ex ante} identical except for their information, which is endogenized in Section 4. Consistent with the literature, we assume “warm-glow” charities that care only about their own provision of the public good (Scharf 2014). This is not critical, however. The qualitative results would hold as long as charities favor their own provision so that there is rivalry. From (1), it is evident that we also assume “warm-glow” givers (Andreoni, 1990). This is a reasonable description of charitable behavior for a continuum of donors. As argued by Glazer and Konrad (1996), suppose that $\Gamma$ is the total provision of the public good by all charities in the market. Let the public good enter the utility in an additively separable way: $x_j^\rho + q_i g_j^\rho + z_j(\Gamma)$. Since with a continuum of donors $d\Gamma/dg_j = 0$, the altruistic motive for giving has no effect on donor behavior.\textsuperscript{11}

\textsuperscript{9}This way of modeling preference for quality mirrors those of Vesterlund (2003) and Andreoni’s (2006). The CES form is assumed for expositional ease as the analysis readily extends to: $u(x_j, g; q_i) = t(x_j) + q_i w(g)$ where both $t$ and $w$ are strictly increasing and strictly concave.

\textsuperscript{10}Since their giving decisions are fixed, we need not specify preferences for attached donors. For consistency, one can, however, assume similar CES preferences for them: $u(x_j, g; a_i) = x_j^\rho + a_i g_j^\rho$, where $a_i = a > 0$ if charity $i$ comes closest to donor $j$’s ideal mission, and $a_i = 0$ otherwise.

\textsuperscript{11}The fact that donations are driven purely by the warm-glow motive also obtains in a large finite economy (e.g., Yildirim, 2014).
3 Equilibrium characterization

To determine charities’ equilibrium quality, we first determine demand for quality by unattached donors. Maximizing the utility in (1) subject to the budget constraint, $x_j + g_j = 1$, donor $j$’s optimal gift for a charity of quality $q$ is easily found to be

$$g_j = g(q) \equiv \frac{q^r}{1 + q^r},$$

where $r = \frac{1}{1-\rho} \in (1, \infty)$ is the elasticity of substitution between private consumption and giving; see Figure 1. It is readily verified that (a) $g(0) = g'(0) = 0$, (b) $g'(q) > 0$ for $q > 0$, and (c) $g''(q) > 0$ for $q < q_c$ and $g''(q) < 0$ for $q > q_c$, where $q_c = \left(\frac{r-1}{r+1}\right)^{1/r}$. In words, an unattached donor would not give to a scam or near scam charity; otherwise, his optimal gift is increasing in the charity’s quality – at an increasing rate for its low levels and at a decreasing rate for its high levels, approaching the donor’s entire unit income as $q \to \infty$. Since an informed donor learns the quality profile of charities, possibly through a rating agency, he optimally picks the highest ranked, denoted by $q_{\text{max}}$. An uninformed donor, on the other hand, gives based on the expected quality $E[\tilde{q}]$ of his randomly selected charity, where $E[\cdot]$ is the usual expectation operator.\footnote{The expected quality $E[\tilde{q}]$ is a sufficient statistic for the giving decision of an uninformed donor owing...} Using (2), the informed and uninformed
gifts are therefore
\[ g_I = g(q_{\text{max}}) \text{ and } g_U = g(E[\tilde{q}]). \]  
(3)

Note that the behavior of informed donors turns the quality competition among charities into an all-pay auction (e.g., Varian 1980; Che and Gale 2003) because a charity can win all the informed donations by investing in quality slightly more than its rivals. It is thus unsurprising that there will also be no pure strategy equilibrium in our setup.

Let each charity mix over quality choices according to the symmetric cumulative distribution \( F(q) \). To characterize, suppose that charity \( i \) sets quality \( q \) and ranks the highest, with probability \( F^{n-1}(q) \). Then, it captures all the informed donations as well as equal shares from the uninformed and the attached donations, resulting in net revenues:

\[ R_{\text{win}}(q) = I g(q) + \frac{U}{n} g(E[\tilde{q}]) + \frac{A}{n} g_A - q. \]  
(4)

If charity \( i \) does not rank the highest, it loses informed donations but continues to receive the rest, generating net revenues:

\[ R_{\text{lose}}(q) = \frac{U}{n} g(E[\tilde{q}]) + \frac{A}{n} g_A - q. \]  
(5)

Clearly, charity \( i \) can always adopt a scam strategy by choosing the minimal quality, \( q = 0 \). Such a strategy, however, can only target the attached and the uninformed donors, yielding the scam profit:

\[ R_{\text{scam}} = \frac{U}{n} g(E[\tilde{q}]) + \frac{A}{n} g_A. \]  
(6)

In order to ensure \( R_{\text{win}}(q) > R_{\text{scam}} \) for some \( q > 0 \) so that scamming is not a dominant strategy, we impose the following condition throughout, which simply requires significant informed giving to justify the cost of quality.

**Condition 1** \( I > I_c \equiv r(r-1)^{\frac{1}{2}} - 1 \).

In equilibrium, the charity that mixes must be indifferent across all quality choices in the support, including \( q = 0 \). In particular, its expected net revenues from choosing \( q > 0 \) to separable utility. Formally, (1) implies that \( E[u(x_i, g_i; \tilde{q})] = u(x_i, g_i; E[\tilde{q}]). \) In this sense, we assume that uninformed donors, like the attached, are “unconditional” warm-glow givers in that they would not be disappointed if they found out \textit{ex post} that their specific charity failed or turned out to be a scam. The qualitative results would, however, continue to hold if warm-glow feelings were “conditional” on the provision, in which case the donor would simply discount the expected quality by the probability of a positive provision. We have opted for the former definition of warm-glow because it appears more consistent with Andreoni (1990) and analytically more transparent.
must equal its scam profit:

\[ E[R_i(q)] = F_1(q)R_{\text{win}}(q) + \left[1 - F_1(q)\right]R_{\text{lose}}(q) = R_{\text{scam}}. \]  

(7)

Inserting (4), (5) and (6) into (7) and canceling terms, we have

\[ F_1(q)I(q) - q = 0, \]  

(8)

or equivalently,

\[ F(q) = \left(\frac{q}{I(q)}\right)^{\frac{1}{r-1}} = \left(\frac{q^1 - r I}{I}ight)^{\frac{1}{r-1}}. \]  

(9)

Note that \( F(0) > 0 \) (since \( r > 1 \)), which suggests a mass point at \( q = 0 \). Our first result completes and formalizes the equilibrium characterization.

Proposition 1 (Equilibrium characterization) Under Condition 1, there is a unique symmetric equilibrium. In equilibrium, each charity continuously mixes over \( q \in [q_L, q_H] \) according to (9) and has a mass point at \( q = 0 \) such that

\[ F(0) = (I_0 / I)^{\frac{1}{r-1}} \]  

where \( q_L = (r - 1)^{\frac{1}{r}} \) and \( q_H > q_L \) is the largest root of \( I(q) - q = 0 \). Moreover, informed and uninformed gifts are as stated in (3).

From (9), it is evident that charities provide quality purely to entice informed donors as the rest are unresponsive: the uninformed donors do not observe and the attached donors do not care about quality. As such, the quality heterogeneity in our model is based solely on donor information – not on preference or income heterogeneity. To understand the mass point at zero quality, or scam, refer to Figure 2.

Note that for a sufficiently low but positive quality, \( q \in (0, q] \), informed donations fall below the cost.\(^{13}\) The charity therefore chooses between scamming the unsuspecting – attached or uninformed – donors and offering a high enough quality, \( q \geq q_L \), to have a chance to grab informed donors, too. This trade-off explains the mass point at \( q = 0 \).\(^{14}\) The standard all-pay auction arguments show that another (interior) mass point is not possible; if it were, a charity would discretely increase the probability of winning by slightly raising its quality. This “race to the top” also accounts for why the lower bound of the quality distribution \( q_L \) exceeds the intermediate break-even quality \( q \) as well as why the upper

\(^{13}\)Formally, \( R_{\text{win}}'(0) = g'(0) - 1 = -1 \).

\(^{14}\)While our model features a linear cost, a mass point at \( q = 0 \) would obtain for more general specifications of the cost of quality, \( C(q) \). In particular, a mass point at zero quality would exist if and only if

\[ \frac{C(0)}{I(0)} > 0. \]

For instance, if \( C(q) = q^\theta, \theta > 0 \), this condition reduces to \( \theta \leq r \), which, given \( r \in (1, \infty) \), does not appear nongeneric.
bound \( q_H \) must obtain at the highest break-even quality. The equilibrium distribution in (9) simply balances the probability of winning informed donations, \( F^{n-1}(q) \), to the cost-to-donation ratio, \( \frac{q}{Ig(q)} \). Since, by definition, the distribution is strictly increasing in the quality level, so is the ratio. In this sense, the cost-to-donation ratio, often utilized by leading watchdogs such as Charity Navigator, appears an unreliable measure for rating charities (Steinberg, 1991; Gneezy et al. 2014).

Armed with the equilibrium characterization, we are ready to evaluate market performance and donor welfare by examining two key comparative statics with respect to the number of charities, \( n \), and informed donors, \( I \). Discounted by the probability of being a scam, note that the expected provision of the public good by charity \( i \) is \([1 - F(0)]E[R_i(q)]\). Using (7), the expected total provision is therefore

\[
TR(I, n) \equiv n[1 - F(0)]E[R_i(q)] = [1 - F(0)][Ug(E[\bar{q}]) + Ag_A].
\]

In particular, given that nonscam charities compete away informed donations when setting quality, the total provision depends on the uninformed and attached donations. Informed

\[\text{Figure 2: Winning and losing payoffs}\]
donors also affect the provision but only through the quality distribution.

As for donor welfare, note from (1) that an unattached donor’s indirect utility from giving to the charity of quality \( q \) is

\[
v(q) = \max_{g \in [0,1]} (1 - g) + qg,\]

which, by employing (2), reveals

\[
v(q) = (1 + q) \frac{1}{r}. \tag{11}\]

It is readily verified that \( v'(q) > 0 \) and \( v''(q) > 0 \). That is, the donor enjoys giving to a higher quality charity and does so at an increasing rate due to the compounded positive effect of quality through the optimal gift. Based on (11), the expected payoffs for the informed and uninformed donors can be expressed as:

\[
v_I(I, n) = E[v(q_{\text{max}})] \quad \text{and} \quad v_U(I, n) = v(E[q]). \tag{12}\]

The following proposition, a key result of this paper, shows that charity competition adversely affects market performance and donor welfare.

**Proposition 2** (Adverse charity competition) In equilibrium, as the number of charities, \( n \), increases,

(a) the expected quality of charity, \( E[q] \), decreases and converges to 0 as \( n \to \infty \),

(b) the expected informed and uninformed donations both decrease, with the former approaching a positive amount and the latter approaching 0 as \( n \to \infty \),

(c) the expected total provision decreases and converges to 0 as \( n \to \infty \),

(d) both informed and uninformed donors become worse off.

According to Proposition 2, competition lowers the expected quality of charity, discourages donations, and reduces donor welfare. To see why, note from (9) that \( F(q) \) is increasing in \( n \): each charity invests stochastically less in quality when competition for informed donors intensifies and the chances of attracting them diminishes. Despite a positive scale effect associated with the number of charities, the negative competition effect

\[16\text{Since an attached donor gives independently of charity quality, his welfare is fixed and not the focus of our analysis.}\]
also dominates for the highest quality charity, \(q_{\text{max}}\), since the probability \(F^a(q)\) is increasing in \(n\), too.\(^{17}\) Together these two facts imply that charity competition discourages both informed and uninformed donors and reduces their welfare as a result. Indeed, in the most competitive market, \(n \to \infty\), only the informed and attached donors remain as contributors, while the uninformed grow too pessimistic about the average quality to contribute. Consistent with the pessimism of the latter, Corollary 1, derived from Proposition 1, shows that most charities would be scams in an unregulated market for donations.

**Corollary 1** The fraction of scam charities in the market, \(F(0) = (I_c/I)^{1/n}\), increases with \(n\) and approaches 1 as \(n \to \infty\). In the limit economy, the expected number of nonscam charities is: \(\lim_{n \to \infty} n[1 - F(0)] = \ln(I/I_c)\).

Corollary 1 obtains because charities are unlikely to provide quality when their chances of winning the informed donations are slim. Together with Proposition 2, Corollary 1 supports the regulation of entry into the nonprofit market. Such regulation may not only lower the fraction of scam charities, but it may also help the nonscam charities recover their costs of quality by enticing uninformed donations, as the next corollary demonstrates.

**Corollary 2** Suppose that \(\frac{U}{\pi} g_U + \frac{A}{n} g_A\bigg|_{n=2} > q_H\). Then, there is a unique threshold \(\pi < \infty\) such that for \(n \leq \pi\), all nonscam charities – winning or losing – receive positive net revenues.

To illustrate, it can be checked that \(\pi = 16\) for parameter values \(r = 2, I = 3, U = 17, A = 80,\) and \(g_A = 0.5\). In addition to entry regulation, Proposition 3 reveals that informed giving can also mitigate the adverse effect of charity competition.

**Proposition 3** (Beneficial informed giving) In equilibrium, each of the following is increasing in \(I\): the expected quality of charity, the expected informed and uninformed donations as well as donor welfare. The expected total provision is, however, non-monotone and maximized at an interior \(I\) if \(A\) is sufficiently small. If both \(A\) and \(n\) are sufficiently large, the expected total provision is increasing in \(I\).

To understand Proposition 3, notice from Proposition 1 that an increase in the mass of informed donors, \(I\), raises the expected quality of charity, both because it extends the quality competition – i.e., \(\partial q_H/\partial I > 0\) – and because it makes the choice of higher quality more

\(^{17}\) If, in contrast to our model, the quality distribution \(F(q)\) were exogenous, \(F^n(q)\) would obviously be decreasing in \(n\) due to the scale effect.
likely – i.e., $\partial[1 - F(q)]/\partial I > 0$. As a result, both informed and uninformed individuals give more generously and by the revealed preference, they are better off. Since, with a fixed population of unattached donors, more informed giving also means less uninformed giving, the total provision may be single-peaked in the former. In particular, when the mass of attached is not large enough to compensate for the lack of uninformed giving, the total provision is maximized when there is an optimal mix of informed and uninformed donors. Otherwise, the total provision is increasing in informed giving if the nonprofit market is sufficiently competitive, which appears to be the case in practice.

### 4 Extensions and variations

Our baseline model identifies the key roles competition and informed giving play on the performance of the nonprofit market. To check its robustness and understand policy issues further, in this section we extend the model in several directions, beginning with a policy of sanctioning charity scams and then considering costly fundraising.

#### 4.1 Sanctioning charity scams

An important message of the baseline analysis is that informed giving alone is insufficient to deter charity scams. In particular, Corollary 1 indicates that in the limit economy, $n \to \infty$, most charities would be scams regardless of the amount of informed giving. In addition to restricting the nonprofit entry, another viable policy is therefore to monitor and sanction those fundraisers that offer too low quality. To examine the efficacy of such an ex post intervention, suppose that at the outset of the baseline model, the regulator publicly announces some $q_0 > 0$ as the minimum quality threshold and some $s \geq 0$ as the (expected) sanction for the offenders.\(^{18}\) Recall from Proposition 1 that without sanctions, charities choose between scamming the unsuspecting – uninformed and attached – donors and offering high quality to also attract informed donors; hence, in equilibrium, $q \in \{0\} \cup [q_L, q_H]$. With sanctions, it is natural to conjecture that charities will be less likely to follow the scam strategy. Our next result confirms this conjecture and establishes that charity scams can be deterred if sanctions are strong enough.

\(^{18}\)Since fundraisers are assumed to maximize their expected net revenues, the expected sanction summarizes the regulator’s monitoring technology and the realized sanction, so we need not model them separately.
Proposition 4 Let $0 < q_0 \leq q_1$. Then, there is a unique symmetric equilibrium, in which each charity continuously mixes over $q \in [q_0, q_1] \cup [q_2, q_3]$ according to the cumulative distribution:

$$F^s(q) = \left( \frac{q - \min\{q_0, s\}}{I^s(q)} \right)^{\frac{1}{n-1}},$$

with unique cutoffs $q_1 \leq q_2 < q_3$ and a mass point at $q = 0$ for $s < q_0$. In equilibrium, the probability of a scam, $F^s(0)$, decreases with $s$ and vanishes for $s \geq q_0$. Moreover, the expected quality of charity increases with $s$, reaching its maximum for $s \geq q_0$.

As noted above, without sanctions, $s = 0$, the probability of a scam is positive in equilibrium. Proposition 4 reveals that a small $s$ would not significantly change this probability or the charity’s trade-off between scamming and trying to grab the informed donations. As $s$ grows moderately high, however, the charity considers a new – intermediate – option in the trade-off: meet the quality requirement and avoid paying the sanction. Once $s$ reaches or exceeds the cost, $q_0$, of the minimum quality, this new option eliminates the incentive to scam, shifting the mass from $q = 0$ to $q \in [q_0, q_1]$ in equilibrium.$^{19}$

Overall, Proposition 4 shows that sanctions are effective in discouraging charity scams. Perhaps more interestingly, sanctions are also effective in raising the quality of nonscam charities; formally, $\partial F^s(q) / \partial s < 0$ for $s < q_0$. The reason is that with the monitoring of scams, uninformed donors give more generously, affording nonprofits to invest in higher quality for a better chance of receiving informed donations. Despite its potential to prevent charity scams, however, (expected) sanctions are likely to be very low in practice. In particular, as we elaborate in Section 5, nonprofit enforcement is likely to be much weaker than its for-profit counterpart due to the significant legal and financial challenges it poses to regulators.

4.2 Fundraising activities

Another key feature of the nonprofit market missing in the baseline model is fundraising. Specifically, in the baseline model, we assume that charities actively compete for informed donations by investing in quality, but they are passive recipients of the uninformed and mission-based donations. In reality, charities, including scams, are rarely passive: they expend substantial resources to sway donors in their favor. Indeed, according to leading industry experts and watchdogs such as Charity Navigator, a 25-35 percent fundraising

$^{19}$More formally, from the equilibrium characterization in the appendix, it can be shown that $[q_0, q_1] = \emptyset$ and thus $q \in \{0\} \cup [q_2, q_3]$ for a small $s$; $q \in \{0\} \cup [q_0, q_1] \cup [q_2, q_3]$ for a moderate $s$, and $q \in [q_0, q_1] \cup [q_2, q_3]$ for $s \geq q_0$. 

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efficiency is considered reasonable – a figure that exceeds 50 percent for those charities that hire for-profit telemarketers.

Much like business advertising, one objective of fundraising is to persuade undecided donors about the charity’s specific mission and/or program; see, e.g., Aldashev et al. (2014). To establish the relationship between such fundraising and quality competition by charities, consider the baseline model and suppose that when simultaneously choosing their quality levels, charities also choose their fundraising efforts – how much time and energy to spend on donors. We assume that at the time of fundraising, charities are unable to distinguish whether a donor is informed, uninformed, or mission-oriented.

Let $x_i \geq 0$ be charity $i$’s fundraising expenditure per donor. If a solicited donor turns out to be mission-oriented or uninformed, we assume that he is persuaded by charity $i$’s appeal with probability $\frac{x_i}{\sum_j x_j}$, which is increasing in $x_i$ and decreasing in $x_j$, $j \neq i$.21,22 If the donor turns out to be informed, however, we assume that he ignores the charity’s appeal and continues to contribute to the highest quality charity. Note that given the quality choices and others’ fundraising efforts, charity $i$ chooses $x_i$ that maximizes its net revenues from fundraising:

$$\max_{x_i} FR_i = \frac{x_i}{\sum_j x_j} [Ug(E[q]) + AgA] - Mx_i.$$ (13)

A close inspection of (13) reveals that for fixed quality decisions, the fundraising game among charities is nothing but the well-known Tullock contest (Tullock 1980), in which charities wastefully compete for the “prize” of the uninformed and attached donations and the winning probability is proportional to their fundraising efforts. It therefore readily follows from (13) that in the unique equilibrium of the fundraising game, we have

$$x_i = x = \frac{n-1}{n^2} \left( \frac{Ug(E[q]) + AgA}{M} \right)$$ and

$$FR_i = FR = \frac{U}{n^2} \frac{8(E[q])}{A} + \frac{A}{n^2} A.$$ (14)

Being symmetric, charities expend equal fundraising efforts and as in the baseline model, each claims an equal fraction, $1/n$, of the uninformed and attached donations in equilibrium. Accounting for the fundraising outlays, however, each charity appears as though it

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20According to Charity Navigator, fundraising efficiency is “the amount spent to raise $1 in charitable contributions”.

21In particular, here we assume that charities’ publicly announced missions are sufficiently vague, and their donor appeals contain some useful but unverifiable information about program quality – at least at the point of solicitation, – so there is room for donor persuasion.

22Though it cannot arise in equilibrium, if $x_j = 0 \forall j$, then one can break ties such that each charity receives $1/n$ share of the donations as in the base model.
received a lower fraction, $1/n^2$, of these donations. This implies that under fundraising, the net revenues from winning, losing, and scamming coincide with those of the baseline model except that $1/n$ is replaced with $1/n^2$ in (4), (5), (6), respectively. As a result, the indifference equation (8) continues to hold and so do Propositions 1-3. In particular, the equilibrium quality distribution found in Proposition 1 remains intact, since informed donors, the driving force behind quality provision, do not respond to fundraising effort.

Fundraising effort, however, does respond to informed giving. From Lemma A1 in the Appendix, we observe that the fundraising effort, $x$, is single-peaked in the number of informed donors: too few informed would discourage the uninformed gift and too many would leave few uninformed donors to appeal to, both reducing the return to a charitable appeal. Hence, we predict fundraising to be done most intensely in a moderately informed population in which the total uninformed donation, $U_{gU}$, is the largest. By the same token, we also predict that nonprofit regulations aimed at raising the quality of charity through imposing high entry barriers and monitoring scams are likely to increase fundraising efforts.

4.3 Quality-adjusted provision

In the baseline model, we also assume that charities care only about the “quantity” of their services, $R_i$; they provide quality to satisfy the donor demand for it. Conceivably, charities may also have an intrinsic preference for quality. For instance, a homeless shelter may value not only how many needy it houses but also how well it houses them – e.g., families, youth, etc. With such an added motive for quality, it is intuitive that charities are less likely to scam and more likely to offer high quality for informed donors. To verify this intuition and demonstrate the robustness of our baseline model, consider the following specification of quality-adjusted provision:

$$R_i = (1 + \alpha q_i)R_i$$

where the parameter $\alpha \geq 0$ measures the charity’s preference for quality. Following similar arguments to the baseline model with $\alpha = 0$, note that if charity $i$ sets quality $q$ and ranks the highest among $n \geq 2$, then it receives net revenues: $R_{win}(q) = (1 + \alpha q)R_{win}(q)$. If it ranks lower or simply scams, then its net revenues are, respectively, $R_{lose}(q) = (1 + \alpha q)R_{lose}(q)$ and $R_{scam} = R_{scam}$. In a mixed strategy equilibrium, the indifference equation

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23 Specifically, it is established in Lemma A1 that $U_{gU}$ is single-peaked in $I$.
24 As an example, for $n = 10$, $r = 2$, and $I + U = 20$, we have that $U_{gU}$ is maximized at $I = 10$. 

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(7) that determines the quality distribution $F$ therefore becomes

$$F^{n-1}(q)R_{win}(q) + [1 - F^{n-1}(q)]R_{lose}(q) = R_{scam}$$

which reveals

$$F(q) = F(q) \times \left( 1 - \frac{\alpha R_{scam}}{1 + \alpha q} \right)^{\frac{1}{n-1}}. \tag{15}$$

For $\alpha = 0$, (15) reduces to $F(q) = F(q)$, as it should. More importantly, for $\alpha > 0$, $F(q) < F(q)$; that is, charities choose stochastically better quality when, like donors, they also care about the quality of their services. Intuitively, a charity that considers scamming donors, $q = 0$, and grabbing a share of the uninformed and attached donations can now earn an additional return on those donations by not scamming, as signified by the fraction $\frac{\alpha R_{scam}}{1 + \alpha q}$ in (15). Interestingly, all else equal, the greater the scam revenue, $R_{scam}$, the less likely a charity is to scam. This mutual dependence, which is absent in the baseline model, between the quality distribution and scam revenue makes equilibrium characterization for $\alpha > 0$ more involved. Nevertheless, when $\alpha$ is not too large, we prove the following result qualitatively similar to Proposition 1.

**Proposition 5** Suppose $\alpha \leq \frac{n}{U + A}$. Then, there is a symmetric equilibrium, in which each charity continuously mixes over $q \in [q_L, q_H]$ according to (15) and has a mass point at $q = 0$. In every equilibrium, charities are more likely to provide higher quality than the baseline model. However, as $n \to \infty$, the effect of $\alpha$ disappears and the fraction of scam charities approaches 1.

The condition for $\alpha$ ensures that the intrinsic return to quality is not too high to rule out the trade-off between scamming and setting high quality to win the informed donations. Note that the condition is more likely to hold as the number of charities grows, countervailing the intrinsic incentive for quality provision. In particular, in the limit economy, the intrinsic incentive vanishes and most charities turn scams as in the baseline model.\(^{25}\)

### 4.4 Market structure in the long run

With an exogenous number of nonprofits, our baseline model can be considered a short-run analysis of the market for donations. The analysis has, however, revealed that the nonprofit market is likely to attract scams and thus further entry.\(^{26}\) To determine the long-run

\(^{25}\)Unlike in the baseline model, we are unable to prove the uniqueness of a symmetric equilibrium in Proposition 5.

\(^{26}\)As discussed in Section 5, charities appear to face low barriers to entry and a light-handed regulation.
market structure, we introduce an entry stage to the baseline model. Specifically, suppose that there are initially no charities in the market, but a large pool of \textit{ex ante} identical organizations simultaneously decide whether or not to begin fundraising for, or in the name of, the charitable cause by incurring a setup cost \( k > 0 \) – e.g., designing a website, advertising, and staffing. Once the number of entrants becomes public, the fundraising game proceeds as in the baseline model: charities and donors simultaneously make their respective quality and giving decisions.

Let \( n^* \) be the equilibrium number of charities in the long run. Since entry will continue until the scam profit in (6) is exhausted, ignoring the trivial integer problem, \( n^* \) solves

\[
\frac{U}{n^*} g(E[\bar{q}]) + \frac{A}{n^*} g_A = k. \tag{16}
\]

Eq. (16) implies that it is primarily the uninformed and attached donors that are responsible for the charity entry. The informed donors also play a role but indirectly through quality provision since their donations are competed away while quality is set. Eq. (16) further implies that a competitive charitable market with \( n^* \geq 2 \) emerges regardless of the unattached – quality-sensitive – donors if there is a large enough mass of the attached donors to cover the fixed cost – i.e., if \( g_A \geq k \). Given this, the following proposition focuses on the complementary, and arguably more interesting, case in which unattached donors and in turn donor information also matter for the existence of a competitive charitable market.

**Proposition 6** Suppose \( g_A < k \). Then, there is a unique long-run equilibrium with \( n^* \geq 2 \) if and only if \( I \in [I_L, I_H] \) and \( k < k_L \), where \( I_c < I_L < I_H < M - A \) and \( k_L > 0 \) is a unique cutoff. In the long run, only the highest quality charity provides the public good.

Proposition 6 makes three points. First, when there are not enough attached donors in the population, the mass of informed donors can be neither too low nor too high to sustain a competitive charitable market: significant informed giving is needed to encourage the costly provision of quality, while significant uninformed giving is also needed to accommodate costly entry. The uniqueness of the equilibrium follows because, from Proposition 2, the expected quality, and thus the uninformed gift \( g(E[\bar{q}]) \), is decreasing in the number of charities, \( n \), resulting in a unique solution \( n^* \) to (16) for a given entry cost \( k \).

Second in the long run the nonprofit market is likely to be highly concentrated: a
monopoly service provider along with many unsuccessful and/or scam charities.\footnote{Of course, it is possible that all charities are scams, with probability $F_n^*(0) = (I_c/I)^{n-1}$, in which case no public good is provided, or said differently, the charitable cause in question is poorly served.} To see why, notice that using (16), the expected net revenues of the winning and losing charities in (4) and (5) reduce to:

$$R_{\text{win}}^*(q) = Ig(q) - q$$ and $$R_{\text{lose}}^*(q) = -q.$$ Clearly, $R_{\text{lose}}^*(q) \leq 0$, and $R_{\text{win}}^*(q) \geq 0$ since $I > I_c$ from Condition 1 above. Hence, at most one – the winning – charity provides the public good in the long run. Intuitively, unlike in the short run, charities exhaust their anticipated revenues from the attached and uninformed donors at the entry stage. This leaves informed donors as the unique source of a positive net revenue, which is grabbed by the highest quality charity. Last, unlike in the baseline model, (16) implies that in the long run attached donors invite entry – mostly scams – into the market and by Proposition 2 lower the average quality and provision of the public good. Hence, the regulators’ attempt to promote informed giving appears a well-founded long-run policy.\footnote{See, for instance, \url{www.usa.gov/donate-to-charity}.}

4.5 Endogenous information

The previous long run analysis demonstrates the importance of informed vs. uninformed giving on the market structure but assumes away information acquisition. To complete that analysis and understand equilibrium incentives to acquire information, suppose that at the outset all donors are uninformed of the charities’ quality levels. Each donor can, however, get privately informed by paying a fixed (utility) cost $c > 0.$\footnote{Our qualitative results in this section extend to heterogenous information costs. Details can be found in our working paper or requested from the authors.} This cost reflects the time and energy spent in learning about the charities’ programs and/or their watchdog ratings. Evidently, focusing solely on the charities’ publicly stated missions, the value of information is zero for an attached donor. For an unattached donor, who cares about the quality of charity, the value of information, denoted by $\Delta(I)$, is simply the difference between his expected informed and uninformed payoffs recorded in (12); that is,

$$\Delta(I) \equiv E[v(\tilde{q}_{\text{max}})] - v(E[\tilde{q}]).$$

Since $v''(q) > 0$ and $E[v(\tilde{q}_{\text{max}})] \geq E[v(\tilde{q})]$, by Jensen’s inequality the value of information is nonnegative. The donor will, however, pay for information if and only if its cost is
justified, namely \( \Delta(I) \geq c \). We assume that donors make their information gathering and giving decisions simultaneously—i.e., without observing others’ choices. Specifically, we say that the triple \((n^*, I^*, F^*(q))\) constitutes a long-run equilibrium under endogenous information if the following conditions are satisfied:

1. **(entry)** \( n^* \) solves the entry condition in (16),
2. **(quality competition)** \( F^*(q) = F(q)|_{n^*,I^*} \), where \( F(q) \) is as characterized in Proposition 1;
3. **(information acquisition)** for \( I^* \in (0, M - A) \),
   \[
   \Delta(I^*) = c. \tag{17}
   \]

The first two conditions are straightforward. The third one requires that the marginal unattached donor is indifferent between informed and uninformed giving if both types of giving exist in equilibrium. For a competitive charitable market, Proposition 6 has revealed that the mass of informed donors must indeed be interior; specifically, \( I^* \in [I_L, I_H] \). In order to determine if there is such a solution to (17), let

\[
\Delta_{\min} \equiv \min_{I \in [I_L, I_H]} \Delta(I) \text{ and } \Delta_{\max} \equiv \max_{I \in [I_L, I_H]} \Delta(I).
\]

It is readily verified that these extreme values of information are well-defined and \( 0 < \Delta_{\min} < \Delta_{\max} < \infty \). From here, Proposition 7 is immediate.

**Proposition 7** Suppose \( \frac{A_2}{2} g_A < k \). Then, under endogenous information, \( n^* \geq 2 \) if and only if the cost of information acquisition for donors is moderate: \( \Delta_{\min} \leq c \leq \Delta_{\max} \).

Intuitively, if the information cost were too high, then all donors would remain uninformed and without the informed, they would expect the minimal quality of service and refrain from giving. If, on the other hand, the information cost were too low, charities would expect a much intense quality competition and with little uninformed giving, they would be unable to recoup their entry costs (however small they are).\(^{30}\) Proposition 7

\(^{30}\) It is interesting to note that for donors to be interested in informed giving, there must be quality uncertainty. But this can only arise under competition; otherwise, having a single-peaked net revenue (see Figure 2), a single charity would choose a perfectly predictable quality, leading all donors to stay uninformed instead. This observation is reminiscent of why the follower may not pay to observe the leader’s action in a duopoly model (e.g., Morgan and Vardy 2007).
thus suggests a limited benefit to freely available charity ratings. To understand this point further, we next examine the effect of the information cost on informed giving and social welfare.

Given the mass of informed donors, $I$, we define social welfare as the sum of expected payoffs for donors and the expected total provision:

$$ W(I) \equiv \{ I[v^*_I(I) - c] + Uv^*_U(I) + Av_A \} + TR^*(I), \quad (18) $$

where inserting the equilibrium number of charities for a fixed $I$, $n^*(I)$, into (10) and (12), we have that $TR^*(I) = TR(I, n^*(I))$, $v^*_I(I) = v_I(I, n^*(I))$, $v^*_U(I) = v_U(I, n^*(I))$. Notice that the expected total provision, $TR^*(I)$, is zero since donations are exhausted at the entry and quality competition stages in the long run.\(^{31}\) This means that only donor payoffs matter for social welfare in (18), which, employing the facts that $U + I = M - A$ and $\Delta(I^*) = c$ simplifies in equilibrium to:

$$ W(I^*) = (M - A)v^*_U(I^*) + Av_A. \quad (19) $$

Since the attached donor’s payoff, $v_A$, is exogenous by assumption, (19) implies that informed giving improves social welfare to the extent that it improves the uninformed payoff, $v^*_U(I)$. In equilibrium, any extra utility from informed giving is outweighed by the information cost – i.e., $\Delta(I^*) = c$. As in Proposition 3, we show below that despite its negative indirect effect on quality provision through the number of charities, $n^*(I)$, both informed and uninformed payoffs are increasing in $I$ under endogenous entry, too. Eq.(19) therefore implies that a higher information cost will lower social welfare if and only if it discourages informed giving. From (17), the latter depends critically on whether $\Delta'(I) > 0$ or $\Delta'(I) \leq 0$ – i.e., whether donors’ information decisions are strategic complements or strategic substitutes. Interestingly, information decisions are strategic complements if, as in practice, entry barriers are low so that there is a large number of nonprofits in the market, as we prove next. For conciseness, we present the result when there are no attached donors, $A = 0$, in the population.\(^{32}\)

\(^{31}\)The observation of the zero expected provision matches that of Rose-Ackerman (1982) who examines a competitive charitable market with costly fundraising and free entry.

\(^{32}\)For $A > 0$, there is an additional but unrealistic equilibrium in which all (unattached) donors give informed (see Footnote 3). The reason is that for $A > 0$, a competitive charitable market, $n^* \geq 2$, would exist without needing uninformed donors to pay for entry.
Proposition 8 Suppose \( A = 0 \). Then, for an arbitrarily small entry cost, \( k \), there is a unique long-run equilibrium for every information cost \( c \in (\Delta_{\text{min}}, \Delta_{\text{max}}) \). In equilibrium, the mass of informed donors, \( I^* \), and social welfare, \( W(I^*) \), are both increasing in \( c \).

Proposition 8 is counter-intuitive at first. It says that a higher – not lower – information cost encourages informed giving and raises social welfare in turn. To see this, suppose that the entry cost is negligible, \( k \approx 0 \) (which seems to be the case in practice). Then a large number of charities floods the market but, by Corollary 1, most of them are scams, completely discouraging uninformed giving, \( g^*_U \approx 0 \). Hence, from (11), \( v^*_U(I) \approx 1 - \) the utility from purely private consumption – resulting in the value of information: \( \Delta(I) \approx v^*_I(I) - 1 \). Evidently, \( \Delta'(I) \approx v''_I(I) > 0 \), which indicates that information acquisition decisions are strategic complements: the more a donor expects others to be informed, the more he wants to be informed.\(^{33}\) Since \( \Delta(I^*) = c \) in equilibrium, this means that \( I^* \approx \Delta^{-1}(c) \), which is clearly increasing in \( c \).

On the face of it, Proposition 8 implies that the widespread availability of charity ratings aimed at encouraging more informed giving may have the unintended consequence of generating less. Hence, a more effective way to ensure informed giving may be a direct and free supply of information about charities. More broadly though, Proposition 8 stresses that in order to understand private incentives for informed giving, one has to understand how donors view others’ efforts: strategic complements or strategic substitutes. Before reaching a firm policy conclusion, it is therefore important to enrich our model with other realistic features such as the possibility of information sharing among donors that may dilute strategic complementarity.

In the next section, we take a stock of our results up to now and discuss their empirical and policy relevance.

5 Discussion

As indicated in the Introduction, the number of nonprofits in the U.S. grew dramatically over the last decade: 50 times faster than small businesses (Bernasek 2014). And chances are that many charitable organizations tackle similar, if not identical, social issues. For example, a quick search online reveals that there are currently about 8500 registered cancer

\(^{33}\)Alternatively, as discussed in Proposition 3, an increase in the mass of the informed extends the nonprofit competition to higher quality levels and widens the quality dispersion, increasing the value of informed giving.
charities in the U.S., of which about 1600 specialize in breast cancer (www.guidestar.org). On the face of it, nonprofit competition, like its for-profit counterpart, should increase the quality of service, weed out poor performers, and ultimately create social value. Our analysis, however, proved otherwise: nonprofit competition is likely to lower the quality of charity, encourage charity scams, and diminish the provision of the public good (Propositions 1-2).

In our model, charities invest in quality to attract informed donors who search for the highest performer. As competition intensifies, the charities’ incentive to win over the informed weakens, resulting in lower quality (Proposition 2). In particular, having low entry barriers and numerous organizations, we predict nonprofit markets to be filled with ineffective or outright scam charities and dominated by one or few service providers (Corollary 1 and Proposition 6). The evidence supports our prediction. Seeman et al. (2014) estimate that charitable markets are indeed highly concentrated, with 21 percent dominated by one organization and 53 percent by a few. In either case, the market contains many small and unsuccessful organizations that seldom exit. Harrison and Laincz (2008) find that only about 12 percent of newly registered public charities exit within five years and 17 percent after ten years. In contrast, Dunne et al. (1988) find that in manufacturing, about 60 percent of new firms die within five years and nearly 80 percent in ten years. Harrison and Laincz attribute this stark difference between the exit rates of the two sectors to the non-distribution constraint for nonprofits, which may lead to undervaluation of assets for an alternative use by internal managers relative to for-profits, and to nonprofit motives other than net revenue maximization.

As noted above, an important implication of our analysis is that most charities end up

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34 For instance, Smith (2014, p. 340), a nonprofit scholar, writes: “The number of nonprofit charitable organizations now exceeds one million [in the US], so extensive oversight is simply impossible. More fundamentally the US has historically been committed to relatively low barriers to entry for individuals and groups interested in forming nonprofits. It is quite straightforward and simple to create a nonprofit charitable organization...In the US, the UK, Canada, Australia, and many European countries, the legal tradition has been to encourage advocacy through nonprofit organizations so the oversight evident in Russia, Turkey, China, and other countries is not considered necessary or appropriate.”

35 In the same study, Seeman et al. estimate only one percent of the markets to be unconcentrated or “atomistic,” with the remaining 25 percent being moderately concentrated or oligopolistic. As usual in such studies, the authors define the market area as Metropolitan Statistical Areas (MSAs) to reflect the presumption that effective competition among nonprofit entities primarily occurs in localized settings, and they limit attention in the dataset to nonprofit organizations that are unaffiliated with larger networks.

36 Taking into account nonreporting charities, another study estimates less than 4 percent of charities to go out of business each year compared to a 50-50 chance of survival for a new small business (Froker Group 2013).
being scams in an unregulated market and tightening nonprofit enforcement might reduce or even solve this problem (Proposition 4). According to Cooper (2015), a former state attorney general, however, nonprofit enforcement presents formidable challenges. First, nonprofit regulation is often spread among multiple offices, requiring a great deal of coordination. Second, due to free speech protection for charitable solicitations, fraudulent solicitation cases are difficult to prove; and third, even if they are proved, financial recoveries tend to be nominal and typically directed to bona fide charities rather than applied to the costs of investigation and litigation. All these challenges for nonprofit enforcement suggest weak sanctions that leave sizable room for charity scams.

Besides monitoring of nonprofit conduct, our investigation also suggests regulation of nonprofit entry to improve market performance. Reich et al (2009), who report nearly perfect approval rates for a nonprofit status by the Internal Revenue Service, agree with such regulation and recommend an increase in the application fee so “it would signal to organizations that they should be more than a fly-by-night organization, minimally staffed or with no revenue stream, before submitting an application.” An alternative policy to control market structure might be to facilitate mergers and acquisitions for nonprofits. Consistent with this policy, Milway et al. (2014) observe a growing support for nonprofit mergers in the industry and attribute the slow pace primarily to a lack of matchmakers between nonprofits and insufficient funding for the merger process such as due diligence and post-merger integration. Like these authors, we therefore see a crucial role that funders and regulators can play in an efficient consolidation of the nonprofit market.

Complementary to nonprofit regulation, our results imply that promoting informed giving can also be an effective, and perhaps less costly, mechanism to increase quality in the charitable sector (Proposition 3). And there is evidence in favor. Summarizing their recent surveys of donors, Ottenhoff and Ulrich (2012) report that 90 percent of donors care about nonprofit performance, but only 3 percent seek the highest performer (see also Footnote 3). These industry experts recommend that nonprofits should supply information to donors and “help themselves by sharing information on their impact and effectiveness on their website, third-party portals, and solicitations.” Given the currently low level of informed giving, we concur with this recommendation but note that there might be a socially optimal mix of informed and uninformed donors in the population (Propositions 3

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37 For example, a recent cancer charity fraud case in the U.S. involved attorneys general for each of the 50 states and it took four years to be brought to fruition (www.cnn.com/2015/05/19/us/scam-charity-investigation).
and 6) – a hypothesis that awaits testing.

Last but not least, our investigation can also shed light on how the charitable market responds to the changes in donor preferences. It is well-documented that people become more generously minded during holiday seasons (McLean and Brouwer 2012; Urban Institute 2014) and in the aftermath of natural disasters (Douty 1972; Brown et al. 2012). And almost concurrent with these events, donors are strongly cautioned against charity scams and advised to research before giving.\textsuperscript{38} To make sense of these facts, note from Proposition 1 that for a fixed number of charities, a charity scam is actually less likely with a higher elasticity of substitution, $r$, between private consumption and giving, whenever $r \geq 2$ since $I_c$ defined in Condition 1 is decreasing in $r$.\textsuperscript{39} The reasoning is analogous to the comparative static with respect to informed giving in Proposition 3 because a greater degree of substitution toward giving also means more generous informed givers, raising the charities’ returns from winning their donations by offering a higher quality service. The number of charities is, however, unlikely to remain fixed: anticipating fewer scams, uninformed donors would give more generously and entice charity scams in turn, validating the concern of regulators. Overall, if, during holiday seasons and disaster reliefs, charity scams find it easy to setup operations and appeal to donors, we predict a lower average quality of charity. This prediction would be reinforced if the increased generosity also meant more giving by mission-oriented donors since, from (16), it is clear that a higher $g_A$ would invite even more scams into the market.

\section{6 Conclusion}

In this paper, drawing upon the all-pay auction literature (e.g., Varian 1980; Che and Gale 2003), we have offered a novel model of charity competition in which donor information alone can explain the vast quality dispersion across similar charities and the demand for information in turn. Our analysis has produced a rich set of results and identified a negative effect of competition and a positive effect of informed giving on the quality of charity. Both effects seem to have empirical and policy relevance, as discussed above.

In closing, we briefly argue that our main findings would generalize to \textit{ex ante} heteroge-


\textsuperscript{39}In fact, even more strongly, it can be verified that the entire quality distribution shifts up with a higher $r$; that is, $\frac{\partial F(q)}{\partial r} < 0$ for $r \geq 2$. For $1 < r < 2$, the shift in the distribution is ambiguous, but given that we are interested in major preference shocks here, this region for $r$ seems less relevant.
nous charities, too. To see this, let the cost of quality for charity $i$ be $C_i(q_i) = c_i q_i$, where $c_i$'s are drawn independently from a continuous distribution. For simplicity, assume that cost realizations are commonly known among charities but unknown to the uninformed donors so they continue to pick a charity at random as in our baseline model. Then, similar to Marceau et al. (2010), who consider tax competition among countries of heterogeneous population sizes, it can be shown that in equilibrium, only the two most efficient charities would choose positive quality, with the rest being scammers with probability one. Thus, as with our analysis of ex ante identical charities, we also predict a widespread problem of scamming in this extension, and it is clear that further entry by less efficient organizations would only exacerbate this problem. Nevertheless, in order to fully understand policy, more work needs to be done. We believe that a dynamic model of charity competition would add to our analysis by allowing organizations to build reputations and distinguish themselves from scammers.

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40 Conceivably, being in the same market for donations, charitable organizations would know each other's technology better than donors.
Appendix

Proof of Proposition 1. As in a standard all-pay auction, no equilibrium in pure strategies exists in quality competition; otherwise, charity $i$ could slightly increase its quality and win all of informed donations. In equilibrium, let each charity mix according to the symmetric c.d.f. $F(q)$, whose support is $S$. Then, for every $q \in S$, $E[R_i(q)] = R$, where $R \geq R_{scam}$ is the expected payoff defined in (6). Using (4), (5), and (7), we find

$$F(q) = \left[ \frac{q + R - R_{scam}}{Ig(q)} \right]^\frac{1}{n-1}.$$ 

Clearly, $F(q) > 0$ for all $q > 0$, requiring a mass point on the support. The mass point, however, cannot be some $q > 0$; otherwise, charity $i$ could shift its weight to $q + \epsilon$ and discretely increase its probability of winning. Hence, $0 \in S$, implying $R = R_{scam}$, and $F(q)$ in (9).

Next, we define $S^+ \equiv \{ q \in S | q > 0 \}$ and establish that $S^+ = [q_L, q_H]$ for some bounds $q_L < q_H$. To do so, we differentiate (9) for $q \in S^+$:

$$F'(q) = \frac{1}{n-1} \left( \frac{1-r+q^r}{q(1+q^r)} \right) F(q).$$

Evidently, $F'(q) \geq 0$ if and only if $q \geq (r-1)^\frac{1}{r}$. Moreover, $q_L \geq (r-1)^\frac{1}{r}$ and $q_H (> q_L)$ uniquely solves $F(q_H) = 1$; or equivalently, $q_H$ is the largest root of:

$$\Omega(q) \equiv Ig(q) - q = 0. \quad (A-1)$$

By definition of $q_H$, there is no incentive to choose $q > q_H$. To show $q_L = (r-1)^\frac{1}{r}$, suppose, to the contrary, that $q_L > (r-1)^\frac{1}{r}$. Then, charity $i$’s expected payoff from a deviation to $q \in (0, q_L)$ is

$$E \left[ R_i^d(q) \right] = (F(q_L))^{n-1} Ig(q) - q.$$ 

Simple algebra shows

$$\frac{\partial}{\partial q} E \left[ R_i^d(q) \right] \bigg|_{q=q_L} = \frac{r}{1+q_L^r} - 1 < \frac{r}{1 + (r-1)^r} - 1 = 0.$$ 

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This means that there exists some \( q \in (0, q_L) \) such that \( E \left[ R_i^d(q) \right] > E \left[ R_i^d(q_L) \right] = 0 \), contradicting \( q_L \in S^+ \). Hence, \( q_L = (r - 1)^{\frac{1}{2}} \). In addition, since \( F'(q) > 0 \) for \( q > q_L \), having no mass at some \( q > 0 \) implies that \( S^+ = [q_L, q_H] \), and \( F(0) = F(q_L) = (I_c / I)^{\frac{1}{2}} \), as desired.

**Proof of Proposition 2.** We prove each part in turn. For part (a), integrating by parts reveals
\[
E[\tilde{q}] = q_H - q_L F(q_L) - \int_{q_L}^{q_H} F(q) dq. \tag{A-2}
\]
From here, \( \frac{dE[\tilde{q}]}{dn} < 0 \) since \( \frac{dF(q)}{dn} > 0 \). Moreover, \( \lim_{n \to \infty} E[\tilde{q}] = 0 \) since \( \lim_{n \to \infty} F(q) = 1 \) for all \( q \in S^+ \).

For part (b), we have
\[
E[g_1] = g(q_H) - g(q_L) F^n(q_L) - \int_{q_L}^{q_H} g'(q) F^n(q) dq, \tag{A-3}
\]
which implies \( \frac{dE[g_1]}{dn} < 0 \) since \( \frac{dF^n(q)}{dn} > 0 \), and \( \lim_{n \to \infty} E[g_1] = 0 \) since \( \lim_{n \to \infty} F^n(q) = \frac{q}{g(q)} \). For the uninformed donation, stated in (3), the result obtains directly from part (a). Similarly, part (c) directly follows from part (a) and Corollary 1.

Finally, for part (d), note that
\[
E[v_1] = v(q_H) - [v(q_L) - v(0)] F^n(q_L) - \int_{q_L}^{q_H} v'(q) F^n(q) dq. \tag{A-4}
\]
Given that \( v'(q) > 0 \) and \( \frac{dF^n(q)}{dn} > 0 \), we have \( \frac{dE[v_1]}{dn} < 0 \). For uninformed donors, the result is immediate from part (a).

**Proof of Corollary 1.** Using \( F(0) = (I_c / I)^{\frac{1}{2}} \) from Proposition 1 and applying L'Hospital's rule, the conclusions are easily verified.

**Proof of Corollary 2.** Suppose that \( \frac{U}{n} g_U + \frac{A}{n} g_A \big|_{n=2} > q_H \). Then, \( R_{\text{win}}(q|n = 2) \geq R_{\text{lose}}(q|n = 2) \) > 0. Next, since \( \frac{\partial g_U}{\partial n} < 0 \) and \( \lim_{n \to \infty} g_U = 0 \) by Proposition 2, \( \frac{U}{n} g_U + \frac{A}{n} g_A \) is strictly decreasing in \( n \), with \( \lim_{n \to \infty} \left( \frac{U}{n} g_U + \frac{A}{n} g_A \right) = 0 \). Hence, there exists a unique \( \pi < \infty \) such that \( \frac{U}{n} g_U + \frac{A}{n} g_A \big|_{n=\pi} = q_H \). Finally, for \( n \leq \pi \), \( R_{\text{lose}}(q) \geq 0 \) for all \( q \), implying that the provision occurs for all \( q \in S^+ \), as claimed.

**Lemma A1** Let \( h(I, n) \equiv \frac{M-A-I}{n} g(E[\tilde{q}]) \). Then, \( h(I, n) \) is continuous and single-peaked in \( I \in [0, M - A] \), with \( h(I, n) = 0 \) for \( I \in [0, I_c] \cup \{ M - A \} \).

**Proof.** Clearly, if \( E[\tilde{q}] \) is continuous in \( I \), so is \( h(I, n) \). For \( I > I_c \), \( E[\tilde{q}] \) is continuous by Proposition 1. For \( I < I_c \), it is straightforward to verify that \( \Omega(q) < 0 \) for all \( q \), where \( \Omega(q) \)
is defined in (A-1). Therefore, \( E[\tilde{q}] = 0 \) and is continuous for \( I < I_c \). For \( I = I_c \), note that

\[
\arg \max_q \Omega(q) = \frac{I}{I_c} I_c = \frac{I}{I_c} q_L, \quad \text{with} \quad \Omega(q_L) = 0.
\]

Hence, \( q_H = q_L \) and \( E[\tilde{q}] = 0 \) at \( I = I_c \). Since \( \Omega(q) \) is continuous in \( I \), we have \( \lim_{I \to I_c} q_H = q_L \), implying \( \lim_{I \to I_c} E[\tilde{q}] = 0 \) and establishing the continuity of \( E[\tilde{q}] \) at \( I = I_c \), too.

To prove single-peakedness, first observe that \( E[\tilde{q}] = 0 \) and thus \( h(I, n) = 0 \) for \( I \leq I_c \). Similarly, \( h(n, M - A) = 0 \). Differentiating \( h(I, n) \) with respect to \( I > I_c \) and taking into account the fact that \( g'(q) = \frac{r x(q)}{q(1 + q')} \), we find

\[
\frac{\partial h(I, n)}{\partial I} = \frac{(E[\tilde{q}])^{-1}}{n (1 + (E[\tilde{q}])^2)} \left( (M - A - I)r \frac{\partial E[\tilde{q}]}{\partial I} - E[\tilde{q}] (1 + (E[\tilde{q}])^2) \right). \tag{A-5}
\]

Furthermore, from (A-2) and \( \frac{\partial E(q)}{\partial I} = -\frac{1}{(n-1)I} F(q) \), we obtain

\[
\frac{\partial E[\tilde{q}]}{\partial I} = \frac{q_H - E[\tilde{q}]}{(n - 1)I} > 0. \tag{A-6}
\]

Recall that \( \lim_{I \to I_c} E[\tilde{q}] = 0 \), revealing that \( \frac{\partial h(I, n)}{\partial I} > 0 \) for \( I \) arbitrarily close to \( I_c \). Moreover, \( \frac{\partial h(M - A, n)}{\partial I} < 0 \) from (A-5), which, by continuity, reveals that \( \frac{\partial h(I, n)}{\partial I} = 0 \) has a solution \( I \in (I_c, M - A) \). For single-peakedness of \( h(I, n) \), it suffices to prove that this solution is unique. Suppose, to the contrary, that it is not and \( I' < I'' \) are the two smallest roots. To ease notation, define \( \Phi(I, n) = \Lambda(I, n) - Y(I, n) \). Then, \( \Phi(I', n) = 0 \). Since \( \Phi(I, n) > 0 \) for \( I \in (I_c, I') \), we have \( \frac{\partial h(I', n)}{\partial I} < 0 \) and that \( h(I, n) \) reaches a local maximum at \( I' \). Simple differentiation yields

\[
\frac{\partial \Lambda(I, n)}{\partial I} = -r \frac{\partial E[\tilde{q}]}{\partial I} + r(M - A - I) \frac{\partial^2 E[\tilde{q}]}{\partial I^2}, \tag{A-7}
\]

and

\[
\frac{\partial Y(I, n)}{\partial I} = \frac{\partial E[\tilde{q}]}{\partial I} \left[ 1 + (r + 1)(E[\tilde{q}])' \right] > 0, \tag{A-8}
\]

where from (A-6),

\[
\frac{\partial^2 E[\tilde{q}]}{\partial I^2} = \frac{1}{(n - 1)I} \left[ -n \frac{\partial E[\tilde{q}]}{\partial I} + \frac{\partial q_H}{\partial I} \right], \tag{A-9}
\]

and from (A-1),

\[
\frac{\partial q_H}{\partial I} = \frac{q'_H}{1 - r + q'H} > 0 \quad \text{(because} \quad q_H > q_L). \tag{A-10}
\]
If \( \frac{\partial \Phi(I_n)}{\partial I} \leq 0 \) for \( I \in (I', M - A) \), then \( \Phi(I, n) < 0 \), and \( I' \) is the unique solution. Hence, 
\( \frac{\partial \Phi(I_n)}{\partial I} > 0 \). Given that \( \frac{\partial \Phi(I_n)}{\partial I} < 0 \), this implies the existence of some \( I'' \in (I', I'') \) such that 
\( \frac{\partial \Phi(I_n)}{\partial I} = 0 \) and \( \frac{\partial^2 \Phi(I_n)}{\partial I^2} > 0 \). In addition, since \( \frac{\partial \Phi(I_n)}{\partial I} > 0 \), we have \( \frac{\partial \Phi(I_n)}{\partial I} > 0 \), which, from (A-7), implies \( \frac{\partial^2 \Phi(I_n)}{\partial I^2} |_{I = I''} > 0 \). Further differentiating (A-7) and (A-8),
\[
\frac{\partial^2 \Phi(I, n)}{\partial I^2} = -2r \frac{\partial^2 \Phi[I]}{\partial I^2} + r(M - A - I) \frac{\partial^3 \Phi[I]}{\partial I^3},
\]
and
\[
\frac{\partial^2 \Phi(I, n)}{\partial I^2} = \frac{\partial^2 \Phi[I]}{\partial I^2} [1 + (r + 1)(E[I])^2] + (r + 1)E[I]^{-1} \left( \frac{\partial^2 \Phi[I]}{\partial I^2} \right)^2.
\]
Since \( \frac{\partial^2 \Phi[I]}{\partial I^2} |_{I = I''} > 0 \), \( \frac{\partial^2 \Phi[I]}{\partial I^2} > 0 \). Moreover, differentiating \( \frac{\partial^2 \Phi[I]}{\partial I^2} \) with respect to \( I \) and simplifying terms yield
\[
\frac{\partial^2 \Phi[I]}{\partial I^2} = -\frac{12n}{n - 1} \frac{\partial^2 \Phi[I]}{\partial I^2} + \frac{1}{(n - 1)I} \frac{\partial^2 \Phi[I]}{\partial I^2},
\]
where \( \frac{\partial^2 \Phi[I]}{\partial I^2} = -\frac{(r - 1)q}{1 - r + 4I} \frac{\partial \Phi[I]}{\partial I} < 0 \). As a result, \( \frac{\partial^2 \Phi[I]}{\partial I^2} |_{I = I''} > 0 \), \( \frac{\partial^2 \Phi[I]}{\partial I^2} |_{I = I''} < 0 \), and 
\( \frac{\partial^2 \Phi[I]}{\partial I^2} |_{I = I''} < 0 \). This, however, implies that \( \frac{\partial^2 \Phi[I]}{\partial I^2} |_{I = I''} < 0 - \) a contradiction. Hence, there is no \( I'' \in (I', M - A) \) such that \( \frac{\partial \Phi[I]}{\partial I} > 0 \), contradicting \( \Phi(I'', n) = 0 \). Hence, there is a unique \( I' \) that solves \( \Phi(I', n) = 0 \), proving that \( h(I, n) \) is single-peaked in \( I \).

**Proof of Proposition 3.** Note that \( E[I] \) is increasing in \( I \) by (A-6). For the informed donor, \( E[I] \), given by (A-3), and \( E[I'] \), given by (A-4), are both increasing in \( I \), as \( \frac{\partial^2 E[I']}{\partial I^2} < 0 \). For the uninformed donor, \( \frac{\partial^2 E[I']}{\partial I^2} = \frac{\partial E[I']}{\partial I} = 0 \) and \( \frac{\partial^2 E[I']}{\partial I^2} = \frac{\partial E[I']}{\partial I} > 0 \), since \( g'(q) > 0, v'(q) > 0 \), and \( \frac{\partial^2 E[I']}{\partial I^2} > 0 \).

Next, note from Lemma A1 that \( TR(I, n) = [1 - F(0)] [nh(I, n) + AgA] \). Differentiating with respect to \( I \), we obtain
\[
\frac{\partial TR(I, n)}{\partial I} = \frac{F(0)}{n - 1} \left[ (M - I - A)g(E[I]) + AgA \right] + [1 - F(0)] n \frac{\partial h(I, n)}{\partial I}.
\]
By Lemma A1, \( \lim_{I \to I_c} \frac{\partial TR(I, n)}{\partial I} > 0 \) because \( h(I, n) \) is increasing in \( I \) for \( I \) arbitrarily close to \( I_c \). Moreover, from (A-5),
\[
\frac{\partial TR(M - A, n)}{\partial I} = \frac{1}{n - 1} \left[ \frac{1}{M - A} F(0)AgA - (n - 1)(1 - F(0))g(E[I]) \right].
\]
Clearly, \( \frac{\partial TR(M - A, n)}{\partial I} \) is increasing in \( A \), with \( \frac{\partial TR(M - A, n)}{\partial I} < 0 \) for \( A = 0 \). Therefore, by continuity, there exists \( A(n) > 0 \) for all \( n < \infty \) such that \( \frac{\partial TR(M - A, n)}{\partial I} < 0 \) for \( A \leq A(n) \). Together,
\[
\lim_{l \to 0} \frac{\partial TR(I, n)}{\partial l} > 0 \text{ and } \frac{\partial TR(M - A, n)}{\partial l} < 0 \text{ prove that } TR(I, n) \text{ is non-monotonic and maximized for } I \in (I_c, M - A) \text{ if } A < \bar{A}(n). \text{ To prove that } TR(I, n) \text{ is monotonically increasing when } A \text{ and } n \text{ are sufficiently large, note that for } A = M - I,
\]
\[
\frac{\partial TR(I, n|A = M - I)}{\partial l} = \frac{1}{n - 1} \left[ F(0) \frac{M - I}{I} g_A - (n - 1) (1 - F(0)) g(E(\bar{q})) \right].
\]
Since \(\lim_{n \to \infty} F(0) = 1, \lim_{n \to \infty} (n - 1) (1 - F(0)) = \ln(1) \in (0, \infty) \text{ (by Corollary 1), and}
\lim_{n \to \infty} g(E(\bar{q})) = 0 \text{ (by Proposition 2), we find } \lim_{n \to \infty} (n - 1) \times TR(I, n|A = M - I) = \frac{M - I}{I} g_A > 0. \text{ Therefore, by continuity of } TR(I, n) \text{ with respect to } A \text{ and } n, \text{ there exist } \bar{n} < \infty \text{ and } \bar{A}(n) < M - I \text{ such that } \frac{\partial TR(I, n)}{\partial l} > 0 \text{ for } n > \bar{n} \text{ and } A > \bar{A}(n). \]

**Proof of Proposition 4.** We first establish that the equilibrium net revenue is: \(R^s = \frac{U}{n} g_A + A \frac{g_A}{n} - \min\{q_0, s\} \equiv R_{\min}. \text{ Clearly, } R^s \geq R_{\min}, \text{ since a charity can guarantee a profit of } R_{\min} \text{ by choosing } q = 0 \text{ for } s < q_0 \text{ or choosing } q = q_0 \text{ for } s \geq q_0. \text{ Suppose } R^s > R_{\min}, \text{ implying that } q = 0 \text{ is not part of the support, denoted by } \Sigma. \text{ Then, for any } q \in \Sigma, \text{ equilibrium indifference across quality choices requires that}
\]
\[
(F^s)^{n-1}(q) I g(q) + \frac{U}{n} g_A + A \frac{g_A}{n} - 1_{(q < q_0)} s - q = R^s,
\]
and in turn,
\[
(F^s)^{n-1}(q) =\frac{R^s - \left(\frac{U}{n} g_A + A \frac{g_A}{n} - 1_{(q < q_0)} s - q\right)}{I g(q)}.
\]
Clearly, \(R^s > R_{\min}\) would imply \(F^s(q) > 0\) for all \(q > 0\), and thus a mass point \(q = 0\), which would, as in the proof of Proposition 1, result in a profitable deviation to \(q + \epsilon\). This means \(R^s = R_{\min}\), and
\[
F^s(q) = \begin{cases} 
\left[\frac{s + q - \min\{q_0, s\}}{I g(q)}\right]^{\frac{1}{n-1}} \equiv F^s_n(q) & \text{if } q < q_0 \\
\left[\frac{q - \min\{q_0, s\}}{I g(q)}\right]^{\frac{1}{n-1}} \equiv F^s_m(q) & \text{if } q \geq q_0.
\end{cases}
\]

Now we argue that only \(q = 0\) can be sanctioned in equilibrium; that is, \(q \in (0, q_0)\) is not in \(\Sigma. \text{ For } q < q_0, \text{ the corresponding p.d.f. is}
\]
\[
f^s_n(q) = \frac{(F^s)^{2-n}}{n-1} \times \frac{1}{I g(q)} \times q \frac{q^r - (r - 1)}{q(1 + q^r)} - r [s - \min\{q_0, s\}] \frac{q^r}{q(1 + q^r)}.
\]
Clearly, \(f^s_n(q) < 0\) for \(q < q_L\) and thus \(q \in (0, q_0)\) cannot be in \(\Sigma. \text{ Next, we derive the equilibrium support for } q \geq q_0, \text{ facilitated by the following lemma.}\)
Lemma A2 Let $q' \in \Sigma^+ = \{q \in \Sigma | q > 0\}$ and $q'' \notin \Sigma^+$, with $F^s(q'') = F^s(q')$ in equilibrium. Then, there is no unilateral incentive by a charity to choose $q''$ if and only if $F_{ns}^s(q'') \geq F_{ns}^s(q')$ for $q'' \geq q_0$ and $F_{s}^s(q'') \geq F_{ns}^s(q')$ for $q'' < q_0$.

Proof. Given that $F^s(q'') = F^s(q')$ and $q' \geq q_0$, no deviation to $q''$ requires

$$\pi_{ns}^{Dev}(q'') = F_{ns}^s(q') - F_{ns}^s(q'') + \frac{U}{n}g_u + \frac{A}{n}g_A - q'' - 1_{(q'' < q_0)}s \leq R^s.$$  \hspace{1cm} (A-12)

For $q'' \geq q_0$, the above inequality reduces to $F_{ns}^s(q'') \geq F_{ns}^s(q')$, while for $q'' < q_0$, it reduces to $F_{s}^s(q'') \geq F_{ns}^s(q')$. $$\square$$

The equilibrium p.d.f. for $q \geq q_0$ is

$$\frac{\partial F_{ns}^s(q)}{\partial q} = \frac{[F_{ns}^s(q)]^{2-n}}{(n-1)q(1+q')I_g(q)\left[q^{r+1} - (r-1)q + r \min\{s,q_0\}\right]}.$$  \hspace{1cm} (A-13)

where

$$J'(q) = (r+1)q^r - (r-1) > 0 \iff q > q_c = \left(\frac{r-1}{r+1}\right)^{\frac{1}{r}},$$

and

$$J''(q) = r(r+1)q^{r-1} > 0.$$  

Therefore, $J(q)$ is minimized at $q = q_c$, with $J(q_c) = r(\min\{s,q_0\} - q_c^{r+1})$. We consider two cases:

$\underline{\min\{s,q_0\} > q_c^{r+1}}$: Then, $J(q) > 0$ for all $q$ and $F_{ns}^s(q)$ is increasing in $q$. Moreover, by Lemma A2, the lower bound of the distribution is $q^s = q_0$; otherwise, if $q^s > q_0$, there would exist a profitable deviation to $q \in [q_0,q^s)$. Let $q_3$ uniquely solve $F_{ns}^s(q_3) = 1$. Then, $q \in [q_0,q_3]$ must be in the support of the distribution since $\frac{\partial F_{ns}^s(q)}{\partial q} > 0$ would otherwise imply a mass point for some $q$ on the support, which, as argued in Proposition 1, would result in a small profitable deviation. Therefore, $q_1 = q_2 = q_0$. For $s < q_0$, $F^s(q_0) > 0$, implying a mass point at 0, with $F^s(0) = F^s(q_0)$. Clearly, $F^s(0)$ is decreasing in $s$. Moreover,

$$E[q^s] = q_3 - q_0F^s(q_0) - \int_{q_0}^{q_3} F^s(q)dq.$$  \hspace{1cm} (A-14)

Straightforward differentiation reveals that $\frac{\partial E[q^s]}{\partial s} > 0$ since $\frac{\partial F^s(q)}{\partial q} < 0$. For $s \geq q_0$, $F^s(q_0) = 0$, implying no scams in equilibrium. Moreover, since $F^s(q)$ is continuous at $s = q_0$, $E[q^s]$ is maximized for $s \geq q_0$.

$\underline{\min\{s,q_0\} \leq q_c^{r+1}}$: Then, $J(q_c) < 0$. Note also that $J(q) > 0$ for all $q \leq \min\{s,q_0\}$ and all $q \geq q_L$. Moreover, $J''(q) > 0$ and $J(q)$ is minimized at $q_c$, implying that it has two roots.
- $q^0 \in (\min\{s, q_0\}, q_c)$ and $q^{00} \in (q_c, q_L)$. $F_{ns}^s(q)$ is increasing in $q$ for $q \in (\min\{s, q_0\}, q^0)$ and reaches a local maximum at $q^0$. $F_{ns}^s(q)$ is decreasing in $q$ for $q \in (q^0, q^{00})$ and reaches a local minimum at $q^{00}$. $F_{ns}^s(q)$ is strictly increasing for $q > q^{00}$. Now we argue that the upper bound of the distribution denoted by $q_3$, solving $F^s(q_3) = 1$, must satisfy $q_3 > q^{00}$. Note that $F_{ns}^s(q^{00}) \leq F_{ns}^s(q_L) < F_{ns}^s(q_L) < 1$, and thus by Lemma A2, any $q_3 < q^{00}$ would result in a profitable deviation to $q^{00}$, implying $q_3 > q_L \geq q^{00}$. Since $F_{ns}^s(q)$ is increasing in $q$ for $q > q^{00}$, there is a unique $q_3 > q^{00}$ that solves for the upper bound of the distribution.

Next, by Lemma A2, $\max\{q_0, q^{00}\}$ has to be in the support too, because there would otherwise be a profitable deviation to $\max\{q_0, q^{00}\}$. Moreover, all $q \in (\max\{q_0, q^{00}\}, q_3)$ belong to the support since the fact that $\Delta F_{ns}^s(q) > 0$ for $q > q^{00}$ would otherwise imply a mass point for some $q$ in the support. Thus, $q_2 = \max\{q^{00}, q_0\}$. If $q_2 = q_0$, we have $q_1 = q_0$ since, as established earlier, $q \in (0, q_0)$ is not in the support of the distribution. If $q_2 = q^{00} > q_0$, then $0 = F_{ns}^s(\min\{s, q_0\}) < F_{ns}^s(q_2) < F_{ns}^s(q^0)$. Since $F_{ns}^s(q)$ is strictly increasing in $q$ for $q \in (\min\{s, q_0\}, q^0)$, by continuity, there exists $q \in (\min\{s, q_0\}, q^0)$ such that $F_{ns}^s(q) = F_{ns}^s(q_2)$ and $F_{ns}^s(q) < F_{ns}^s(q_2)$ for all $q \in [\min\{s, q_0\}, q]$. Then, by Lemma A2, all $q \in [q_0, q]$ are in the support of the distribution. Thus, $q = q_1$. Moreover, since $F_{ns}^s(q) > F_{ns}^s(q_2)$ for all $q \in (q_1, q_2)$, and $F^s(q)$ must be nondecreasing in $q$, $q \in (q_1, q_2)$ is not in the support of the distribution. To prove no profitable deviation to $q \in (0, q_0)$, note from (A-11) that $F^s(q)$ is minimized at $q_{\text{min}}^{s} \geq q_L$. Moreover, $F_{ns}^s(q) < F_{ns}^s(q_0)$ for all $q$. Therefore, $F_{ns}^s(q) > F_{ns}^s(q_{\text{min}}^{s}) > F_{ns}^s(q_2)$ for all $q < q_0$, which, by Lemma A2, establishes no profitable deviation. A similar argument establishes no profitable deviation to $q \in [q_1, q_2]$. For $s < q_0$, $F^s(q) > 0$ for all $q > s$. This implies a mass point at $q = 0$, with $F^s(0) = F^s(\hat{q}^s) > 0$ where $\hat{q}^s$ denotes the lower bound of the distribution for $\Sigma^+$. For $q_0 < q_1$ or $q_0 > q^{00}$, $q^s = q_0$ and $F^s(q^s)$ is clearly decreasing in $s$. For $q_1 < q_0 < q^{00}$, $q^s = q^{00}$. From (A-12) and $J(q^{00}) = 0$, $\frac{\partial q^{00}}{\partial s} = -\frac{r}{(r+1)(q^{00})-(r-1)} < 0$ since $q^{00} > q_c$, establishing that $F^s(\hat{q}^s)$ is decreasing in $s$. For $s \geq q_0, q_1 > q_0$ and $q^s = q_0$. Then, $F^s(q_0) = 0$ and thus the probability of a scam vanishes.

Last, consider $E[\hat{q}^s]$. For $q_0 < q_1$,

$$E[\hat{q}^s] = \int_{q_0}^{q_1} qdF^s(q) + \int_{q_2}^{q_3} qdF^s(q) = -[q_2 - q_1]F^s(q_2) - q_0F^s(q_0) + q_3 - \int_{q_0}^{q_1} F^s(q)dq - \int_{q_2}^{q_3} F^s(q)dq,$$
Lemma A1. Then, for all $q$ following equation:

$$
\frac{\partial E[\tilde{q}^*]}{\partial s} = -[q_2 - q_1] \left[ \frac{\partial F^s(q_2)}{\partial s} + f^s(q_2) \frac{\partial q_2}{\partial s} \right] - q_0 \frac{\partial F^s(q_0)}{\partial s} - \int_{q_0}^{q_1} \frac{\partial F^s(q)}{\partial s} dq - \int_{q_2}^{q_3} \frac{\partial F^s(q)}{\partial s} dq > 0,
$$

because $\frac{\partial F(q)}{\partial s} < 0$ and $\frac{\partial q_2}{\partial s} \leq 0$ (given that $\frac{\partial q_0}{\partial s} < 0$). Analogously, it can be shown that $\frac{\partial E[\tilde{q}^*]}{\partial s} > 0$ for $q_0 > q_1$. Moreover, since $F^s(q)$ is continuous at $s = q_0$, $E[\tilde{q}^s]$ is maximized for $s \geq q_0$.

**Proof of Proposition 5.** To prove the existence of an equilibrium, we find $\overline{q}_L$, $\overline{q}_H$, and $R_{\text{scam}}$ such that (15) is a proper c.d.f. Note first that $R_{\text{scam}} = \frac{U}{n} + \frac{A}{n} \frac{S_A}{U} < \frac{U + A}{n}$. Therefore, the assumption that $\alpha \leq \frac{n}{U + A}$ implies $1 - \frac{\alpha R_{\text{scam}}}{1 + aq} > 0$ and $F(q) \geq 0$ for all $q \geq 0$. Next, differentiating (15),

$$
\overline{F}'(q) = \frac{F(q) \left( 1 - \frac{\alpha R_{\text{scam}}}{1 + aq} \right)^{\frac{2-n}{n+1}}}{(1 + q')^n} Z(q),
$$

where $Z(q) = [(q' - r + 1) \left( 1 - \frac{\alpha R_{\text{scam}}}{1 + aq} \right) + \frac{\alpha^2 q(1+q')}{(1+aq)^2} R_{\text{scam}}]$. Evidently, $Z(0) = -(r-1)(1-\alpha R_{\text{scam}}) < 0$, $Z(q_L) > 0$, and that $Z(q)$ is increasing in $q$. Hence, there is a unique $\overline{q}_L < q_L$ that solves $Z(\overline{q}_L) = 0$ and $\overline{F}'(q) > 0$ if and only if $q > \overline{q}_L$. Analogous to the proof of Proposition 1, it can be verified that $\overline{q}_L$ is the lowest positive $q$ on the support of the distribution. Moreover, since $\overline{F}(\overline{q}_L) < 1$, there exists a unique $\overline{q}_H > q_L$ such that $\overline{F}(\overline{q}_H) = 1$, which solves for the upper bound of the distribution. Proving the lack of an incentive to deviate to $q$ outside the support is also analogous to the proof of Proposition 1. Finally, to establish the existence of an equilibrium, we establish the existence of $R_{\text{scam}}$ that solves the following equation:

$$
\Psi(R_{\text{scam}}) = R_{\text{scam}} - \frac{U}{n} \frac{[E(\overline{q})]^r}{1 + [E(\overline{q})]^r} - \frac{A}{n} \frac{S_A}{U} = 0. \quad (A-13)
$$

Note that $\Psi(0) < 0$, $\Psi(U + A \frac{S_A}{U}) > 0$, and that $\Psi(\cdot)$ is a continuous function. Hence, there exists $R_{\text{scam}} \in (0, U + A \frac{S_A}{U})$ that solves $\Psi(R_{\text{scam}}) = 0$. It is easily verified that $\overline{F}(q) < F(q)$ for all $q \in (0, \overline{q}_H)$, and thus higher quality levels are more likely with a positive $\alpha$, as claimed. To complete the proof, we observe from (A-13) that $\lim_{n \to \infty} R_{\text{scam}} = 0$, which, from (15), implies that $\lim_{n \to \infty} \overline{F}(0) = \lim_{n \to \infty} F(0) = 1$.

**Proof of Proposition 6.** Let $H(I, n) = h(I, n) + \frac{A}{n} \frac{S_A}{U}$, where $h(I, n)$ is defined in Lemma A1. Then, $n^*$ solves $H(I, n^*) = k$. Let $k_L = \max_{k \in [k_c, M - A]} H(I, 2)$. By Lemma
A1, \( H(I, 2) = \frac{A}{2} g_A < k \) for \( I \notin (I_c, M - A) \). Moreover, since \( H(I, n) \) is single-peaked by Lemma A1 and \( k < k_I \) by assumption, there exist critical values of \( I \) such that \( I_c < I_k < I_H < M - A \), where \( H(I, 2) = H(I_H, 2) = k \), \( H(I, 2) < k \) for \( I \notin [I_L, I_H] \) and \( H(I, 2) > k \) for \( I \in (I_L, I_H) \). Note that \( \frac{\partial H(I, n)}{\partial n} < 0 \), because \( \frac{\partial E(q)}{\partial n} < 0 \) by Proposition 2. Therefore, \( H(I, n) < H(I, 2) < k \) for \( n > 2 \) and \( I \notin [I_L, I_H] \), proving that there is no equilibrium for \( I \notin [I_L, I_H] \). For \( I \in [I_L, I_H] \), \( H(I, 2) \geq k \). Hence, by the continuity of \( H(I, n) \) and the fact that \( \frac{\partial H(I, n)}{\partial n} < 0 \), there exists a unique \( n^* \geq 2 \) such that \( H(I, n^*) = k \) for all \( I \in [I_L, I_H] \), establishing a unique symmetric equilibrium.

To show that only the highest quality charity has a positive net revenue, note that \( R_{\text{win}}^*(q) = R_{\text{win}}^*(q|n = n^*) - k \) and \( R_{\text{lose}}^*(q) = R_{\text{lose}}^*(q|n = n^*) - k \), where \( R_{\text{win}}^*(q) \) and \( R_{\text{lose}}^*(q) \) are given by (4) and (5), respectively. Then, \( H(I, n^*) = k \) implies that \( R_{\text{win}}^*(q) = I g(q) - q \geq 0 \) for \( q \in [q_L, q_H] \) and \( R_{\text{lose}}^*(q) = -q \leq 0 \).

**Proof of Proposition 7.** Immediate from Proposition 6 and the continuity of \( \Delta(I) \).

**Proof of Proposition 8.** Suppose \( A = 0 \). It suffices to prove that \( \lim_{k \to 0} \frac{d\Delta(I)}{dt} > 0 \) for all \( I \in [I_L, I_H] \). To do so, recall from Lemma A1 that \( h(I, n^*) = k \). By Proposition 2, \( \lim_{n \to \infty} h(I, n) = 0 \), which means that \( \lim_{k \to 0} n^* = \infty \) and \( \lim_{n \to \infty} \frac{d\Delta(I)}{dt} = \lim_{n \to \infty} \frac{d\Delta(I)}{dt} \). Since, by definition, \( \frac{d\Delta(I)}{dt} = \frac{dE[q]}{dt} - \frac{dE[q]}{dt} \), we sign each term on the r.h.s. in turn as \( n^* \to \infty \).

Using (12), \( \frac{dE[q]}{dt} = v'(E[q^*]) \frac{dE[q^*]}{dt} \). Moreover, differentiating both sides of \( h(I, n^*) = k \) with respect to \( I \), we find

\[
\frac{dE[q^*]}{dt} = \frac{g(E[q^*])}{(M - I - A)g'(E[q^*])} = \frac{E[q^*](1 + (E[q^*])^r)}{r(M - I - A)},
\]

which, given \( \lim_{n^* \to \infty} E[q^*] = 0 \) from Proposition 2, implies that \( \lim_{n^* \to \infty} \frac{dE[q^*]}{dt} = 0 \), and thus \( \lim_{n^* \to \infty} \frac{dE[q]}{dt} = 0 \). Next, notice from (A-4) that

\[
\frac{dv^*_I(I)}{dt} = -[v(q_L) - v(0)] \frac{dF^*(q_L)}{dt} - \int_{q_L}^{q^*} v'(q) \frac{dF^*(q)}{dt} dq,
\]

where

\[
\frac{dF^*(q)}{dt} = -F^*(q) \left( \frac{n^*}{I(n^* - 1)} + \ln \left( \frac{1 + q^r}{q^r - 1} \right) \frac{1}{(n^* - 1)^2} \frac{\partial n^*}{\partial I} \right).
\]

Hence, in order to show \( \frac{dv^*_I(I)}{dt} \), it suffices to show that \( \lim_{n^* \to \infty} \frac{dF^*(q)}{dt} < 0 \) for all \( q \in [q_L, q_H] \). Note that \( \lim_{n^* \to \infty} F^*(q) = \frac{1 + q^r}{q^r - 1} \), and \( \lim_{n^* \to \infty} \frac{n^*}{I(n^* - 1)} = \frac{1}{I} \). To complete the proof,
we prove that \( \lim_{n^* \to \infty} \frac{1}{(n^*-1)^2} \frac{\partial n^*(I)}{\partial I} = 0 \). Clearly, \( \frac{\partial n^*(I)}{\partial I} = -\frac{\partial h(I,n^*)/\partial I}{\partial h(I,n^*)/\partial n^*} \). And taking into account the fact that \( g'(q) = \frac{r_\xi(q)}{q(1+q')^r} \), we have

\[
\frac{1}{(n^*-1)^2} \frac{\partial n^*(I)}{\partial I} = -\frac{(M-I-A) r \frac{\partial E[\tilde{q}^*]}{\partial I} - E[\tilde{q}^*](1 + (E[\tilde{q}^*])')}{(M-I-A) \left[ r(n^*-1)^2 \frac{\partial E[\tilde{q}^*]}{\partial n^*} - \frac{(n^*-1)^2}{n^*} E[\tilde{q}^*](1 + (E[\tilde{q}^*])') \right]}, \tag{A-17}
\]

The numerator converges to 0 as \( n^* \to \infty \), because \( \lim_{n^* \to \infty} E[\tilde{q}^*] = 0 \), and \( \lim_{n^* \to \infty} \frac{\partial E[\tilde{q}^*]}{\partial I} = 0 \) by (A-6). The denominator converges to a strictly negative value, since by (A-2),

\[
\lim_{n^* \to \infty} \frac{(n^*-1)^2}{n^*} E[\tilde{q}^*](1 + (E[\tilde{q}^*])') = \lim_{n^* \to \infty} \frac{(n^*-1)^2}{n^*} E[\tilde{q}^*] = -\lim_{n^* \to \infty} \frac{(n^*-1)^2}{n^*} \frac{\partial E[\tilde{q}^*]}{\partial n^*}.
\]

where the last equality follows from L'Hospital’s rule. Hence, \( \lim_{n^* \to \infty} \frac{1}{(n^*-1)^2} \frac{\partial n^*}{\partial I} = 0 \), which implies that \( \lim_{n^* \to \infty} \frac{dE^r(q)}{dt} = -\frac{1+q'}{r^2 q^r} < 0 \) for all \( q \in [q_L, q_H] \), and establishes \( \lim_{n^* \to \infty} \frac{d\sigma^r(I)}{dt} > 0 \) and in turn, \( \lim_{k \to 0} \frac{d\Delta(I)}{dt} > 0 \), as desired. Given that \( \Delta(I^*) = c \), this means that \( I^* \) is increasing in \( c \) for arbitrarily small \( k > 0 \), as claimed. Last, recall from (19) that \( \frac{dW(I^*)}{dc} = \frac{d\sigma^r(I^*)}{dt} \frac{dI^*}{dc} \). Hence, \( \frac{dW(I^*)}{dc} > 0 \). □
References


