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A Theory of Charitable Fund-raising with Costly Solicitations

Alvaro Name Correa, *Duke University*
Huseyin Yildirim, *Duke University*

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Abstract

We present a theory of charitable fund-raising in which it is costly to solicit donors. We fully characterize the solicitation strategy that maximizes donations net of fund-raising costs. It is optimal for the fund-raiser to target only the "net contributors" – donors who would give more than their solicitation costs. We show that as the income inequality increases, so does the level of the public good, despite a (potentially) non-monotonic fund-raising strategy. This implies that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and government grants often found in empirical studies. (*JEL* H00, H30, H50)

Charitable fund-raising[1] is a costly endeavor. Andreoni and Payne (2003, 2011) indicate that an average charity spends 5 to 25 percent of its donations on fund-raising activities, including direct mailing, telemarketing, door-to-door solicitations, and staffing.[2] For instance, every year more than 115,000 nonprofit organizations hire fund-raising staff and consultants, paying them 2 billion dollars (Kelly 1998).[3] Despite its significance, however, fund-raising costs have not been fully incorporated into the theory of charitable giving. This is the gap we aim to fill in this paper.

*Name-Correa: Department of Economics, Duke University, Box 90097, Durham, NC 27708 (email: alvaro.namecorrea@duke.edu); Yildirim: Department of Economics, Duke University, Box 90097, Durham, NC 27708 (email: yildirh@econ.duke.edu). We thank three referees, Silvana Krasteva, Thomas Nechyba, Richard Romano, and participants of the Duke Theory lunch and the Midwest Theory Conference for comments. All errors are ours.

[1] The charitable sector is a significant part of the U.S. economy. For instance, in 2008, total donations amounted to $307 billion. $229 billion of this total came from individuals, corresponding to 1.61 percent of GDP (Giving USA, 2009). See Andreoni (2006a) and List (2011) for an overview of this sector.

[2] Various watchdog groups such as BBB Wise Giving Alliance and Charity Navigator regularly post these cost-to-donation percentages for thousands of charities in the U.S. They often recommend a benchmark of 30-35 percent for a well-run charity.

[3] In 2004, the estimated number of paid workers employed by charities was 9.4 million, which is more than 7 percent of the U.S. workforce (Sherlock and Gravelle, 2009).

We build on the "standard" model of giving in which donors care only about their private consumption and the total supply of the public good.[4] Unlike the standard model, we assume that each donor becomes aware of the charitable fund-drive and thus participates in the "contribution game" only if solicited by the fund-raiser.[5] The solicitation is, however, costly. Our first observation is that the charity will contact an individual if he is expected to give more than his solicitation cost, or become a "net contributor" in equilibrium. We show that identifying these net contributors in our model is equivalent to identifying the contributors in the standard model (without fund-raising costs) except that each donor's wealth is reduced by his solicitation cost. This important equivalence allows us to appeal to Andreoni and McGuire's (1993) elegant algorithm to solve for the latter. Without requiring any equilibrium computation, our optimal fund-raising strategy pinpoints the exact set of donors to be targeted based on their preferences, incomes, and solicitation costs.[6]

When income is the only source of heterogeneity, the fund-raising strategy attaches each individual a cutoff cost that depends on the incomes of wealthier others. Intuitively, the charity first contacts the richest donor; and once this donor is in the "game", it becomes more conservative about contacting the second richest donor in order to curb the free-riding incentive. Iteratively applied, this logic implies that unlike the well-known neutrality result predicted by the standard theory (e.g., Warr 1983; Bergstrom, Blume, and Varian 1986), an income redistribution is likely to affect the fund-raising strategy and thus the provision of the public good. In particular, as the income distribution becomes more unequal in the sense of Lorenz dominance (defined below), we find that the level of the public good strictly increases in the presence of costly fund-raising despite a (potentially) non-monotonic fund-raising effort. Such non-neutrality of the public good provision also manifests itself in response to a government grant to the charity. We show that a more generous grant partially crowds out fund-raising effort, leaving some donations unrealized, as well as reducing the amount of the realized donations. The importance of this fund-raising channel for crowding-out has been recently evidenced by Andreoni and Payne (2003, 2011).

[4] See, e.g., Warr (1983), Roberts (1984), Bergstrom, Blume, and Varian (1986), Andreoni (1988), and Fries, Golding, and Romano (1991).

[5] For instance, the fund-raiser may be running occasional fund drives, and a solicitation, much like advertising, informs the donor of the current one. We elaborate on this point in the next section.

[6] This is consistent with the fact that fund-raising professionals often recommend a careful study of the potential donor base for an effective campaign (Kelly 1998). For instance, several software companies such as DonorPerfect (www.donorperfect.com), DonorSearch (http://donorsearch.net), and Target Analytics (www.blackbaud.com/targetanalytics) compile donor databases and sell them to charities along with programs to identify the prospective donors.

We should note that in order to make our results transparent, we use the standard but highly stylized model of giving. Our results, however, extend to a more realistic model of "warm-glow" giving (Andreoni 1989). In two other extensions, we show how our results can be modified when the fund-raiser is uncertain about donors' incomes and when she grows to be a more "efficient" solicitor over time.

Aside from the papers mentioned above, our work relates to a relatively small theoretical literature on strategic fund-raising as a means of: providing prestige to donors (Glazer and Konrad 1996; Harbaugh 1998; Romano and Yildirim 2001), signaling the project quality (Vesterlund 2003; Andreoni 2006b), and organizing lotteries (Morgan 2000). Our work is more closely related to the models of strategic fund-raising under non-convex production (Andreoni 1998; Marx and Matthews 2000). Unlike these models, the production threshold in ours is endogenous to solicitation costs.

Our work is most closely related to Rose-Ackerman (1982), and Andreoni and Payne (2003). Rose-Ackerman introduces the first model of costly fund-raising but does not construct donors' responses from an equilibrium play. Andreoni and Payne (2003) endogenize both the fund-raiser and donors' responses as in our model; however, they assume solicitation letters to be randomly distributed.[7]

In addition to the theoretical literature, there is a more extensive empirical and experimental literature on charitable giving, to which we will refer below. For recent surveys of the literature, see the reviews by Andreoni (2006a) and List (2011).

The rest of the paper is organized as follows. In the next section, we set up the basic model. In Section II, we characterize the optimal fund-raising strategy as a modified Andreoni-McGuire algorithm. In Sections III and IV, we consider income distributions and government grants, respectively. We present the extensions in Section V, and conclude in Section VI. The proofs of the main results are provided in the Appendix, while others can be found in an Online Appendix.

I. Basic Model

Our basic setup extends the standard model for private provision of public goods (e.g., Bergstrom, Blume, and Varian 1986), which we briefly review before introducing fund-raising costs.

[7] Such indiscriminate solicitations can be optimal in the absence of donor information (see Section V.B).

Standard Model. There is a set of individuals, $N = \{1, ..., n\}$, who each allocates his wealth, $w_i > 0$, between a private good consumption, $x_i \geq 0$, and a gift to the public good or charity, $g_i \geq 0$. Units are normalized so that $x_i + g_i = w_i$. Letting $G = \sum_{i \in N} g_i$ be the supply of the public good, individual i's preference is represented by the utility function $u_i(x_i, G)$, which is strictly increasing, strictly quasi-concave, and twice differentiable. Individual i's (Marshallian) demand for the public good, denoted by $f_i(w)$, satisfies the strict normality: $0 < f_i'(w) \leq \theta < 1$ for some parameter θ.[8] The equilibrium gifts, $\{g_1^*, ..., g_n^*\}$, are made simultaneously (without observing others); and under strict normality, there is a unique Nash equilibrium. We further assume that $f_i(0) = 0$ for all i so that $G^* > 0$.

Costly Fund-raising. Since everyone is already in the "contribution game", there is no role for (strategic) fund-raising in the standard model.[9] Thus, similar to Rose-Ackerman (1982), and Andreoni and Payne (2003), we assume that person i enters the game and considers giving only if solicited by the fund-raiser.[10] Doing so, however, costs $c_i > 0$ to the fund-raiser in the form of telemarketing, direct mails, or door-to-door visits.[11] For simplicity, we assume that c_i is not too large; in particular, $c_i < \widehat{C}_i$, where $\widehat{C}_i \in (0, w_i]$ is the unique cutoff cost for person i if he were to pay for the entire fund-raising cost himself.[12]

Let $F \subseteq N$ be the set of donors contacted by the fund-raiser, or the *fund-raiser set*. In the basic model, we assume that the fund-raiser set is commonly known by the contacted donors, though we relax this assumption in Section II.B.[13] As in the standard setup, let $g_i^*(F)$ be donor i's equilibrium gift engendered by the simultaneous play in F. Then, the total fund-raising cost and the gross donations are defined, respectively, by $C(F) = \sum_{i \in F} c_i$

[8] The existence of parameter θ is not essential to our analysis but eases it by ensuring a finite G_i^0 below. It is also commonly assumed in the literature (e.g., Andreoni 1988; Fries, Golding, and Romano 1991).

[9] Alternatively, absent fund-raising costs, the fund-raiser would trivially ask everyone for donations since the equilibrium provision can never decrease by including a new donor (Andreoni and McGuire 1993).

[10] We envision that the charity organizes occasional fund-raising campaigns, and a solicitation, much like advertising, informs the donor of the current one. Alternatively, the donor may procrastinate in giving (O'Donoghue and Rabin 1999), and this procrastination is minimized by the fund-raiser's asking. See Yoruk (2009), and Meer and Rosen (2011) for evidence on "the power of asking". We, however, do not allow the fund-raiser to "pressure" people to give. In particular, a solicited person can choose not to give, though this is unlikely to occur in equilibrium.

[11] While, to break the fund-raiser's indifference, we do not allow for $c_i = 0$ (e.g., a repeat donor), our results do extend to this possibility up to a trivial non-uniqueness in the fundraiser's strategy.

[12] See the online appendix for a formal derivation of $\widehat{C}_i$. For the CES utility: $u_i = (x_i^{\rho_i} + (\overline{G})^{\rho_i})^{1/\rho_i}$, with $\rho_i < 1$, it is easily verified that $\widehat{C}_i = [1 - (1/2)^{(1-\rho_i)/\rho_i}]w_i$ for $\rho_i \in (0, 1)$, and $\widehat{C}_i = w_i$ for $\rho_i \leq 0$ (including the Cobb-Douglas specification).

[13] The fact that prior to giving, donors may know the fund-raiser set is not completely unrealistic. For instance, universities often organize alumni re-unions and fund-raising events where contacted donors meet each other.

and $G^*(F) = \sum_{i \in F} g_i^*(F)$, where $C(\varnothing) = 0$ and $g_i^*(\varnothing) = 0$ by convention.[14] The charity chooses F that maximizes the supply of the public good (or net revenues):

$$\overline{G}^*(F) = \max\{G^*(F) - C(F), 0\}. \tag{1}$$

Eq. (1) implies that if insufficient funds are received to cover the cost, then no public good is provided.[15] We assume that when indifferent between two sets, the charity strictly prefers the one with the lower fund-raising cost. Our solution concept is subgame perfect Nash equilibrium.

II. Optimal Fund-raising

A. Characterization

Our first observation is that an optimizing charity classifies donors into *net* contributors and *net* free-riders depending on how their equilibrium gifts compare with their respective solicitation costs.

LEMMA 1. *The optimal fund-raiser set, F^o, is uniquely identified by these two conditions:*

(C1) *every individual i in F^o is a "net contributor" in the sense that $g_i^*(F^o) - c_i > 0$;*

(C2) *any individual i outside F^o would be a "net free-rider" if added to F^o, in the sense that $g_i^*(F^o \cup \{i\}) - c_i \leq 0$.*

Lemma 1 says that the optimal set should exactly identify the set of net contributors. A similar identification problem would arise in the standard model if one were to detect the contributors, or equivalently the (pure) free-riders. For that case, Andreoni and McGuire (1993) offer an elegant algorithm. Lemma 1 permits us to build on their work here.[16]

When finding the optimal set, the fund-raiser can tentatively consider individual i paying for his own cost, c_i, even though all individuals care about the total fund-raising cost. The optimal set problem then reduces to identifying the net contributors with residual incomes,

[14] We could also include a fixed setup cost of fund-raising; but its analysis would be similar to that of a (negative) government grant considered in Section IV.

[15] We assume that donations are not refunded in the case of a failed fundraising, or they are used for other causes that donors do not care about.

[16] Note that much like in Andreoni (1998), some fund-raising may never start because, given the cost, the fund-raiser believes that donors would give zero. In general, $F^o \neq \varnothing$ if and only if $\overline{G}^*(F^o) > 0$.

$w_i - c_i$, by using the Andreoni-McGuire algorithm. Let G_i^0 be the "drop-out" level of the public good for person i, which uniquely solves:

$$f_i(w_i - c_i + G_i^0) = G_i^0. \tag{2}$$

One interpretation of G_i^0 in our context is that person i becomes a net contributor if and only if he expects the sum of others' net contributions to stay below G_i^0. Without loss of generality, index individuals in a descending order of their dropout levels: $G_1^0 \geq G_2^0 \geq ... \geq G_n^0$. Next, define $\Phi_i(G) \equiv \sum_{j=1}^{i}(\phi_j(G) - G) + G$, where $\phi_j \equiv f_j^{-1}$ and thus $\Phi_i'(G) > 0$ by the strict normality. The following result fully characterizes the optimal fund-raising strategy.

PROPOSITION 1. *Define* $\Delta_i \equiv \Phi_i(G_i^0) - \sum_{j=1}^{i}(w_j - c_j)$. *Then,* $\Delta_1 \geq \Delta_2 \geq ... \geq \Delta_n$, *with* $\Delta_1 > 0$. *Moreover, letting* $k \in N$ *be the largest index such that* $\Delta_k > 0$, *the optimal fund-raiser set is* $F^o = \{1, ..., k\}$. *This set generates the public good,* $\overline{G}^* = \Phi_k^{-1}(\sum_{j=1}^{k}(w_j - c_j))$.

To understand how the optimal strategy works, note that Δ_i can be interpreted as a measure of person i's incentive to pay for his solicitation cost. In particular, as in Bergstrom, Blume, and Varian (1986), $\Phi_i(G)$ is the minimum level of total wealth needed to sustain public good G as an equilibrium among agents, $1, ..., i$. This means that if the actual total wealth available to these agents is strictly less than $\Phi_i(G_i^0)$, namely $\Delta_i > 0$, then the dropout value of person i, G_i^0, cannot be reached, making him a net contributor and thus a *candidate* for the fund-raiser set. Given that $\Phi_i'(G) > 0$, these incentives are monotonic in that $\Delta_i \geq \Delta_{i+1}$, and therefore, the fund-raiser considers the largest set of individuals with a positive incentive. This set will be optimal if, given the total fund-raising cost, $\sum_{j=1}^{k} c_j$, incurred, each individual decides to contribute rather than consume only the private good; i.e., if, in equilibrium, his net cost, $\sum_{j=1}^{k} c_j - G_{-i}^*$, is strictly less than his cutoff, $\widehat{C}_i$. Since everyone else in the set is expected to give more than his solicitation cost, this net cost cannot exceed his own cost, c_i, which is less than $\widehat{C}_i$ by assumption.[17] It is worth observing that if the charity could *force* each contacted donor to pay for his solicitation cost, then everyone would be contacted without the concern for the incentive constraint, $\Delta_k > 0$. Thus, providing donors with the incentives to be net contributors is the reason why some in the population may not be solicited in our model.

[17] The online appendix partially weakens the cost condition, $c_i < \widehat{C}_i$.

The optimal fund-raising strategy has some intuitive comparative statics. The fund-raiser is more likely to contact a person if, all else equal, he is richer; he has a greater demand for the public good; and/or he has a lower solicitation cost.[18] This is consistent with the anecdotal evidence that schools often exclusively solicit alumni and parents; religious organizations first target their members; and health charities primarily ask former patients and their families for donations.

The optimal fund-raising strategy is also easy to apply because it does not require any equilibrium computation. To illustrate, take $u_i = x_i^{1-\alpha}\overline{G}^{\alpha}$, with $\alpha = 0.0342$ (which is cited to be realistic by Andreoni 1988), and consider three agents such that: $(w_1, w_2, w_3) = (87, 87, 90)$ and $(c_1, c_2, c_3) = (0.1, 1, 5.5)$. Then, $G_1^0 = 3.08$, $G_2^0 = 3.05$, and $G_3^0 = 2.99$. Moreover, $\Delta_1 = 3.08$, $\Delta_2 = 2.15$, and $\Delta_3 = -0.91$, which imply that $F^o = \{1, 2\}$, resulting in $\overline{G}^* = 3$ and $g_1^* = g_2^* = 2.05$.

B. Unobservability of the Fund-raiser Set

While our assumption that the fund-raiser set is observable to donors is reasonable in some settings, it may be less so in others. In particular, it may be difficult or infeasible for donors to monitor the charity's solicitations, in which case they can only hold beliefs about them. Given the unique optimal set F^o, one natural belief system is as follows: if a donor in F^o is contacted, he learns about the fund-drive and believes that the rest of F^o will also be contacted, whereas if a donor outside F^o is contacted, he attributes this to a mistake and believes that he is the only one contacted besides F^o.[19] Under these beliefs, the following result shows that the unobservability of the fund-raiser set is of no consequence in equilibrium.

PROPOSITION 2. *Suppose that the fund-raiser set is unobservable to donors. Then, under the beliefs described above, F^o is sustained as a perfect Bayesian equilibrium.*

Proposition 2 mainly obtains from Lemma 1, and says that the fund-raiser does not necessarily have a commitment problem about its targeting strategy.

[18] In each case, eq.(2) implies that G_i^0 is higher for such a person. We say that person i has a greater demand for the public good than j if $f_i(w) \geq f_j(w)$ for all w.

[19] These beliefs are similar to "passive" beliefs often used in bilateral contracting in which one party privately contracts with several others (e.g., Cremer and Riordan 1987; McAfee and Schwartz 1994). One justification for such beliefs in our context is that the fund-raiser assigns a different staff member to contact different donors so that mistakes are perceived to be uncorrelated.

Armed with the optimal fund-raiser behavior, we next address two policy-related issues, the first one being the role of an income redistribution.

III. Income Redistribution and Non-neutrality

Suppose that individuals differ only in incomes, namely $c_i = c$ and $u_i = u$. Without loss of generality, rank incomes as $w_1 \geq w_2 \geq .. \geq w_n$, which, from (2), implies that $G_1^0 \geq G_2^0 \geq ... \geq G_n^0$. Applying Proposition 1, the fund-raising strategy then simplifies to a cutoff solicitation cost for each donor.

LEMMA 2. *Let $\overline{\phi}(G) \equiv \phi(G) - G$, and donor i's cost cutoff be given by*

$$\overline{c}_i = w_i - \overline{\phi}(\textstyle\sum_{j=1}^{i}(w_j - w_i)). \tag{3}$$

Then, $\overline{c}_1 \geq \overline{c}_2 \geq ... \geq \overline{c}_n$, and $F^o = \{i \in N | c < \overline{c}_i\}$.

In general, since $\overline{\phi}'(G) > 0$ by the strict normality, the cutoff cost in (3) is strictly less than one's income except for the richest; and the gap increases for lower income individuals.[20] The reason is that for a given c, the charity first contacts the richest person, and upon informing him of the fund-drive, the charity becomes more conservative in contacting the second richest person to alleviate the free-rider problem, which is a function of their wealth difference. Applied iteratively, this logic explains why person i's cutoff in (3) is decreasing in the sum of wealth differences between him and the wealthier others. One important implication of this observation is that a redistribution of income is likely to affect the fund-raising strategy and thus the equilibrium provision of the public good.

As first observed by Warr (1983), if the set of contributors and their total wealth do not change by an income redistribution, then neither does the level of the public good in the standard model of giving.[21] This striking theoretical prediction has, however, been at odds with empirical evidence on private charity.[22] As such, several researchers have modified the standard model, but these modifications have been mostly confined to the donor side – the most prominent one being "warm-glow" giving in which people also receive a direct benefit

[20] In fact, $\overline{c}_i < 0$ is possible and signifies that i would contribute nothing if solicited.

[21] Subsequent work showed the robustness of this result with varying generality. See, e.g., Bergstrom, Blume, and Varian (1986), Bernheim (1986), Roberts (1987), Andreoni (1988), and Sandler and Posnett (1991).

[22] See, e.g., Clotfelter (1985), Kingma (1989), Steinberg (1991), Brunner (1997), and Ribar and Wilhelm (2002).

from contributing (see Section V.A).[23] Here, we show that costly fund-raising can provide a complementary explanation as to the endemic breakdown of neutrality.

To develop some intuition, suppose that individuals have identical Cobb-Douglas preferences: $u_i = x_i^{1-\alpha}\overline{G}^{\alpha}$, and consider these two income distributions: $\mathbf{w}' = (w, w, ..., w)$ and $\mathbf{w}'' = (\varepsilon + n(w-\varepsilon), \varepsilon, ..., \varepsilon)$, with $1/[1+\alpha/(n(1-\alpha))] < \varepsilon/w < 1$. It is readily verified that in the standard model, all individuals contribute under both income distributions and thus in equilibrium, $G^{*\prime} = G^{*\prime\prime} > 0$. This neutrality result should extend to costly fund-raising as long as c is small so that everyone is still contacted. For a sufficiently large c, however, the fund-raising strategy, and thus the public good provision, is likely to be affected by the income distribution. For instance, when $\varepsilon < c < w$, it is clear that whereas everyone is contacted under the egalitarian income distribution, $\mathbf{w}'$, only the richest individual is contacted under the unequal income distribution, $\mathbf{w}''$. This means that although there are more contributors under $\mathbf{w}'$, there are also more fund-raising expenses. Trivial algebra shows that equilibrium public good levels are given respectively by $\overline{G}^{*\prime} = [n\alpha/(n(1-\alpha)+\alpha)](w-c)$ and $\overline{G}^{*\prime\prime} = \alpha(\varepsilon + n(w-\varepsilon) - c)$, and comparing them reveals $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$. Note also that if the fund-raising were even costlier, $w \leq c < \varepsilon + n(w-\varepsilon)$, then the fund-raising effort would be reversed: no individual would be solicited under $\mathbf{w}'$, whereas the richest person under $\mathbf{w}''$ would still be solicited. Nevertheless, the public good provision would again imply that $0 = \overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$. Of course, if $c \geq \varepsilon + n(w-\varepsilon)$, then no fund-raising takes place in either case.

Overall, it seems that when fund-raising cost is significant, the neutrality result is *unlikely* to hold. It also seems that while the equilibrium number of solicitations responds non-monotonically to a more unequal distribution of income, the public good provision will always increase. To prove these observations generally, we employ the well-known concept of Lorenz dominance for income inequality (e.g., Atkinson 1970).

DEFINITION (Lorenz Dominance) *Let $\mathbf{w} = (w_1, w_2, ..., w_n)$ be a vector of incomes whose elements are indexed in a descending order, and define $L_i(\mathbf{w}) = \sum_{j=1}^{i} w_j$. Consider two income vectors $\mathbf{w}' \neq \mathbf{w}''$ such that $L_n(\mathbf{w}') = L_n(\mathbf{w}'')$. It is said that $\mathbf{w}''$ is more unequal than $\mathbf{w}'$, if $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$, i.e., $L_i(\mathbf{w}'') > L_i(\mathbf{w}')$ for all $i < n$.*

Intuitively, an income distribution $\mathbf{w}''$ is more unequal than $\mathbf{w}'$ if the total income is more concentrated in the hands of the few. In particular, the egalitarian income distribution

[23] See, e.g., Cornes and Sandler (1984), Steinberg (1987), and Andreoni (1989).

Lorenz dominates all the others, whereas a perfectly unequal income distribution in which one person possesses all the wealth is dominated by all the others. Based on this inequality concept, we reach,

PROPOSITION 3. *Let* $\mathbf{w}' \neq \mathbf{w}''$ *be two income vectors such that* $\mathbf{w}''$ *is more unequal than* $\mathbf{w}'$ *in the sense of Lorenz. Moreover, suppose that with the standard model, every person is a contributor under both* $\mathbf{w}'$ *and* $\mathbf{w}''$ *so that* $G^{*\prime} = G^{*\prime\prime} > 0$. *Then,* $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} > 0$ *for* $c \in [0, \overline{c}''_n)$, *and* $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$ *for* $c \in [\overline{c}''_n, \overline{c}''_1)$. *For* $c \geq \overline{c}''_1$, *no fund-raising takes place, yielding* $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} = 0$.

Proposition 3 generalizes our intuition from the above discussion. For a sufficiently small cost of fund-raising, every donor is solicited regardless of the income redistribution, resulting in the same level of the public good. When the cost is significant, however, the fund-raising strategy, and the level of public good, are influenced by the income redistribution. In particular, a more unequal income distribution produces a higher level of the public good. Note from (3) that the interval $[\overline{c}''_n, \overline{c}''_1)$ is likely to be wide because $\overline{c}''_1 = w''_1$, and $\overline{c}''_n$ can be much smaller than w''_n.[24]

We should point out that strategic costly fund-raising offers a complementary explanation for the non-neutrality to those identified in the literature. In particular, as with Bergstrom, Blume, and Varian (1986), we draw attention to the endogenous nature of the contributor set to the income distribution; but unlike in their study of the standard model, the contributor set in ours is optimally chosen by the fund-raiser. This means, for instance, that the non-contributors in our model are not necessarily pure free-riders; rather they are not asked for donations due to solicitation costs. We should also point out that in their Theorem 1d, Bergstrom, Blume, and Varian also observe that "Equalizing income redistributions that involve any transfers from contributors to non-contributors will decrease the equilibrium supply of the public good."[25] However, as is clear from Proposition 3, under strategic costly fund-raising, the non-neutrality exists even when everyone remains a contributor under both income distributions in the standard model.

IV. Government Grants

[24] For instance, in the numerical example above, if solicitation costs were taken *equal*, then 90 and 2.3 would be the respective cutoffs for the incomes, 90 and 87.

[25] Bergstrom, Blume, and Varian use direct transfers among donors, but it is well-known that such Daltonian transfers are equivalent to Lorenz dominance (Atkinson 1970).

A long-standing policy question in public economics is that if the government gives a grant to a charity, to what degree will it displace private giving? While, in light of the neutrality result, the standard model of giving predicts a complete (dollar-for-dollar) crowding out, there is overwhelming evidence that this is not the case (see n.22). The empirical studies have, for the most part, attributed any crowding-out to the donors' responses. Recently however, Andreoni and Payne (2003, 2011) have empirically showed that a significant part of the crowding out can be explained by reduced fund-raising. By simply modifying our model, we can theoretically support their findings. Let $R > 0$ be the amount of the government grant, and F_R^o and F_0^o denote the optimal fund-raiser sets with and without the grant, respectively.

PROPOSITION 4. *Suppose that, without a grant, some public good is provided, i.e.,* $\overline{G}_0^* > 0$. *Then, with the grant, donor i is solicited if and only if* $\Delta_i > R$. *Moreover,*

(a) *there is less fund-raising with the grant, i.e.,* $F_R^o \subseteq F_0^o$;

(b) *each donor gives strictly less with the grant, i.e.,* $g_i^*(F_R^o) < g_i^*(F_0^o)$ *for* $i \in F_R^o$;

(c) *private giving is partially crowded out, i.e.,* $\overline{G}^*(F_R^o) < R + \overline{G}^*(F_0^o)$, *but* $\overline{G}^*(F_R^o) > \overline{G}^*(F_0^o)$.

Since a government grant directly enters into public good production, part (a) implies that the charity optimally solicits fewer donors. Under a linear production, this reduced fund-raising is, however, not because the charity has diminishing returns to funds, but because it anticipates that donors will be less willing to give, as reflected by the optimal strategy. While, all else equal, cutting back fund-raising increases the public good provision by cutting costs, it also leaves some donations unrealized. Moreover, despite a smaller fund-raiser set, and thus less severe free-riding, with the grant, part (b) indicates that each contacted donor gives strictly less than he would without the grant. This is due to diminishing marginal utility from the grant that simply overwhelms the small group effect. Part (c) shows that the two effects of a government grant, namely lower fund-raising and fewer donations, never neutralize its direct production effect on the public good. That is, the crowding-out is partial because of both the fund-raiser's and the donors' behavioral responses.[26]

[26] Note that Proposition 4 ignores the financing issue of the government grant and thus may be underesti-

V. Extensions

In this section, we briefly discuss three extensions: (A) warm-glow giving, (B) fund-raising with income uncertainty, and (C) "learning-by-fundraising". Many technical details are relegated to the online appendix.

A. Warm-Glow Giving

It is well-documented in the literature that a model of warm-glow giving in which individuals also receive a private benefit from contributing explains the data better than the purely altruistic model employed so far (see n.22). Our results, however, easily generalize to such added realism. Following Andreoni (1989), let $u_i = u_i(x_i, G, g_i)$ be person i's utility function, which is increasing and strictly quasi-concave. In the absence of fund-raising costs, the Nash supply of person i's gift can be written: $g_i = \max\{\widehat{f}_i(w_i + G_{-i}, G_{-i}) - G_{-i}, 0\}$, where partial derivatives satisfy $0 < \widehat{f}_{i1} < 1$ and $\widehat{f}_{i2} \geq 0$ by normality of goods. If, in addition, $0 < \widehat{f}_{i1} + \widehat{f}_{i2} \leq \theta < 1$, then a unique Nash equilibrium obtains. Note that for $\widehat{f}_{i2} = 0$, the warm-glow model reduces to the standard model.

Building on Andreoni's characterization, we define the inverse Nash supply, $\widehat{\phi}_i(G, w_i)$ such that $\widehat{f}_i(w_i + \widehat{\phi}_i, \widehat{\phi}_i) = G$. It is readily verified that $\widehat{\phi}_{i1} > 1$ and $-1 \leq \widehat{\phi}_{i2} < 0$. Analogous to Φ_i above, we also define $\widehat{\Phi}_i(G, \mathbf{w}) = \sum_{j=1}^{i}(\widehat{\phi}_j(G, w_j) - G) + G$, which is strictly increasing in G and strictly decreasing in w_j. In the presence of fund-raising costs, it can be shown that Lemma 1 continues to hold (see the online appendix).[27] Thus, slightly modifying eq.(2), let G_i^0 be uniquely determined by: $\widehat{f}_i(w_i - c_i + G_i^0, G_i^0) = G_i^0$. Next, similar to Δ_i in Proposition 1, set $\widehat{\Delta}_i = \widehat{\Phi}_i(G_i^0, \mathbf{w} - \mathbf{c})$. Then, our results in Propositions 1 and 4 obtain by simply replacing Δ_i with $\widehat{\Delta}_i$. In particular, since, without a warm-glow motive, $\widehat{\phi}_i(G, w_i) = \phi_i(G) - w_i$, our previous results under pure altruism follow.

In order to perform comparative statics, we consider a general CES utility for all i:

$$u_i = [(1-\alpha)x_i^\rho + \alpha((1-\omega)G + \omega g_i)^\rho]^{1/\rho},$$

where $\rho \in (-\infty, 1)$, $\alpha \in (0, 1)$, and $\omega \in [0, 1]$. Clearly, as ω increases, person i cares more about the warm-glow and less about the altruistic giving. From Proposition 4, person i is

mating the crowding out. In particular, having a reduced wealth $w_i - t_i$ by a tax t_i toward the grant, person i would have a lower G_i^0, which would, in turn, lead to less fund-raising and a lower equilibrium gift than without taxation. Note also that any cost of receiving the grant by the fund-raiser could be absorbed by R.

[27] We assume that warm-glow is felt by the net contribution, $g_i - c_i$, so that no extra utility is received by simply covering the solicitation cost. We also assume that $c_i < \widehat{C}_i$ for a cutoff similarly defined for the warm-glow model.

contacted if and only if $\widehat{\Delta}_i > R$, or the solicitation cost, c is less than his cutoff:

$$\bar{c}_i = w_i - \eta(1-\omega)R - \frac{\eta(1-\omega)}{1+\eta\omega}\sum_{j=1}^{i}(w_j - w_i), \tag{4}$$

where $\eta = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1-\rho}}$. Eq.(4) implies that $\bar{c}_i$ is increasing in ω, and decreasing in R at the rate of $\eta(1-\omega)$. That is, as warm-glow giving becomes more pronounced, the fund-raiser solicits more people; and she is less discouraged by an outside grant.

These observations suggest that with warm-glow giving, both the fund-raiser's and the donors' diminished response to a government grant are responsible for the partial crowding out. It is, however, an empirical matter to quantify them. In a recent paper, Andreoni and Payne (2011) measure 73 percent crowding out and attribute all to the reduced fund-raising. We believe that the absence of the (classic) donor crowding out can be evidence of a strong warm-glow motive in their data. Given this, the high fund-raiser reaction to government grants seems inconsistent with net revenue maximization. That is, at the margin, the fund-raiser could increase net revenues by contacting more donors. This conclusion firmly supports Andreoni and Payne's empirical finding.[28] As a policy remedy, they propose (and we agree) that "...requirements that charities match a fraction of government grants with increases in private donations could be a feasible response to crowding out." (p.342)

In a related paper, Andreoni and Payne (2003) find that government grants crowd out fund-raising efforts in social services organizations much less than they do in the arts. In light of our analysis, this evidence points to a stronger warm-glow giving toward social services than toward the arts. This inference appears reasonable because the contributors to the arts are more likely to be the beneficiaries than the contributors to the social services. In another paper, Ribar and Wilhelm (2002) present clear evidence of warm-glow giving to international relief and development organizations. Together with our theory, we should expect substantial fund-raising by these organizations despite sizable governmental aids to international relief programs.

B. Fund-raising with Income Uncertainty

Up to now, we have maintained the strong assumption that the fund-raiser fully knows donors' incomes and preferences. We partially relax this assumption here by introducing

[28] In general, the evidence on fund-raisers' objectives is mixed (Andreoni 2006a). The net revenue maximization is, however, often adopted in theoretical studies.

income uncertainty to our basic model. Suppose that depending on its demographics, the fund-raiser divides the population into $m \geq 1$ groups of donors. She believes that each member of group i independently draws his income from a discrete distribution, $\widetilde{w}_i$, with mean $E[\widetilde{w}_i]$. The fund-raiser's strategy is to choose the number of donors to be contacted from each group. To focus the analysis on the fund-raiser, we continue to assume that donors have no uncertainty about the income profile in the population. Moreover, to simplify the analysis, we consider identical homothetic preferences so that $f(w) = \alpha w$ for some $\alpha \in (0, 1)$. Then, given the cost c per solicitation and the ranking of the mean group incomes: $E[\widetilde{w}_1] \geq ... \geq E[\widetilde{w}_m]$, we can write the cutoff cost for group i as:

$$\bar{c}_i = E[\widetilde{w}_i] - \frac{1-\alpha}{\alpha} \sum\nolimits_{j=1}^{i} n_j \left(E[\widetilde{w}_j] - E[\widetilde{w}_i]\right), \tag{5}$$

where n_j is the size of group j (see the online appendix). We show that it is optimal to solicit all members of group i if $c < \bar{c}_i$; and no members, otherwise. Note that if each group contains a single donor, say demographics are sufficiently informative, eq.(5) reduces to eq.(3), as it should. With income uncertainty, however, the fund-raiser optimally treats each group member as having its mean income. We show that the use of such coarse information for solicitations hurts the fund-raiser: if members of any two groups become "indistinguishable" by the fund-raiser, the equilibrium supply of the public good decreases. The reason is that with increased uncertainty, the fund-raiser is more likely to contact net free-riders and leave out net contributors. This means that information is valuable to the fund-raiser, which may explain the existence of a market for donor research (see n. 6).

C. Learning-by-Fundraising

As in many service and manufacturing sectors,[29] the fund-raiser may also learn and become a more efficient solicitor over time. This raises the interesting issue that the fund-raiser may "invest" in learning by initially contacting net free-riders. To illustrate this point, we re-consider our basic model with identical individuals. Let $c(i)$ be the marginal cost of soliciting ith individual in sequence such that $c(1) > c(2) \geq ... \geq c(n)$ due to learning. Also, let $a_n = (1/n) \sum_{i=1}^{n} c(i)$ be the average cost of solicitation where $a_n < \widehat{C}$. Clearly, a_n is decreasing in n and thus converges to some $a_\ell < c(1)$. We show that it is optimal to contact every donor in this case. More importantly, in the unique equilibrium, each (symmetric)

[29] See Benkard (2000) and the references therein.

gift, g_n^*, is decreasing in n and converges to a_ℓ.[30] This implies that $g_n^* - c(1) < 0$ for a sufficiently large n. That is, with learning, the fund-raiser may initially solicit some net free-riders to lower future costs – a benefit that was absent in the basic setup.

VI. Conclusion

As part of doing business, charities often spend money to raise money. Thus, a careful planning of whom to ask for donations should be paramount for a charity aiming to control its fund-raising costs while maximizing donations. Perhaps this is why the charitable sector has grown to be highly professional and innovative.[31] Yet, the theory of charitable fund-raising has mostly ignored its cost side. In this paper, we take a first stab at filling this void. We fully characterize the optimal fund-raising strategy that can be easily computed from the donors' preferences, incomes, and the solicitation costs. Among other results, we show that costly fund-raising can provide a novel explanation for the non-neutrality of income redistributions and the crowding-out hypothesis often encountered in empirical studies. For future research, it may be worthwhile to consider sequential solicitations where donations are revealed in each visit. Another promising, and perhaps more challenging, direction would be to investigate the competition between charities where donors' responses are fully accounted for.

[30] The supply of the public good, $\overline{G}_n^*$ is increasing in n and converges to $\overline{\phi}^{-1}(w - a_\ell)$.

[31] For instance, the Association of Fundraising Professionals (AFP) represents 30,000 professional fund-raisers.

Appendix

This appendix contains the proofs of Lemmas 1, 2 and Propositions 1, 3. The remaining proofs as well as the formal details of the extensions are relegated to an online appendix. In what follows, $\Phi_F(G) \equiv \sum_{i\in F}(\phi_i(G) - G) + G$; $F_C = \{i \in F | g_i^*(F) > 0\}$; and $F_{-i} \equiv F\backslash\{i\}$. For Lemma 1, we first prove Lemma A1.

LEMMA A1: *If* $\overline{G}^*(F) > 0$, *then* $\Phi_{F_C}(\overline{G}^*(F)) = \sum_{i\in F_C} w_i - C(F)$ *and* $\Phi_F(\overline{G}^*(F)) \geq \sum_{i\in F} w_i - C(F)$.

PROOF: Suppose $\overline{G}^*(F) > 0$. If $i \in F_C$, then $\phi_i(\overline{G}^*(F)) = w_i + G^*_{-i}(F) - C(F)$. Summing over all $i \in F_C$, and arranging terms yield $\Phi_{F_C}(\overline{G}^*(F)) = \sum_{i\in F_C} w_i - C(F)$. Moreover, since $\phi_i\ (\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i$ for any $i\ \in F\backslash F_C$, summing over all $i \in F$ yields $\Phi_F(\overline{G}^*(F)) \geq \sum_{i\in F} w_i - C(F)$, as desired.

PROOF OF LEMMA 1: ($\Longrightarrow$): Let F^o be the unique optimal fund-raiser set. Suppose that $i \in F^o$ but, contrary to C1, $g_i^*(F^o) \leq c_i$. Since $F^o \neq \varnothing$, clearly $\overline{G}^*(F^o) > 0$. Next, we show that $F^o = F^o_C$. Since $F^o_C \subseteq F^o$ by definition, we only show that $F^o \subseteq F^o_C$. Suppose not. Then, $j \in F^o$ but $j \notin F^o_C$ for some j. That is, person j is contacted even though $g_j^*(F^o) = 0$. Then, Lemma A1 reveals that $\Phi_{F^o_C}(\overline{G}^*(F^o)) < \sum_{i\in F^o_C} w_i - (C(F^o) - c_j) \leq \Phi_{F^o_C}(G^*(F^o_C) - C(F^o_{-j}))$. Since $\Phi'_{F^o_C} > 0$, we have $G^*(F^o_C) - C(F^o_{-j}) > \overline{G}^*(F^o)$. Given this, note that if $i \notin F^o_C$ under cost $C(F^o)$, then $i \notin F^o_C$ under cost $C(F^o_{-j})$. Thus, $F^o_{-j,C} \subseteq F^o_C$, which implies $\overline{G}^*(F^o_{-j}) = G^*(F^o_C) - C(F^o_{-j})$, and in turn, $\overline{G}^*(F^o_{-j}) > \overline{G}^*(F^o)$, contradicting the optimality of F^o. Hence, $F^o = F^o_C$.

Now, recall our hypothesis that $i \in F^o$ and $g_i^*(F^o) \leq c_i$. We also know that $g_i^*(F^o) > 0$, and thus $\phi_i(\overline{G}^*(F^o)) - \overline{G}^*\ (F^o) = w_i - g_i^*(F^o)$. Inserting this into the equilibrium condition in Lemma A1: $\Phi_{F^o}(\overline{G}^*(F^o)) = \sum_{j\in F^o} w_j - C(F^o)$, we obtain

$$\begin{aligned}
\Phi_{F^o_{-i}}(\overline{G}^*(F^o)) &= \textstyle\sum_{j\in F^o_{-i}} w_j - C(F^o) + g_i^*(F^o) \\
&= \textstyle\sum_{j\in F^o_{-i}} (w_j - c_j) - (c_i - g_i^*(F^o)) \\
&\leq \textstyle\sum_{j\in F^o_{-i}} (w_j - c_j) \\
&\leq \Phi_{F^o_{-i}}(\ \overline{G}^*(F^o_{-i})),
\end{aligned}$$

where the last inequality is due to Lemma A1. Then, given that $\Phi'_{F^o_{-i}} > 0$, we have $\overline{G}^*(F^o) \leq \overline{G}^*(F^o_{-i})$. But, this contradicts the optimality of F^o either because $\overline{G}^*(F^o) < \overline{G}^*(F^o_{-i})$, or because $\overline{G}^*(F^o) = \overline{G}^*(F^o_{-i})$ and $C(F^o) > C(F^o_{-i})$. As a result, $g_i^*(F^o) > c_i$.

To prove that F^o must also satisfy C2, suppose, by way of contradiction, that individual i is not in F^o, but that if added to F^o, i's contribution would satisfy $g_i^*(F^o \cup \{i\}) - c_i > 0$. To economize on notation, let $F^o \cup \{i\} \equiv F^+$ and $F_{C,-i}^+ \equiv F_C^+ \setminus \{i\}$. By definition, $F_{C,-i}^+ \subseteq F^o$. Moreover, since $c_i > 0$, we have $g_i^*(F^+) > 0$, which means that $\phi_i(\overline{G}^*(F^+)) - \overline{G}^*(F^+) = w_i - g_i^*(F^+)$. Inserting this into the equilibrium condition, $\Phi_{F_C^+}(\overline{G}^*(F^+)) = \sum_{j \in F_C^+} w_j - C(F^+)$, we obtain $\Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) = \sum_{j \in F_{C,-i}^+} w_j - C(F^o) + (g_i^*(F^+) - c_i)$. If $F_{C,-i}^+ = F^o$, then

$$\Phi_{F^o}(\overline{G}^*(F^+)) = \sum_{j \in F^o} w_j - C(F^o) + (g_i^*(F^+) - c_i) > \sum_{j \in F^o} w_j - C(F^o) = \Phi_{F^o}(\overline{G}^*(F^o)),$$

where the last equality follows because $F^o = F_C^o$. But, given that $\Phi'_{F^o} > 0$, we then have $\overline{G}^*(F^+) > \overline{G}^*(F^o)$, which contradicts the optimality of F^o.

If $F_{C,-i}^+ \neq F^o$, or equivalently $F_{C,-i}^+ \subset F^o$, then, by definition of Φ_{F^o},

$$\Phi_{F^o}(\overline{G}^*(F^+)) = \Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) + \sum_{j \in F^o \setminus F_{C,-i}^+} (\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+)).$$

Since $\Phi_{F_{C,-i}^+}(\overline{G}^*(F^+)) = \sum_{j \in F_{C,-i}^+} w_j - C(F^o) + (g_i^*(F^+) - c_i)$ and $\phi_j(\overline{G}^*(F^+)) - \overline{G}^*(F^+) \geq w_j$ (because $j \in F^o \setminus F_{C,-i}^+$ and thus a free-rider in the set F^+), it follows that

$$\Phi_{F^o}(\overline{G}^*(F^+)) \geq \sum_{j \in F^o} w_j - C(F^o) + (g_i^*(F^+) - c_i) > \sum_{j \in F^o} w_j - C(F^o).$$

Note again that $\sum_{j \in F^o} w_j - C(F^o) = \Phi_{F^o}(\overline{G}^*(F^o))$ because $F^o = F_C^o$. This implies that $\Phi_{F^o}(\overline{G}^*(F^+)) > \Phi_{F^o}(\overline{G}^*(F^o))$, which, in turn, implies that $\overline{G}^*(F^+) > \overline{G}^*(F^o)$, contradicting the optimality of F^o. As a result, i is in F^o, which means F^o also satisfies C2.

($\Longleftarrow$): We prove the uniqueness of the equilibrium fund-raiser set. Suppose, on the contrary, that there are two distinct sets F and F' each satisfying C1 and C2. Note that $F \subset F'$ or $F' \subset F$ cannot be the case: otherwise, either C1 or C2 would be violated for at least one set. Next, take any i such that $i \in F'$ but $i \notin F$. By C2, i would be a net free-rider in $F \cup \{i\} = F^+$, i.e, $g_i^*(F^+) - c_i \leq 0$, which implies that $G_i^0 \leq G_{-i}^*(F^+) - C(F)$. Therefore,

$$G_i^0 = f_i(w_i - c_i + G_i^0) \leq f_i(w_i - c_i + G_{-i}^*(F^+) - C(F)) \leq \overline{G}^*(F^+).$$

Note also that $\overline{G}^*(F^+) \leq \overline{G}^*(F)$ because, by the first part, removing a net free-rider increases the equilibrium public good. Together, $G_i^0 \leq \overline{G}^*(F)$. In addition, since i is a net contributor in F' by C1, i.e., $g_i^*(F') - c_i > 0$, we have $G_i^0 > G_{-i}^*(F') - C(F'_{-i})$ and thus,

$$G_i^0 = f_i(w_i - c_i + G_i^0) > f_i(w_i - c_i + G_{-i}^*(F') - C(F'_{-i})) = \overline{G}^*(F'),$$

implying that $G_i^0 > \overline{G}^*(F')$. Together, the two inequalities reveal that $\overline{G}^*(F) \geq G_i^0 > \overline{G}^*(F')$, which, in turn, reveals $\overline{G}^*(F) > \overline{G}^*(F')$. But, a symmetric argument shows that $\overline{G}^*(F) < \overline{G}^*(F')$, yielding a contradiction. Hence, $F = F'$.

PROOF OF PROPOSITION 1: We first claim that if $\overline{G}^*(F) > 0$ for some F, then $g_i^*(F) - c_i > 0$ if and only if $G_i^0 > \overline{G}^*(F)$. Note that $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = w_i - g_i^*(F)$, or equivalently $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = (w_i - c_i) - (g_i^*(F) - c_i)$ if $g_i^*(F) > 0$; and $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i$ if $g_i^*(F) = 0$. Since $\phi_i(G_i^0) - G_i^0 = w_i - c_i$ by eq.(2), and $\phi_i' > 1$, the claim follows.

Next, for $G_i^0 \geq G_{i+1}^0$, it easily follows that $\Delta_i \geq \Delta_{i+1}$ and $\Delta_1 = G_1^0 > 0$. Let $k \in N$ be the largest index with $\Delta_k > 0$. Since $\Phi_k(0) = 0$, $\Phi_k' > 0$, and $\sum_{j=1}^k (w_j - c_j) > 0$, there is a unique solution, $\overline{G}^* = \Phi_k^{-1}(\sum_{j=1}^k (w_j - c_j)) > 0$ to $\Phi_k(\overline{G}^*) = \sum_{j=1}^k (w_j - c_j)$. $\overline{G}^*$ is an equilibrium because $\sum_{j=1}^k c_j - G_{-i}^* \leq \sum_{j=1}^k c_j - \sum_{j \neq i}^k c_j = c_i$, and $c_i < \widehat{C}_i$ by assumption. Moreover, each $i = 1, ..., k$ is a net contributor because $G_i^0 > \overline{G}^*$, and thus must be solicited by Lemma 1. By the same token, each $i = k+1, k+2, .., n$ is a net free rider because $G_i^0 \leq \overline{G}^*$, and thus must be left outside the fundraiser set.

PROOF OF LEMMA 2: From Proposition 1, define $\widetilde{\Delta}_i(c) \equiv \Phi_i(G_i^0(c)) - \sum_{j=1}^i w_j + ic$ such that $i \in F^o$ if and only if $\widetilde{\Delta}_i(c) > 0$. Substituting for $\phi_i = \phi$, it follows that $\widetilde{\Delta}_i'(c) = -1/[\phi'(G_i^0) - 1] < 0$ since $\phi' > 1$. Hence, $i \in F^o$ if and only if $c < \overline{c}_i$, where $\widetilde{\Delta}_i(\overline{c}_i) = 0$. Simplifying terms, $\overline{c}_i$ solves: $i[\phi(G_i^0) - G_i^0] + G_i^0 - \sum_{j=1}^i w_j + ic = 0$. Since $\phi(G_i^0) - G_i^0 = w_i - c$ from (2), we have $G_i^0(\overline{c}_i) = \sum_{j=1}^i (w_j - w_i)$. In addition, given that $\overline{\phi}(G) \equiv \phi(G) - G$, we also have $\overline{\phi}(G_i^0(\overline{c}_i)) = w_i - \overline{c}_i = \overline{\phi}\left(\sum_{j=1}^i (w_j - w_i)\right)$, which reduces to

$$\overline{c}_i = w_i - \overline{\phi}(\textstyle\sum_{j=1}^i (w_j - w_i)).$$

To prove the last part, note from (2) that $\overline{c}_i - \overline{c}_{i+1} = w_i - w_{i+1} + \overline{\phi}(\sum_{j=1}^{i+1} (w_j - w_{i+1})) - \overline{\phi}(\sum_{j=1}^i (w_j - w_i))$. Since $w_i \geq w_{i+1}$ and $\overline{\phi}' > 0$, it follows that $\overline{c}_i \geq \overline{c}_{i+1}$, as desired.

For Proposition 3, we first prove the following result.

LEMMA A2: *Let $u_i = u$ and $c_i = c$ for all $i \in N$. Moreover, let $\mathbf{w}' \neq \mathbf{w}''$ be two income distributions such that $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$. Then, $\overline{G}^{*\prime} \leq \overline{G}^{*\prime\prime}$. In addition, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$, if one of the following conditions is satisfied: (1) $\varnothing \neq F^{o\prime}$, $F^{o\prime} = F^{o\prime\prime}$, and $F^{o\prime\prime} \neq N$; (2) $F^{o\prime\prime} \subset F^{o\prime} \neq N$; or (3) $F^{o\prime} \subset F^{o\prime\prime}$.*

PROOF: Let $|F^{o\prime}| = m'$ and $|F^{o\prime\prime}| = m''$. First, consider condition (1). Since, $L_{m'}(\mathbf{w}') < L_{m''}(\mathbf{w}'')$, it follows that $\Phi_{m'}(\overline{G}^{*\prime}) = \sum_{i=1}^{m'}(w_i' - c) < \sum_{i=1}^{m''}(w_i'' - c) = \Phi_{m'}(\overline{G}^{*''})$, which implies that $\overline{G}^{*\prime} < \overline{G}^{*''}$. Next, assume condition (2), and by way of contradiction, that $\overline{G}^{*\prime} \geq \overline{G}^{*\prime\prime}$. Then, $\sum_{i=1}^{m'}(w_i' - c) = \Phi_{m'}(\overline{G}^{*\prime}) \geq \Phi_{m'}(\overline{G}^{*\prime\prime})$. Moreover, $\Phi_{m'}(\overline{G}^{*\prime\prime}) \geq \sum_{i=1}^{m''}(w_i'' - c) + \sum_{i=m''+1}^{m'}(w_i'' - c)$ because individuals $\{m''+1, ..., m'\}$ are net free- riders under $\mathbf{w}''$. Thus, $\sum_{i=1}^{m'}(w_i' - c) \geq \sum_{i=1}^{m'}(w_i'' - c)$, or equivalently, $L_{m'}(\mathbf{w}') \geq L_{m'}(\mathbf{w}'')$, which contradicts our hypothesis that $\mathbf{w}'$ Lorenz dominates $\mathbf{w}''$. Thus, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$.

Finally, consider condition (3). Let $\overline{G}_{m'}^{*\prime\prime}$ be the equilibrium level of the public good if agents $1, ..., m'$ constituted the whole economy under $\mathbf{w}''$. Since individuals $m'+1, .., m''$ are also contributors under $\mathbf{w}''$, it follows that $\Phi_{m''}(\overline{G}_{m'}^{*\prime\prime}) < \sum_{i=1}^{m'}(w_i'' - c) + \sum_{i=m'+1}^{m''}(\phi(G_i^{0\prime\prime}) - G_i^{0\prime\prime}) = \sum_{i=1}^{m''}(w_i'' - c) = \Phi_{m''}(\overline{G}^{*\prime\prime})$. Then, $\overline{G}_{m'}^{*\prime\prime} < \overline{G}^{*\prime\prime}$. Now, assume, by way of contradiction, that $\overline{G}^{*\prime} \geq \overline{G}^{*\prime\prime}$. It follows that$\sum_{i=1}^{m'}(w_i'' - c) = \Phi_{m'}(\overline{G}_{m'}^{*\prime\prime}) < \Phi_{m'}(\overline{G}^{*\prime\prime}) \leq \Phi_{m'}(\overline{G}^{*\prime}) = \sum_{i=1}^{m'}(w_i' - c)$, which implies that $L_{m'}(\mathbf{w}') > L_{m'}(\mathbf{w}'')$, contradicting the Lorenz dominance hypothesis. Hence, $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$.

PROOF OF PROPOSITION 3: From (3), our hypothesis that $L_i(\mathbf{w}'') > L_i(\mathbf{w}')$ for every $i < n$ implies that $\overline{c}_n'' < \overline{c}_n'$ and $\overline{c}_1'' > \overline{c}_1'$. Next, $\overline{c}_n'' > 0$ since he is assumed a contributor for $c = 0$ in the standard model. Hence, for $c \in [0, \overline{c}_n'')$ all individuals are net contributors and thus $F^{o\prime} = F^{o\prime\prime}$. Since $L_n(\mathbf{w}'') = L_n(\mathbf{w}')$, this means that $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime}$. For $c \in [\overline{c}_n'', \overline{c}_1'')$, we clearly have one of the three conditions in Lemma A2 satisfied, implying that $\overline{G}^{*\prime} < \overline{G}^{*\prime\prime}$. Finally, for $c \geq \overline{c}_1''$, no fund-raising takes place and so $\overline{G}^{*\prime} = \overline{G}^{*\prime\prime} = 0$, completing the proof.

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