Connectivity of Wireless Ad Hoc Networks: Impacts of Antenna Models

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Abstract—This paper concerns the impact of various antenna models on the network connectivity of wireless ad hoc networks. Existing antenna models have their pros and cons in the accuracy reflecting realistic antennas and the computational complexity. We therefore propose a new directional antenna model called Approx-real to balance the accuracy against the complexity. We then run extensive simulations to compare the existing models and the Approx-real model in terms of the network connectivity. The study results show that the Approx-real model can better approximate the best accurate existing antenna models than other simplified antenna models, while introducing no high computational overheads.

Keywords—Connectivity; Ad Hoc Networks; Directional Antennas; Beamforming

I. INTRODUCTION

As one of the most important measures of the reliability of wireless ad hoc networks, the network connectivity has received extensive research attentions recently. Essentially, the network connectivity is a necessity to ensure the network is connected so that each source station (node) can successfully communicate with its destination station (node). However, different researchers adopt different antenna models on the study of the network connectivity, whereas different antenna models have their respective pros and cons in the accuracy reflecting realistic directional antennas and the computational complexity.

Existing antenna models are roughly classified into omni-directional and directional antennas. Most of current studies on the network connectivity of wireless ad hoc networks assume that each node is equipped with an omni-directional antenna, which radiates radio signals in all directions including some undesired directions [1]–[3]. As a result, the networks equipped with such an antenna suffer from poor performance (such as the low throughput and the long delay) due to the interference on some undesired directions.

Recently, many studies [4]–[10] show that using directional antennas instead of omni-directional antennas in wireless ad hoc networks can significantly improve the network performance (particularly for the network connectivity). The performance improvement mainly owes to the reduction in the interference from undesired directions and the extension in the transmission range because directional antennas concentrate radio signals on the desired directions.

To investigate the performance of wireless ad hoc networks with directional antennas, we often need to choose a proper antenna model to simulate a directional antenna. Specifically, realistic directional antenna models were used in [8], [9] to study the network connectivity. However, realistic directional antenna models are too complicated to be tractable in analysis. Thus, two simplified directional models, i.e., a sector model [4], [5], [11] and a keyhole model [12], [13] were proposed to approximate a directional antenna. But, these simplified directional models are too idealistic to be realistic, although they are theoretically tractable. Therefore, a novel antenna model, which can accurately approximate a realistic directional antenna without significantly increasing the computational complexity, is expected to be proposed.

Besides, to the best of our knowledge, there is no study on the comparison of the impacts of various antenna models on the network connectivity of wireless ad hoc networks. Therefore, the goal of this paper is to investigate the network connectivity under various existing antenna models. Specifically, the major research contributions of this paper are summarized as follows.

• Propose a new approximate directional antenna model, namely, approximately realistic directional antenna (Approx-real). The Approx-real model balances the accuracy against the complexity.

• Provide a comprehensive comparative study on the existing popular antenna models and the proposed Approx-real model in terms of the network connectivity under different environments. The extensive experiments we conducted show that the Approx-real model can better approximate the realistic directional antenna models [8], [9] than others, such as the sector model [4], [5], [11] and the keyhole model [12], [13].

The rest of this paper is organized as follows. Section II introduces the existing popular antenna models and proposes our Approx-real model. Section III presents the link model. Section IV gives the problem formulation and defines the path probability that measures the network connectivity. In Section V, we compare via extensive simulations the popular antenna models and the Approx-real model in terms of the network connectivity. Finally, Section VI concludes this paper.

II. ANTENNA MODELS

A. Antenna gain

An antenna is a device that is used for radiating/collecting radio signals into/from space. An omni-directional antenna,
which can radiate/collect radio signals uniformly to all directions in space, is typically used in conventional wireless ad hoc networks. Different from an omni-directional antenna, a directional antenna can concentrate transmitting or receiving capability to some desired directions so that it has better performance than an omni-directional antenna.

To model the transmitting or receiving capability of an antenna, we often use the *antenna gain*, which is the directivity of an antenna in 3-D space. The antenna gain of an antenna can be expressed in *radiation pattern* in a spherical coordinate system as follows [14].

\[
G(\theta, \phi) = \frac{U(\theta, \phi)}{U_o} \tag{1}
\]

where \( \theta \) is the elevation angle from z-axis \((\theta \in (0, \pi))\), \( \phi \) is the azimuth angle from the x-axis in the \( xy \)-plane \((\phi \in (0, 2\pi))\), and \( \eta \) is the efficiency factor, which is set to be 1 since an antenna is often assumed to be lossless. In Eq. (1), \( U(\theta, \phi) \) is the *radiation intensity*, which is defined as the power radiated from an antenna per unit solid angle, and \( U_o \) denotes the radiation intensity of an omni-directional antenna with the same radiation power \( P_{rad} \) as a directional antenna.

We next describe various existing antenna models and introduce our proposed new antenna model.

### B. Isotropic Antenna

In this paper, we consider an idealistic antenna - isotropic antenna to model the antenna gain of an omni-directional antenna. Thus, an omni-directional antenna is denoted by an isotropic antenna interchangeably throughout the paper. Since an isotropic antenna radiates the radio power uniformly in all directions in 3-D space, it is obvious that an isotropic antenna has gain \( G_o = 1 \) since \( U(\theta, \phi) = U_o \). In this paper, we need to conduct simulation experiments on a 2-D plane. Thus, we project the 3-D antenna gain to the \( xy \)-plane. Fig. 1 (a) shows the radiation pattern of an isotropic antenna.

### C. Directional Antennas

Different from an isotropic antenna, a directional antenna can radiate or receive radio signals more effectively in some directions than in others. A directional antenna consists of the *main-lobe* (or *main-beam*) with the largest radiation intensity and the *side-lobes* and *back-lobes* with the smaller radiation intensity, as shown in Fig. 1 (b).

In order to compute the antenna gain of a directional antenna, we firstly compute the radiation power \( P_{rad} \) of an antenna, which is given by

\[
P_{rad} = \iint_{\Omega} U(\theta, \phi) \, d\Omega = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta \, d\theta \, d\phi \tag{2}
\]

where \( \Omega \) is the steradian used to measure the solid angle subtended by a particular spherical surface \( S \) and the element of solid angle \( d\Omega \) of a sphere is \( d\Omega = \sin \theta \, d\theta \, d\phi \).

Since an isotropic antenna radiates power in all directions with a constant radiation intensity \( U_o \), we have \( P_{rad} = 4\pi U_o \) after integrating on Eq. (2). In other words, \( U_o = \frac{1}{4\pi} P_{rad} \). After replacing \( U_o \) in Eq. (1) by \( \frac{1}{4\pi} P_{rad} \) and replacing \( P_{rad} \) by the integration of Eq. (2), we have

\[
G(\theta, \phi) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta \, d\theta \, d\phi \tag{3}
\]

Note that Eq. (3) is applied for the calculation of the antenna gain of any types of directional antenna models, which will be described as follows.

1) **Uniform Circle Array (UCA) Antenna**: One of the most commonly used directional antennas is a Uniform Circle Array (UCA) antenna, which consists of \( M \) isotropic antenna elements equally spaced on the \( xy \)-plane along a circle of radius \( a \), as shown in Fig. 4. In this structure, \( r \) is the distance between the antenna and the observation position, \( \Delta \) is the distance between two neighboring elements, which is usually chosen as \( \lambda/2 \) and \( \lambda \) is the wavelength of electromagnetic wave radiated from elements. As shown in [14], the radiation intensity of a UCA antenna fulfills the following formula.

\[
U(\theta, \phi) \propto |E(\theta, \phi)|^2 \tag{4}
\]

where \( E(\theta, \phi) \) denotes the electric field strength at a given direction \((\theta, \phi)\), which can be obtained by

\[
E(\theta, \phi) = \sum_{m=1}^{M} I_m e^{jka[\sin \theta \cos(\phi-\phi_m) - \sin \theta_o \cos(\phi_0-\phi_m)]} \tag{5}
\]

where \( j \) is the imaginary unit for which \( j^2 = -1 \), \( k = 2\pi/\lambda \), \( \lambda \) is the wavelength of the propagating signal, \( \phi_m = 2\pi m/M \) is the angular position of \( m \)th element on \( xy \)-plane, \( I_m \) is the amplitude excitation of the \( m \)th element, which is set to be 1, similar to [9]. We let \( \theta_0 = \pi/2 \) (i.e., the \( xy \) plane) and \( \phi_0 \in [0, 2\pi] \) is the azimuth angle of the desired main beam.
After replacing $U(\theta, \phi)$ in Eq. (3) by combining Eq. (4), we then compute the gain $G(\theta, \phi)$ as follows.

$$G(\theta, \phi) = \frac{|E(\theta, \phi)|^2}{\frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi |E(\theta, \phi)|^2 \sin \theta d\theta d\phi}$$  \hspace{1cm} (6)

We next obtain the radiation pattern of the UCA antenna in 3-D space, as shown in Fig. 4 (a). Since we will conduct simulations on a 2-D plane, we project the UCA gain in 3-D space to a 2-D plane by setting $\theta = \pi/2$ (in $xy$-plane). After that, we have the different radiation patterns based on different values of $\phi_0 \in [0, 2\pi]$. Fig. 4 (b) shows the gain patterns of a UCA antenna with $M = 8$ elements when $\phi_0 = 0$ in a 2-D plane.

2) Uniform Line Array (ULA): Another common directional antenna is a ULA antenna, which consists of antenna elements equally placed along a line. Fig. 5 shows the structure of a ULA antenna with $M$ isotropic elements, where $\Delta$ is the distance of any two neighboring elements. The radiation intensity of a ULA antenna fulfills the following formula [8].

$$U(\theta, \phi) \propto \left(\frac{\sin(m \psi)}{m \sin \psi}\right)^2$$  \hspace{1cm} (7)

Due to the rotational symmetry of a ULA antenna (as shown in Fig. 3 (a)), the antenna gain is independent of $\phi$. Besides, $\psi$ in Eq. (7) can be obtained by

$$\psi = \frac{\pi \Delta}{\lambda} \left(\cos \theta - \cos \theta_0\right)$$  \hspace{1cm} (8)

where $\lambda$ denotes the wavelength of electromagnetic wave radiated from antenna elements., $\Delta$ is usually chosen as $\lambda/2$ and $\theta_0$ is the azimuth angle of the desired main beam.

Thus, the gain of ULA antennas can be computed by

$$G(\theta) = \frac{\left(\frac{\sin(m \psi)}{m \sin \psi}\right)^2}{\frac{1}{2} \int_0^{\pi} \left(\frac{\sin(m \psi)}{m \sin \psi}\right)^2 \sin \theta d\theta}$$  \hspace{1cm} (9)

The gain of a ULA antenna is a function of $\theta$ with the fixed $\theta_0$. We then obtain the radiation pattern of a ULA antenna in 3-D space, as shown in Fig. 5 (a) with $\theta_0 = 0$. Similar to a UCA antenna, we obtain a 2-D radiation pattern of a ULA antenna by projecting the radiation pattern in 3-D space to a 2-D plane, as shown in Fig. 5 (b). Different from a UCA antenna, a ULA antenna consists on two main-lobes.

D. Simplified Directional Antenna Models

The realistic directional antenna models (e.g., UCA and ULA antennas) are too complicated to be used in practice. Thus, several simplified directional antenna models are proposed to approximate the realistic directional antennas. In particular, there are two typical simplified directional antenna models described as follows.

- **Keyhole Antenna Model** [12], [13] consists one main-lobe with beamwidth $\theta_d$ and side/back-lobes to other directions, as shown in Fig. 6 (a).
- **Sector Antenna Model** [4], [5] consists only one main-lobe with beamwidth $\theta_d$ after ignoring back/side-lobes, as shown in Fig. 6 (b).

We first derive the antenna gain of the keyhole model in 3-D space, as shown in Fig. 6 (a). The radiation power $P_{rad}$ consists of the main-lobe part denoted by $P_m$ and the side/back-lobe part denoted by $P_s$, fulfilling the following equation.

$$P_{rad} = P_s + P_m$$  \hspace{1cm} (10)

where $P_{rad} = 4\pi U_0$, $P_m$ and $P_s$ can be calculated by the following integral equations, respectively.

$$P_m = \int_0^{2\pi} \int_0^{\pi} G_m U_0 \sin \theta d\theta d\phi$$  \hspace{1cm} (11)

$$P_s = \int_0^{2\pi} \int_0^{\pi} G_s U_0 \sin \theta d\theta d\phi$$  \hspace{1cm} (12)

where $G_m$ and $G_s$ are the gains of main-lobe and side-lobe respectively.

We then have

$$G_s = \frac{2 - G_m(1 - \cos(\frac{\theta_d}{2}))}{1 + \cos(\frac{\theta_d}{2})}$$  \hspace{1cm} (13)

As shown in Eq. (13), $G_s$ is a function of the antenna gain of the main-lobe $G_m$ and the beamwidth $\theta_d$. When $G_s = 0$, this keyhole model becomes a sector model. Note that either the keyhole model or the sector model consists of only one main-lobe. Thus, they only apply for a UCA antenna, which has only a single main-lobe. However, these two simplified directional
models do not apply for a ULA antenna, which often consists of two main-lobes. We need another new approximated model to represent both UCA antennas and ULA antennas.

E. Approx-real Model

As shown above, realistic directional antenna models (i.e., UCA and ULA) are too complicated to be used in practice, while the simplified directional antenna models, such as the keyhole model and the sector model are too simple to accurately depict an antenna. Besides, both the keyhole model and the sector model cannot depict a ULA antenna consisting of two main-lobes. Therefore, we propose a novel antenna model that approximates both UCA and ULA antennas without increasing the computational complexity significantly.

Observing that the radiation pattern of a realistic directional antenna consists of multiple lobes (main-lobes and side/back-lobes), we approximate each lobe as a sector, which leads to our Approx-real antenna model.

Definition 1: Approx-real Model. The antenna gain \( G(\theta) \) at a certain direction can be calculated by

\[
G(\theta) = \begin{cases} 
G_{\text{max}}(i) & \text{within the half-power beamwidth of lobe } i \\
0 & \text{otherwise} 
\end{cases}
\]

(14)

where \( G_{\text{max}}(i) \) denotes the maximal antenna gain of lobe \( i \) and the half-power beamwidth is the subtend angle of the directions when the radiation power falls the half of the maximal radiation power of lobe \( i \).

Fig. 7 (a) and Fig. 7 (b) show the corresponding Approx-real models to the UCA antenna and the ULA antenna shown in Fig. 4 and Fig. 5, respectively.

III. LINK MODEL

We now define the link model, which determines whether two nodes in an ad hoc network can establish a link. We first assume that a node transmits with power \( P_t \). Then, the receiving power at the receiver denoted by \( P_r \) can be calculated by

\[
P_r = \frac{P_t G_t G_r}{d^\alpha}
\]

(15)

where \( d \) is the distance between the transmitter and the receiver, \( \alpha \) is the pathloss exponent (usually \( 2 \leq \alpha \leq 4 \) [15]), \( G_t \) and \( G_r \) are the antenna gains of a transmitter and a receiver, respectively. In practice, we usually compute the power attenuation between two nodes instead of computing the received power \( P_r \). We then introduce the power attenuation \( \delta \) as follows

\[
\delta = \frac{P_t}{P_r} = \frac{d^\alpha}{G_t G_r}
\]

(16)

Any two nodes can establish a link if \( \delta \) is less than a threshold \( \delta_0 \). If we substitute \( \delta \) in Eq. (16) with \( \delta_0 \), we can calculate the maximum distance \( d_{\text{max}} \) between two nodes with a link.

\[
d_{\text{max}} = \sqrt{G_r G_t \delta_0}
\]

(17)

As shown in Eq. (17), the maximum distance \( d_{\text{max}} \) depends on the antenna gains of the transmitter and the receiver, i.e., \( G_r \) and \( G_t \). Note that \( d_{\text{max}} \) varies with the different directions of antennas since \( G_r \) and \( G_t \) vary in different directions.

We then have the following link condition.

Definition 2: Link Condition. Any node pair can establish a link if and only if the distance \( d \) between the two nodes is less than the maximum distance \( d_{\text{max}} \).

IV. NETWORK CONNECTIVITY

A. Problem Formulation

We consider the following problem: \( n \) nodes are randomly placed in a 2-D plane. Each node can determine whether it can establish a link with one of its neighbors by computing the link condition as defined in Section III. The connectivity problem is to evaluate the number of the linked nodes. Note that we are interested in investigating the impacts of different antenna models on the network connectivity of wireless ad hoc networks. Thus, we consider the realistic directional antennas (i.e., UCA and ULA antennas), the simplified directional models (i.e., the keyhole model and the sector model) and our proposed Approx-real model in the study of the network connectivity. However, it is difficult for a node to find its neighbors with directional antennas since a node has no location knowledge of other nodes [16]. Thus, we take the random beamforming strategy [8] to help nodes find each other. In particular, in this scheme, the main beam of a directional antenna at each node should be chosen from directions with the uniform distribution on the interval \([0, 2\pi]\).
B. Connectivity

In this paper, we adopt the path probability as a metric to measure the level of the network connectivity [8]. The path probability is a statistical average (in percentage) of the number of connected node pairs over that of the total node pairs. In particular, we define the path probability $P$(path) of a network as follows.

$$P\text{(path)} = \frac{\# \text{ connected node pairs}}{\# \text{ node pairs}} = \frac{\sum_{i=1}^{v} \frac{1}{2} n_i (n_i - 1)}{\frac{1}{2} n(n-1)} \quad (18)$$

where $\#$ denotes the number of, $v$ denotes the number of connected components of a network and $n_i$ denotes the number of nodes in the $i$-th connected component. For example, there are 5 connected components in Fig. 8, where each connected node pair is connected by a link (denoted by a blue line).

It is shown in Eq. (18) that the higher path probability $P$(path) implies the better network connectivity. In particular, $P$(path) = 1 if the whole network is completely connected. If all the nodes are isolated (i.e., $v = 0$), $P$(path) = 0.

V. Empirical Study

We conduct extensive simulations to investigate the path probability of the network with various antenna models, such as the isotropic model, the keyhole model, the sector model, the UCA model, the ULA model, the Approx-real (UCA) and the Approx-real (ULA) model. Our simulations were conducted in an area of $l \times l$ with ignoring the impacts of the border effects [3], where $l$ is chosen as 200m, 400m and 1000m. Each result point is an average value over 1000 topologies. Note that we consider the pathloss exponent $\alpha$ ranging from $\alpha = 2.5$ to $\alpha = 4$ and the fixed threshold attenuation $\delta_0 = 50$dB.

A. Connectivity Comparison with UCA antenna, Isotropic antenna, keyhole, sector and Approx-real antenna

Fig. 9 gives the comparisons of the path probability of the networks with UCA antenna, isotropic antenna, keyhole model, sector model and Approx-real model.

As shown in Fig. 9, the connectivity in wireless ad hoc networks varies with different antennas. In particular, Fig. 9 shows that the network with the sector model has much lower path probability than those with other antenna models when the pathloss exponent $\alpha$ is ranging from 2.5 to 4 (see Fig. 9 (a)-(d)). In other words, the sector model is “over-pessimistic” to estimate the network connectivity. This is because a sector antenna may lead to the lost number of neighbors when the neighbors do not reside in the beamwidth of a sector antenna even if a sector antenna has a high gain without side/back-lobes. In contrast to the sector model, the path probabilities with the keyhole model are always higher than those with the UCA antenna (see Fig. 9 (a)-(d)). This may owe to the property of the keyhole model that has the circular radiation pattern of side/back-lobes in all directions. As a result, it has more chances to connect with other nodes than a sector antenna.

Compared with the sector model and the keyhole model, our proposed Approx-real (UCA) model offers a better approximation of the network connectivity to a realistic directional antenna with $\alpha = 2.5$, $\alpha = 3$, $\alpha = 3.5$. But, with the increased pathloss factor $\alpha$, our proposed Approx-real (UCA) model also becomes “pessimistic”. For example, as shown in Fig. 9 (d), when $\alpha = 4$ and $n = 250$, the path probability of Approx-real (UCA) model is about 73% while that of the UCA antenna is about 91% and that of the keyhole is about 99%.

Note that the path probabilities $P$(path) of a realistic UCA antenna are significantly higher than that of an isotropic antenna. The reason lies the fact that using directional antennas in wireless networks can establish some long links to connect the nodes that are far away and even out of the transmission range of an isotropic antenna. This result further confirms the results [8]–[10].

B. Connectivity Comparison with ULA antenna, Isotropic antenna, and Approx-real antenna

We then conducted the second set of simulations with consideration of ULA antenna, Approx-real (ULA) antenna and isotropic antenna. Note that the keyhole model and the sector model cannot be used to denote a ULA antenna since a ULA antenna has two main-lobes while the keyhole model and the sector model can only model one main-lobe. Fig. 10 shows the results.

As shown in Fig. 10, from $\alpha = 2.5$ to $\alpha = 4$, the network with our Approx-real (ULA) antenna has an approximated network connectivity (the path probability $P$(path)) to a realistic ULA antenna. Specifically, when $\alpha = 2.5$ in Fig. 10 (a), the path probability of ULA antenna is 89% when the number of nodes $n = 60$, while the path probability of approx-real antenna is 85%. When $\alpha = 4$ in Fig. 10 (d), the path probability of ULA antenna is 98% when the number of nodes $n = 300$, while the path probability of Approx-real (ULA) antenna is 96%. The difference between the network connectivity of a ULA antenna and that of an Approx-real (ULA) antenna is much smaller than that between the network connectivity of a UCA antenna and that of an Approx-real (UCA) antenna.
In this paper, we conduct a study on the network connectivity of wireless ad hoc networks with different antenna models. In particular, we originally propose an Approx-real antenna, which offers a better approximation to realistic directional antennas (UCA and ULA antennas) than other existing models, such as the keyhole model and the sector model. The Approx-real antenna model, which has no restriction on the number of main-lobes and side/back-lobes, can be applied to all realistic directional antennas.

More precisely, we conducted extensive simulations to compare the network connectivity (the path probability) with various antenna models under different environments. We found that the sector antenna model, which is usually used in MAC layer design of wireless ad hoc networks with directional antenna, is not suitable to denote a realistic directional antenna.

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