Tying, Price Discrimination and Antitrust Policy

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Introduction

A tying arrangement is a seller’s requirement that before a customer may purchase the seller’s “tying” product, she must also take one or more units of the seller’s “tied” product. The tying and tied products are typically complements in use, which means that they are consumed together.1 Anticompetitive tying arrangements have been illegal under United States antitrust law ever since the Motion Picture Patents case in 1917,2 which prohibited the owner of a patented movie projector from forcing purchasers to agree that they would use the projector only to show the projector seller’s films. Most ties are contractual, in the sense that the thing that binds the tying and tied product together is a contract or perhaps an intellectual property license. Some ties are “technological,” which means that the two products are tied together by virtue of product design. For example, the owner of a Kodak Instamatic camera may be able to use only Kodak’s own film cartridges designed to fit that camera,3 or the owner of a Lexmark computer printer may be limited by virtue of product design to the use of Lexmark ink cartridges.4

At least since the 1950s it has been clear that tying arrangements can be used as price discrimination devices – that is, as devices for obtaining different prices or

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1 The products can also be complements in production or distribution; that is, costs are lower when two products are produced or distributed together. E.g., Times-Picayune Pub. Co. v. United States, 345 U.S. 594, 613 (1953) (refusing to condemn tying of advertising in morning and evening newspapers; defendant set type a single time for both editions). See Brief on Behalf of 98 Newspaper Publishers as Amicus Curiae, 1953 WL 78355, *13 (Feb. 26, 1953) (once type is set, cost of printing additional issues is “hardly any more expense than the cost of ink and paper”).


different rates of return from different customers.\(^5\) In the case of variable proportion
ties, the seller has a monopoly in a tying product, such as a printer, which uses some
consumable product such as ink that consumers purchase as they need it. The seller
then reduces the price of the tying product, sometimes to cost or even to zero,\(^6\) but
requires purchasers to use its tied product and sells it at a premium over the market
price. The seller then earns varying amounts of profit from different customers,
depending on the amount of the tied product that they use.

The economic effects of price discrimination ties has provoked considerable
debate and there is some confusion about what type of price discrimination results from
ties. The answer to that question is critical, because differing types of price
discrimination produce very different effects on general or consumer welfare. As
developed below, the literature on the effects of price discrimination strongly
distinguishes between second and third degree price discrimination.\(^7\) Third degree
price discrimination that does not increase output necessarily decreases welfare.\(^8\) This
is not true of second degree price discrimination, and the economic consensus is that
most instances of it are probably welfare increasing,\(^9\) particularly in the presence of
fixed costs.\(^10\) As we show below, variable proportion ties are a form of second degree
price discrimination. Further, they can be shown to harm consumer welfare in only the
most flagrant situations, and they often increase welfare even if output falls.

The term “welfare” has a relatively fixed meaning in economics. It equals the
sum of consumer and producer surplus, assuming no one else is affected.\(^11\) The

\(^5\) Ward Bowman, *Tying Arrangements and the Leverage Problem*, 67 YALE L.J. 19, 21-23 (1957). See also Richard
\(^6\) See *discussion infra*, text at notes 61-62.
\(^7\) See *discussion infra*, text at notes 32-42.
\(^8\) E.g., Stephen K. Layson, *Third-Degree Price Discrimination with Interdependent Demands*, 46 J. INDUS. ECON. 511
91998); Richard Schmalensee, *Output and Welfare Implications of Monopolistic Third-degree Price Discrimination*,
71 AM.ECON.REV. 242 (1981); Hal R. Varian, *Price Discrimination*, in 1 HANDBOOK OF INDUSTRIAL
ORGANIZATION 600 (Richard Schmalensee & Robert D. Willig, eds., 1989);
\(^9\) See FREDERIC M. SCHERER & DAVID ROSS, *INDUSTRIAL MARKET STRUCTURE AND ECONOMIC PERFORMANCE* 495
(3d ed. 1990) (“First- or second-degree discrimination usually leads to larger output than under simple monopoly,
and from there to lower dead-weight losses and improved allocative efficiency”); Massimo Motta, *Competition
Policy* 494-495 (2004) (second degree price discrimination tends to be welfare improving). But see our own
discussion, *infra*, text at notes 69-71; and Appendix.
\(^10\) On the relevance of fixed costs, see *discussion infra*, text at notes 77-78.
\(^11\) A “surplus” is the difference between the amount that someone is willing to accept or pay for something, and the
amount he or she must actually accept or pay. For example, if a consumer is willing to pay $3.00 for a loaf of bread
but the grocery store price is $2.00, the consumer receives a surplus of $1.00.
antitrust literature has seen a great deal of debate, however, over whether “total surplus,” which is the same thing as the economist’s “welfare,” should govern antitrust policy, or whether antitrust should limit its concern to “consumer welfare.” A consumer welfare standard seeks to maximize consumer surplus without regard to effects on producer surplus. For example, a merger that simultaneously increases productive efficiency and raises price would be unlawful, even if the profit gains to the merging firms were greater than the higher prices to consumers. In that case the merger increases total welfare but reduces consumer welfare. We do not express any opinion on this issue, but throughout this paper make several observations about both the general welfare effects and the consumer welfare effects of ties. Clearly, however, a practice that increases both total welfare and consumer welfare should not be condemned under either of these tests, while a practice that reduces both might be. Most importantly, one must keep in mind that a firm imposes tying only if it is profitable to do so. As a result, a tie that increases consumer welfare necessarily increases general economic welfare as well, assuming third parties are not affected.

Significantly, the antitrust laws do not speak of either measure of welfare and, indeed, never use the term “welfare” at all. The provisions that are most relevant to tying are §1 of the Sherman Act and §3 of the Clayton Act. The Sherman Act provision extends to conduct “in restraint of trade,” and the Clayton Act provision reaches conduct where “the effect may be substantially to lessen competition.” A natural meaning of conduct that “restrains trade” is conduct that reduces output below the level it would otherwise be. In the case of variable proportion ties, however, we show that output effects are not the same as either general welfare or consumer welfare effects. A tie that reduces output might increase both general welfare and consumer

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15 15 U.C.C. §14 (2006). In the case of a monopolist §2 of the Sherman Act, 15 U.S.C. §2, could also be relevant. However, §2’s requirement of “monopolizing” conduct presumably refers only to “foreclosing” ties, or ties that cause harm by excluding rivals. These are not the subject of this paper.
welfare, while a tie that increases output might conceivably harm both, although this is less likely. Further, the term “output” itself requires further clarification because a variable proportion tie typically does two things. First, it typically increases the output of the tying product, such as a printer. Second, in some cases it decreases the volume of the tied product, such as ink cartridges, although in other cases it increases tied product volume as well.

Nonforeclosing ties, or those that do not cause competitive harm by excluding rivals, may extract higher prices from some customers but they also charge lower prices to others and typically bring new customers into the market. As a result, the case for condemning them is very weak. The means of extraction is or at least resembles price discrimination. While the economic literature on price discrimination and tying focuses on monopolists, most challenged ties occur in oligopoly markets where the defendant typically has no more market power than results from product differentiation. Indeed, many franchise ties, which are of variable proportions, occur in competitive or even highly competitive product differentiated markets and involve nondominant firms. In those cases a tie that includes a substantial price reduction in the tying product can increase the number of sales significantly. The true monopoly case is the rare, but hardly unheard of, worst case scenario. However, even if output of the tied product falls under variable proportion tying, it is generally impossible to demonstrate that the tie harms welfare. This is because many of the consumers who buy fewer units under tying are nevertheless better off as a result of the tie. For these consumers, the price cut applied to the tying product contributes more to consumer surplus than is extracted by the increase in the tied product’s price. If the market

16 See the Appendix.
17 The foreclosure rationale for condemning ties is that they exclude, or foreclose, rivals in the tied product market. For example, by requiring all those who use its surgical facilities to purchase its anesthesiologist services, a hospital might be able to exclude rival anesthesiologists from the market. See Jefferson Parish Hospital Dist. v. Hyde, 466 U.S. 2 (1984) (refusing to condemn a hospital’s surgical facility/anesthesiologist tie where hospital did not have dominant market share for surgical admissions; plaintiff was excluded rival anesthesiologist). By contrast, when a maker of salt injection machines requires users to purchase its salt, foreclosure cannot be a threat, since such machines process only a miniscule percentage of the salt market. See International Salt Co. v. United States, 332 U.S. 392 (1947) (condemning such a tie). See also PHILLIP E. AREEDA & HERBERT HOVENKAMP, ANTITRUST LAW ¶¶1704-1709 (2d ed. 2004) (foreclosure and its assessment); ¶¶1722-1726 (nonforeclosing ties).
20 See, e.g., Siegel v. Chicken Delight, Inc., 448 F.2d 43 (9th Cir. 1971) (minor fast food fried chicken franchisor; condemning tying of spices and supplies); Kypta v. McDonald’s Corp., 671 F.2d 1282 (11th Cir. 1982) (fast food; hamburgers and related products; tying of lease of location); Little Caesar Enter., Inc. v. Smith, 34 F.Supp.2d 459 (E.D.Mi. 1998) (pizza; tying of paper plates and other products bearing franchisor’s logo).
includes a relatively high number of these customers, it is possible that output of the tied product falls while consumer welfare increases.\footnote{See discussion infra, text at notes 72-73.}

The traditionally stated concern of tying arrangements was “leverage.” The fear was that a firm with a monopoly in one product could create a second monopoly by requiring purchasers or lessees of the first product to purchase a second product from that firm as well. Historically the concern emerged in patent law.\footnote{See Christina Bohannan, \textit{IP Misuse as Foreclosure} (Iowa Legal Studies Working Paper, Sep, 2009, available on SSRN).} For example, in the \textit{Carbice} decision the Supreme Court condemned an arrangement under which the seller of a patented ice box required those who used it to purchase its dry ice as well. The tie was nonforeclosing, since dry ice, which occurred naturally and was readily manufactured, was not patentable.\footnote{Dry ice had been discovered in the 1830s by Charles Thilorier, a French chemist, as the residue from rapid evaporation of liquid carbon dioxide. See Duane H.D. Roller, \textit{Thilorier and the First Solidification of a “Permanent Gas} (1835), 43 Isis 109 (1952).} Nevertheless, Justice Brandeis wrote for the Supreme Court, the requirement was an unlawful leveraging of the ice box patent, because it enabled the patentee to “derive its profit, not from the invention on which the law gives it a monopoly, but from the unpatented supplies with which it is used.”\footnote{Carbice Corp. v. American Patents Dev. Corp., 283 U.S. 27, 31-32 (1931).} If a monopoly could be contractually expanded in this way, a patentee “might conceivably monopolize the commerce in a large part of the unpatented materials used in its manufacture. The owner of a patent for a machine might thereby secure a partial monopoly on the unpatented supplies consumed in its operation.”\footnote{Id.}

Many antitrust theories prior to the 1980s were based on exaggerated views of the anticompetitive possibilities of leverage. In general, however, leverage was never a significant component in Harvard School antitrust analysis\footnote{See Herbert Hovenkamp, \textit{United States Competition Policy in Crisis: 1890-1955}, \textit{Minn. L. Rev.} \textbf{___} (2009), currently available at \url{http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1156927}.} and it was enthusiastically rejected by the Chicago School of antitrust,\footnote{See Richard A. Posner, \textit{The Chicago School of Antitrust Analysis}, 127 \textit{Univ. Pa. L. Rev.} 925, 933-935 (1979).} particularly after Ward Bowman’s article came out in 1957.\footnote{See Bowman, note 5 at 23-24.} While no one disputes that a monopolist can design contractual mechanisms that exploit its monopoly position by price discrimination, attaching strong implications to this for competition policy has proven to be all but impossible.

In Bowman’s price discrimination model the tying arrangement served as an \textit{alternative} to selling the machine itself at different prices to different customers. As
Bowman observed, such an attempt would encounter two different problems. First, the seller would have a difficult time identifying the users who valued the product by more. Second, those who paid a lower price would arbitrage the machine to higher value users, thus defeating the scheme. In fact, the two strategies are often used simultaneously. For example, a printer manufacturer might engage in cartridge tying while also offering different packages to commercial and residential users, or discounts to educational institutions. This would be a combination of second and third degree price discrimination.

One problem with the leverage argument is its ambiguity. A tie cannot create a second “monopoly” in the tied product unless the latter has no untied uses. For example, even if Justice Brandeis’ ice box manufacturer had an ice box monopoly, tying ice would not create a second monopoly as long as there were numerous uses of dry ice that did not involve the monopolist’s ice box. Fundamentally, the leverage theory concerns “extraction,” not monopoly. The monopolist is obtaining a higher price for the dry ice that it sells, but the rest of the dry ice market remains unaffected, assuming it is competitive.

Nonforeclosing Ties and Second Degree Price Discrimination

Ever since the time of Cambridge economist Arthur Cecil Pigou, price discrimination has been divided into three classes, or “degrees.” First degree, or “perfect,” price discrimination, involves selling each unit of a good at the highest price any consumer is willing to pay for that unit. Output in that case rises to the competitive level because every sale is made right down to marginal cost. However, all of the surplus goes to the seller rather than to the customers. Strictly speaking, neither variable proportion tying nor any other real world practice constitutes first degree price discrimination, because sellers cannot practically extract the highest price that the consumer is willing to pay on each sale.

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29 Id. at 23 (using as an example Heaton-Peninsular Button-Fastener Co. v. Eureka Specialty Co., 65 Fed. 619 (C.C.W.D. Mich. 1895), in which the defendant required users of each button fastening machine to purchase its buttons).
30 See Alan C. Deserpa, A Note on Second Degree Price Discrimination and its Implications, 2 REV. INDUS.ORG. 368 (1985) (real world practices often involve a combination of second and third degree discrimination).
34 The closest situation would be an auction in which each unit available in the entire market is sold to the highest bidder until every unit is sold. Even here, however, the winning bid does not represent the winner’s willing to pass, but only the fact that no other bidder was willing to pay more. For example, if bidders bid against each other until
Even an exceptionally finely tuned variable proportion tie will not come very close to first degree price discrimination. While a well executed printer/ink tie could accurately make prices proportional to the number of copies a person prints, it could not control for the fact that different purchasers place different values on each copy. For example, both a law firm drafting legal opinions on securities offerings and a printer of handbills about garage sales might print 1000 pages weekly. As a result, if they purchased identical printers under the same tying arrangement they would pay the same amount per print. But given what is at stake the law firm might value the printouts at many dollars per page, while the handbill printer values them at only a few cents. The variable proportion tie will not capture these differences in valuation and will thus permit at least some consumers to retain surpluses.

By contrast to first decree discrimination, second and third degree price discrimination are quite common. Although they are very different practices, some complex schemes may contain attributes of both. In third degree price discrimination the seller divides customers into discrete groups based on observations about their willingness to pay, and each group is charged a unique price. Prices offered to one group are not made available to the other group. For example, the manufacturer of computer software might license it to commercial users at a higher rate, and to home users at a lower rate. This sort of discrimination is profitable only when consumer valuations are concentrated into two or more distinct price intervals. If the monopolist were to charge a single monopoly price, it would very likely set it somewhere between the high and low prices used to discriminate between groups. The discrimination scheme excludes consumers whose valuations lie below the price they have been offered even if that price is higher than the non-discriminatory monopoly price. On the other hand, the scheme ordinarily draws in some consumers who would have been unwilling to pay the monopoly price. As a result, third degree price discrimination has the very important effect of redirecting output from consumers with relatively high

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35 See Deserpa, note 30.

36 In the words of Arthur Cecil Pigou:

This degree, it will be noticed, differs fundamentally from either of the preceding degrees, in that it may involve the refusal to satisfy, in one market, demands represented by demand prices in excess of some of those which, in another market, are satisfied.” Pigou, note 32, II.17.6.


38 In some instances the monopolist might discriminate only between customers willing to pay the monopoly price and some group of high value customers willing to pay more.
valuations to those with relatively low ones. Hence, consumer welfare will be harmed even if output levels are maintained but do not increase.

To take a simple example, suppose a monopolist identifies and segregates two groups of customers, offering the first group a price of $8 and the second a price of $5. Buyers in the high price group will purchase until their marginal valuations of the good fall to $8 and then stop, because they cannot purchase at a price of, say, $7.90, even if they wish to. The $7.90 price is profitable to the seller, and the seller is actually selling to others at a profitable price of $5. As a result, the discrimination scheme takes a sale away from a high valuation customer, willing to pay $7.90, and shifts it to a low valuation customer. This has led economists since the time of Pigou\textsuperscript{39} and Joan Robinson\textsuperscript{40} to infer that third degree price discrimination reduces welfare whenever it fails to generate more output than simple monopoly pricing.\textsuperscript{41}

By contrast, in second degree price discrimination everyone is offered the same price schedule, with different unit prices corresponding to different quantities or product varieties.\textsuperscript{42} A quantity discount scheme is one example. Another is division of transportation tickets by classes. For example, airlines might offer first class and coach tickets, or advance purchase and immediate purchase fares. The same fare structure is available to everyone, but different customers make different choices based on willingness to pay, and profitability is higher for some classifications than for others. For example, the lawyer accustomed to flying first class but facing an economic recession might choose to shift all or part of her air travel to coach. When conditions improve she may switch back.

To be sure, second degree price discrimination may lead to its own inefficiencies, but they are much different from those produced by third degree price discrimination. One problem second degree price discrimination does not typically create is a discontinuity in the levels of marginal value at which consumers stop purchasing additional units.

\textsuperscript{39} In Pigou’s words: “This degree [third], it will be noticed, differs fundamentally from either of the preceding degrees, in that it may involve the refusal to satisfy, in one market, demands represented by demand prices in excess of some of those which, in another market, are satisfied.” Pigou, note 32, II.17.6, at 254-255.

\textsuperscript{40} See JOAN ROBINSON, THE ECONOMICS OF IMPERFECT COMPETITION 205-206 (1933) (making the same observation).


\textsuperscript{42} GORDON MILLS, RETAIL PRICING STRATEGIES AND MARKET POWER 26 (2002).
So what about variable proportion ties? Do they constitute second degree price discrimination, third degree, or perhaps some hybrid that resists classification? Professor Einer Elhauge believes that they constitute third degree price discrimination and as a result can be quite harmful to consumer welfare. His argument is contrary to the position taken in the Antitrust Law treatise, which he criticizes, that they are examples of second degree price discrimination. Economists generally take the position argued in Antitrust Law that such ties constitute second degree discrimination.

As noted above, third degree price discrimination involves a seller’s prior segregation of groups of customers based on willingness to pay. Tying does not; rather, the same price schedule is available to everyone, as is typical of second degree price discrimination. In second degree discrimination, as in tying, the dominant firm selects the products and places them on the market, with the same price schedule offered to all. Moreover, the profitability of a tying strategy is not affected by its ability to distinguish between consumers with different valuations. Tying can be a viable strategy even when consumers’ preferences are too idiosyncratic to be discerned with any information available ex-ante.

Consider the example of a durable electronic printer which consumes ink in cartridges as an example of a variable proportion tying arrangement. Professor Elhauge states:

The crucial difference between second and third degree price discrimination is that the former employs a pricing schedule that allows each consumer to choose whatever package provides him with the most consumer surplus. Conversely second-degree price discrimination involves charging all buyers the same price schedule, and varying prices with the units bought of the product over which the

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44 Id. at ___ [TAN 74], criticizing 9 PHILLIP E. AREEDA & HERBERT HOVENKAMP, ANTITRUST LAW ¶1711b(B) (2d ed. 2004).
45 E.g., JEAN TROLE, THE THEORY OF INDUSTRIAL ORGANIZATION 147 (1998). See also Richard A. Posner, Vertical Restraints and Antitrust Policy, 72 UNIV. CHI. L.REV. 229, 236 (2005) (ties are a form of second-degree price discrimination; conjecturing that they might have negative welfare effects but that the conjecture is an insufficient basis for condemning them).
46 On this point see MILLS, note 42 at 26 (difference between second and third degree price discrimination is that in second degree discrimination seller cannot distinguish customers into diverse groups, but rather they self select according to a pricing schedule that is the same for all).
seller has market power. That is not what it happening when tying is used to meter. Buyers are not paying less per printer if they buy more printers. Buyers are instead effectively paying more for the same printer if they fall into a category of buyers who use more cartridges with it.47

Printers and ink cartridges are near-perfect complements. Two goods are complements if the value of the pair exceeds the values each good retains in the other’s absence. Perfect complements are an extreme case of complementarity in which each good has no value unless used with the other. For the most part consumers use printers and ink cartridges together,48 although they differ in the amounts of printing they wish to do. So while professor Elhauge is correct when he points out that variable proportion ties do not discriminate among consumers according to the number of printers they buy, he is not on point. To be sure, all customers buy a single printing machine, but the volume of prints they consume differs according to the number of ink cartridges they purchase. In particular, the customer’s average cost of using the printer (i.e. the per-print price) decreases as the total amount of use increases. No two consumers who do the same amount of printing will be made to pay different per-print prices, even if one consumer derives more total value from his prints. Rather, the per-print price a consumer pays is determined only by the total volume of prints he produces. Specifically, the per-print price gets progressively smaller as the total volume of prints increases.49 And this is exactly the sort of situation that occurs under second degree price discrimination.

Pigou’s point in distinguishing between second and third degree price discrimination was to differentiate situations where customers’ valuations could be identified ex ante and offers made to them separately (third degree), from situations where the seller knew something about demand generally but could not identify the specific buyers at the time of the sale.50 As a result the seller used the price schedule to enable buyers to self select. For these reasons the economics literature identifies two part tariffs, which strongly resemble variable proportion ties, as forms of second degree price discrimination.51 In a two-part tariff a seller requires consumers to pay a “fixed”

47 Elhauge, note 43 at ___.
48 The value of a printer without a cartridge might be a little greater than zero. For example, a printer without an ink cartridge might be used as a doorstop, or perhaps on the set of a television show such as The Office.
49 See the table, infra, text at note 60.
50 See Pigou, note 32 at 249-255.
51 See, e.g., Jean Tirole, THE THEORY OF INDUSTRIAL ORGANIZATION§3.3.1 (1979); Jean-Jacques Laffont & Jean Tirole, A THEORY OF INCENTIVES IN PROCUREMENT AND REGULATION 175-177 (1993); Satya R.
fee before they can begin purchasing individual units of the good at a constant "marginal" price. Neither the fixed fee nor the marginal price differ among consumers, regardless of how many units they buy or what valuations they maintain. However, when the onetime fee is factored into the total price, the average price of each unit falls as more units are purchased. This causes two-part tariffs to resemble quantity discounting, which explains why they too are classified as second degree price discrimination mechanisms. For example, a water company might charge home users a rate of $10 per month plus $1 per hundred gallons of water consumed. Such tariffs are typically used in situations where fixed costs are too high to be covered under marginal cost pricing. Further, building a fixed cost component into the usage charge would result in higher volume users paying much more. So the tariff effectively segregates the fixed cost component by means of the fixed fee, while the variable costs are billed on the basis of usage.52

Professor Elhauge’s argument that variable proportion ties reduce welfare even if output is constant or increases depends on his premise that variable proportion ties price discriminate in the third degree. He argues that even if a price discrimination tie should increase output, welfare consequences are negative because the discrimination scheme switches output from high value purchasers (that is, high intensity buyers) to low value purchasers.53 This is clearly true of third degree price discrimination, and it is an important reason for its inefficiencies. To return to the previous example, suppose a discrimination scheme divides customers into high and low classes where arbitrage is impossible, charging prices to the two classes of $8 and $5, respectively. Buyers in the first group will purchase down to the point that the marginal value they place on the incremental purchase (i.e., their marginal valuation) is $8, but they will not purchase more. As a result, a potential sale to someone in this group at a price of $7.90 is left unmade, even as sales are being made to the lower price group at a price of $5. So to the extent that third degree price discrimination shifts output away from the higher value group and toward the lower value group, the discontinuity guarantees that the value of the marginal sale that is lost to the higher priced group is considerably greater than the value of the marginal sale that is made to the lower price group.54

53 See Elhauge, note 43 at ____ [TAN n. 75] (“reallocates some output from high value buyers to low value buyers”).
54 For example, if 100 units are lost to buyers in the high priced group who were willing to pay $7.90 but no more, and these same hundred units were picked up in the lower price group at a price of $5, to someone who valued them at $5.10, then output would be the same but welfare would be reduced. See also Schmalensee, note 8 at 242-243:
However, this is not the case with the variable proportion ties, and this is where Professor Elhauge’s argument seems to founder. To be sure, the variable proportion tie reduces fixed costs to the buyer and increases marginal costs, and any marginal cost increase is a distortion. But under the variable proportion tie the distortion is continuous across the demand curve and is the *same for everyone*. For example, suppose that the monopoly price for a digital photo printer is $400 and the competitive ink price is 2¢ per print. The monopolist uses a variable proportion tie, cutting the printer price to $300 but tying ink and increasing the price to 4¢ per print. To the customer, the printer is a fixed cost and the ink cost is variable, so the tie has the effect of reducing fixed costs but increasing variable costs. Significantly, however, the marginal cost of 4¢ per print is the same for all buyers at all places on the demand curve, from those that print the most to those that print the least. Each buyer will make prints until the marginal value of the next print drops to 4¢. As a result, in equilibrium the less intensive user and the more intensive user both have marginal valuations of 4¢ for their next prints, and there is no transfer at the margin from higher to lower value customers.

Of course, tying may shift purchases from high-intensity buyers to lower intensity ones, as it reduces average prices at low quantities and increases them at high quantities. But, unlike third degree price discrimination, the marginal price of the next unit is the same for everyone; no consumer is denied sales at a marginal price available to someone else. Hence there is no reason to believe that selling an additional unit to high intensity buyers would benefit consumer welfare more than selling an additional unit to lower intensity ones. Further, the fact that purchases are reallocated under tying does not prima facie imply that consumer welfare is harmed. In fact, because the price cut applied to the tying product is more significant to lower use customers, they often benefit from tying even if they purchase fewer units of the tied product. As a result, even when a tie reduces output of the tied product, it may increase consumer welfare.

**A Closer Look at Price Discrimination Ties**

For any fixed total output of the monopolized product, efficiency requires that all buyers have the same marginal valuation of additional units. (If all buyers are households, they must have the same marginal rate of substitution between the good involved and any numeraire good.) Selling the same product at different prices to different buyers induces different marginal valuations and produces what Robinson terms "a maldistribution of resources as between different uses.”

(quoting JOAN ROBINSON, THE ECONOMICS OF IMPERFECT COMPETITION 206 (1933).)

55 This illustration is developed further infra, text at notes 60-61.

56 See appendix for a proof.
Returning to the definitions of price discrimination and the variable proportion tie, exactly how should such ties be characterized? First, the term “price discrimination” must be defined. Both economists and others often use it to mean charging different prices to two different groups, or for two different classes of sales. More technically, it is commonly defined as sales at differing ratios of price to marginal cost, or as prices that have different percentage markups in relation to cost. While economists seem to prefer the latter definitions as a technical matter, the models generally define third degree price discrimination as the charging of different prices to different classes of consumers. In many of the models marginal cost is simply assumed to be zero. All of this is complicated by the fact that real world practices contain attributes of both definitions, often within the same scheme. For example, consider the airline that practices second degree price discrimination by selling first class and coach seats at different prices. At least part of the differential may be explained by differences in costs: first class passengers receive more costly treatment. But to the extent that the airline earns more on first class passengers notwithstanding these extra costs, differential returns are present as well. The same thing can be true of third degree price discrimination. For example, the seller who provides software to commercial and residential customers at different prices might be earning different returns, but it might also be supplying some services to the higher price commercial customers that the lower price residential customers do not receive. In sum, price discrimination in practice is a more complex phenomenon than Pigou’s original formulation indicated.

Variable proportion ties are also complex price discrimination arrangements. First, the components of a variable proportion tie involve price discrimination only when they are considered together. When the goods are viewed separately, the ratio of price to marginal cost is the same for all customers. However, because the components of a variable proportion tie are nearly always used together, and often have little value when they are separated, it is much more helpful to consider the prices paid for the entire tie. This allows for comparison of the different amounts consumers pay for each unit of the tying product’s use, which can be measured in units of the tied good. For

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57 See discussion supra, text at notes 60-61.
example, if the tie consists in a printer and ink cartridges, we should consider the different amounts consumers pay per print, which will vary depending on the total amount of printing a consumer does. The table below illustrates three different pricing scenarios for a monopoly seller of digital photo printers and their cartridges. Because customers place little or no value on printers or cartridges separately, the relevant price is the one that they pay for a print, which is the thing they value.

Whenever the tying product’s price is above zero the average cost of using that good falls as the total amount of use increases. Specifically, the average cost of using the tying product converges to the price of the tied product. Holding the price of the tied product constant, the variation in average cost of using the combination is smaller as the price of the tying product decreases. One can view the difference between the highest and lowest average costs within this range as a measure of how extensively a tie discriminates.

In scenario A in the table, which is non-tying, the monopolist charges all purchasers its standalone profit maximizing price for the photo printer, which is $400. Cartridges are sold under competition at a marginal cost price that comes out to 2¢ per printed photo. All buyers pay the same amount for the printer and the same amount for each cartridge. In scenario B the monopolist drops the price of the printer to $300 but ties cartridges at a price of 4¢ per photo. Once again, everyone pays the same price for printers and for ink. Under the third scenario the monopolist charges a price of zero for the printer but ties cartridges at a constant price of 8¢/photo. Or alternatively, it could keep the printers and simply print the photos itself from customers’ emailed files, at a price of 8¢ per photo. Mail order sites such as Snapfish.com or Shutterfly.com offer such services.60 Price/marginal cost ratios are the same for all.

<table>
<thead>
<tr>
<th>Total photo quantity</th>
<th>1K</th>
<th>2K</th>
<th>3K</th>
<th>4K</th>
<th>5K</th>
<th>6K</th>
<th>7K</th>
<th>8K</th>
<th>9K</th>
<th>10K</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*</td>
<td>42¢</td>
<td>22¢</td>
<td>15.3¢</td>
<td>12¢</td>
<td>10¢</td>
<td>8.7¢</td>
<td>7.7¢</td>
<td>7¢</td>
<td>6.4¢</td>
<td>6¢</td>
</tr>
<tr>
<td>B*</td>
<td>34¢</td>
<td>19¢</td>
<td>14¢</td>
<td>11.5¢</td>
<td>10¢</td>
<td>9¢</td>
<td>8.3¢</td>
<td>7.7¢</td>
<td>7.3¢</td>
<td>7¢</td>
</tr>
<tr>
<td>C</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
<td>8¢</td>
</tr>
</tbody>
</table>

60 See www.Snapfish.com (offering prints as low as 9¢); www.shutterfly.com (offering prints as low as 10¢).
Scenario A: $400 for the printer, which is its standalone profit-maximizing price; cartridges not tied and are sold at a competitive price of 2¢ per photo

Scenario B: $300 for the printer, which is marginal cost; cartridges are tied and sold for 4¢ per photo

Scenario C: $0 for the printer; cartridges are tied and sold for 8¢ per photo

*costs per print are calculated by taking the price of the printer and dividing it by the output, and then adding the cartridge costs per photo.

For each scenario, the table shows the different prices per printed photo that consumers pay at different quantities of photos, which range from 1000 to 10,000 photos. We assume that the printer is an upfront cost, needs no maintenance, and is worn out and must be discarded after it prints 10,000 photos or within a finite time period such as two years. In scenario A the customer who makes 1000 prints ends up paying 2¢ in variable costs per print, plus 400/1000, or 40¢ for the amortized costs of using the printer. If that customer increases its usage to 5000 prints, then variable costs are still 2¢ but now the amortized printer costs are 400/5000, or 8¢.

The principal effect of tying in the printer/cartridge story (or numerous similar stories in the litigated cases) is to change the consumers’ cost structure by making a larger portion of their costs variable rather than fixed. Charging the standalone monopoly price for the printer plus the competitive price for ink causes fixed costs to play a larger role in a consumer’s cost structure, because printer costs do not vary with use. At the other extreme, charging zero for the printer and a high price for the cartridges makes all consumer costs variable.

The result, which needs to be appreciated, is that the range of discriminatory prices becomes smaller as the price cut in the tying product increases. As a result tying actually serves to reduce the disparity in what consumers pay to use the tying product. The non-tying case (scenario A) produces printing costs that range from a high of 42¢ per photo to a low of 6¢ per photo over the output ranges in question. Scenario B, the “moderate” tying case, which entails a marginal cost price for the printer plus 4¢ per photo for the cartridge, yields a range of 34¢ down to 7¢ per photo. And the “aggressive” tying case, which involves a price of zero for the printer and 8¢ per photo for the cartridges, produces constant costs of 8¢ per photo at all output levels. This makes it cheaper for low intensity consumers to use the tying product, because total costs are lower at low quantities of total use.
To generalize, the more of the price that the monopolist transfers from the tying product (printer) to the tied product (photos), the less discrimination will result in the price of the tied product to the monopolist’s customers. The limiting case occurs when the tying product price is reduced to zero. In that case, the price of a photo is the same at all output levels.

Intellectual property licenses and franchise ties tend to have these same characteristics. For example, a patentee might license a patent at a fixed rate of, say, $1000 per year and the licensee could produce as little or as much as it pleased during that time period. Or it could engage in two part pricing – say, $500 up front plus a 2% royalty on sales (similar to “moderate” tying in the above illustration). Or, as is most typical, it could charge zero up front but a higher royalty on sales (similar to Scenario C in the table). The straight royalty increases the patentee’s revenue from high volume users, but it also serves to bring into the market low volume users who are unable to pay a high fixed price up front. If the licensor’s marginal costs are zero, even a licensee who produces one unit is profitable to the licensor.

If the monopolist simply printed the photos itself and mailed them to customers, then the cost structures faced by consumers become nondiscriminatory for the same reason that they are nondiscriminatory under tying when the price of the tying product is zero. In this case, the per-print price paid by consumers is the same regardless of how many prints they buy. In both scenarios A and B in the table, welfare could theoretically be improved by a form of arbitrage. Low volume purchasers could ask higher volume purchasers to print for them. As more printing was aggregated on fewer printers, per unit costs would decline, perhaps until every printer was fully utilized at a price of 6¢ per photo in scenario A. Of course, transaction costs might defeat such a scheme.

Assessing Output in Litigated Tying Cases

Variable proportion ties typically reduce the price of the tying product from its standalone profit maximizing price. Indeed, in many variable proportion ties of complementary products, such as printers and cartridges, the tying product is priced at or below marginal cost, leaving the monopoly overcharge and even part of the

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61 For example, in one of the earliest variable proportion tying cases Henry v. A.B. Dick Co., 224 U.S. 1 (1912), the patentee sold its mimeograph machine at less than its costs but tied ink, stencils and other supplies and assessed a high markup on those. See A.B. Dick Co. v. Henry, 149 F.424, 425(C.C.N.Y. 1907) (“The evidence establishes that the complainants sell the machines at a loss, less than the actual cost of making, relying on sales of supplies therefor for a profit. The complainants have sold about 11,000 of these machines under this license restriction.”). See also Motion Picture Patents Co. v. Universal Film Mfg. Co., 243 U.S. 502, 516 (1917) (noting patentee’s argument that the public benefitted “by the sale of the machine at what is practically its cost”); Static Control Components, Inc. v. Lexmark Int’l, Inc., 487 F. Supp. 2d 830 (E.D. Ky. 2007) (printer manufacturer received lower price for cartridges
competitive return to be earned on the tied product. In some case the tying product is even sold at a price of zero. The result is typically to increase the number of consumers using the tying combination, but to decrease the number of units that previously existing customers purchased. For example, when a firm ties printers and ink cartridges, buyers do less printing on average, but the number of buyers increases. This is also typically the case under franchise tying, where the entry price of the franchise is typically relatively low or occasionally zero, but the tied products (very common staple products or services) are sold at an overcharge. The result of such arrangements is that many more potential franchisees can afford a franchise. The franchisor’s profits are changed from a fixed up front entry fee to an overcharge that varies with output. As a result, the higher the output of the franchise the more profitable it is.

**Variable Proportion Ties: A Preliminary Welfare Analysis**


62 See, e.g., Kentmaster Mfg. Co. v. Jarvis Products Corp., 146 F.3d 691 (9th Cir. 1998), amended, 164 F.3d 1243 (9th Cir. 1999) (defendant provided durable meat cutting equipment at no charge to meat cutters but charged high prices for aftermarket parts). Cf. a common distribution mechanism of soft drink dispensing machines, which provides the machines to owners of locations where vending occurs at a price of zero, but the machine may stock only that supplier’s brand of soft drinks. See [http://www.vendingsolutions.com/coke-vending-machines/](http://www.vendingsolutions.com/coke-vending-machines/) (Coca-Cola; free dispensing machine to plant locations containing 40 employees or more, but only Coca-Cola products can be dispensed in the machine).

E.g., Siegel v. Chicken Delight, Inc., 448 F.2d 43 (9th Cir. 1971) (franchisor charged no franchising fee or royalty, but required franchisees to purchase tied products at higher-than-market prices).

64 See ROGER D. BLAIR AND FRANCINE LAFONTAINE, _THE ECONOMICS OF FRANCHISING_, ch. 3 (2005) (most up front franchise fees very low in relation to value of business); Steven C. Michael, _The Extent, Motivation, and Effect of Tying in Franchise Contracts_, 21 MANAGERIAL AND DECISION ECON. 191 (2000) (tying in restaurant franchises less a function of market power than of nature of equipment employed); Benjamin Klein & Lester Saft, _The Law and Economics of Franchise Tying Contracts_, 28 J.L. & ECON. 345 (1985); Roger D. Blair and David L. Kaserman, _Vertical Integration, Tying and Antitrust_, 68 AM. ECON. REV. 397 (Jun., 1978) (on equivalence of variable proportion tying and vertical integration; results in a more optimal use of downstream inputs and probable output increases); see also F.R. Warren-Boulton, _Vertical Control with Variable Proportions_, 82 J.POL.ECON. 783 (1974).
One misconception about variable proportion ties is that they harm all consumers who purchase fewer units of the tied product, and that the only consumers who benefit are those who would not purchase either good under untied monopoly pricing. It is true that increasing the tied product’s price reduces the surplus achieved on each tied unit, but consumer surplus is also increased by the reduction in the tying product’s price. A consumer is worse off under tying only if the tie subtracts more surplus than it provides. We can better understand this by distinguishing between a buyer’s surplus realization and the amount of surplus received but for the price of the tying product. In effect, but-for surplus is the total amount of consumer surplus a buyer would receive if the price of the tying product were zero. The relevant measure of a consumer’s welfare is not merely but-for surplus, but rather the surplus realization that remains when the tying product’s price is subtracted from it.

This approach allows us to contrast the two ways in which tying affects the welfare of consumers. First, the price increase applied to the tied product reduces the but-for surplus levels achieved by consumers who are willing to buy the goods even under monopoly pricing. This is because increasing the tied price is tantamount to increasing the marginal cost of using the tying product, which compels most buyers to use the tying product less (i.e. to buy fewer tied units). The extent of a tie’s impact on but-for surplus is greater among consumers who desire to use the tying product more. On the other hand, the price of the tying product falls, so that a smaller amount of a but-for surplus is subtracted upon buying the tie. Many consumers will be better off under tying, even though it causes them to purchase fewer tied units.65

Tying impacts consumers in three different ways depending on their status under non-tying monopoly. First, there are low intensity consumer types who buy the two goods under tying, but not under monopoly pricing. For these consumers the tie is an unambiguous welfare improvement. Second, are medium intensity consumer types who achieve more consumer surplus under tying even though it leads them to buy fewer units of the tied product. This occurs because tying reduces their but-for surplus levels by less than it cuts the price of the tying product; that is, they gain more from the reduction in the tying product price than they lose from the increase in the tied product price. Finally, there are high intensity consumer types who buy fewer tied units and achieve less surplus under tying. For these buyers, the price cut applied to the tying good is too small to cover the tie’s reduction of but-for surplus. From the seller’s side, the seller generally earns greater surplus from the low intensity group, because these are sales that are not made at all prior to tying; and also from the high intensity group, because it earns more on the higher volume of tied product that they purchase.

65 For a comprehensive proof and graphical analysis, see the appendix.
However, the seller loses money on the medium intensity group because the losses on the tying product (printer) price cut is greater than the gains on the higher tied product (ink cartridges) price. As a result, the tie is profitable if the gains from the first and third group exceed the losses from the intermediate group.

In this situation the relationship between consumer welfare and output of the tied product is uncertain. Unlike situations involving only one good, a reduction in output does not imply a reduction in consumer welfare. Welfare can increase even though output of the tied product falls, provided that the number of medium intensity consumers is sufficiently large. If, on the other hand, output levels are maintained or increased, the tie should be assumed to enhance consumer welfare unless there is reason to believe that the injuries to high intensity buyers outweigh the improvements obtained by low and medium intensity buyers. Finally, all this is aside from any realization of production efficiencies attending higher output, which should further support the conclusion that a tie is welfare increasing.

The one case where a welfare improvement will not occur is when the tie fails to serve any low intensity customers. For example, if a printer-ink tie increased the price of a single ink cartridge by the same amount that it cut the printer’s price, then a buyer who buys only one ink cartridge is no better off under tying. Further, every consumer who buys more than one ink cartridge is worse off, because the average cost of printing is higher at all print quantities requiring two or more ink cartridges. In this case, tying fails to benefit any consumers, and it leaves existing customers either indifferent or worse off. Such situations are probably rare, and they can be distinguished using a simple “consumer benefit” test, which asks whether a tie succeeds in serving any low intensity consumers, who are defined to be those who will not buy under monopoly pricing. This test is passed whenever output of the tying product increases upon tying. The test is also passed whenever output of the tied product does not fall as a consequence of tying, as low and medium intensity buyers will all reduce the number of tied units they purchase, meaning that low intensity buyers must account for the difference. However, an increase in output of the tied product is merely sufficient for passing the consumer benefit test, it is not necessary. When a tie passes the consumer benefit test, one can be sure that it serves both low and medium intensity consumers, though this does not determine the proportion of buyers who maintain low or medium

66 Id.
67 On the manifold sources of cost savings and product improvement that results from ties, see 9 PHILLIP E. AREEDA & HERBERT HOVENKAMP, ANTITRUST LAW ¶¶1712-1718 (2d ed. 2004).
68 This is also likely to be true if the manufacturer increases the price of the tying good upon tying; however, we have not been able to find any such cases.
intensity levels. Rather, passage of this test demonstrates that two consumer types benefit from tying, one of which contributes to the output of the tied product, and one of which detracts from it.

A tie that passes the consumer benefit test increases the surplus levels achieved by both low and medium intensity consumers, and decreases the surplus levels of high intensity buyers. And, because medium intensity buyers reduce the quantity of tied units they purchase, the tie’s net impact on welfare is not necessarily negative when it causes output of the tied product to fall. For this reason, if a tie passes the consumer benefit test, it is generally not possible to show that welfare falls without relying on empirical evidence showing that a sufficiently high portion of buyers are high intensity consumers. This is necessary to ensure that the injuries suffered by high intensity consumers outweigh the welfare improvements enjoyed by low and medium intensity buyers.

Graphical analysis of variable proportion ties is somewhat similar to that used to illustrate the effects of two-part tariffs. When two-part tariffs are graphed, there is typically a lump sum payment that is reflected by some area under the demand curve. This payment reduces the consumer surplus received for purchasing the good. In the case of variable proportion ties, the tying product’s price is analogous to the lump sum payment used by a two-part tariff.

Consider the diagram in Figure One, which illustrates how consumers choose whether or not to buy a printer-ink tie, and how different consumers arrive at different decisions:

Figure One

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69 Id.
70 See, e.g., Viscusi, Harrington and Vernon, note 52 at 412-416 (graphical and mathematical illustrations of two-part tariffs).
Figure One indicates the marginal price of a print as given by the price of ink, plus an assumption about printer costs. The Figure considers two consumer types, high intensity and low intensity. The marginal value curves are demand curves that reflect consumers’ optimal print quantities. The price of the printer, which is a fixed cost to the consumer, is given by the area of the shaded region, or $A_1 + A_2$. This area must be subtracted from a consumer’s but-for surplus, which is given by the area under the marginal value curve and above price. (Recall that this is the surplus a consumer achieves but for the price of the printer.) A consumer’s surplus realization would equal his but-for surplus only if the printer was free, in which case the high intensity buyer would earn consumer surplus of $A_1 + A_2 + A_3$. However, when the printer's price is subtracted, the high intensity buyer achieves a surplus realization of only $A_3$. This amount is positive, so he will elect to buy the tie. Conversely, even at his optimal quantity of prints, the low intensity buyer does not achieve a level of but-for surplus that exceeds the price of the printer. The low intensity buyer will not buy either the printer or the ink.

Assessing the impact of a tie requires comparison with the untied situation – namely, where the printer is sold at its standalone monopoly price and ink is priced competitively. We draw this comparison from the perspective of a single consumer type, so that the marginal value curves are the same in both situations. Further, we assume that the consumer benefit test is passed, so that we can see how a consumer might achieve more surplus in spite of purchasing fewer prints. Some consumers will benefit, while others are injured. Figure Two illustrates:

Figure Two

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<sup>71</sup> This is the price of an ink cartridge divided by the number of prints that can be produced from one. We ignore the price of paper, electricity and other collateral inputs.
The printer prices in the monopoly and tying situation are given by the areas MP ("monopoly price") and TP ("tying price"), respectively. In the tying scenario the price of the printer (TP) is lower than the monopoly price of MP, but the price of the ink is higher. Consumer surplus in the two situations is given by the respective areas MS and TS, where MS includes the white region above the dotted line. Because the price of the ink is the only variable cost, and thus equivalent to marginal cost, the consumer purchases less ink under the tying arrangement. Nevertheless, from this consumer's perspective, the tie is preferable if the area of the region TS exceeds that of MS. Thus, as the figure illustrates, a consumer can be better off with the tie even if it leads him to consume fewer prints. These "medium intensity" consumers benefit from the tie because the amount saved on the printer exceeds the tie’s reduction of surplus resulting from higher ink prices.

Comparing the two situations becomes easier when they can be shown in a single diagram. This allows one to calculate exactly the welfare transfers that take place between the two situations, as Figure Three illustrates:
This diagram superimposes the tying situation onto the monopoly situation.
Before tying is introduced, this buyer earns a consumer surplus of $S_1 + S_2$. If the printer price under tying decreased by only $A_3$ dollars, the consumer would lose $S_2$ in surplus and gain nothing in return. However, if the printer price under tying falls by $A_2 + A_3$, the consumer receives a surplus of $S_1 + A_2$ under the tie. Thus the consumer prefers the tie if $A_2$ is larger than $S_2$, even though he buys fewer prints.

Area $A_2$ is what incentivizes new consumers to buy printers and ink when the tie is introduced. Upon tying, the depicted consumer’s but-for surplus on ink purchases falls by $A_3 + S_2$, while the price of the printer falls by $A_2 + A_3$. Because the difference between but-for surplus and the printer’s price determines the consumer’s surplus realization, he is better off if $A_2 > S_2$. Low intensity consumers are those whose marginal value curves run through the area $A_2$, as these are defined to be the consumers who buy printers and ink only in the tying situation.

Finally, figure Four depicts a low intensity consumer. In this diagram the monopoly printer price is still equal to the sum of all shaded regions, while the tied printer price is still only $A_1$. But this graph illustrates a consumer who achieves a positive amount of consumer surplus only in the tying situation. In the monopoly situation, the consumer achieves a negative surplus of $B_2 + C_2$, and hence does not purchase. Under tying, however, the consumer achieves a positive surplus realization of $B_1$ and therefore enters the market. It should be noted that all consumers whose marginal value curves run through the regions $B_1$ or $B_2$ will buy printers and ink only in the tying situation. Area $B_1$ is a pure welfare gain.

**Figure Four**

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72 Suppose the tied printer price were equal to $A_1 + A_2$. The consumer depicted in the diagram is necessarily worse off in the tying situation, because he loses $S_2$ in surplus and gains none back, even though the price of the printer has fallen by $A_3$. For example, suppose that the printer manufacturer cut the price of a printer by $4$ but increased the price of each cartridge by $4$. This tie would reduce output to everyone except the person who never used more than the first cartridge, who would be indifferent.
We can demonstrate how a tie affects different consumer types differently by plotting the surplus realizations under both tying and monopoly pricing, and by contrasting them over a continuum of consumer intensity levels. This makes it easy to distinguish low, medium, and high intensity consumers, and to see how each type is affected by tying.

Figure Five
In Figure Five, the gray curve represents surplus under monopoly pricing, while the black curve represents surplus under tying. The horizontal axis plots consumer intensity levels, increasing as one moves from the origin to the right. The black curve lies above the gray curve over the ranges of low and medium intensity buyers, which reflects the fact that both of these types benefit from tying. Conversely, the gray curve is highest over the range of high intensity buyers, as these consumers are injured by the tie. Every tie that passes the consumer benefit test will produce a graph similar to this, because the test is used to demonstrate that low and medium intensity consumers are served by the tie. If the tie failed to pass the consumer benefit test, the black curve would not intersect the horizontal axis at a lower intensity level than the grey curve, and the gray curve would be above the black curve at all intensity levels where it is above zero. This is because when a tie fails the consumer benefit test, no consumers are better off, and most are injured. By contrast, when a tie passes the consumer benefit test it necessarily increases the output of the tying product.

Figure Five does not account for how many buyers or how many purchases are at each intensity level or, more importantly, how aggregate welfare is affected by a tie. In general, showing whether a tie that passes the consumer benefit test is a net benefit or a net harm to consumers is extremely difficult. If buyers are sufficiently concentrated at low and medium intensity levels, then their consumer surplus gains will outweigh the consumer surplus losses derived from high intensity buyers. Moreover, if buyers are sufficiently concentrated over the range of medium intensity levels, then it is possible that consumer welfare increases and yet output of the tied product falls. If output of the tied product increases as a result of the tie, one can be sure that there are significant numbers of buyers in the low intensity range. Their entirely new purchases of the tied product outweigh reduction in consumption by high intensity buyers. This indicates that numerous consumers benefit from tying.

Unfortunately, an antitrust tribunal will almost never have information about how consumers are distributed over the various intensity levels. As a result welfare effects are probably unclear unless the gains are clearly positive among all three groupings. This could happen in a situation in which all existing users benefit from the tie because the price cut in the tying product is greater for each of them than the price increase in the tied product. This would be most likely to occur when the price cut on the tying product is significant, the price increase on the tied product is fairly modest, and the tie brought in a large number of new tying product customers.73

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73In such a case the seller would be losing money from all existing customers but earning additional profits from all customers brought into the market by the tie. That situation would create a gain to everyone – i.e., all customers who purchased the tying product previously, all new customers who come into the market in response to the tying product price cut, and the manufacturer. That is, it would increase both general welfare and consumer welfare.
Incidentally, the assumption that medium intensity consumers are costly to the manufacturer assumes that the tie itself does not yield any production efficiencies from economies of scale or economies of joint provision of the tying and tied product. To the extent that a reduced printer price reflects reduced printer production costs when printers are tied medium intensity consumers could be better off and those sales could be profitable to the manufacturer. For example, if tying increased the volume of printers substantially, reducing costs by, say, $40 per printer, then a $100 price cut to the buyer would represent only a $60 revenue loss to the seller. Then, if medium intensity buyers contribute more than $60 in profits upon buying ink cartridges, these customers will have become more profitable under the tie. That is, medium intensity consumers would be profitable to the tying firm even though they buy fewer tied units and achieve more consumer surplus.

If the tying product is sold cost under tying, then a small fraction of high intensity consumers may also be less profitable under tying, while the rest increase the manufacturer’s profits. All of the low intensity consumers are more profitable under tying, because they do not purchase the tying product at all under monopoly pricing. Importantly, depending on how customers are distributed among different intensity levels, it is possible that a tie increases profits, improves consumer welfare, and yet fails to increase output of the tied product. This is because a consumer’s profitability does not stipulate the way in which tying affects his surplus or the number of tied units he buys. That is, there is significant ambiguity in the relationships between profits, welfare, and output of the tied product. For example, some profitable consumers buy more tied units and achieve more surplus under tying (low intensity buyers), while others buy fewer tied units and achieve less surplus (high intensity buyers). On the other hand, all unprofitable consumers buy fewer tied units under tying, but some achieve more surplus (medium intensity buyers), while others achieve less (high intensity buyers). In sum, a tie that reduces output could increase consumer welfare, while one that increases output could reduce consumer welfare.

Some price discrimination strategies permit a ready assessment of welfare effects, because output levels typically serve as an indicator of consumer welfare. For example, third degree price discrimination that fails to increase output reduces

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For example, suppose the monopoly price of the printer is $200 and the tying price is $100. When the manufacturer ties it also increases the cartridge price from $8 to $10. Over the life of the tying product the “break even” point for all prior users would be 50 units of the tied product; for new customers everyone is profitable to the seller. If they are sufficiently numerous the gains they produce for the manufacturer could easily exceed the losses from the pre-existing users.

See the discussion in the Appendix.
welfare. Variable proportion tying presents an unusual challenge, because some consumers will typically benefit from it even though they purchase fewer tied units. Observing changes in the output of the tying product is equally unhelpful. Indeed, output of the tying product will increase under any tie that benefits at least one consumer (i.e. any tie that passes the consumer benefit test).

Nevertheless, output effects in the tied market may serve to make certain welfare effects more likely than others. The fact that a tie increases tied product output indicates that low intensity customers brought into the market by the tie purchase more tied units than are given up by medium and high intensity customers. This implies that a large portion of consumers are necessarily better off under the tie. As such, the tie should be presumed welfare-increasing unless there is reason to believe that high intensity buyers lose more surplus than is gained by low and medium intensity ones.

On the other hand, when tied product output is unchanged or decreased, there is no necessary reason to believe the tie reduces consumer welfare. A tied product output reduction suggests only that low intensity buyers are relatively few, but this does not indicate that they are more concentrated at high intensity levels than medium intensity levels, or vice versa. As such, there is still no reason to believe that high intensity customers lose more surplus than is gained by the sum of low and medium intensity ones. So the consumer welfare effects of ties that reduce or maintain output levels should generally be considered ambiguous. In fact, the only ties that can be categorically shown to harm consumer welfare are those that fail the consumer benefit test, which means that they do not benefit any consumers at all.

To summarize, under a variable proportion tie of printers and ink cartridges that passes the consumer benefit test:

- Low intensity consumers will begin buying the tie, though they would not buy either good under monopoly pricing; for new customers brought into the market by the tie, the result is an unambiguous increase in consumer surplus;

- Both medium and high intensity users buy fewer ink cartridges than under monopoly pricing, but medium intensity buyers achieve more surplus under tying, while high intensity buyers are injured by the tie; the medium intensity buyers experience greater gains from the tying product price reduction than losses from the tied product price increase.

75 See discussion and citations supra note 8.
• Whether the tie increases or decreases consumer welfare depends on the distribution of buyers among the three intensity levels. This ambiguity still exists if output of the ink cartridges falls, because medium intensity buyers buy fewer ink cartridges and are nevertheless benefitted by the tie. Rather, a decrease in ink sales implies only that buyers are relatively unconcentrated at low intensity levels. Welfare could still increase if they are relatively concentrated among medium intensity levels.

• And, of course, since the firm will not tie unless it earns more by doing so, producer surplus is increased as well. The seller gains by tying from both the low intensity buyers, who do not buy at all under the tie; and the high intensity buyers, who now pay more for ink. The seller loses from the medium intensity buyers because for them the cut in the printer price exceeds the increased profits on the ink cartridge price. As a result tying is profitable if the gains from the high and low intensity group exceed the losses from the medium intensity group.

Other Effects

The preceding analysis deals only with the price and demand effects that can result from variable proportion ties. It assumes that a tie produces no cost savings or product improvements whatsoever. As a result, the analysis very significantly underestimates welfare improvements that come from other sources, namely:

• To the extent a tie lowers the purchaser’s initial fixed cost investment and converts the investment costs to variable, it serves to reduce investment risk;

• Economies of scale in tying product production can yield significantly reduced costs, at least some of which will be passed on to consumers;

• Economies of scale in tied product production can have similar effects;

• By bringing smaller firms into the market variable proportion ties can serve to deconcentrate the downstream market, benefitting both the monopolist and consumers; this is particularly true of franchise ties and, relatedly, of IP licensing.

• Many ties reflect production or provision cost savings, improvements in product quality, or overall product satisfaction.

Changing Fixed/Variable Cost Ratio; Risk Reduction
As noted earlier, ties tend to result in lower fixed costs to buyers, but higher variable costs. This can be particularly important when high, unrecoverable upfront costs might serve to deter investment. Franchising is a good example. The market value of a popular fast food franchise might easily be $1,000,000. However, the franchisor might sell the franchise for, say, $200,000 and tie an overcharge on various goods distributed through the franchise. The result is to reduce the costs of a business failure to the franchisee and make entry more attractive to small, typically undiversified entrepreneurs. Alternatively, if the franchise is young the franchisor may have a more optimistic opinion of the franchisor’s value than the franchisee. The tie effectively reduces the franchisee’s upfront investment.\footnote{The literature on these various points is large. See, e.g., Banjami Klein, The Economics of Franchise Contract, 2 J.Corp.Fin. 9 (1995); G. Frank Mathewson & Ralph A. Winter, The Economics of Franchise Contracts, 28 J.L. & Econ. 503 (1985); Paul H. Rubin, The Theory of the Firm and the Structure of the Franchise Contract, 21 J.L. & Econ. 223 (1978). See also Oliver E. Williamson, The Economic Institutions of Capitalism 21 (1985).}

Fixed Costs and Economies of Scale

Fixed costs in the tying product can result in dramatic welfare improvements from tying.\footnote{Accord Richard A. Posner, Vertical Restraints and Antitrust Policy, 72 Univ. Chi.L.Rev. 229, 236 (2005), who observes that price discrimination ties are more likely to be welfare enhancing as the proportion of fixed costs rises, and that this is particularly true of intellectual property rights).} If a product is subject to significant fixed costs an output increase leads to lower product costs because fixed costs can be allocated over a larger number of units. For example, suppose development of a printer and the fixed cost components in production cost $1,000,000. Variable costs are $100 per unit. In that case the break-even point will be $1100 per printer if 1000 units are sold, or $200 per printer if 10,000 units are sold. It is hardly a coincidence that many variable proportion ties involving durable tying products occur in markets that are subject to significant research and development costs as well as relatively short production runs (quick obsolescence). This could include computers, printers, and medical devices. Ties in these cases can dramatically increase output of the tying product, leading to equally dramatic reductions in primary product costs. For example, with linear demand output at the competitive price is double output at the monopoly price.\footnote{See Alan A. Fisher, Robert H. Lande and Walter Vandaele, Afterword: Could a Merger Lead to Both a Monopoly and a Lower Price?, 71 Cal.L.Rev. 1697, 1699 (1983).} Using the above assumption of $100 in variable costs, if the tying monopolist dropped the tying product price to the competitive level – a fairly common scenario – the production cost of this printer could drop from, say, $300 if 5000 units were sold at the monopoly price, to $200
at the competitive tying price, which produces sales of 10,000 units. This result is particularly strong in markets with a large upfront R & D component, because such costs can generally be allocated over the full production run. To the extent that the tie increases tied product output the same thing can be also true in that market. If the tied product is a pure commodity cost increases at higher individual output levels may not be all that dramatic; but if the tied product is something that is highly engineered, such as a printer cartridge, then these cost savings could be substantial as well.

The impact of fixed costs is also likely to be large for franchise ties.\(^{79}\) In franchising the franchisor’s fixed costs typically include intellectual property rights of various sorts and a business method. By tying and building the overcharge into the tied products the franchisor typically reduces the size of the upfront franchise fee very considerably, sometimes even reducing it to zero. The result is that many more franchise locations are opened. The fixed costs can then be amortized across all these franchises, and their increased output yields a lower profit-maximizing product price.

*More Competitive Downstream Markets*

Variable proportion ties can also address the problem of downstream markets that are prone to oligopoly or collusion. As noted previously, bundling tends to change customer costs from fixed to variable and also to make them more constant.\(^{80}\) By contrast, unbundled pricing tends to favor large volume users because it is characterized by large upfront fixed costs and declining marginal costs. For example, looking at scenario A in the table given above,\(^ {81}\) if only three of the purchasing firms attain the 8000 mark, they will have per unit costs of 7¢ each or less, while smaller firms could have total costs as high as 42¢. The three firms could then collude, thus injuring both the supplier and consumers. In contrast, bundling tends to equalize downstream costs, making markets more competitive even if they have diverse firm sizes.

For example, a gasoline refiner may contemplate franchising gasoline stations into a particular community. The cost of building a station, coupled with a reasonable return on intellectual property rights, might well amount to a million dollars, and the

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\(^{79}\) E.g., Siegel v. Chicken Delight, Inc., 448 F.2d 43 (9th Cir. 1971); Kypta v. McDonald’s Corp., 671 F.2d 1282 (11th Cir. 1982); Little Caesar Enter., Inc. v. Smith, 34 F.Supp.2d 459 (E.D.Mi. 1998).

\(^{80}\) See discussion *supra*, text at notes 55.

\(^{81}\) See discussion *supra*, text at note 60.
franchisor might have difficulty finding very many potential franchisees willing to make that investment. As an alternative, however, it might finance a large portion of the fixed cost investment itself, charging the franchisees a sum sufficient to guarantee their commitment. It would then make up the rest by tying gasoline and charging a few cents more per gallon than the wholesale price. The result could be many more franchisees in the community – say, eight instead of two or three – with the result that they behave more competitively and sell more gasoline overall.

**Economies Resulting from Joint Provision**

Finally, while we do not develop the rationales here, it is worth remembering that an important and perhaps the most important reasons for tying are reductions in cost and improvements in quality.82

**Conclusion**

Variable proportion tying arrangements injure some buyers while they benefit others. Most of those that increase output very likely also increase welfare. Further, because they are a type of second degree rather than third degree discrimination, reduced welfare cannot be inferred from reduced output. As a result a court is probably wasting its time by considering the output effects of a tie unless they are dramatic and obvious.

Also important is the fact that variable proportion ties are found in both highly competitive and monopolized markets. That in itself suggest a great deal about consumer welfare effects. For example, Lexmark, a manufacturer of computer printers, has been embroiled in litigation concerning technological and contractual ties of its printers and cartridges.83 But Lexmark is hardly a monopolist. It does not dominate any segment of the market, and is far smaller than Hewlitt-Packard, Canon and Epson. Consistently it is fourth or fifth place in market share both in the United States and worldwide.84 Assuming printer makers are not colluding, this means that a high

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82 See 9 PHILLIP E. AREEDA & HERBERT HOVENKAMP, ANTITRUST LAW ¶¶1716-1717 (2d ed. 2004).
84 On worldwide computer printer market shares, see http://www.outlookseries.com/N1/Infrastructure/3049_Color_Laser_Multifunction_Peripherals_MFP_Shipments_Up_6%25.htm (multifunction printers, 2009); and http://seekingalpha.com/article/37170-are-printer-companies-chasing-the-wrong-target (all printers, 2007). And see http://on-
volume user who believes it is paying too much has plenty of competitive alternatives. In such a situation, serious welfare losses cannot be expected. So Lexmark’s strategy must be profitable for some other reason than monopoly, most likely because it increases output. A monopolist’s strategy need not be any different.

As a result we recommend that courts in antitrust cases forget about price discrimination or leveraging as anticompetitive concerns and focus on foreclosure, or anticompetitive exclusion. The one empirically identifiable exception we have been able to find occurs when the tie benefits no one other than the seller, but such cases will almost never occur unless the tie is accompanied by a higher price for the tying product, an event that appears to be uncommon and should never be presumed. Finally, we note that even the noneconomic rationale for condemning ties on the ground that price discrimination itself is an unappealing practice disappears once we consider that the true impact of variable proportion ties is to reduce rather than increase the extent of price disparities to buyers.

Finally, this analysis suggests that the current tests for evaluating ties under the antitrust laws is wrong on two different counts. That test requires tying of separate products, substantial market power in the tying product, and a “not insubstantial” volume of tied product commerce. First, by failing to assess market foreclosure in the tied product market the prevailing test contains no device by which anticompetitive exclusion can be assessed. Second, by misunderstanding the nature of price discrimination ties it incorporates exaggerated concerns about leverage.

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85 Contrast Jefferson Parish Hospital Dist. V. Hyde, 466 U.S. 2, 14-15 (1984) (suggesting that ties “can increase the social costs of market power by facilitating price discrimination....”).

86 On this point, see Christina Bohannan, *IP Misuse*, supra note 2.

87 See discussion supra, text at notes 68-69.

88 See discussion supra, text at note 60 and accompanying table.

Appendix

The model developed here for assessing the impact of variable proportion tying on consumer welfare assumes a tying arrangement containing one fixed tying product and one tied product that can be used with the tying product in variable proportions. Consumers buy one unit of the fixed good, if at all, and they purchase the variable good only to facilitate use of the tying good. Hence consumers will never buy a unit of the fixed good without also buying some quantity of the variable good, and they will never buy units of the variable good without buying one unit of the fixed good. The model speaks of the monopoly situation, in which the manufacturer monopolist sells the tying good at the standalone monopoly price and the tied good is priced competitively; and also of the tying situation, in which the price of the fixed good is decreased from the monopoly situation and a markup is applied to the variable good. That is, in the tying situation, profits are always earned on sales of the variable good, while they may or may not be earned on sales of the fixed good.

Consumer preferences are defined over the use of the fixed good. As in real world instances of tying, the variable good serves as little more than an input that must be regularly purchased to facilitate consumption of the fixed good. To that end, we will let $\mathbf{\tilde{c}}$ denote the quantity of the fixed good’s use, which is described in units of the variable good. This is appropriate because the fixed good is generally useless without the variable good, and each unit of the variable good allows the same amount of consumption to be derived from the fixed good. For that reason, the model assumes that each consumer who buys a positive quantity of $\mathbf{\tilde{c}}$ also purchases one unit of the fixed good, so that the prices of both goods are always considered when a consumer determines his optimal purchase. Finally, price vectors will be given in the form $\mathbf{P} = (p_v, p_f) \in \mathbb{R}_+^2$, where $p_v$ and $p_f$ give the prices of the variable and fixed goods, respectively. Eventually, different price vectors will be distinguished using superscripts, which will allow us to compare the monopoly and tying situations.

A set of consumer types is given by $\Sigma = [0, \mathbf{\bar{c}}] \in \mathbb{R}$, where $\mathbf{\bar{c}} > 0$. The distribution of consumers among types is described by the probability density function $\nu(\cdot)$ and corresponding cumulative density function $F(\cdot)$. When we begin using the model, we will typically assume that this distribution is unknown. A consumer’s type determines the intensity level of his preferences over $\mathbf{\tilde{c}}$, which is effectively a measure of how many
units of the consumer demands at a given price vector. The terms "intensity level" and "consumer type" are thus used interchangeably. Preferences of all consumer types are described by a surplus function, which is defined as the difference between utility (value) and expenditure. To define this surplus function, we will use the utility function
\[ u(x, \sigma) = \begin{cases} 
\sigma x - \frac{1}{2} \Delta x^2 & \text{if } x \leq \sigma/\Delta \\
\frac{\sigma^2}{2\Delta} & \text{otherwise}
\end{cases} \]
where \( \Delta > 0 \).

The utility function's non-traditional form is intended to create a more realistic degree of heterogeneity among consumer purchasing decisions. This occurs because, marginal utility does not converge to zero as does. For that reason, we can be sure that, under any price vector, some mass of consumer types (of positive measure) will choose not to buy either good. This will allow the model to more realistically describe the influx of new consumers that typically results under variable proportion tying. Moreover, marginal utility diminishes in at the same constant rate \( \Delta > 0 \) for all consumers, which allows for easy comparisons of consumer surplus realizations. This causes the graph of each consumer’s marginal utility to resemble a linear demand curve, the slope of which is the same for all consumer types. In this way, a consumer’s type serves to determine the height of his marginal utility curve or, more accurately, its y-intercept. This is why optimal quantities of increase over the range of intensity levels, which is given by \( \Sigma \). Of course, for a given consumer type \( \sigma \in \Sigma \), marginal utility is equal to zero for all quantities \( x \geq \sigma/\Delta \), so that the utility function is only once differentiable. But this was merely a precautionary measure intended to prevent marginal utility from becoming negative, and because the resulting marginal utility function is very intuitive. Consumers will always choose quantities of over which their utility functions retain the standard properties, including strict concavity. Finally, it should be noted that functions of both utility and marginal utility are continuous.

We are now in a position to define individual realizations of consumer surplus, which will be given by the function
\[ \mathbb{R}_+^{\Sigma} \rightarrow \mathbb{R} \]
where for all \( \sigma \in \Sigma \) we define:
We will assume that each consumer chooses the quantity of his purchase by maximizing this function, which is strictly concave over the relevant quantities of \( x \) (i.e. those at which marginal utility is positive), and weakly concave everywhere. As such, we must assume that consumer preferences are quasilinear in wealth, but this is not problematic. We can assume that all consumers are endowed with the same amount of wealth, and that this amount is large enough for all consumer types to purchase \( x \) in any quantity at which marginal utility is positive. We can then further assume that consumers are always indifferent between one unit of utility and one dollar. In this way, the surplus function represents preferences over both \( x \) and wealth, while the utility function represents only the portion of surplus that is derived from the consumption of \( x \). And, because all consumers are endowed with the same amount of wealth, we need not include it as a variable of the surplus function.

We will define the demand for \( x \) had by a particular consumer type using the function \( q: \mathbb{R}_{+}^{2} \rightarrow \mathbb{R}_{+} \). Using first order conditions, it is easy to show that for all \( \sigma \in \Sigma \) this function is given by:

\[
q(P, \sigma) = \begin{cases} 
\frac{1}{2}(\sigma - p_{v}) & \text{if } \sigma \geq \sqrt{2\Delta p_{f} + p_{v}} \\
0 & \text{otherwise} 
\end{cases}
\]

The condition that \( \sigma \geq \sqrt{2\Delta p_{f} + p_{v}} \) ensures that a consumer purchases the tie only if the maximal level of surplus she can achieve is nonnegative. It should be noted that because \( x \) is described in units of the variable good, the function \( q(P, \sigma) \) can also be considered as the Walrasian demand for that good had by consumers of type \( \sigma \in \Sigma \).

We can further use this function to determine the unique consumer type that achieves zero surplus when behaving optimally, but whose optimal quantity of \( x \) is positive. This consumer type serves as the boundary between consumers who do not buy \( x \) (and hence achieve zero surplus), and those whose optimal quantities of \( x \) are positive and engender positive amounts of surplus. Of all consumer types that buy \( x \) in positive
quantities, this is the one of lowest intensity (i.e. the one who desires to use the fixed good the least). This consumer type can be determined as a function of prices, and will be denoted as $\gamma(P)$. As the condition stipulated by the above demand function suggests, this type is defined by $\gamma(P) = \sqrt{2\Delta p_f + p_v}$. We can use this to define the mass of consumer types who buy positive quantities of $\Box$ as $\Gamma(P) = [\gamma(P), \bar{\sigma}] = [\sqrt{2\Delta p_f + p_v}, \bar{\sigma}]$.

It will be helpful to define an indirect surplus function, as this will allow us to compare the optimal surplus realizations of different consumer types, and to determine how different consumer types are affected by tying. We will denote this indirect surplus function as $V: \mathbb{R}^2_+ \cdot \Sigma \rightarrow \mathbb{R}$, which is defined for all $\sigma \in \Sigma$ as:

$$V(P, \sigma) = \mathcal{S}(q(P, \sigma), P, \sigma) = \begin{cases} \frac{1}{2}\sigma - [p_v] - p_F & \text{if } \sigma \geq \gamma(P) \\ 0 & \text{otherwise} \end{cases}$$

It will also be helpful to provide a graphical interpretation of optimal surplus realizations in both situations. To do this, we will let $P_T$, $P_M \in \mathbb{R}^2_+$ denote the price vectors maintained in the monopoly and tying situations, respectively. To maintain generality, we assume only that $P_T > P_M$ and $P_T < P_M$. The following graph illustrates the marginal utility function of a single consumer type, and can be treated as a demand function in order to determine the optimal quantity of his purchase, which varies between situations. This also allows us to see the optimal level of surplus that this consumer achieves in each situation.

[Diagram A: Unit Price vs. Consumption](image)
The curve labeled \( \varphi \in (\varphi, \sigma) \) describes the marginal utility of the consumer type \( \sigma \in \Sigma \). The prices of the variable good resulting under the two situations are plotted along the vertical axis. Conversely, the prices of the fixed component are described as different combinations of the areas \( A_1, A_2, \) and \( A_3 \). The optimal quantities purchased by this consumer in the two situations are found at the intersections of variable price and marginal utility. In this case, these are given by \( q(P_T, \sigma) = q_T \) and \( q(P_M, \sigma) = q_M \) in the tying and monopoly situations, respectively. The utility a consumer derives from a given quantity \( \varphi \) is given by the total area under the marginal utility curve, taken over the interval \([0, \varphi]\). To determine the consumer’s surplus realization, the net expenditure on the variable good (given by the area under the variable price curve, taken up to the quantity purchased) and the price of the fixed good are subtracted from the consumer’s utility. In this way, the consumer depicted here achieves optimal surplus levels of \( V(P_T, \sigma) = A_2 + S_1 \) in the tying situation, and \( V(P_M, \sigma) = S_1 + S_2 \) in the monopoly situation.

As mentioned earlier, a consumer’s type indicates the point at which his marginal utility function intersects the vertical axis. Because this curve has the same slope for all consumer types, Diagram A also indicates what consumer types purchase positive quantities of \( \varphi \) in the two situations. As the diagram implies, a consumer type can receive a positive amount of surplus only if her marginal utility curve intersects the vertical axis at a point above the area of the fixed good’s price. Hence any consumers of type \( \sigma \lt \varphi^T \) will never buy a positive quantity of \( \varphi \), because they would achieve a negative amount of surplus in both situations. Conversely, any consumer with type \( \varphi^T \leq \sigma \lt \varphi^M \) will purchase positive quantities of \( \varphi \) only in the tying situation. All remaining consumer types buy positive quantities of \( \varphi \) in both situations, though these quantities are always smaller under tying. In fact, all of these consumers reduce their optimal quantities of \( \varphi \) by the same amount, as their marginal utility curves all maintain the same constant slope.
It will be useful to compare how different consumer types are affected by the transition from monopoly pricing to variable proportion tying. The following diagram describes how surplus realizations vary among consumer types, and how those realizations compare under the different pricing situations:

As this graph demonstrates, this particular tie causes a larger number of consumer types to buy positive quantities of $\square$. This is equivalent to saying that tying creates positive surplus realizations at lower levels of $\sigma$, so that $\gamma(P^T) < \gamma(P^M)$. For that reason, we will refer to consumer types located in the interval $[\gamma(P^T), \gamma(P^M)]$ as low intensity consumer types, which the text defines as those who buy only under tying. Trivially, all low intensity consumers buy more units of $\square$ under tying, because they buy zero units otherwise. Also, there is a unique consumer type $\sigma^*$ that achieves the same positive level of surplus in both situations. However, Diagram A demonstrates that all consumers in $\Gamma(P^M)$ reduce the quantities of $\square$ they purchase in the tying situation, and by the same amount. Thus, consumer types in $\Gamma(P^M)$ and to the left of $\sigma^*$ buy fewer units of $\square$ under tying, but strictly benefit from the tie. Conversely,
consumer types in $\Gamma(P^M)$ and to the right of $\sigma^*$ also buy fewer units under tying and are strictly harmed by the tie.

The tie depicted in diagram B causes new consumers to begin buying positive quantities of $\Box$. These new consumers are obviously benefited by the tie. But the tie also provides more surplus to some consumers who already bought positive quantities of $\Box$ in the monopoly situation (i.e. some consumers in $\Gamma(P^M)$). Finally, some consumer types achieve strictly less surplus in the tying situation, though these consumers still achieve positive levels of surplus in both situations. As such, the welfare effects of the tie are not obvious, because some consumers are better off, while others are harmed. Rather, the welfare effects of a tie are obvious only in the most flagrant instances of tying, or those in which no consumers are better off. Fortunately, an easy test can be implemented to determine whether or not this is the case. This test, which we describe as the consumer benefit test, queries whether a variable proportion tie benefits any consumer types. Because the indirect surplus function merely shifts its position between the two situations, it is clear that $V(P^T, \sigma) > V(P^M, \sigma)$ at some $\sigma \notin \Sigma$ if and only if it is also true that $\gamma(P^T) < \gamma(P^M)$. This means that the consumer benefit test is equivalent to the question of whether the tie brings any low intensity consumer types into the market. Accordingly, the consumer benefit test is necessarily passed if output of the fixed good increases. Importantly, if the tie passes the test, then we cannot infer that the tie decreases welfare. The following account provides a more explicit definition of the test:

**Consumer benefit test:** Let $P^M = (P^M_v, P^M_f) \in \mathbb{R}^2_+$, where $P^M_v$ is the (perfectly) competitive price of the variable good, and where $P^M_f$ is the monopoly price of the fixed good. Let $T$ be a variable proportion tie that imposes the price vector $P^T = (P^T_v, P^T_f) \in \mathbb{R}^2_+$ in place of $P^M$, where $P^T_v > P^M_v$ and $P^T_f < P^M_f$. Then $T$ passes the consumer benefit test if and only if $\Gamma(P^M)$ forms a nontrivial subset of $\Gamma(P^T)$.

This test is useful, because it implies that two sorts of consumer types are strictly better off under tying. First, the consumer types that buy positive quantities of $\Box$ only under tying are obviously better off, as this allows them to achieve a positive level of surplus, which would be impossible otherwise. Second, the tie benefits some consumer types that buy positive quantities of $\Box$ in both situations, even though these consumers
buy fewer units of $\square$ in the tying situation. This latter sort of consumers is particularly important because those who favor a law condemning variable proportion ties have often argued that all of the consumers who consume less under tying are made worse off by the tie. In fact, this assertion is false whenever the consumer benefit test is passed, as the following proposition illustrates:

**Proposition 1:** Fix $P^T, P^M \in \mathbb{R}_+^2$, where $P^T > P^M$ and $P^T < P^M$. Assume that $\Gamma(P^M)$ is nonempty. Then there is a mass of consumer types in $\Gamma(P^M)$ that achieve strictly more surplus under tying if and only if the consumer benefit test is passed.

**Proof:** (□) Assume the consumer benefit test is passed, so that $\Gamma(P^M) \cap \Gamma(P^T)$.

$V(P^T, \sigma)$ is strictly increasing in $\sigma$, so $\sigma \in \Gamma(P^T)$ implies $\sigma' \in \Gamma(P^T)$ for all $\sigma' > \sigma$, for any $\square = T, M$. This implies $\gamma(P^T) < \gamma(P^M)$.

$V(P^T, \gamma(P^T)) = 0$, by the definition of $\gamma(P^T)$, and implies $V(P^T, \gamma(P^M)) > 0$, since $V(\cdot, \cdot)$ is strictly increasing in $\sigma$. Hence $V(P^T, \gamma(P^M)) > V(P^M, \gamma(P^M)) = 0$.

$\square \in \epsilon > 0$ such that $\sigma \in (\gamma(P^T), \gamma(P^T) + \epsilon)$ implies $V(P^T, \sigma) > V(P^M, \sigma)$, since $V(\cdot, \cdot)$ is continuous in each of its arguments.

(□) Assume the set $\mathcal{D} = \{\sigma \in \Gamma(P^M) | V(P^T, \sigma) > V(P^M, \sigma)\}$ is nonempty, and has positive measure.

We have that $V_\sigma(P^T, \sigma) < V_\sigma(P^M, \sigma)$ at all $\sigma$, where $V_\sigma(\cdot, \cdot)$ denotes the partial derivative of $V(\cdot, \cdot)$ with respect $\sigma$. This implies that, for all $\sigma, \sigma' \in \Gamma(P^M)$ with $\sigma' < \sigma$, $\sigma' \in \mathcal{D}$ whenever $\sigma \in \mathcal{D}$, because the difference $V(P^T, \cdot) - V(P^M, \cdot)$ is strictly diminishing over $\Gamma(P^M)$. Thus $\square$ is an interval with lower bound $\gamma(P^M)$.

$\square V(P^T, \gamma(P^M)) > 0$, since $V(P^T, \gamma(P^M)) - V(P^M, \gamma(P^M)) > 0$.

$\square \in \epsilon > 0$ such that $\sigma \in (\gamma(P^M) - \epsilon, \gamma(P^M))$ implies $V(P^T, \sigma) > 0 = V(P^M, \sigma)$, because $V(\cdot, \cdot)$ is continuous.

$\square \gamma(P^T) < \gamma(P^M)$, which implies $\Gamma(P^M) \cap \Gamma(P^T)$.

□.
This proof demonstrates that, whenever a tie persuades new consumer types to buy positive quantities of $\Diamond$, some consumer types who already bought $\Box$ in positive quantities under monopoly pricing are better off. At this point it will be helpful to distinguish between different groups of consumer types according to the way in which their surplus realizations are impacted by tying. We will define $\mathcal{L}(P_T, P_M) = \{ \sigma \mid \gamma(P_T) < \sigma < \gamma(P_M) \}$ to be the set of low intensity consumer types who are drawn in by the tie, or those who (i) buy positive $\Box$ only in the tying situation; and (ii) achieve positive surplus only in the tying situation. One corollary of proposition 1 is that, when a tie passes the consumer benefit test, there is a unique consumer type that achieves the same positive level of surplus in both situations, provided that that the upper bound of $\Sigma$ is sufficiently high. This follows because, as the proof of proposition 1 states, the difference $V(P_T, \cdot) - V(P_M, \cdot)$ is decreasing in $\sigma$, and it is doing so at an increasing rate. For that reason, we will define $\sigma^*(P_T, P_M)$ to be the unique consumer type in $\Gamma(P_M)$ at which the aforementioned difference is equal to zero, but which achieves positive surplus in both situations. This consumer type is indifferent between the tying and monopoly situations, even though consumers of this type buy fewer units of $\Box$ under tying. With this, we can define the set $\mathcal{M}(P_T, P_M) = [\gamma(P_M), \sigma^*(P_T, P_M))$ to be the set of medium intensity consumer types who benefit from the tie, but nevertheless buy fewer units under tying. These consumer types comprise the portion $\Gamma(P_M)$ to the left of the indifferent consumer type. Finally, we will define the set of consumers who are injured by the tie. Explicitly, we will define $\mathcal{H}(P_T, P_M) = (\sigma^*(P_T, P_M), \Box]$ to be the set of high intensity consumer types buy fewer units and achieve less surplus under tying.

Proposition 1 is useful because it demonstrates that the positive effects of tying are not limited to the surplus realizations derived from the low consumer types who only enter the market under tying. This casts additional doubt on any claim that variable proportion ties are inherently welfare reducing. Indeed, when a tie passes the consumer benefit test, such a claim would be justified only if we have reason to believe that the welfare gains resulting from low and medium intensity consumers are outweighed by the welfare reductions accrued from high intensity buyers. Of course, this determination would seem to require detailed knowledge concerning the distribution of consumers among consumer types. In fact, the following proof demonstrates that, when the distribution of consumers among types is unknown, a tie
that passes the consumer benefit test has an ambiguous impact on total consumer welfare.

**Proposition 2:** \( \text{Fix } P^T, P^M \in \mathbb{R}^2_+ \), where \( P^T > P^M \) and \( P^T < P^M \). Assume that the consumer benefit test is passed, and that the distribution of consumers among consumer types, denoted as \( \sigma^* \), is unknown. Then the welfare effects of the tie are ambiguous, meaning that we cannot determine whether the tie caused total consumer welfare to increase or decrease.

**Proof:** Assume the consumer benefit test is passed, and let \( \mathcal{G} \) be the space of possible (continuous) distributions of consumers among types.

Given a distribution \( \sigma \in \mathcal{G} \), let \( \mathcal{W}(\sigma) \) define the effect of tying on total consumer welfare when consumers are distributed among types according to \( \sigma \). Explicitly:

\[
\mathcal{W}(\sigma) = \int_{\gamma(P^T)} \sigma(\sigma)[V(P^T, \sigma) - V(P^M, \sigma)]d\sigma
\]

\( \mathcal{W}(\sigma) \) gives the expected value of the difference \( V(P^T, \sigma) - V(P^M, \sigma) \) under the distribution \( \sigma \), and given that \( \sigma \in \Gamma(P^T) \).

\( \mathcal{W}(\sigma) \) is bounded above by \( V(P^T, \gamma(P^M)) - V(P^M, \gamma(P^M)) > 0 \), because the consumer type \( \gamma(P^M) \) benefits most from tying. (This follows from the proof of proposition 1.) Likewise, \( \mathcal{W}(\sigma) \) is bounded below by \( V(P^T, \overline{\sigma}) - V(P^M, \overline{\sigma}) < 0 \), because the consumer type \( \overline{\sigma} \) is most injured by tying.

For simplicity, let \( W^+ = V(P^T, \gamma(P^M)) - V(P^M, \gamma(P^M)) \), and \( W^- = V(P^T, \overline{\sigma}) - V(P^M, \overline{\sigma}) \).

\( \mathcal{W}(\sigma) \) can get arbitrarily close to \( W^+ \) and \( W^- \) by choosing different distributions, which is possible even if density is positive at all consumer types.

\[ \mathcal{W}(\sigma^*) \subseteq (W^+, W^-) \subseteq \mathcal{G}, \text{ where } W^+ > 0 > W^- \]
The welfare effects of the tie are ambiguous, as it is impossible to tell whether it caused total consumer welfare to increase or decrease.

This result is important because it implies that, if a tie passes the consumer benefit test, its effect on consumer welfare can generally not be determined unless one has information concerning the distribution of consumers among consumer types. That is, depending on whether consumers are relatively more or less intensive, a tie might decrease or increase total consumer welfare. Moreover, this result holds regardless of any production efficiencies that might result from the tie. As such, this result would tend to suggest that government intervention is unwarranted when a tie passes the consumer benefit test, particularly if the tie results in production efficiencies.

As described earlier, new consumer types buy more units of the variable good under tying and low intensity consumer types buy fewer units of variable good under tying but, nevertheless, both classes of consumer types benefit from the tie. This implies that one cannot determine a tie’s impact on consumer welfare by merely observing what impact it has on output of the variable good. For example, it is possible most consumers are distributed along medium and high intensity ranges, and each range has the same cumulative density. Further, suppose that the medium intensity consumers benefit by the same amount that high intensity consumers are injured. Then, if there is even a small number of low intensity consumers, consumer welfare is improved and output of the variable good will fall. (Of course, a tie increases total output of the fixed good if and only if it passes the consumer benefit test, because the set of consumers who buy positive quantities of is equivalent to the set of consumers who buy the fixed good.) It should be noted that this example does not suggest the tie would be less profitable than monopoly pricing, even if it does not result in production efficiencies. Indeed, it is easy to use figure A to show that, when the fixed good is sold at cost under tying, low intensity consumers are profit increasing, medium intensity consumers are profit decreasing, and high intensity consumers are split between these possibilities. The relative profitability of all consumer types only increase when the fixed good is sold above cost under tying. In this way, it is easy to construct a
distribution that makes tying profitable and welfare increasing, even though it causes output of the variable good to fall.