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Tables for dihedral angles between the adjacent faces of platonic solids & various uniform polyhedra

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Tables for dihedral angles between the adjacent faces with a common edge of various uniform polyhedra

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These tables have been prepared by the author Mr H.C. Rajpoot using his data tables of the various polyhedra for determining the dihedral angles for different uniform polyhedral shells. These are very useful for the construction & preparing the wire-frame models of the uniform polyhedral shells having different regular polygonal faces. A polyhedral shell can be easily constructed/framed by continuously fixing all its adjacent (flat) faces each two as a pair at their common edge at an angle equal to the dihedral angle between them. **The dihedral is the angle of inclination between any two adjacent faces, having a common edge in a uniform polyhedron, measured normal to their common edge**. Mathematically, the dihedral angle ($\boldsymbol{\theta_{m-n}}$) between any two **regular m-gonal** & **n-gonal faces**, having a common edge, at the normal distances H_m & H_n respectively from the centre of a uniform polyhedron with edge length a is given by the **generalized formula** as follows

$$\boldsymbol{\theta_{m-n} = \tan^{-1}\left\{\frac{H_m}{\left(\frac{a}{2}\cot\frac{\pi}{m}\right)}\right\} + \tan^{-1}\left\{\frac{H_n}{\left(\frac{a}{2}\cot\frac{\pi}{n}\right)}\right\}} \qquad \forall\ m,n \in N\ \ \&\ \ m,n \geq 3$$

Thus by setting the values of no. of sides m & n and the normal distances H_m & H_n of the adjacent regular polygonal faces of a uniform polyhedron with edge length a in the above expression, we can easily calculate the dihedral angle between the corresponding adjacent regular m-gonal & n-gonal faces each with an equal edge length a. Although, the dihedral angle does not depend on the edge length a of any uniform polyhedron but it merely depends on the geometry & the no. of the faces of a polyhedron. There are the tables for the dihedral angles of some important regular & uniform polyhedra having congruent regular polygonal faces each with an equal edge length.

Table for the dihedral angles between the adjacent faces of all five platonic solids

Platonic solid	**Pair of the adjacent congruent faces having a common edge**	**Dihedral angle between the corresponding pair of the adjacent faces**
Regular tetrahedron	Equilateral triangles	$\theta_{T-T} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^o$
Regular hexahedron (cube)	Squares	$\theta_{S-S} = 90^o$
Regular octahedron	Equilateral triangles	$\theta_{T-T} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$
Regular dodecahedron	Regular pentagons	$\theta_{P-P} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$
Regular icosahedron	Equilateral triangles	$\theta_{T-T} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$

Tables for dihedral angles between the adjacent faces with a common edge of various uniform polyhedra

Table for the dihedral angles between the adjacent faces of uniform polyhedra

Uniform Polyhedron	**Dihedral angle between the adjacent regular polygonal faces with a common edge**
Truncated Tetrahedron having 4 equilateral triangular faces & 4 regular hexagonal faces	Dihedral angle between equilateral triangular face & regular hexagonal face $$\theta_{T-H} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$
	Dihedral angle between two regular hexagonal faces $$\theta_{H-H} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^o$$
Truncated cube having 8 equilateral triangular faces & 6 regular octagonal faces	Dihedral angle between equilateral triangular face & regular octagonal face $$\theta_{T-O} = \pi - \tan^{-1}(\sqrt{2}) \approx 125.2643897^o$$
	Dihedral angle between two regular octagonal faces $$\theta_{O-O} = 90^o$$
Truncated octahedron having 6 square faces & 8 regular hexagonal faces	Dihedral angle between square face & regular hexagonal face $$\theta_{S-H} = \pi - \tan^{-1}(\sqrt{2}) \approx 125.2643897^o$$
	Dihedral angle between two regular hexagonal faces $$\theta_{H-H} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$
Truncated dodecahedron having 20 equilateral triangular faces & 12 regular decagonal faces	Dihedral angle between equilateral triangular face & regular decagonal face $$\theta_{T-D} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$
	Dihedral angle between two regular decagonal faces $$\theta_{D-D} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$$
Cuboctahedron having 8 equilateral triangular faces & 6 square faces	Dihedral angle between equilateral triangular face & square face $$\theta_{T-S} = \pi - \tan^{-1}(\sqrt{2}) \approx 125.2643897^o$$
Icosidodecahedron having 20 equilateral triangular faces & 12 regular pentagonal faces	Dihedral angle between equilateral triangular face & regular pentagonal face $$\theta_{T-P} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$

Tables for dihedral angles between the adjacent faces with a common edge of various uniform polyhedra

Small rhombicuboctahedron having 8 equilateral triangular faces & 18 square faces	Dihedral angle between equilateral triangular face & square face $$\theta_{T-S}=\pi-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\approx 144.7356103^o$$
	Dihedral angle between two square faces $$\theta_{S-S}=2\tan^{-1}\left(1+\sqrt{2}\right)=135^o$$
Small rhombicosidodecahedron having 20 equilateral triangular faces, 30 square faces & 12 regular pentagonal faces	Dihedral angle between equilateral triangular face & square face $$\theta_{T-S}=\pi-\tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right)\approx 159.0948426^o$$
	Dihedral angle between square face & regular pentagon $$\theta_{S-P}=\pi-\tan^{-1}\left(\frac{3+5\sqrt{5}}{29}\right)\approx 153.9424193^o$$

Table for the dihedral angles between the adjacent faces of a great rhombicuboctahedron

Pair of the adjacent faces having a common edge	Square & regular hexagon	Square & regular octagon	Regular hexagon & regular octagon
Dihedral angle of the corresponding pair (of the adjacent faces)	$\theta_s+\theta_h=\pi-\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\approx 144.7356103^o$	$\theta_s+\theta_o=135^o$	$\theta_h+\theta_o=\pi-\tan^{-1}\left(\sqrt{2}\right)$ $\approx 125.2643897^o$

Table for the dihedral angles between the adjacent faces of a great rhombicosidodecahedron

Pair of the adjacent faces having a common edge	Square & regular hexagon	Square & regular decagon	Regular hexagon & regular decagon
Dihedral angle of the corresponding pair (of the adjacent faces)	$\theta_s+\theta_h=\pi-\tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right)$ $\approx 159.0948426^o$	$\theta_s+\theta_D$ $=\pi-\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ $\approx 148.2825256^o$	$\theta_h+\theta_D=\pi-\tan^{-1}\left(3-\sqrt{5}\right)$ $\approx 142.6226319^o$

Note: Above articles had been derived & *illustrated by Mr H.C. Rajpoot (B Tech, Mechanical Engineering)*

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