Ziv–Zakai Bounds on Image Registration

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Abstract—Image registration is a fundamental and important task in image processing. The goal essentially is to estimate the transformation that aligns two images. We focus on the general rigid body transformation case. In this paper, we derive the Ziv–Zakai bounds (ZZB) on image registration by assuming an uncertainty model for the rotation and translation errors, and propose to use the ZZB as a benchmark to evaluate the registration ability of an image pair. We also compare the performance of several image registration algorithms with the derived bounds when applied to several datasets.

Index Terms—Image registration, parameter estimation, Ziv–Zakai bound.

I. INTRODUCTION

IMAGE registration is a crucial step in all image analysis tasks in which image data is acquired from multiple sources. Examples of such tasks are image fusion, change detection, and multichannel image restoration [1], [2]. Some of the application areas where image registration is required include remote sensing, medical image analysis, cartography, pattern recognition and computer vision [3]. The primary objective of image registration is to match two images that differ in certain aspects, e.g., translation, scaling and rotation (assuming a rigid-body misalignment), but essentially represent the same scene. A transformation is to be found so that the points and objects in one image can be related to their corresponding points and objects in the other image.

Current registration approaches may broadly be classified as feature based, Fourier based and intensity based methods. The feature based approach is most widely used and involves extraction of geometric features, also known as ground control points (GCP) such as intersections and landmarks that are clearly visible in both images. Image registration is then performed by matching of GCPs in the two images and estimating the transformation parameters. The Fourier based methods work with the images in the frequency domain and utilize the translation and rotational properties of Fourier transforms to calculate the transformation parameters. Intensity based methods involve maximization of a similarity measure computed from the intensity values of the two images. Some commonly used similarity

measures are the cross correlation or correlation ratio, sum of squared differences and mutual information. Overviews of these image registration approaches can be found in [1]–[3].

Although many approaches have been proposed for image registration [1]–[3], very little attention has been paid to evaluate their performance [4], [5]. Performance analysis for image registration is usually performed visually. However, a visual inspection is not a completely satisfactory evaluation method and hence one needs to explore other evaluation techniques such as the use of performance bounds for the estimation of transformation parameters. Analytical evaluation of the performance of registration algorithms is often difficult because the transformation parameters appear in the image data in a nonlinear manner. The registration ability of a given image pair and suitability of different methods is image dependent and depends on factors such as image content. Image registration is essentially a parameter estimation problem. Because there is no systematic method to select the best registration method for a given image pair, investigation of the performance bounds, which is directly associated with image content, will help us evaluate the efficiency of image registration techniques [4], determine parameter regions where good or poor registration is expected [4], and interpret the effect of translation and rotation uncertainty on the registration error. A tight performance bound for image registration tells us what the best achievable performance for any image registration algorithm is for the specific image pair and no image registration algorithm can produce registration error smaller than the bound. Thus, performance bounds, which are independent of the algorithm, can serve as a benchmark for the image dataset and registration ability. They can also be used to study how the image content or some image preprocessing operations affect registration performance. The distance or gap between the actual registration error of an image registration approach and a tight bound indicates the efficiency of that registration approach and the room for improvement. Moreover, without implementing any image registration approach, we are able to get some useful insights about the achievable performance of image registration for the particular pair of images from the bounds. Thus, it is an important problem to obtain tight and robust performance bounds for image registration.

Robinson and Milanfar [5] proposed that mean-square error (MSE) be used as a standard performance measure to provide a fair comparison between different algorithms for image registration. Cramér–Rao lower bound (CRLB) for image translation was obtained which evaluates the translation estimator performance from a statistically meaningful perspective [5]. Yetik and Nehorai [4] extended the results in [5] to a wide variety of geometric transformations including translation, rotation, scaling, shearing, rigid body and more general affine transformations. However, in their results, the CRLBs of some transformation parameters such as the rigid body transformation parameters

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are functions of the unknown rotation angle and may be difficult to obtain in practice. Further, the derived CRLB in [4] requires the estimators to be unbiased. This restriction may limit the applicability of the bounds derived in [4] when the estimates of the transformation parameters are biased. For this reason, in this paper, we assume that prior information regarding the uncertainty of transformation parameters is known and investigate the use of Bayesian performance bounds for the image registration problem.

Bayesian bounds assume that the parameter to be estimated is a random variable with known *a priori* distribution and provide bounds on the global MSE averaged over the *a priori* probability density function (pdf) [7]. The group of Bayesian performance bounds consists of Bayesian Cramér–Rao lower bound, Bayesian Bhattacharyya bound, Weiss–Weinstein bound, and Ziv–Zakai bound (ZZB) [7]. Bayesian Cramér–Rao lower bound and Bayesian Bhattacharyya bound are both based on the Fisher information matrix, which contains the unknown parameter rotation angle in the rigid body transform distortion case. The solution of these two bounds involves a numerical integration and a closed form solution is often not available. Weiss–Weinstein bound involves choosing test points and inverting a matrix [7], which makes the computation of the bound very complicated. ZZB relates the MSE corresponding to the estimation problem to the probability of error of a binary detection problem [7], which enables us to derive its analytical expression for the image registration problem. In this paper, we incorporate knowledge of the transformation parameter space via the *a priori* pdf and bound the performance of image registration using ZZB. Our goal is to derive the performance bound, compare our derived bound with the performance of various registration algorithms when applied to several image datasets and investigate the effect of factors such as image size and image texture on registration via the bounds. It gives us an insight into the achievable accuracy of a registration algorithm. The closed form expression of the ZZB is more informative and offers an insight into the nature of the image registration problem. Unlike the Cramér–Rao bound, ZZB is tighter and able to capture the threshold effect in the low SNR region. By analyzing the details of ZZB, we can study the effect of transformation uncertainty on the MSE of transformation estimation. Further, ZZB can serve as an indicator for the best achievable registration performance for a given image pair (independent of algorithm) and therefore it can serve as a benchmark for different image registration algorithms and image reference dataset.

The paper is organized as follows. In Section II, we formulate the image registration problem and derive the ZZB for the general rigid body transformation case in image registration. These performance bounds are compared with our experimental results in Section III. Concluding remarks are provided in Section IV.

II. BOUNDS FOR THE RIGID BODY TRANSFORMATION CASE

A. Image Registration Problem

Image registration is defined as the process to determine a mapping between two images spatially so that they are aligned.

The objective is to estimate the appropriate transformation parameters given the two images. Thus, image registration is essentially an estimation problem. Let $I_1(x, y)$ and $I_2(x, y)$ denote the two images to be registered. Each image can be considered as a noise-free image plus noise. The misalignment transformation is assumed to be applied to the noise-free image. Here, we only consider the registration of images of the same modality. Thus, their corresponding noise-free images are assumed to only have geometric distortion and the corresponding pixels in the two noise-free images are assumed to have the same intensities. For the registration of images of different modalities, the intensity of a pixel in one image is a nonlinear function of that of the corresponding pixel in the other image. This makes the derivation of the bound more complicated and is not considered in this paper. Assuming $u(x, y)$ and $v(x, y)$ to be the transformed coordinates, the reference image and the floating image can be modeled as [4], [5], [8]

$$\begin{aligned} I_1(x, y) &= f(x, y) + n_1(x, y) \\ I_2(x, y) &= f(u(x, y), v(x, y)) + n_2(x, y). \end{aligned} \quad (1)$$

In this paper, we assume that the noises $n_1(x, y)$ and $n_2(x, y)$ are i.i.d Gaussian noises with zero mean and variance N . Thus, each pixel intensity is considered as a Gaussian random variable with the mean equal to the noise-free pixel intensity.

For different types of misalignment transformations, the mapping function $u(x, y)$ and $v(x, y)$ would have different expressions. Rigid body transformation misalignment is a popularly used model in image registration. A typical rigid body transformation is composed of a combination of rotation and translations along the two axes. We assume that scaling does not need to be estimated. It would be a straightforward extension if we were to estimate scaling also. Misalignment error defined as a rigid body transformation in this case has the following mapping function:

$$\begin{aligned} u(x, y) &= \cos \theta_0 x + \sin \theta_0 y + x_0 \\ v(x, y) &= -\sin \theta_0 x + \cos \theta_0 y + y_0. \end{aligned} \quad (2)$$

The parameters to be estimated for registration are rotation θ_0 , and translations x_0 and y_0 . In practice, when u and v correspond to nonintegers, an interpolation technique is used to obtain the intensity value of the pixel at the location (u, v) , which may produce artifacts. We do not consider the effect of artifacts produced by interpolation on the performance bound in this paper. In the following, we introduce the ZZB and derive it for the rigid body transformation case.

B. Ziv–Zakai Bound

Considering the estimation of a random variable ϕ based upon noisy observations $\mathbf{x}$, the Ziv–Zakai bound is a Bayesian bound, which assumes that the parameters are random variables with a known *a priori* distribution. Assume that the parameter ϕ has *a priori* pdf $p_\phi(\phi)$. The estimation error is $\epsilon = \hat{\phi}(\mathbf{x}) - \phi$. Thus, the MSE $\bar{\epsilon}^2 = \mathbf{E}\{|\epsilon|^2\} = \mathbf{E}\{|\hat{\phi}(\mathbf{x}) - \phi|^2\}$ can be written as [7], [9], [10]

$$\bar{\epsilon}^2 = \frac{1}{2} \int_0^\infty \Pr\left(|\epsilon| \geq \frac{h}{2}\right) h dh. \quad (3)$$

Based on it, the scalar ZZB for estimation of ϕ can be derived as [6]

$$ZZB = \int \left\{ \int \min\{p_\phi(\phi), p_\phi(\phi + h)\} P_e(\phi, \phi + h) d\phi \right\} h dh \quad (4)$$

where $P_e(\phi, \phi + h)$ is the probability of error associated with the hypothesis testing problem

$$\begin{aligned} H_0 : \hat{\phi} &= \phi \\ H_1 : \hat{\phi} &= \phi + h \end{aligned} \quad (5)$$

where the two hypotheses are equally likely.

For a parameter vector $\vec{\phi} = \{\phi_1, \dots, \phi_K\}$, we assume that all of the parameters are random variables. When we compute the bound for one component of the parameter vector, the second approach mentioned in ([6], page. 627) is used, where we first compute the conditional ZZB using (4) and then take the expectation with respect to the other parameters. That is to say, assuming $p(\phi_1, \dots, \phi_{i-1}, \phi_{i+1}, \dots, \phi_K)$ to be the pdf of the parameters $\phi_1 \dots \phi_{i-1}, \phi_{i+1}, \dots, \phi_K$, the ZZB of a parameter ϕ_i is obtained by

$$\begin{aligned} ZZB(\hat{\phi}_i) &= \int ZZB(\hat{\phi}_i | \phi_1, \dots, \phi_{i-1}, \phi_{i+1}, \dots, \phi_K) \\ &\times p(\phi_1, \dots, \phi_{i-1}, \phi_{i+1}, \dots, \phi_K) \\ &\times d\phi_1, \dots, d\phi_{i-1}, d\phi_{i+1}, \dots, d\phi_K. \end{aligned} \quad (6)$$

In the case of rigid body transformation, the parameters to be estimated are $\vec{\phi} = \{\theta_0, x_0, y_0\}$. We assume the transformation parameters, θ_0, x_0 and y_0 to be uniformly distributed random variables in $[0, \Delta_\theta]$, $[0, \Delta_x]$ and $[0, \Delta_y]$, respectively. We obtain bounds for translation error and rotation error separately.

1) *Translation Error:* For rigid body transformation, the hypothesis testing problem for the translation along the x axis x_0 is

$$\begin{aligned} H_0 : \hat{x}_0 &= x_0 \\ H_1 : \hat{x}_0 &= x_0 + h. \end{aligned} \quad (7)$$

In this case, the probability of error is given by [6]

$$P_e(x_0, y_0, \theta_0, h) = P_e(\theta_0, h) = Q\left(\sqrt{\frac{E(1 - \rho(h))}{2N}}\right) \quad (8)$$

where E is the energy function of the image and ρ is the correlation coefficient of two images. The definitions of E and ρ are included in Appendix A.

Let $F(w_x, w_y)$ be the Fourier transform of the image $f(x, y)$ and we define

$$\begin{aligned} M_1 &= \frac{1}{\pi^2} \int_0^\infty \int_0^\infty |F(w'_x, w'_y)|^2 w'^2_x dw'_x dw'_y; \\ M_2 &= \frac{1}{\pi^2} \int_0^\infty \int_0^\infty |F(w'_x, w'_y)|^2 w'^2_y dw'_x dw'_y; \end{aligned}$$

$$\begin{aligned} M_3 &= \frac{1}{\pi^2} \int_0^\infty \int_0^\infty |F(w'_x, w'_y)|^2 w'_x w'_y dw'_x dw'_y; \\ r &= \sqrt{M_3^2 + (M_1 - M_2)^2 / 4}; \\ \delta &= \arctan\left(\frac{2M_3}{M_1 - M_2}\right); \quad c = \frac{M_1 + M_2}{2r}; \\ \beta &= \sqrt{r(c + \cos(2\theta_0 + \delta)) / E}. \end{aligned} \quad (9)$$

Assuming that Δ_x is given to be larger than $(\sqrt{2})/(\beta)$, we show in Appendix A that $P_e(\theta_0, h)$ can be bounded by

$$P_e(\theta_0, h) \geq \begin{cases} Q\left(\frac{h}{2} \sqrt{\frac{E\beta^2}{N}}\right), & 0 \leq h \leq \frac{\sqrt{2}}{\beta} \\ Q\left(\sqrt{\frac{E}{2N}}\right), & h > \frac{\sqrt{2}}{\beta}. \end{cases} \quad (10)$$

Because the function $P_e(\theta_0, h)$ is associated with the rotation angle θ_0 , according to (4), the ZZB for the translation error along the x axis in the rigid body transformation case is

$$ZZB(\hat{x}_0) = \int_0^\infty \left\{ \int_{-\infty}^\infty \int_{-\infty}^\infty A(\theta_0, x_0, h) P_e(\theta_0, h) dx_0 d\theta_0 \right\} h dh \quad (11)$$

where $A(\theta_0, x_0, h) = \min\{P(\theta_0, x_0), P(\theta_0, x_0 + h)\}$.

Substituting (10) into (11), we get

$$\begin{aligned} ZZB(\hat{x}_0) &= \int_{-\infty}^\infty \int_{-\infty}^\infty \int_0^\infty A(\theta_0, x_0, h) Q\left(\sqrt{\frac{E}{2N}}\right) dh d\theta_0 dx_0 \\ &+ \int_{-\infty}^\infty \int_{-\infty}^\infty \left\{ \int_0^{\frac{\sqrt{2}}{\beta}} A(\theta_0, x_0, h) B(\theta_0, h) h dh \right\} d\theta_0 dx_0 \end{aligned} \quad (12)$$

where

$$B(\theta_0, h) = [Q((h\beta)/(2)\sqrt{(E)/(N)}) - Q(\sqrt{(E)/(2N)})].$$

Using a property in [6],

$$\frac{1}{2} \int_0^\infty \left\{ \int_{-\infty}^\infty \int_{-\infty}^\infty A(\theta_0, x_0, h) d\theta_0 dx_0 \right\} h dh = \frac{\Delta_x^2}{12} \quad (13)$$

and because $\int_{-\infty}^\infty A(\theta_0, x_0, h) dx_0 = P(\theta_0)(1 - (h)/(\Delta_x))$ [6], (12) becomes

$$\begin{aligned} ZZB(\hat{x}_0) &= \frac{\Delta_x^2}{6} Q\left(\sqrt{\frac{E}{2N}}\right) \\ &+ \int_0^{\Delta_\theta} \frac{1}{\Delta_\theta} \left\{ \int_0^{\frac{\sqrt{2}}{\beta}} B(\theta_0, h) \left(1 - \frac{h}{\Delta_x}\right) h dh \right\} d\theta_0 \\ &= \frac{\Delta_x^2}{6} Q\left(\sqrt{\frac{E}{2N}}\right) + g_1(h) + g_2(h) \end{aligned} \quad (14)$$

where

$$\begin{aligned} g_1(h) &= \int_0^{\Delta_\theta} \frac{1}{\Delta_\theta} \left\{ \int_0^{\frac{\sqrt{2}}{\beta}} B(\theta_0, h) h d\theta_0 \right\} d\theta_0 \\ g_2(h) &= - \int_0^{\Delta_\theta} \frac{1}{\Delta_\theta} \left\{ \int_0^{\frac{\sqrt{2}}{\beta}} B(\theta_0, h) \frac{h^2}{\Delta_x} d\theta_0 \right\} d\theta_0. \end{aligned} \quad (15)$$

The solution of $g_1(h)$ can be obtained as

$$g_1(h) = \frac{N\Gamma_{1.5}\left(\frac{E}{4N}\right)}{\Delta_\theta r \sqrt{c^2 - 1}} \left[\tan^{-1} \left(\frac{\tan(\Delta_\theta + \delta/2)}{\sqrt{\frac{c+1}{c-1}}} \right) - \tan^{-1} \left(\frac{\tan(\delta/2)}{\sqrt{\frac{c+1}{c-1}}} \right) \right] \quad (16)$$

where, $\tan^{-1}(\cdot)$ is defined in $[0, \pi]$. Note here that the derivative of $g_1(h)$ in (16) with respect to Δ_θ is equal to $(1)/(\cos(2\Delta_\theta + \delta) + c)$, and therefore is always positive, because c is greater than 1. Also the absolute value of derivative of g_2 with respect to Δ_θ is much less than that of g_1 when the translation distortion Δ_x is more than a pixel. Thus, the derivative of ZZB in (14) is always positive, which indicates that translation error increases as the rotation distortion Δ_θ increases.

When Δ_θ is small, g_1 can be simplified by the Taylor series expansion as

$$g_1(h) = N\Gamma_{1.5}\left(\frac{E}{4N}\right) \left(\frac{1}{M_1} + \frac{M_3}{M_1^2} \Delta_\theta + O(\Delta_\theta^2) \right). \quad (17)$$

The derivations to obtain (16) and (17) are detailed in Appendixes B and C, respectively.

In the same way, when Δ_θ is small, we can calculate the term $g_2(h)$ as

$$\begin{aligned} g_2(h) &= - \frac{32N^{1.5}\Gamma_2\left(\frac{E}{4N}\right)}{3\sqrt{2\pi}\Delta_x} \\ &\quad \times \left(\frac{1}{M_1^{1.5}} + \frac{3M_3\Delta_\theta}{2M_1^{2.5}} + O(\Delta_\theta^2) \right). \end{aligned} \quad (18)$$

Then substituting (17) and (18) into (14), the result of ZZB for translation error along the x axis, when Δ_θ is small, is given by

$$\begin{aligned} \text{ZZB}(\hat{x}_0) &= \frac{\Delta_x^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + N\Gamma_{1.5} \left(\frac{E}{4N} \right) \left(\frac{1}{M_1} + \frac{M_3}{M_1^2} \Delta_\theta \right) \\ &\quad - \frac{32N^{1.5}\Gamma_2\left(\frac{E}{4N}\right)}{3\sqrt{2\pi}\Delta_x} \left(\frac{1}{M_1^{1.5}} + \frac{3M_3\Delta_\theta}{2M_1^{2.5}} \right) + O(\Delta_\theta^2). \end{aligned} \quad (19)$$

Similarly, we can obtain the ZZB for translation error along the y axis, which is given by

$$\begin{aligned} \text{ZZB}(\hat{y}_0) &= \frac{\Delta_y^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + N\Gamma_{1.5} \left(\frac{E}{4N} \right) \left(\frac{1}{M_2} + \frac{M_3}{M_2^2} \Delta_\theta \right) \\ &\quad - \frac{32N^{1.5}\Gamma_2\left(\frac{E}{4N}\right)}{3\sqrt{2\pi}\Delta_y} \left(\frac{1}{M_2^{1.5}} + \frac{3M_3\Delta_\theta}{2M_2^{2.5}} \right) + O(\Delta_\theta^2). \end{aligned} \quad (20)$$

Note that in the absence of rotation misalignment, $\Delta_\theta = 0$, and therefore, the ZZBs for the translation only case reduce to

$$\begin{aligned} \text{ZZB}(\hat{x}_0) &= \frac{\Delta_x^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + \frac{N}{M_1} \Gamma_{1.5} \left(\frac{E}{4N} \right) - \frac{32N^{1.5}\Gamma_2\left(\frac{E}{4N}\right)}{3\sqrt{2\pi}\Delta_x M_1^{1.5}} \\ \text{ZZB}(\hat{y}_0) &= \frac{\Delta_y^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + \frac{N}{M_2} \Gamma_{1.5} \left(\frac{E}{4N} \right) - \frac{32N^{1.5}\Gamma_2\left(\frac{E}{4N}\right)}{3\sqrt{2\pi}\Delta_y M_2^{1.5}}. \end{aligned} \quad (21)$$

In the translation only distortion case, the translation problem in image processing can be considered as a time delay estimation problem. Bell *et al.* [6] have derived the ZZB for time delay estimation. The result is exactly equivalent to the result in (21). In the presence of rotation distortion, registration error of translations increases as Δ_θ increases, i.e., as the uncertainty of rotation angle increases. Thus, according to the above result, the ZZB for the translations depends on the uncertainty of the rotation as well as the image content, M_1, M_2, M_3 , and the noise variance.

When the uncertainty of the translation is not within subpixel level, the term g_2 can be normally omitted. So when Δ_θ is small, the ZZB of translations x_0 and y_0 can be simplified as

$$\begin{aligned} \text{ZZB}(\hat{x}_0) &= \frac{\Delta_x^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + N\Gamma_{1.5} \left(\frac{E}{4N} \right) \left(\frac{1}{M_1} + \frac{M_3}{M_1^2} \Delta_\theta + O(\Delta_\theta^2) \right) \\ \text{ZZB}(\hat{y}_0) &= \frac{\Delta_y^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + N\Gamma_{1.5} \left(\frac{E}{4N} \right) \left(\frac{1}{M_2} + \frac{M_3}{M_2^2} \Delta_\theta + O(\Delta_\theta^2) \right). \end{aligned} \quad (22)$$

Because the image energy E is the total energy of all the pixels in the image, even for a small image such as an image of size 30×30 , we still have 900 samples, and therefore E is usually very large, compared with the variance of noise N for each pixel in the mid-to-high-SNR case. This implies that $Q(\sqrt{E}/(2N))$ tends to 0 and $\Gamma_{1.5}(E)/(4N)$ tends to 1.

Thus, the ZZB for the translation errors in (22) are dominated by the second term, which can be approximated as

$$\text{ZZB}(\hat{x}_0) \approx N \left(\frac{1}{M_1} + \frac{M_3}{M_1^2} \Delta_\theta + O(\Delta_\theta^2) \right). \quad (23)$$

$$\text{and} \\ \text{ZZB}(\hat{y}_0) \approx N \left(\frac{1}{M_2} + \frac{M_3}{M_2^2} \Delta_\theta + O(\Delta_\theta^2) \right). \quad (24)$$

When Δ_θ is large, e.g., larger than 45 degrees, the ZZB of translations x_0 and y_0 can not be simplified by the Taylor series expansion and are given by

$$\begin{aligned} \text{ZZB}(\hat{x}_0) &= \frac{\Delta_x^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + \frac{N \Gamma_{1.5} \left(\frac{E}{4N} \right)}{\Delta_\theta r \sqrt{c^2 - 1}} \left[\tan^{-1} \left(\frac{\tan(\Delta_\theta + \delta/2)}{\sqrt{\frac{c+1}{c-1}}} \right) \right. \\ &\quad \left. - \tan^{-1} \left(\frac{\tan(\delta/2)}{\sqrt{\frac{c+1}{c-1}}} \right) \right] \\ \text{ZZB}(\hat{y}_0) &= \frac{\Delta_y^2}{6} Q \left(\sqrt{\frac{E}{2N}} \right) \\ &\quad + \frac{N \Gamma_{1.5} \left(\frac{E}{4N} \right)}{\Delta_\theta r \sqrt{c^2 - 1}} \left[\tan^{-1} \left(\frac{\tan(\Delta_\theta + \delta/2)}{\sqrt{\frac{c-1}{c+1}}} \right) \right. \\ &\quad \left. - \tan^{-1} \left(\frac{\tan(\delta/2)}{\sqrt{\frac{c-1}{c+1}}} \right) \right]. \quad (25) \end{aligned}$$

In addition, because the terms M_1 , M_2 and M_3 can be written using Parseval's theorem as

$$\begin{aligned} M_1 &= \sum_{\mathbf{r}} f_x^2(\mathbf{r}) \\ M_2 &= \sum_{\mathbf{r}} f_y^2(\mathbf{r}) \\ M_3 &= \sum_{\mathbf{r}} f_x f_y(\mathbf{r}) \quad (26) \end{aligned}$$

where f_x and f_y denote the gradients of the noise free image along the x axis and y axis, respectively. The gradient of the image can be obtained by using a derivative filter, such as $[1 \ -1]$ and $[1 \ -1]^T$ for the horizontal axis and vertical axis, respectively. Thus, it can be concluded that the ZZB of the translations are mainly determined by the gradients of the images. Images with smaller gradients will produce larger registration errors, which is very reasonable because larger image gradients correspond to higher frequencies in the image. An image with higher frequencies contains more features and is easier to register resulting in smaller registration errors. Thus, the more sensitive the image is to the unknown transformation parameters, the better we should be able to estimate it. It implies that images with a smooth texture, such as those containing

lakes and sky, will yield a lower registration accuracy as registration of textured images with many edges will achieve higher registration accuracy.

2) *Rotation Error:* We derive the ZZB of rotation error in the frequency domain of the images. In the image model of (1), each pixel intensity is modeled as a Gaussian random variable with the mean equal to the noise-free pixel intensity and variance equal to N . Because Gaussian distribution in the time domain is still a Gaussian distribution in the Fourier domain due to the linear operation of Fourier transformation, each pixel in the Fourier domain of the image is also a Gaussian random variable. Then a polar transform is applied on the coordinate system in the Fourier domain of the image, which does not change the value of any pixel in the Fourier domain of the image. Thus, each pixel in the polar coordinate system of the Fourier domain of the image is still a Gaussian random variable. In the polar coordinate system for the images in the Fourier domain, $P_F(\rho, \theta)$, the rotation can be represented as a translation displacement [11]. This is an important feature used to estimate the rotation in the Fourier based image registration approach. Thus, we have [11], [12]

$$\begin{aligned} F_1(\rho, \theta) &= P_F(\rho, \theta) + n(\rho, \theta), \\ F_2(\rho, \theta) &= P_F(\rho, \theta - \theta_0) e^{-j\rho \cos(\theta)x_0 - j\rho \sin(\theta)y_0} + n(\rho, \theta) \end{aligned} \quad (27)$$

where $n(\rho, \theta)$ is Gaussian noise.

By applying the complete results of translation error in the translation only case in (21) here to obtain the ZZB on the rotation error, we have

$$\begin{aligned} \text{ZZB}(\hat{\theta}_0) &= \frac{\Delta_\theta^2}{6} Q \left(\sqrt{\frac{E_r}{2N}} \right) \\ &\quad + \frac{N}{M_r} \Gamma_{1.5} \left(\frac{E_r}{4N} \right) - \frac{32N^{1.5} \Gamma_2 \left(\frac{E_r}{4N} \right)}{3\sqrt{2\pi} \Delta_\theta M_r^{1.5}} \end{aligned} \quad (28)$$

where

$$\begin{aligned} E_r &= \frac{1}{4\pi^2} \int \int_{w_\rho w_\theta} |F_P(w_\rho, w_\theta)|^2 dw_\rho dw_\theta \\ M_r &= \frac{1}{4\pi^2} \int \int_{w_\rho w_\theta} |F_P(w_\rho, w_\theta)|^2 w_\rho^2 dw_\rho dw_\theta. \end{aligned} \quad (29)$$

$F_P(w_\rho, w_\theta)$ is the Fourier transform of $P_F(\rho, \theta)$.

When the uncertainty of rotation misalignment is not very small, the third term can be omitted due to the exponent of M_r in its denominator being more than 1 and therefore, the ZZB on the rotation error is simplified as

$$\text{ZZB}(\hat{\theta}_0) \approx \frac{\Delta_\theta^2}{6} Q \left(\sqrt{\frac{E_r}{2N}} \right) + \frac{N}{M_r} \Gamma_{\frac{3}{2}} \left(\frac{E_r}{4N} \right). \quad (30)$$

For most images, E is much larger than N in mid-high-SNR case, and therefore rotation error can be approximated as

$$\text{ZZB}(\hat{\theta}_0) \approx \frac{N}{M_r}. \quad (31)$$

Fig. 1. Example Images 1. (a) Lena image. (b) Brain image.

From the result of ZZB for rotation misalignment, we observe that the estimation accuracy of rotation misalignment does not depend on translation misalignment uncertainty. It is mainly determined by the image content itself as well as the noise power while the ZZB of translation misalignment also depends on the uncertainty of the rotation misalignment besides the image content and noise power. Next we will compute ZZBs and compare them with the MSE of registration achieved by two registration methods for a few datasets.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Experiment 1

The performance bounds derived above serve as performance limits of image registration algorithms. For illustration purposes, we conduct Monte Carlo (MC) simulation experiments where we compute the actual MSE for a set of image registration algorithms and compare them to the ZZB. Two images, “Lena” and “Brain,” in Fig. 1 are used for simulations. The image “Lena” is of size 53×53 and the image “Brain” is of size 90×108 . For each image, we generated a pair of images with a random translation along the x axis and random translation along the y axis uniformly distributed in $[0, T]$ using bilinear interpolation for the translation only case, where the parameter T is 30 for the “Lena” image and 70 for the “Brain” image. In the rigid body transformation case, we generated a pair of images with random translations along the x and y axes and a random rotation angle, uniformly distributed in $[0, T]$ and $[0, 90^\circ]$, respectively. Then white Gaussian noise was added to the image pair prior to registration. The mutual information (MI) based method [13], [14] and the mean-square difference (MSD) based method [3] were used to estimate the translation and rotation errors. For the MI based method and the MSD based method, we used an exhaustive search in the search space of $[0, T]$ and $[0, 90^\circ]$ to find the optimal parameters which maximized the MI and minimized the MSD respectively. The whole process was repeated 500 times at each noise power value. Then we computed the MSE of the translation and rotation estimates and compared them with the ZZBs given by (25) and (30). The results for the two cases, translation only and rigid body transformation, are shown in Figs. 2–4. Since the results for translation error along x axis and the translation error along y axis are very similar, we only show the results for translation error along x axis in our paper. The SNR in the figures is defined as the normalized image energy divided by the noise power. It is observed that the ZZB is able to capture the threshold effect and tracks the actual MSE performance fairly well. Among the two registration methods, the MSD based method obtains smaller registration error than

Fig. 2. Performance results for translation error along the x axis in the translation only case. (Translations are uniformly distributed in $[0, 30]$ for Lena and $[0, 70]$ for Brain, respectively.) (a) Lena image. (b) Brain image.

the MI based method. The bounds give us the best achievable performance. In other words, no image registration algorithm can achieve MSE for the image pair that is less than this bound. The gap between the bounds and the registration error implies that there may exist better registration algorithms for these image pairs. Also, there may exist tighter bounds based on more accurate image models.

B. Experiment 2

As discussed in Section II and stated in [5], image registration performance is highly influenced by the image content. In this experiment, we conducted experiments using images with different textures and show how image registration performance using different image content is predicted by ZZB. We generated three synthetic images in this experiment. Image (a) is a sine wave embedded in a homogeneous background with size 300×300 ; image (b) is a sine wave embedded in a textured background filled with random points with size 300×300 ;

(a)

(b)

Fig. 3. Performance results for translation error along the x axis in the rigid body transformation case. (Translations are uniformly distributed in $[0, 30]$ for Lena and $[0, 70]$ for Brain, respectively. Rotation angles are uniformly distributed in $[0, 90]$ for both images.) (a) Lena image. (b) Brain image.

(a)

(b)

Fig. 4. Performance results for rotation error in the rigid body transformation case. (Translations are uniformly distributed in $[0, 30]$ for Lena and $[0, 70]$ for Brain, respectively. Rotation angles are uniformly distributed in $[0, 90]$ for both images.) (a) Lena image. (b) Brain image.

Image c is a sine wave embedded in a grid background with size 300×300 . The three images are shown in Fig. 5. The translations are assumed to be uniformly distributed in $[0, 100]$ and the rotation angle is assumed to be uniformly distributed in $[0, 90^\circ]$. We calculate the ZZB for the three images at different SNR levels and compare them with the actual registration errors achieved by the MSD method, which produced the registration error closest to the bound in experiment 1. We use the same MC procedure as described in experiment 1 to obtain the actual registration error. As shown in Fig. 6, the actual registration error approaches the ZZB. Note that for translation misalignment, ZZBs of the image with the grid background and the image with the random-point background are smaller than the image with homogeneous background, and the ZZB of image with the grid background is the smallest among the three in the mid to high-SNR range. This implies that adding texture in the

background results in a decrease for the translation misalignment estimation error and particularly, adding the grid texture achieves higher registration accuracy than adding the texture of random points. However, for the rotation misalignment, adding the random point texture does not affect the ZZB of the rotation estimation while adding the grid texture decreases the ZZB. It is because the random point texture is invariant to the rotation misalignment while the grid texture is sensitive to the rotation misalignment. The actual registration error calculated by the MSD method supports this conclusion. As observed in Fig. 6, registration error of the three images with different texture backgrounds have the same ordering as the ZZBs of the three images. Thus, adding the grid texture to the image can significantly improve the registration performance of the image with both translation misalignment and rotation misalignment while adding the random point texture to the

Fig. 5. Example images 2. (a) Size: 300×300 , without texture, (b) Size: 300×300 , with random point texture, (c) Size: 300×300 , with grid texture.

image can only improve the registration performance of the image with translation misalignment. When the SNR is below -35 dB, registration errors are controlled by the uncertainty of transformation parameters and registration errors of all the images with different texture backgrounds tend to converge. Thus, adding texture to images does not provide any value when the SNR is below -35 dB.

In addition, to evaluate the registration performance of images with different sizes, we generated three other synthetic images which are the shrunk version of images (a), (b), (c) with the reduced size of 30×30 . The translations are assumed to be uniformly distributed in $[0, 10]$ and the rotation angle is assumed to be uniformly distributed in $[0, 90^\circ]$. We plot the ZZBs for these three shrunk images and the three original images (a), (b), (c). The results are shown in Fig. 7. It is observed that image resolution is an important factor in image registration. Images (a), (b), and (c) have ten times higher resolution than the three shrunk images, which results in their translation error being 5%–10% of that of the image with lower resolution and their rotation error is about 0.1% of that of images with lower resolution in the high-SNR range. Adding texture to the image with low resolution does not improve its registration performance as much as that to the image with high resolution.

C. Discussion

In our experiments, we have used some approximations to obtain the ZZB values. First, the image data was assumed to be continuous while deriving the ZZB. In practice, an image is sampled to yield a discrete data set leading to a violation of the assumption. We used a gradient filter to approximate the gradient of the image, which may produce certain errors. In addition, note that when the misalignment is not an integer, the interpolation

Fig. 6. Ziv-Zakai bounds and registration errors for synthetic images of size 300×300 . (Translations and rotation angle are uniformly distributed in $[0, 100]$ and $[0, 90]$, respectively.) (a) Translation along the x axis. (b) Rotation.

used while generating the misaligned image pair introduces artifacts, which produces a certain model bias. In the registration experiments, the same interpolation operation is used to calculate the MI and MSD, which minimizes the error due to interpolation in our experiments but is still a source of errors. In addition, since both images contain noise, when we applied transformation to one noisy image, the interpolation caused the noise correlation among neighboring pixels and introduces the registration error at the subpixel level. This factor also contributes to the gap between the MSE of the MSD method and the bound in the mid to high-SNR range, e.g., $\text{SNR} \geq -10$ dB. Second, when we calculate the ZZB of rotation misalignment, interpolation error is introduced during the computation of the polar transform, which is ignored in practice. The above approximations contribute to the gap in the experiments. Although the exact ZZB is hard to obtain, we can still use it as a benchmark for different registration methods.

Fig. 7. Ziv-Zakai bounds for synthetic images. (For 300*300 image, translations and rotation angle are uniformly distributed in $[0, 100]$ and $[0, 90]$, respectively; for 30*30 image, translations and rotation angle are uniformly distributed in $[0, 10]$ and $[0, 90]$, respectively.) (a) Translation along the x axis. (b) Rotation.

IV. CONCLUSION AND FUTURE WORK

In this paper, we considered the problem of image registration performance evaluation from a statistical estimation point of view. We focused our attention on the general rigid body transformation cases. Performance bounds have been evaluated to determine the performance limits of image registration algorithms for a given pair of images. When the image pair is given and the type of transformation is known, one can obtain the minimum achievable registration error in terms of the MSE of the transformation parameters by this technique. This paper assumed knowledge of the *a priori* distribution of the transformation parameters and obtained a closed-form solution for the ZZB for the rigid body transformation distortion case. Our experimental results demonstrate that our developed bounds can predict the performance of the image registration problem. From our ZZB results, we can conclude that the registration errors

mainly depend on the image content and the noise power in the translation only distortion case. However, in the presence of rotation distortion, registration errors are spread over a rotation interval and the registration error increases as the uncertainty of the rotation increases. Thus, if we know the rotation is limited in a small range in advance, a more accurate registration is expected while the prior information regarding the translation does not benefit the estimation of rotation misalignment.

In practice, the derived bound can be used to evaluate how well the image registration approaches are likely to perform for a particular image pair. It offers guidance to evaluate the registration ability of an image pair. When the bound is compared with the actual MSE of a registration algorithm, a small gap between them implies the superiority of this registration algorithm while a large gap indicates the need for improvement of this registration algorithm or selection of another registration algorithm. In addition, in certain applications, one may have a set of reference images available for registration of an acquired image. The bounds can help us select a reference image which can best perform the image registration. The one which produces the smallest ZZB should be selected because it may produce the smallest registration error. Comparing the ZZBs for images with different textures provides us an idea on how the content of the image affects the image registration performance. According to our experimental results on ZZB, we conclude that images with textured backgrounds achieve higher registration accuracy than those with homogeneous backgrounds in the high-SNR range. In the low SNR case, registration error is controlled by the uncertainty of the transformation parameters, and therefore, images with textured background do not have much advantage over those with homogeneous background. For images with low resolution, the textured background may have less effect on the registration performance. In addition, the image with grid textured background is more sensitive to the rotation distortion and, therefore, it produces significantly higher registration accuracy for images with rotation distortion.

Note that the image model considered here was an ideal statistical model. We assumed that each pixel in the image has i.i.d Gaussian distribution as in [5], [4]. However, as we know, the pixels in an image are spatially correlated. In particular, in high-resolution images, the neighboring image pixels are highly correlated. Thus, it is important to incorporate the spatial correlation into an image model in the future. The development of performance bounds based on this model is desired. Moreover, when images are sampled below the Nyquist rate, aliasing noise will be introduced [15]. We did not consider the aliasing effect in our model. It would be interesting to study this issue based on the bounds.

APPENDIX A

In this Appendix, we detail the steps involved to obtain (10). Assuming $F(w_x, w_y)$ to be the Fourier transform of the image $f(x, y)$, the Fourier transform of the image $f(u(x, y), v(x, y))$ is

$$F(w'_x, w'_y)e^{-j\Phi(w_x, w_y)} \quad (32)$$

where

$$\begin{aligned} w'_x &= \cos(\theta_0)w_x + \sin(\theta_0)w_y \\ w'_y &= -\sin(\theta_0)w_x + \cos(\theta_0)w_y \end{aligned} \quad (33)$$

$u(x, y)$ and $v(x, y)$ are given in (2) and $\Phi(w_x, w_y)$ is the spectrum phase of the image. The energy function E is defined by $E = \int_x \int_y f^2(x, y) dx dy = 1/(4\pi^2) \int_0^{+\infty} \int_0^{+\infty} 4|F(w, v)|^2 dw dv$. Thus, the correlation coefficient is expressed as

$$\begin{aligned} \rho(h) &= \frac{1}{E} \int_x \int_y f(u(x, y), v(x, y)) \\ &\quad f(u(x, y) - h, v(x, y)) dx dy \\ &= \frac{1}{\pi^2 E} \int_0^{+\infty} \int_0^{+\infty} |F(w'_x, w'_y)|^2 \cos(w_x h) dw_x dw_y. \end{aligned} \quad (34)$$

By using the inequality $1 - \cos(x) \leq x^2/2$, $1 - \rho(h)$ is bounded by

$$1 - \rho(h) \leq \frac{1}{2\pi^2 E} \int_0^{+\infty} \int_0^{+\infty} |F(w'_x, w'_y)|^2 w_x^2 h^2 dw_x dw_y. \quad (35)$$

From (33), it is easy to obtain

$$\begin{aligned} w_x &= \cos(\theta_0)w'_x - \sin(\theta_0)w'_y \\ w_y &= \sin(\theta_0)w'_x + \cos(\theta_0)w'_y. \end{aligned} \quad (36)$$

Thus, substituting (36) into (35), we can obtain

$$\begin{aligned} 1 - \rho(h) &\leq \frac{1}{2\pi^2 E} \int_0^{+\infty} \int_0^{+\infty} |F(w'_x, w'_y)|^2 (\cos(\theta_0)w'_x \\ &\quad - \sin(\theta_0)w'_y)^2 h^2 dw'_x dw'_y \\ &= \frac{h^2}{2E} (\cos^2(\theta_0)M_1 + \sin^2(\theta_0)M_2 \\ &\quad - 2\sin(\theta_0)\cos(\theta_0)M_3) \\ &= \frac{h^2}{2E} \left(\frac{M_1 + M_2}{2} + \frac{M_1 - M_2}{2} \cos(2\theta_0) \right. \\ &\quad \left. - \sin(2\theta_0)M_3 \right) \\ &= \frac{h^2}{2E} r(c + \cos(2\theta_0 + \delta)). \end{aligned} \quad (37)$$

Since $1 - \rho(h) \leq 1$, $1 - \rho(h)$ is lower bounded by the minimum of (37) and 1, that is, $1 - \rho(h) \leq \min\{(h^2\beta^2)/(2), 1\}$ assuming $\beta = \sqrt{r(c + \cos(2\theta_0 + \delta))}/E$. And therefore, it yields

$$P_e(\theta_0, h) \geq \begin{cases} Q\left(\frac{h}{2}\sqrt{\frac{E\beta^2}{N}}\right), & 0 \leq h \leq \frac{\sqrt{2}}{\beta} \\ Q\left(\sqrt{\frac{E}{2N}}\right), & h > \frac{\sqrt{2}}{\beta}. \end{cases} \quad (38)$$

APPENDIX B

In this Appendix, we show the derivation to obtain the closed-form solution for term g_1 . Since $B(\theta_0, h)(h^2)/(2)|_0^{(\sqrt{2})/(\beta)} = 0$, $g_1(h)$ can be expressed as

$$\begin{aligned} g_1(h) &= \int_0^{\Delta_\theta} \frac{1}{\Delta_\theta} \left\{ B(\theta_0, h) \frac{h^2}{2} \Big|_0^{\frac{\sqrt{2}}{\beta}} \right. \\ &\quad \left. + \int_0^{\frac{\sqrt{2}}{\beta}} \frac{1}{2\sqrt{2\pi}} \beta \sqrt{\frac{E}{N}} \exp\left\{-\frac{h^2\beta^2 E}{8N}\right\} \frac{h^2}{2} dh \right\} d\theta_0 \\ &= \frac{1}{\Delta_\theta 4\sqrt{2\pi}} \int_0^{\Delta_\theta} \int_0^{\frac{\sqrt{2}}{\beta}} \beta \sqrt{\frac{E}{N}} \exp\left\{-\frac{h^2\beta^2 E}{8N}\right\} h^2 dh d\theta_0. \end{aligned} \quad (39)$$

Let $h' = (h^2\beta^2 E)/(8N)$, and $g_1(h)$ can be written by

$$g_1(h) = \frac{1}{\Delta_\theta} \int_0^{\Delta_\theta} \int_0^{\frac{E}{4N}} \frac{2N\sqrt{h'} \exp\{-h'\}}{\sqrt{\pi}\beta^2 E} dh' d\theta_0. \quad (40)$$

The incomplete gamma function $\Gamma_a(b)$ is equal to $(1)/(\Gamma(a)) \int_0^b t^{a-1} \exp\{-t\} dt$ and $\Gamma(a) = (\sqrt{\pi})/(2)$. Thus, (40) can be simplified as

$$\begin{aligned} g_1(h) &= \frac{1}{\Delta_\theta} \int_0^{\Delta_\theta} \frac{N\Gamma_{1.5}\left(\frac{E}{4N}\right)}{\beta^2 E} d\theta_0 = \frac{N\Gamma_{1.5}\left(\frac{E}{4N}\right)}{\Delta_\theta r\sqrt{c^2 - 1}} \\ &\quad \times \left[\tan^{-1}\left(\frac{\tan(\Delta_\theta + \delta/2)}{\sqrt{\frac{c+1}{c-1}}}\right) \right. \\ &\quad \left. - \tan^{-1}\left(\frac{\tan(\delta/2)}{\sqrt{\frac{c+1}{c-1}}}\right) \right]. \end{aligned} \quad (41)$$

APPENDIX C

In this Appendix, we use the Taylor series expansion to simplify term g_1 .

According to Taylor's theorem, a function $q(t) = \tan^{-1}((\tan(t))/K)$ can be expressed in a Taylor series as

$$\begin{aligned} q(t) &= q(a) + \frac{\partial q(t)}{\partial t} \Big|_{t=a} (t-a) \\ &\quad + \frac{\partial^2 q(t)}{\partial t^2} \Big|_{t=a} \frac{(t-a)^2}{2} + O(t-a)^3 \\ &= \tan^{-1}\left(\frac{\tan(a)}{K}\right) + \frac{K}{K^2 \cos^2 a + \sin^2 a} (t-a) \\ &\quad + \frac{K(K^2 - 1) \sin(2a)}{(K^2 \cos^2 a + \sin^2 a)^2} \frac{(t-a)^2}{2} + O(t-a)^3. \end{aligned} \quad (42)$$

Let $K = \sqrt{(c+1)/(c-1)}$, then $t = \Delta_\theta + \delta/2$ and $a = \delta/2$. When t is small, $\tan^{-1}((\tan(\Delta_\theta + \delta/2))/(\sqrt{(c+1)/(c-1)}))$ can be written in a Taylor series expansion as

$$\tan^{-1}\left(\frac{\tan(\Delta_\theta + \delta/2)}{\sqrt{\frac{c+1}{c-1}}}\right) = \tan^{-1}\left(\frac{\tan(\delta/2)}{\sqrt{\frac{c+1}{c-1}}}\right) + \frac{r\sqrt{c^2-1}}{M_1}\Delta_\theta + \frac{r\sqrt{c^2-1}M_3}{M_1^2}\Delta_\theta^2 + O(\Delta_\theta^3). \quad (43)$$

Substituting (43) into (16), we can obtain

$$\begin{aligned} g_1(h) &= \frac{N\Gamma_{1.5}\left(\frac{E}{4N}\right)}{\Delta_\theta r\sqrt{c^2-1}} \left(\frac{r\sqrt{c^2-1}}{M_1}\Delta_\theta + \frac{r\sqrt{c^2-1}M_3}{M_1^2}\Delta_\theta^2 + O(\Delta_\theta^3) \right) \\ &= N\Gamma_{1.5}\left(\frac{E}{4N}\right) \left(\frac{1}{M_1} + \frac{M_3}{M_1^2}\Delta_\theta + O(\Delta_\theta^2) \right). \quad (44) \end{aligned}$$

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REFERENCES

- [1] B. Zitov'a and J. Flusser, "Image registration methods: A survey," *Image Vision Comput.*, vol. 21, pp. 977–1000, 2003.
- [2] L. G. Brown, "A survey of image registration techniques," *ACM Comput. Surv.*, vol. 24, no. 4, pp. 325–376, Dec. 1992.
- [3] P. K. Varshney, B. Kumar, M. Xu, A. Drozd, and I. Kasperovich, "Image registration: A tutorial," presented at the NATO ASI, Bulgaria, 2005.
- [4] I. S. Yetik and A. Nehorai, "Performance bounds on image registration," *IEEE Trans. Signal Process.*, vol. 54, no. 5, pp. 1737–1749, May 2006.
- [5] D. Robinson and P. Milanfar, "Fundamental performance limits in image registration," *IEEE Trans. Image Process.*, vol. 13, no. 9, pp. 1185–1199, Sep. 2004.
- [6] K. L. Bell, Y. Steinberg, Y. Ephraim, and H. L. Van Trees, "Extended Ziv-Zakai lower bound for vector parameter estimation," *IEEE Trans. Inf. Theory*, vol. 43, pp. 624–637, Mar. 1997.
- [7] H. L. Van Trees and K. L. Bell, *Bayesian Bounds for Parameter Estimation and Nonlinear Filtering/Tracking*. New York: Wiley-IEEE Press, Aug. 2007.
- [8] M. Xu and P. K. Varshney, "Tighter performance bounds on image registration," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Toulouse, France, May 2006.
- [9] J. Ziv and M. Zakai, "Some lower bounds on signal parameter estimation," *IEEE Trans. Inf. Theory*, vol. IT-15, no. 3, pp. 386–391, 1969.
- [10] M. Zakai, D. Chazan, and J. Ziv, "Improved lower bounds on signal parameter estimation," *IEEE Trans. Inf. Theory*, vol. IT-21, no. 1, pp. 90–93, Jan. 1975.
- [11] B. S. Reddy and B. N. Chatterji, "An FFT-based technique for translation, rotation and scale-invariant image registration," *IEEE Trans. Image Process.*, vol. 5, no. 8, pp. 1266–1271, Aug. 1996.
- [12] H. Liu, B. Guo, and Z. Feng, "Pseudo-log-polar Fourier transform for image registration," *IEEE Signal Process. Lett.*, vol. 13, no. 1, pp. 17–20, Jan. 2006.
- [13] H. Chen and P. K. Varshney, "Mutual information based CT-MR brain image registration using generalized partial volume joint histogram estimation," *IEEE Trans. Med. Imag.*, vol. 22, no. 9, pp. 1111–1119, Sep. 2003.
- [14] H. Chen, "Mutual information based image registration with applications," Ph.D. dissertation, Elect. Eng. and Comput. Sci. Dept., Syracuse Univ., Syracuse, NY, May 2002.
- [15] H. S. Stone, M. Orchard, and E.-C. Chang, "Subpixel registration of images," in *Proc. 33rd Asilomar Conf. Signals, Systems, Computers*, 1999, vol. 2, pp. 1446–1452.

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