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APPLYING GROUP THEORY FOR SOLVING MACHINES' TURNOVERS WITH CAPITAL BUDGETING APPROACH

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Abstract— *Applying and utilizing technology in modern manufacturing systems needs to update and upgrade facilities repetitively by efficient ways to stay with great productivity along efficiency. Capital Budgeting Problem (CBP) is one of the most important issues in decision making about capital in manufacturing management; CBP determines the best investments in capital goods such as new plans, replacements, research development projects and other worth pursuing authorization subjects. This problem deals with the amount of revenues as the result of investing in the previous periods in order to maximize the amount of Net Present Value (NPV) on the decision time. Making discussions about investments and its related effects on revenues is the purpose of this paper. Group theory with network approach as an exact mathematical discipline is used to solve such an Integer Programming Model (IPM). An example with predicted maintenance or improvement options of distinctive machines is supposed in order to clear this model.*

Keywords: *Machine's Turnovers, Capital Budgeting, Integer Programming Model, Group Theory*

I. Introduction

Each operation system consists of software, intelligent to unintelligent resources such as buildings, machines, processing tools, human resource and some other needed installations. By agile worldwide modernization direction in manufacturing, most assets in the producing services have converted to buildings, productions' facilities and functions. Therefore, machines as specified assets in the production processes have become as pulse for organizations. It might occur different usual (planned) or unusual (unplanned) cases for machines in the manufacturing system that cause to face with choosing appropriate options among varied alternatives to stay with flexibility steadily. Some conditions may not provide enough knowledge about decision making for critical situations. Since manufacturing systems try to use efficient scheduling procedures with minimum cost, so a view on the future horizon could bring more efficiency and less expenditure for making decisions about such situations. We could refer to some articles in the literature, which highlighted the importance of scheduling for machinery processes. Qi et al. [9] tracked a rescheduling on anticipated disruption for a shortest processing time in single and parallel machines. Akturk and Gorgulu [1] introduced a new rescheduling strategy through a feedback mechanism by determining the time critical decision making concept for machine breakdowns. The difficulty and time consuming of these rescheduling problems encountered in the large size manufacturing industry aroused applying some heuristic, metaheuristic and simulation-based methods to solve these kinds of problems with partly good solutions in acceptable run time. In this case, Smith [12] surveyed designing manufacturing's operations with simulation. While, there is a great need for improvements in scheduling operations in complex and turbulent manufacturing environments, it is often difficult to measure the quality of the scheduling without knowing the optimum.

In addition to breakdowns, more participated matters related to machines with the need to investigate about the current conditions and the effects of them on the future situations should be considered. It could be said that if there is any breakdown in a system, a planning about buy, repair, update, add on, remove or in other words, some rescheduling is needed. The purpose of our work in this paper is to maximize the amount of revenues earned from making decisions about alternative reactive scheduling methods in different periods. This approach is similar to financial management that follows the mentioned objective entitled Capital Budgeting (CB) which have control on investments. The aim of investments is to produce future cash flows or gaining more assets; this profit could be tangible (like the earned profit from using buildings or machinery) or intangible (the usefulness of schooling or training). As Kemp and Dunbar [8] mentioned budgeting is more than a job for financial management in manufacturing system or a firm, this kind of planning prevents to catch the errors in the plans and will typically reduce the costs of a project by about a factor of ten in any kind of organizations.

Generally, the assets and liabilities are divided into three categories; accounts receivable, accounts payable and inventory. Regarding to this fact that most works in nowadays modern systems are depended on machines, we suppose the machine as special possesses of the system. So we could plan on assets and modifying their usage to gather more profit of possessions. Related decisions of this plan majority are divided into short-term and long-term classes. The short term decisions are entitled as Net Working Capital (NWC) which considers the balance operating liquidity with the current assets minus the current liabilities in a short period like in upcoming next year. Besides the short term, the long term decisions are well known as Capital Investment Decisions (CID) which determines whether to invest with equity or debt, or to pay dividends to shareholders and when is the suitable time for these two ways. There are variety methods for evaluating CB, among them NPV regards the Time Value of Money (TVM) which is an important concept in financial management. As the first time Dean [4] expressed that accounting implies a historical record, whereas economic problems do it with determining the future, so NPV is a proper method among the other methods that involves TVM.

The objective function of CB in current case is similar to the objective of scheduling problem in Operational Fixed Job Scheduling (OFJS) where each job has a weight representing the value for the decision making. Different models of budgeting approaches were arisen in the literature. The Weingartner's model as the essence for mathematical model of capital budgeting is developed in Weingartner's research [13]. This model had been followed by large researchers, ere we point out the most significant works of them. Elton [5] discussed about the importance of the external discount rate in capital rationing in the cases of no external borrowing opportunities but the access to the capital market. Weingartner [14] gathered some methods in this field. Afterwards Badell et.al [2] provided a mixed integer linear programming model of cash flow management in short period for scheduling and budgeting. In the uncertain and ambiguous environment, a stochastic framework for zero-one goal programming in this problem is developed by De et al. [3]. Huang [7] implied NPV in the random fuzzy project selection frame and used a hybrid genetic algorithm for solving it. A fuzzy multi-objective decision making containing the information system, research and development and project selection problems in CB is designed in Rabban and Seraj model [10].

Here, we try one exact approach that has received less attention among the other exact mathematical algorithms to solve CB. We believe that his approach named group theory with network base design can proceed in small size of CBP.

The rest of this paper is organized as follows; a mathematical model is discussed in the next section then, a group theory concept is defined in section III. Afterwards, section IV shows a numerical example of proposed problem. Finally, the points of this paper are concluded in section V.

II. Mathematical Model of CB

The essence of CB mathematical model of Weingartner for linear programming of capital budgeting could be written as follow [13]:

$$max \sum_{j=1}^{J} \sum_{t=0}^{T} \left[a_{jt} / (1+k)^t \right] x_j \qquad (1\text{-}1)$$

$$s.t.$$

$$-\left[\sum_{j=1}^{J} a_{jt} x_j \right] \leq M_t \quad , \quad t = 0,1,\dots,T \qquad (1\text{-}2)$$

$$x_j \geq 0 \; , \; j = 1,\dots,\mathrm{J} \qquad (1\text{-}3)$$

Where k is fixed discount rate (cost of capital) which should be internally determined and independent of monetary phenomena, because our knowledge is about capital rationing. $(1+k)^t$ is internal discounting factor. a_{jt} is the net cash flow which possibly negative and obtained from project j in period t. x_j is the number of jth project units that could be considered. M_t is the fixed amount of available cash at period t.

This model can be interpreted as a rescheduling model for maximizing machines' turnover when there are re-planning or fixing alternatives in different periods as it is presumed in the current paper.

Weingartner model did not determine the related detail of future expenditure based on current returns from pervious spent alternatives. Moreover, there is a fixed amount of internal rate of return for all periods that mentioned rate could be modified for all periods. On the other hand, we developed a model that is an expanded version of Weingartner model. Prepared mathematical model formulas are as follows.

Notifications:

h, t: Planning horizon $h, t = 1,2,\dots,T$

i: Alternatives index $i = 1,2,\dots,N$

t_i: Identifier of ith alternative in tth period

CI_{t_i}: Needed capital to invest in t_i

b_t: Interest rate in tth period

$R^h_{t_i}$: Revenue of t_i that would be backed in hth period

FF: The first capital

RFF_{t_i}: Returned profit of t_i to the decision time

$$RFF_{1_0} = FF \ , \ RFF_{t_0} = RFF_{(t-1)_N} \ , \qquad \forall t \geq 2$$

CF_{t_i}: Fixed capital in t_i (we supposed it consistent for all periods)

M: Enough large number

X_{t_i}: Decision variable, would be invested on t_i project

$$max\left(\sum_t \sum_i \sum_{h>t} \left(R^h_{t_i} \times \frac{1}{(1+b_t)^{h-t}} - CI_{t_i} - CF_{t_i}\right) \times \frac{X_{t_i}}{(1+b_0)^t}\right) + RFF_{T_N} \qquad (2\text{-}1)$$

$s.t.$

$$\left(RFF_{t_i} \times (1+b_0)^t - CI_{t_i} - CF_{t_i}\right) \times \left(\sum_{h>t} R^h_{t_i} \times \frac{1}{(1+b_t)^{h-t}} - CI_{t_i}\right)$$

$$.< MX_{t_i} \quad \forall t, \forall i \qquad (2\text{-}2)$$

$$RFF_{t_i} = \left(RFF_{t_{i-1}} \times (1+b_0)^t - \left(CI_{t_i} + CF_{t_i}\right) \times X_{t_i}\right) \frac{1}{(1+b_0)^t} \ , \forall t, \forall i \qquad (2\text{-}3)$$

$$X_{t_i} \geq 0 \ and \ integer \ , \ \forall t, \forall i \qquad (2\text{-}4)$$

(2-1) as the objective function maximizes NPV regarding to all returned profits and all investments. (2-2) demonstrates that if there are both enough cash flow (the first parenthesis) and positive NPV in budgeting time (the second parenthesis), then the related alternative would be selected. (2-3) calculates the returned amount of profits, which are invested respect to the plan period and TVM. Fig.1 shows an example of our study in limited periods. For example, P_{11} is the needed cost for selecting the first project on the first period, and R_{112} is paying back the amount of P_{11} on the second period.

III. Group Theory Background

Gomory [6] showed that any IPM may be solved by first solving its linear program adjoining a new constraint which cuts off the current solution and optimizes it. He indicated that the coefficient vectors of the derived inequalities from a finite set are closed under the operation of addition when the arithmetic operations are taken module 1, namely integer parts are removed. It could be said that this set forms a group. By relaxing non negativity, but not integrality constraints on certain variables, an integer program may be transformed to ones whose columns of constraint coefficients and right hand side are elements of an Abelian group which has the commutative property (If g_1, g_2 are in the group, then $g_1 + g_2 = g_2 + g_1$). As it was described in Salkin's research [11]; group theory is concerned with systems that always there is an unique solution for $a \bullet x = b$. The theory does not concern itself what a and b and operation symbolized by $\bullet$ actually are. By taking this abstract approach, group theory requires only a mathematical system obey a few simple rules, like:

a) Closure: If a and b are in the group, then $a \bullet b$ is also in the group,
b) Associativity: If a, b and c are in the group, then $(a \bullet b) \bullet c = a \bullet (b \bullet c)$,
c) Identity: There is an element e of the group such that for any element a of the group $a \bullet e = e \bullet a = a$,
d) Inverse: For any element a of the group, there is an element a^{-1} such that $a \bullet a^{-1} = e$ and $a^{-1} \bullet a = e$.

It can be indicated that if a and b are in the group then $a \bullet b$ is also in the group. Group theory is a clear example of abstraction in modern mathematics. The

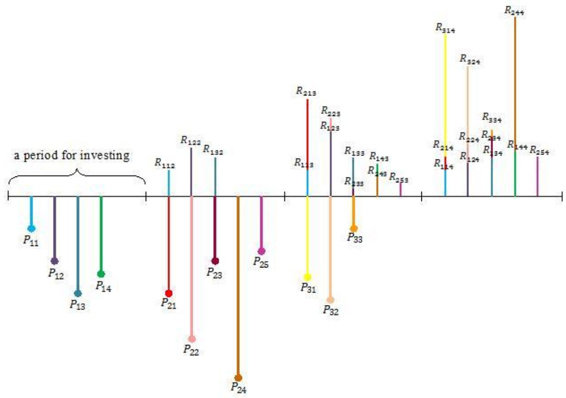

Figure1. An example of rescheduling alternatives and related revenues.

basic quality of groupness that is common in all groups demonstrates the validity for all groups. There are variety numbers of examples for showing the concept of group theory. The most familiar of them come from elementary arithmetic. For instance, the integers $\mathbb{Z}^+$ form a group under the operation of addition; 0 is the identity and each negative number plays the inverse characteristic for each positive number. Another example is non zero rational numbers with the group operation multiplication $(\mathbb{Q}^+, +)$, obviously negative rational numbers does not form a group under multiplication. Similarly, the real numbers $(\mathbb{R}^+, +)$, the complex numbers are groups under addition and their none-zero elements form a group under multiplication $(\mathbb{R}^+, \times)$. Also, even set $\{1, -1\}$ together with the operation multiplication is a group.

If a group problem is solved and its solution yields non negative values for the variables of the original problem, then it could be shown that the integer program is solved. Discussion on the geometry of the feasible region of the group problem indicated that its bounding hyper planes will yield cutting planes that are often very strong. As [11] [11] illustrated the group problem can be treated as the integer program problem with one constraint and also as the network problem. The algorithms designed to solve the integer programming formulation are usually the dynamic programming type. However, the network derived from the group problem has the special structure and attracts a lot of researchers' interests. Noteworthy when the number of elements in the group increases, every solving algorithm naturally becomes substantially less efficient, but it could be guaranteed that the optimal solution to the group problem procedures an optimal solution to the integer program.

Salkin [11] explained forming Group Mathematic Problem (GMP) and its relationship with Gomory cutting tools. Different procedures to solve this model are expanded in this reference that readers are referred to it for reading more. We drew some basic information to form a GMP that is described incoming lines. Suppose an integer program formed in the model set (3).

$$max\ c_B B^{-1}b - (c_B B^{-1}N - c_N)x_N \qquad (3\text{-}1)$$
$$s.t.$$
$$x_B = B^{-1}b - B^{-1}Nx_N \qquad (3\text{-}2)$$
$$x_B \geq 0,\ x_N \geq 0 \ and\ integer \qquad (3\text{-}3)$$

Hence constant term $c_B B^{-1}b$ does not influence the maximization, so we could drop it. Moreover x_B is as an integer variable that is equivalent to $x_B \equiv 0\ modulo\ 1$. Subsequently, the problem set (3) could be transferred to minimizing model (4).

$$min(\ c_B B^{-1}N - c_N)x_N \qquad (4\text{-}1)$$
$$s.t.$$
$$B^{-1}Nx_N \equiv B^{-1}b \quad (mod1) \qquad (4\text{-}2)$$
$$x_N \geq 0\ and\ integer \qquad (4\text{-}3)$$
$$and \quad x_B = B^{-1}b - B^{-1}Nx_N \geq 0 \qquad (4\text{-}4)$$

If $B^{-1}Nx_N = \sum_{j=1}^{n} \alpha_j\ x_{J(i)}$ where α_j is the jth column of $B^{-1}N$, and $x_{J(i)}$ is jth $(1 \leq j \leq n)$ nonbasic variable and if $B^{-1}b = \alpha_0$, then the congruence relationship becomes $\sum_{j=1}^{n} \alpha_j\ x_{J(j)} \equiv \propto_0\ (mod1)$. Then the GMP is derived from set (4) as follow:

$$min \sum_{j=1}^{n} \bar{c_j} x_{J(j)} \qquad (5\text{-}1)$$
$$s.t.$$
$$\sum_{j=1}^{n} \bar{\propto}_j\ x_{J(j)} \equiv \bar{\propto}_0 \quad (mod1) \qquad (5\text{-}2)$$
$$x_{J(j)} \geq 0\ and\ integer \quad j = (1,2,\ldots,n) \qquad (5\text{-}3)$$

We can represent any group minimization problem in the terms of a network. Since a network is a collection of nodes and arcs that joins them, so that each element in the group $G(\bar{\alpha})$ corresponds to a distinct node. A directed arc (k,l) joins two nodes representing group elements g_k and g_l. Whenever $g_l - g_k\ (mod\ 1)$ is equal to some $\bar{\alpha}_j$ $(j \geq 1)$. Traversing an arc from node k to node l corresponds to increase $x_{J(j)}$, so the cost $\bar{c}_j$ is assigned to arc (k,l), so GMP can reduce to finding shortest route.

The presented CB mathematics model of the paper with the short period tendency is an integer programming model that could be solved with the simple approach of GPM that is not regarded in the literature. The main reason for utilizing GPM strategy is, firstly, determining the slack variables and opportunity costs and secondly the calculating procedure simpler rather than cutting methods. Moreover, the network approach for solving CBP with look back to the pervious options seems adaptive for such a scheduling problem. In the next section, we developed a sample model to explain how it works for solving a short term capital budgeting problem.

IV. Numerical Example

Here a simple sample with one period is assumed in order to clarify the solving process. There is a mathematical model that wishes to maximize profits earned from two projects simply symbolized x_1 and x_2.

$$max\ 5x_1 + 2x_2$$
$$s.t.$$
$$2x_1 + 2x_2 \leq 9$$
$$3x_1 + x_2 \leq 11$$
$$x_1, x_2 \geq 0\ and\ integer$$

The amount of profit based on some prediction methods is predetermined and now we should decide about how much should be invested regarding to limitation on the resources to expend on the projects. The fractions of the first phase of integer solving with simplex method are displayed in Table 1.

Table1. The fractional solution for the example

Z	0.25	1.5	18.75
x_2	0.75	-0.5	1.25
x_1	-0.25	0.5	3.25

Where $B = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}$, so with calculating the determinant of B, the degree of group is $D = 4$. GMP is constructed based on the descriptions of section III:

$$max \frac{1}{4} x_{j(1)} + \frac{2}{4} x_{j(2)}$$
$$s.t.$$
$$\begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix} x_{j(1)} + \begin{pmatrix} \frac{2}{4} \\ \frac{2}{4} \end{pmatrix} x_{j(2)} = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \end{pmatrix} \quad (mod1)$$
$$x_{j(1)}, x_{j(2)} \geq 0\ and\ Integer$$

The resulted tableau of GMP is revealed in Table 2. As it is clear g is the symbol of the group. μ and β are integer numbers up to the degree of the group that are multiplied in $\bar{\alpha}_1 = \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix}$ and $\bar{\alpha}_2 = \begin{pmatrix} \frac{2}{4} \\ \frac{2}{4} \end{pmatrix}$ respectively to show the group relations. The matched results of each multiplying are exhibited in the individual columns. For instance, multiplying $\beta =$ or $\beta = 3$ in $\bar{\alpha}_2$ yields $\begin{pmatrix} \frac{2}{4} \\ \frac{2}{4} \end{pmatrix}$ in $mod\ 1$ that is presented in column 2 of Table 2.

Table2. Tableau for supposed example

$\mu\bar{\alpha}_1$	*1*	*2*	*3*	*4*
$\beta\bar{\alpha}_2$		*1,3*		*2,4*
	g_1	g_2	g_3	$g_4 = g_0 = g_D$
g	$\begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \frac{2}{4} \\ \frac{2}{4} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

The current network for this model is schematically shown in Fig.2. $Source = g_0$ and $Sink = g_3$ with the adapted motion cost on the arcs. There is a movement caused within nodes from subtract $\alpha_{J(j)}\ for\ j = 1,2,3,4$ between predecessor and successor regarding to network procedure. The cost of one step is equal to $1/4$ and the cost of two steps is equal to $2/4$.

When the above network is solved, two routes with the lengths less than $D - 1$ would be obtained. Two non-negative solutions are earned for the basic variables that both are counted as optimum answers. As Fig.3 shows there are two different ways from source to sink that both took a same cost with$\left(x_{J(1)}, x_{J(2)}\right) = (1,1)$. Now with determined slack variables the amount of basic variables can be calculated.

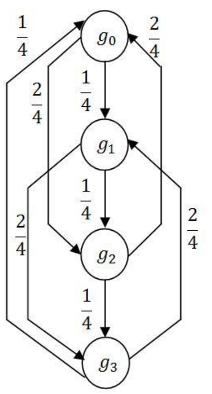

Figure2. The Network of supposed example

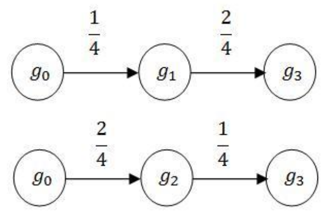

Figure3. A Scheme of different ways to answer the example

Eventually, the optimum answer would be $(x_1, x_2, s_1, s_2) = (3,1,1,1)$ that suggests to chose three units for the first option and one unit for the second one.

V. Conclusion

Flexible manufacturing systems have to make decisions about components that make them more compatible in different situations either planned or unplanned. This paper has given an account of the capital budgeting problem adapted to the machine's turnover periods in ordinary and critical situations. An exact method of grouping theory with network design approach was applied to solve this problem with integer decision making variables about rescheduling alternatives. This exact method can be implemented for small scale problems with the short time period. Although there are large numbers of varied heuristic and metaheuristic methods for solving to counter with this problem, but these algorithms that belong to approximation category methods could never be applicable as much as the exact methodology can find proper answers. It is suggested to survey implying this method for some other IPM having common characteristics like CBP.

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