Prepare for Landing

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The 2006 Winter Meeting of the AAPT Was Over …

and the flight home from Anchorage to Cleveland was just about to end—eight hours in the air, only two complimentary beverages, no meals, a jump across four time zones, a one-year-old baby daughter, and a wife whose motto for the week was, “Why did they choose to have a winter meeting in Alaska?” made for a mentally and physically taxing airborne ordeal. As we entered the last hour of flight, my small family was exhausted and the pilot’s decision to dim the interior cabin lights mixed with the soothing hum of the Airbus® A320’s engines quickly put us to sleep. Fading in and out of my delirium, I eventually heard the pilot’s voice crackle over the intercom with a seemingly innocent comment: “We are going to begin our final descent into Cleveland … we should have you on the ground in exactly eight minutes.” Something about the pilot’s use of the word “exactly” must have triggered a reaction in my brain, because his remarks initiated a series of calculations: So how fast are we flying? How high are we flying? What’s our angle of descent? With only eight minutes until touchdown, my curiosity to determine the descending airplane’s motion led me to conduct a hasty constructed experiment.

A Hasty Experiment

I quickly concocted a simple pendulum to measure our angle of descent. I remembered the memory stick in my laptop’s USB port. The stick would make an adequate mass and it was conveniently attached to a 3-in length of string that connected it to a key ring. A Post-it® note on the plane’s window would make a perfect backdrop for measuring the two necessary positions of the pendulum: first, while the plane was descending and second, when the plane eventually came to rest on the ground. I quickly mounted my crude accelerometer to the plane’s window with a piece of medical tape. Instead of marking the pendulum’s positions with a pencil, I took pictures with a digital camera so that I could later measure the angular displacement on a computer. I took several pictures to determine how much angular variation occurred so I could later gauge the uncertainty in my calculations. Once we landed and came to rest at the gate, I took a final picture of the pendulum to get a reference as to its equilibrium position.

Statement of the Problem

After the flight, the pilot was kind enough to give me a spec sheet with the various performance and flight parameters of the Airbus A320. Also, I had a detailed conversation with some airline representatives who were able to share some of their insights regarding the landing of a jet aircraft. Obviously, many of these parameters depend on the weather, landing conditions, and air traffic, so my colleague and I have chosen mid-range values for use in our calculations. From the spec sheet we know that before final descent, the plane is flying at its most economical cruising speed between 525 and 575 mph; therefore, we will estimate our cruising speed as 550 mph. In order to land, the pilot executes two maneuvers resulting in additive effects that are measured by the onboard pendulum: (1) he tilts the nose of the plane so that its angle of attack $\theta_{\text{tilt}}$ is $3.3^\circ$ below the horizontal (also on the spec sheet), thus flying the plane at velocity $v_0$; and (2) by
reducing engine power and changing wing configurations, he decelerates the plane from this velocity \( \mathbf{v}_0 \) to its landing velocity \( \mathbf{v} \). We also know from the spec sheet that the landing speed is between 135 and 155 mph, so we will use a landing speed of 145 mph. To keep our calculations at the introductory level, we will assume this deceleration is uniform (this would also minimize the discomfort to passengers), and the time interval over which this deceleration occurs is indeed the eight minutes announced by the pilot. We are not concerned with more sophisticated effects like the varying accelerations during landing or the changing orientations of the plane as it lands (i.e., obviously the pilot does not lift the nose of the plane before landing, known as “flaring,” to dissipate momentum and to touch down on the rear wheels). Thus, given the cruising speed of a plane, its angle of descent, its landing speed, and the time until landing, will a calculation of the total angle of deflection of a pendulum be close to that measured by our make-shift accelerometer? The situation is depicted in Fig. 1.

**Solution**

The total measured angle of deflection of the pendulum will be the sum of the angles produced by the pilot’s two maneuvers: the tilt of the aircraft as the pilot rotates it about an axis along its wingspan and the deceleration of the aircraft as the pilot reduces engine power:

\[
\theta_{\text{measured}} = \theta_{\text{tilt}} + \theta_{\text{accelerated frame}}. \tag{1}
\]

According to airline representatives, the angle of descent rarely exceeds \( 4^\circ \) so we will set \( \theta_{\text{tilt}} = 3.3^\circ \) to align with the spec sheet. To compute \( \theta_{\text{accelerated frame}} \), we use our assumption of uniformly accelerated motion to calculate the motion of the plane as it descends to the runway:

\[
a_x = \frac{v_x - v_0_x}{t} = \frac{145 \text{ mph} - 550 \cdot \cos(3.3^\circ) \text{ mph}}{\left(\frac{8}{60}\right) \text{ h}}
\]

\[
= -3030 \text{ mi/h}^2,
\]

and

\[
a_y = \frac{v_y - v_0_y}{t} = \frac{0 - [-550 \cdot \sin(3.3^\circ)] \text{ mph}}{\left(\frac{8}{60}\right) \text{ h}}
\]

\[
= 240 \text{ mi/h}^2.
\]

Armed with the acceleration of our noninertial reference frame and temporarily ignoring its tilt, we construct a free-body diagram of the pendulum to compute \( \theta_{\text{accelerated frame}} \). We incorporate the acceleration of the reference frame into Newton’s second law by treating the mass as if it were in equilibrium while adding a pseudoforce (i.e., a fictitious or inertial force) to the pendulum as shown in Fig. 2. Newton’s law therefore gives us the expression for \( a_{\text{accelerated}} \) as:

\[
\theta_{\text{accelerated frame}} = \arctan \left( \frac{a_{\text{pseudo}_y}}{g + a_{\text{pseudo}_x}} \right). \tag{2}
\]
Plugging in our acceleration values $a_{\text{pseudo}} = 3030 \text{ mi/h}^2$ and $a_{\text{pseudo}} = 240 \text{ mi/h}^2$ and noting that $g = 78,900 \text{ mi/h}^2$, we calculate $\theta_{\text{accelerated frame}}$ to be $2.2^\circ$. Thus, using Eq. (1) and the value of $\theta_{\text{accelerated frame}}$ from Eq. (2), our expected value for $\theta_{\text{measured}}$ is:

$$\theta_{\text{measured}} = \theta_{\text{tilt}} + \theta_{\text{accelerated frame}} = 3.3^\circ + 2.2^\circ = 5.5^\circ.$$

**Measurements and Discussion**

We downloaded the images from the digital camera, set the equilibrium image (i.e., taken when the plane was at rest) to 70% transparency, and superimposed the two images. We tried multiple images to gauge the uncertainty in our calculations. The composite image is shown in Fig. 3. Pasting this image into Microsoft Word, we drew lines on the image to form the legs of a right triangle. Finally, using MS Word’s “Format Autoshape” function, we were able to accurately measure the lengths of the lines and thus determine the deflection of the pendulum to be $5.9^\circ \pm 0.9^\circ$, which nicely matches our calculated value. To check for consistency, we used our estimated velocities and calculated acceleration to compute the height and range from which the plane would have started its final descent. The resulting altitude of 11,140 ft and range of 46 miles initially had us worried since the typical cruising altitude of most commercial airliners is 30,000–35,000 ft. However, the airline representatives notified us that final descents usually occur 50 miles from the airport at altitudes under 18,000 ft. Pilots descend planes in a series of steps so even though an aircraft may cruise at 35,000 ft, it does not start its final descent from that altitude. Instead, the plane might initially descend from 35,000 ft to 25,000 ft, then from 25,000 ft to 15,000 ft, and finally from 15,000 ft to ground. In other words, the plane goes through a series of descents starting with the initial descent and ending with the final descent. Our experiment was performed in this final stage. The pilot does not lift the nose of the plane for landing until that last ¼-mile of the flight, well after our measurement was taken.

The demonstration can be done whenever students fly aboard a plane, but once the teacher has his/her own set of photographs, the pictures can be used for a demonstration of uniformly accelerated motion and show that such motion has applications to everyday life.

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Fig. 3. Displacement of a hastily constructed accelerometer.