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Irreversibility and Interest Rates
Giuseppe Travaglini*
Faculty of Economics, University of Urbino “Carlo Bo”, Italy

Abstract
The literature on irreversible investment fails in exploring the relationship between the present value of alternative strategies and the appropriate risk-adjusted interest rates. We will attempt to fill this gap showing that to avoid arbitrage opportunities the real option’s rate must be higher than the rate of the immediate strategy. Further, we explain how irreversibility influences the risk-return combination of competing strategies acting as a pure factor of risk.

Key words: irreversibility; real option; risk-adjusted rate
JEL classification: G12; D92; E22

1. Introduction
In traditional project evaluation the net present value (NPV) of any investment plan is computed discounting profits at some interest rate, and accepting project whether its current value exceeds the direct cost.

There are, however, a number of problems arising from this criterion. In fact, only when returns are certain there is a widespread agreement that the riskless interest rate must be employed to compute the NPV. Yet, if rewards are uncertain matters become more complex and the technique suggests to estimate expected profits and discounting at the appropriate risk-adjusted interest rate.

But, may this criterion assure a correct decision whether investment is at least partially irreversible? And, since any irreversible project gives to the owner a set of real options, what is the appropriate interest rate of any option?

Most investment projects share three important characteristics: the investment may be partially or completely irreversible; the stream of future profits may be uncertain; any investor can find advantageous to defer action to get information about the future. These three features interact to determine the value of any investment strategy, and the criterion for asset pricing must be capable of handling these three considerations.

Received September 7, 2004, revised November 29, 2005, accepted October 17, 2006.
*Correspondence to: Università di Urbino Carlo Bo, Facoltà di Economia, Istituto di Scienze Economiche, Via Saffi 42, 61029, Urbino (PU) - Italy. E-mail: travaglini@uniurb.it., Tel.: +39 0722 305510, Fax: +39 0722 305555. The author would like to thank Paolo Carnazza, Paolo Liberati, Enrico Saltari, and two anonymous referees for insightful comments and suggestions.
We show that the no-arbitrage principle must be satisfied to figure out the appropriate interest rates of an irreversible project. Two are the results of our analysis. First, the interest rate of the real option is always higher than the rate of the immediate investment. Second, irreversibility affects the risk-return combination of the competing strategies as a pure factor of risk.

This problem has received a scant attention in literature. Our analysis starts from the empirical evidence discussed by Carnazza and Travaglini (2001). They asked a sample of 4000 manufacturing Italian firms to quantify the minimum interest rate necessary to implement a new project, under the alternative assumption of reversible versus irreversible investment. From their survey comes out that irreversibility increases substantially the expected rate necessary to carry out irreversible projects. Table 1 illustrates this result: with irreversibility the distribution of answers (expressed in percentage value) is skewed towards the highest rates, implying the existence of a trade-off between irreversibility and interest rates.

Unfortunately, the standard literature on investment with irreversibility and uncertainty fails in exploring this topic in depth. As an example of the standard way of studying this relationship let us consider investment models.

McDonald and Siegel (1986) employ a simulation to estimate the expected rate of return at which it is optimal to pay a sunk-cost to get an irreversible investment. They find that the expected rate of return rises from 4% of the stock to 16% of the real option. Later on, Dixit (1992) shows that if the expected interest rate from a reversible project were 5%, the rate for the corresponding irreversible project would be at least 9%.

On the theoretical ground, Baldwin and Trigeorgis (1993), and more recently Trigeorgis (1997) observe that the use of a single risk-adjusted interest rate can lead to significant errors in the evaluation of competing irreversible strategies since asymmetric claims on an asset do not generally have the same discount rate as the asset itself. They conclude that finding the appropriate interest rate via standard NPV technique may be difficult in situations involving real options.

Further, the certainty equivalent technique - often employed in its refined version of the risk neutral evaluation - introduces two difficulties in presence of irreversibility. It remains doubtful why it may be optimal to postpone an irreversible investment when its risk is fully diversifiable. Indeed, if a project can be treated as a riskless asset, the decision to postpone the commitment can only produce a loss of profit at the current time without advantages to the firm. Hence, whether it is optimal to defer the project, the future returns - and the corresponding interest rate - would be higher than the one gained investing immediately, and the single interest rate will not, in general, evaluate the competing strategies correctly.

In the next section we explore a simple framework to explain why it is necessary to use different interest rates when the project is irreversible. Section 3 generalizes this finding, showing how to calculate the value of competing strategies when investment is a commitment. Section 4 analyses the risk structure of interest rates focusing on the risk premium. Section 5 concludes.
Table 1. The minimum interest rate required by firms to implement a new project under the alternative assumption of reversible versus irreversible investment. (Numbers in table are percentage values of the total answers; Pavitt taxonomy is used to classify firms by sector).

<table>
<thead>
<tr>
<th>Interest rate:</th>
<th>between 1% and 5%</th>
<th>between 5% and 10%</th>
<th>between 10% and 15%</th>
<th>between 15% and 20%</th>
<th>higher than 20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>type of project:</td>
<td>Revers</td>
<td>Irrevers</td>
<td>Revers</td>
<td>Irrevers</td>
<td>Revers</td>
</tr>
<tr>
<td>Total firm:</td>
<td>19</td>
<td>17</td>
<td>40</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>Traditional</td>
<td>23</td>
<td>17</td>
<td>38</td>
<td>27</td>
<td>19</td>
</tr>
<tr>
<td>High tech.</td>
<td>1</td>
<td>3</td>
<td>55</td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>Specialized supplier</td>
<td>15</td>
<td>12</td>
<td>31</td>
<td>16</td>
<td>33</td>
</tr>
<tr>
<td>Scale intensive</td>
<td>20</td>
<td>20</td>
<td>43</td>
<td>21</td>
<td>18</td>
</tr>
</tbody>
</table>

Figure 1. The binomial process of $V$ (stock) and $F$ (real option)

2. A Basic Example

This example provides a good baseline case from which we can depart later. Note that this example differs from others with irreversibility (see for example Dixit and Pindyck 1994) for an important detail: using the arbitrage principle we derive the appropriate risk-adjusted interest rates of the alternative strategies.

Consider a firm with the monopoly right to start or defer a project. If the commitment is made at the current time, the firm loses the opportunity to resell the project in the future and the chance of deferring the investment decision to some future date. In other words, the investment is irreversible. The firm maximizes the NPV of the project by choosing the most profitable investment schedule. It can invest either at the current time or in the next time period. Let us assume that the investment spending is a sunk-cost, $I = 70$, and that the project is risky with returns governed by a binomial process. Indicate with $\delta = 0.1$ the dividend rate paid on the stock.

The value of the stock at period $t$ is the present value of all dividends from that
time on. At the end of period \( t \), the value of the project may be \( V_t = 150 \) with probability \( q = 0.7 \) or \( V_t = 70 \) with probability \( 1 - q = 0.3 \) (see figure 1). Of course, the firm does not know the current value \( V \) of the project.

The firm can decide to initiate the project immediately. Alternatively, it decides to postpone the commitment until after the state of nature is known, renouncing to the current dividend (see figure 1). This decision introduces an asymmetry in the pay-offs of the NPV: the deferred strategy has a payoff of \( F_\text{d} = 65 \) in the best state, and \( F_\text{w} = 0 \) in the worst state. As before, the present value \( F \) of this strategy is unknown at the current time.

### 2.1 The Value of the Stock

The aim of the firm is to choose one of the two investment plans, comparing the NPVs. To begin with, let’s assume that the stock strategy can be modelled as a twin security traded on the capital market. This security is a risky asset with a current price of \( P = 10 \). In the next period it will be worth \( P_{t+1} = 15 \) in the best state, with probability \( q = 0.7 \), and \( P_{t+1} = 7 \) in the worst state. The price will then remain at this new level forever. Then, assume that a riskless security with interest rate equal to \( r = 8\% \) is exchanged on the market. Given these two assets what are the appropriate risk-adjusted interest rate of the alternative strategies?

Note that returns on \( P \) are proportional to returns on the stock strategy, and that the payoffs are governed by the same binomial distribution. We can therefore argue that they are equivalent in risk. Thus, we can use the interest rate of the underlying asset to calculate the NPV of the immediate investment strategy. The required rate \( \mu \) for the underlying asset is given by the expression:

\[
10 = \frac{(0.7)15 + (0.3)7}{(1 + \mu)} \Rightarrow \mu = 0.26
\]

\( \mu = 0.26 \) is the rate at which the firm will discount the expected payoff from the stock. Employing this risk-adjusted rate, we obtain the present value \( V \):

\[
V = \frac{(q)V_\text{d} + (1-q)V_\text{w}}{1 + \mu} = \frac{(0.7)150 + (0.3)70}{1.26} = 100
\]

and the corresponding net present value \( W \):

\[
W = -I + V = -70 + 100 = 30
\]

As said above, the ability to postpone the commitment alters the payoffs of the strategy, changing the risk attached to the project. Thus, it follows that the rate \( \mu \) would be inappropriate to evaluate the deferred strategy. However, let us apply the rate \( \mu \) to the real option. We obtain:

\[
F = \frac{(q)F_\text{d} + (1-q)F_\text{w}}{1 + \mu} = \frac{(0.7)65}{1.26} = 36.1
\]
Hence, since $F > W$ the firm finds optimal to postpone the investment.

2.2 The Value of the Real Option

In this section we explain why the previous computation of the real option is wrong. The basic idea enabling the exact pricing of options is that one can construct a portfolio of traded securities which replicates the future payoffs of the deferred strategy. We begin by constructing a portfolio of traded securities calculating the payoffs of the option at time $t + 1$. Let $z$ and $n$ indicate shareholdings respectively of the risky asset $P$ and the risk-free security. If shareholdings are to give equivalent returns in the best and the worst states of nature we have:

\[
\begin{align*}
    z(15) + n(1.08) &= 65 \\
    z(7) + n(1.08) &= 0
\end{align*}
\]

Solving this system yields $z' = 8.125$ and $n' = -52.662$. Hence, to obtain the same return of the option strategy on the two securities an investor would have to purchase 8.125 shares in the risky asset and go short on the riskless asset. Of course, these same quantities of securities are in the portfolio at the initial time $t$. As a result, the NPV of the option strategy is given by:

\[
F_nP = z(15) + n(1.08) = 65
\]

In short, the value $F = 28.58$ is smaller than it would have been if we had employed the rate $\mu$. Further, note that it is even smaller than the value of the immediate strategy $W = 30$, implying that the best choice is to invest immediately. This result reverses the previous choice.

Now, using the no-arbitrage argument we can see that the appropriate rate $\gamma$ of the option must be higher than $\mu$. Since $F = 28.58$ and knowing its values at $t + 1$, we can calculate the corresponding risk-adjusted interest rate $\gamma$:

\[
F = \left[ \frac{(q)F_{t} + (1-q)F_{t+1}}{1+\gamma} \right]
\]

\[
28.58 = (0.7)\left[ \frac{65}{1+\gamma} \right] \Rightarrow \gamma = 0.59
\]

Thus, this procedure yields two different interest rates: $\mu = 0.26$ for the stock, and $\gamma = 0.59$ for the option. To complete our argument, note that risk-neutral probabilities yield the same value for $F$. The importance of this synthetic probabilities is that expectations calculated with them equal the NPV of $F$ once discounted by the riskless rate $r$. We indicate this probability with $\hat{q}$. To obtain such a value, assume the firm is risk neutral. In this scenario, all that the firm requires is the riskless rate on the stock strategy. Hence:
\[ (1 + r)V = \hat{q}V_s + (1 - \hat{q})V_c \]
\[ (1 + 0.08)100 = \hat{q}150 + (1 - \hat{q})70 \]

Solving for \( \hat{q} \) we obtain \( \hat{q} = 0.475 \). Using this value to compute the current value \( F \) we get:

\[ F = \hat{q}F_s + (1 - \hat{q})F_c \]
\[ F = \hat{q} \left[ \frac{F_s}{1 + r} \right] = \frac{(0.475)65}{1.08} = 28.588 \]

which confirms our previous result. This does not mean, of course, that the appropriate rate of interest for the deferred strategy is the riskless rate \( r \).

The above example provides a simple illustration of the difficulty of applying traditional calculations of NPV when assessing real options. Unless the rate of interest is adjusted upwards to reflect the change of risk brought about by the firm's investment decisions, the traditional approach over-estimates the value of real options, and can induce the firm to choose the investment plan which does not maximizes the NPV. This is an important implication for the asset pricing of competing irreversible projects.

But, note that the absence of arbitrage does not imply necessary that the choice of the optimal investment strategy must be reversed once plans are correctly priced. Actually, the main point here is that no external investor would be willing to finance the investment plan if the interest rate offered by the firm is smaller than the one required by the market. Indeed, the realization of the option strategy cannot be made without knowledge of the appropriate cost of capital.

If the interest rate were equal for both the strategies, the supply of financial funds would be infinitely elastic irrespective of the riskiness of the project. But, in a risky environment the stock and the option have different degrees of risk and, consequently, different risk-adjusted rates. Hence, the example shows how the risk-return combination affects the interest rate, pointing out how to extend the concept of risk-adjusted rate to the situation of irreversibility where opportunities do not all have the same risk.

### 3. Pricing Competing Strategies: The General Case

In this section we generalize the results presented above considering that any investment opportunity offers to the owner two products, expected return and risk. We assume that the operating profit \( P \) given by an investment is a stochastic variable obeying the geometric Brownian motion:

\[ dP = \alpha P \, dt + \sigma P \, dz \]  \hspace{1cm} (1)

where \( \alpha \) is the constant drift, \( \sigma \) the constant variance, and \( dz \) is a normally
distributed random variable, with \( E(dz) = 0 \) and \( E(dz)^2 = dt \).

Our way of approaching the problem is treating immediate and deferred investments as two distinct assets - a stock and a real option - whose value depends on the more fundamental variable \( P \). When investment is irreversible this approach makes it possible to benchmark alternative plans using the features of the underlying asset \( P \).

3.1 The Stock

We can start from the stock writing the value of the immediate strategy as:

\[
V(P) = E\left\{ \int_{-\infty}^{t} e^{-\mu s} (P_s) ds |_{t_0} \right\} \tag{2}
\]

where \( \mu \) is the appropriate rate of interest of the stock. The corresponding NPV is \( W(P) = V(P) - I \), where \( I \) is the direct cost of the irreversible investment. For the moment we assume \( \mu \) as given.

It is primary to show that the stock \( V(P) \) has both the same expected growth rate and the instantaneous variance of \( P \). To see this, we use dynamic programming to write the previous problem (2). It breaks a whole sequence of decisions into just two components: the value of the operating profit \( P \) over the interval \((t, t + dt)\) and the expected value beyond \( t + dt \) which encapsulates the consequences of all subsequent decisions from that time on:

\[
V(P) = Pdt + E\left[ V(P + dP)e^{-\mu s} \right]
\]

Applying Ito's lemma to relate changes in \( V \) to those in \( P \) we obtain the expression:

\[
V(P) = Pdt + \left[ \alpha V_r P + \frac{1}{2} \sigma^2 V_{rr} P^2 \right] dt + (1 - \mu dt)V(P)
\]

where \( V_r \) and \( V_{rr} \) are respectively the first and the second derivatives of \( V \) with respect to \( P \). This equation can be rewritten as:

\[
\frac{P}{V(P)} + \frac{1}{2} \sigma^2 V_{rr} P^2 = \mu V + \frac{\alpha V_r P}{V(P)} \tag{3}
\]

This is an arbitrage equation: in equilibrium the expected return \( \mu \), is equal to the instantaneous rate of dividend \( P/V(P) \), plus the expected cash appreciation rate \((\alpha V_r P + (1/2)\sigma^2 V_{rr} P^2)/V(P)\). Thus \( \mu \) is the actual rate of return for the immediate investment. From (3) we obtain the stochastic partial differential equation:

\[
\frac{1}{2} \sigma^2 V_{rr} P^2 + \alpha V_r P - \mu V + P = 0 \tag{4}
\]
which has the general solution \( V(P) = A_1 V^{d_1} + A_2 V^{d_2} + (P/\delta) \), where \( d_1 > 1 \) and \( d_2 < 0 \) are the roots of the characteristic equation, and where \( A_1 \) and \( A_2 \) remain to be determined. Given that \( V(0) = 0 \), we must impose that \( A_1 = 0 \) and \( A_2 = 0 \) to avoid speculative bubbles. Hence, the solution of this problem is:

\[
V(P) = \frac{P}{\delta} \tag{5}
\]

which is the expected present value of the profit obtained investing at the current time. Further, note that from (5) the dividend rate is given by \( \delta = P/V(P) \), and that substituting in (3) the expected cash appreciation rate reduces to \( \alpha \), so that \( \mu = \delta + \alpha \). Finally, a basic consequence of this result is that the changes of \( V \) can be written as:

\[
dV = V dp + \frac{1}{2} V^{d_2}(dp)^2
= \frac{1}{\delta}[dV dt + \sigma d\zeta] + \frac{1}{2} V^{d_2} \sigma^2 dt
\]

Using (5) to solve the previous expression we find that the dynamic of \( V(P) \) is:

\[
dV = \alpha V dt + \sigma V d\zeta
\]

This means that \( P \) and \( V \) are perfectly correlated and equivalent in risk.

3.2 The Real Option

Let us now turn to the deferred strategy with \( F(P) \) representing the value of the real option. So long as nothing has been invested, the life of the investment program remains the same and the firm sacrifices the rewards associated with the investment. This is the opportunity cost of keeping the right to invest in the future. The expected present value of this opportunity is:

\[
F(P^*) = E \left[ \int_0^\infty \left[ v(P^*, \tau) - I \right] e^{-\gamma \tau} d\tau \right]
= E \left[ e^{-\gamma \tau} \int_0^\infty \left( e^{\gamma \tau} P^* ds \right) - e^{-\gamma \tau} I \right] \tag{6}
\]

The investment decision depends on both the time \( \tau \) and the critical value \( P^* \), where the firm finds optimal to undertake the irreversible project. With \( \gamma \) we indicate the discount rate of the real option.

Now, we show that the real option is riskier than the stock. Given that the operating profit \( P \) has the same dynamics as \( V \) we can substitute \( F(V) \) for \( F(P) \), rewriting the problem (6) in the form of the corresponding Bellman equation:

\[
\gamma F(V) dt = E \left[ dF(V) \right] \tag{7}
\]
The instantaneous profit $P$ does not appear in this expression because the real option yields no cash flow in the inactive region. As it stands, equation (7) can be seen as an application of Ito's Lemma. The derived dynamic pattern of $F(V)$ is the process:

$$dF = \gamma F dt + sF dz$$

where:

$$\gamma = \frac{\alpha F_v V + \frac{1}{2} \sigma^2 V_v V^2}{F} = \frac{E(dF)}{dt}$$

and:

$$s = \alpha F_v V$$

is the corresponding risk, with $F_v(V/F)$ measuring the elasticity of $F$ with respect to $V$. From (9) we obtain the differential equation:

$$\frac{1}{2} \sigma^2 V_v V^2 + \alpha F_v V - \gamma F = 0$$

and to ensure convergence it must be that $\gamma > \alpha$. In addition $F(V)$ must satisfy the following boundary conditions: $F(0) = 0$, $F(V^*) = V^* - I$ and $F_{v} = 1$, where $V^*$ is the critical value at which firm finds optimal to undertake the irreversible project. Note that from the second boundary condition we have that $V^* = I + F(V^*)$, setting the value of the project equal to the full cost - direct cost plus opportunity cost - of making the commitment. For the positive root $b > 1$ this gives the solution:

$$F(V) = BV^b$$

where $b = b$. This expression is the value of the real option. Using this solution, we can solve the elasticity term in (10) finding that:

$$F_v V = bBV^{b-1} V = b > 1$$

Thus, from (10) we get that $s > \sigma$; in other words, the real option is riskier than the stock.

4. Asset Pricing and Interest Rates

To clarify the meaning of the previous result note that in our solution we use the assumption that real and financial assets are exchanged in the market. So, let's assume
that the intertemporal CAPM is a valid description of asset returns at equilibrium. Applying this framework we can explain why optimal investment decisions require $\gamma > \mu > r$. We refer to this relationship as the risk structure of interest rates.

As said above, the actual interest rate of the stock is $\mu$. In a CAPM framework this is the required interest rate which satisfies the no-arbitrage condition:

$$\mu = r + \lambda \beta_r$$  \hspace{1cm} (13)

As usual, $\beta_r = \sigma_{rm} / \sigma_r$ is the systematic component of risk, $\lambda = [E(R_m) - r]$ is its price, and $m$ is the market portfolio. Now, since the standard deviation of the real option is $s = b\sigma$ then the covariance between the option and the market portfolio is $\sigma_{rm} = b\sigma_{mx}$, so that the corresponding beta coefficient for the option is given by:

$$\beta_r = b\beta_r$$  \hspace{1cm} (14)

This last equality is of key importance: since $b > 1$ the option is riskier than the stock, and the deferred strategy will have a higher risk premium. Of course, for excluding arbitrage this must imply that $\gamma > \mu$. To prove that this result is a consequence of the arbitrage principle take the CAPM equation of the real option $\gamma = r + \lambda \beta_r$, that from equation (13) and (14) can be rewritten as:

$$\gamma = r + b(\mu - r)$$  \hspace{1cm} (15)

Finally, using equations (9) and (12), and the condition $\mu = \delta + \alpha$ we obtain the following evaluation equation:

$$\frac{1}{2} \sigma^2 F_r V + (r - \delta) F_r V - r F = 0$$  \hspace{1cm} (16)

This differential equation is identical to (11), but with the significant difference that now we adopt an explicit no-arbitrage argument to identify the relationship between interest rates. To show that (16) satisfies the no arbitrage condition, equate equation (15) to the actual expected rate of the option $F(V) = BV^r$. Applying Ito's lemma we get:

$$\frac{dF}{F} = \left[ b\alpha + \frac{1}{2} b(b-1) \sigma^2 \right] dt + b \sigma dZ$$  \hspace{1cm} (17)

with expected value equal to $E(dF)/dtF = b\alpha + (1/2)b(b-1)\sigma^2$. Equating this expression to equation (15) we get the characteristic equation:

$$b\alpha + \frac{1}{2} b(b-1) \sigma^2 = r + b(\mu - r)$$  \hspace{1cm} (18)

which has the solution:
But, this expression is the positive root $b_1 > 1$ of equation (16) which ensures that:

$$\gamma > \mu > r$$

Hence, in a portfolio approach the risk-adjusted rate of the real option incorporates a risk premium which is higher than the one of the stock.

5. Concluding Remarks

Irreversibility plays a particularly important role for investment decisions. The firm must look ahead to plan its current choice taking into account irreversibility and uncertainty on future profits. If the firm has the opportunity to postpone the irreversible project, the appropriate interest rate of any single strategy must capture the characteristics of the corresponding investment plan, and the current value of any strategy requires an appropriate rate revealing not only the cost of uncertainty but also the market value of irreversibility.

Some of the results presented in this paper may appear in contrast with the traditional wisdom of economic modelling. In fact, it is common to investigate the exchange of fixed quantities of risk for a given return; it is frequent to evaluate alternative irreversible strategies for the same project using a single interest rate. On the other hand, the key point of our analysis is that irreversibility introduces peculiar elements into investment decisions, leading to the result that it is necessary to compute the risk structure of the interest rates to correctly evaluate competing irreversible investment opportunities.

This result is based on the general property that an investment is correctly priced only if its market value does not allow opportunities for arbitrage. When this condition is violated it is likely that incorrect decisions can be made. Hence, any given interest rate will not, in general, evaluate the alternative strategies of a single irreversible project correctly.

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