THE POLITICAL POWER OF THE RETIREES IN A TWO-DIMENSIONAL VOTING MODEL

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Abstract

We show that the retirees are able to obtain favorable pension policies whereas they belong to a minority in the population. The argument relies on the multidimensional nature of the political process. We consider a two-dimensional collective choice problem. The first of these choices is the level of the contribution rate to the Pay-As-You-Go pension system. The second is a noneconomic decision, unrelated to the pension system. Using a political agency model, we show that, as soon as the retirees are sufficiently numerous, the equilibrium tax rate may be higher than the tax rate preferred by the young, who yet constitute a majority over the pension issue.

1. Introduction

Due to the aging of the population, there is now a huge debate over the evolution of the pay-as-you-go (PAYG) pension system. As the size of the working population relative to the number of retirees is decreasing, financing of the PAYG pension system becomes problematic. In this period of declining fertility and rising life expectancy, the intergenerational conflict between the young and the old is exacerbated. On the one hand, the current young generations do not expect a high rate of return from the PAYG system.

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Consequently, they would like to limit the size of the system and rely more heavily on private savings. On the other hand, the retirees and the people close to retirement are strongly opposed to such a downsizing. In this paper we want to analyze how this conflict is resolved through the political process.

Traditional political economy models of social security deal with a unidimensional policy choice, the only issue to be voted on being the contribution tax rate to the PAYG system. In such a framework, the majority coalition imposes its choice to the losing minority. In a two-period overlapping generations model, without heterogeneity within age classes, the young constitute a majority (as soon as the rate of population growth is positive) and can then choose their preferred tax rate. In particular, if they are in favor of turning down the system, because for example the rate of return of private savings dominates the rate of return of the PAYG system, the political process is very detrimental to the retirees who receive no pension.

In this paper we extend this model to a multidimensional setting. More precisely, we consider a two-dimensional collective choice problem. The first dimension is the choice of the contribution rate to the PAYG system. The second dimension is a noneconomic, discrete, decision, unrelated to the pension system. We call it loosely a reform. For convenience, we assume that only the young are interested in this second dimension. Part of them are in favor of the reform whereas the others are opposed to it. The old are indifferent and only consider the value of the tax rate, which determines the pension they receive. To determine the outcome of the political process, we use a political agency type model. There is an incumbent government who seeks to be reelected and maximizes the number of votes obtained. Voters derive electoral power by deciding whether to retain or vote out of office the incumbent government.

We first determine the subgame perfect Nash equilibria of the game. We obtain that the equilibrium tax rates depend crucially on the relative size of each group of voters. When the number of retirees is smaller than the number of both types of workers, the equilibrium tax rate is always the preferred tax rate of the workers. However, when the retirees outnumber at least one group of workers, a tax rate strictly higher is sustainable at equilibrium. This is the case when both groups of workers try to make an alliance with the retirees and sustain the tax rate asked by the retirees. None of these groups should deviate. In particular, if the individuals belonging to the largest workers group decide to ask for a higher utility level, the only way for the government to be reelected is then to satisfy the coalition of the retirees and the other group of workers. Moreover, when the retirees constitute the largest group in society, only tax rates strictly larger than the preferred tax rate of the workers may

\[1\] This branch of the literature has been initiated by Browning (1975). Subsequent developments include Hu (1982), Sjoblom (1985), Boadway and Wildasin (1989), and Cooley and Soares (1999).
occur at equilibrium. In this case, the maximal possible tax rate, 1, belongs to the set of possible equilibrium outcomes.

Refining the set of equilibria by iteratively deleting weakly dominated strategies, we obtain two main changes. First, the preferred tax rate of the workers is not anymore a possible equilibrium outcome when the retirees constitute the intermediate group. Second, when the retirees are the largest group, the maximal tax rate sustainable at equilibrium is strictly lower than 1.

These results contrast with the one-dimensional model in which the tax rate chosen is always the preferred tax rate of the workers. We obtain here that the equilibrium tax rate is always larger in our model. The political power of the retirees is therefore larger in this two-dimensional framework. Nevertheless, the tax rate they are able to obtain is strictly bounded above. In other words, their political power is not unlimited. Previous attempts to explain the success of the retirees in the political process, and more generally that of minorities, follow.

Grossman and Helpman (1998) argue that the different groups in society are able to lobby the government by offering it monetary contributions contingent on the policy adopted. When only the old lobby the government, they are, not surprisingly, successful. However, when two groups are active interest groups, they engage in a fierce competition that benefits only the government.

Lohmann (1998) develops an informational theory of the political power of minorities. In Lohmann’s paper, smaller groups possess better information about what the politicians do (because of the free-rider problem associated with costly information acquisition) and consequently the policy chosen is biased in their favor. Indeed, those better-informed individuals are more able to infer the incumbent government “quality” from the policy outcome they experience. The incumbent government, which wishes to be reelected, has then an incentive to bias policy in their favor in order to improve its reelection prospects.

Campbell (1999) shows that, in a majority vote between two alternatives, minorities may be decisive when the electorate is large enough. This result holds when members of the minority have a larger stake in the outcome than do members of the majority. As the number of voters becomes large, the probability of being pivotal for a given individual becomes small and, assuming that voting is costly, only the individuals with the largest stake in the outcome have an incentive to go to the polls.

The papers closest to ours are Roemer (1998) and Besley and Coate (2000). Roemer is not concerned with pensions but tries to explain, when there are no tax distortions and the median income is below the average income, why the marginal tax rate of a linear tax schedule is not set to 1. The explanation given is that political competition is essentially bidimensional, with one dimension called the tax rate and the other, a nonwealth issue, called, for simplicity, “religion.” A leftist party can then favor a tax rate
less than 1 in order to attract richer people who have “moderate” religious views in the electoral competition against a rightist party. Roemer uses an electoral competition setting between two partisan political parties that represent rich and poor individuals and want to maximize the expected utility of their constituents.

Besley and Coate also explore an electoral competition model in a two-dimensional context. In this model parties select candidates with policy preferences rather than platforms. They identify three reasons that could lead to a nonmajoritarian policy outcome over a given issue: the issue is not politically salient, it is salient only for the minority group, and interest group is organized on this (nonsalient) issue. In the first case, individuals’ voting behavior depends only on the position of the parties with respect to the salient issue. Parties therefore choose their position on the nonsalient issue according to the preferences of their members, which may differ from the preferences of the electorate at large. In the second case, parties are willing to compromise and choose the position of the minority in order to improve their chances of being elected. Not doing so would lead to the loss of all the votes coming from this minority. The logic in the third case is quite similar, the difference being that the party not pandering to the interest group loses its support (instead of votes) and has a lower probability of being elected.

In the next section we describe the model. In section 3 we analyze the different equilibria of this game. Finally, in section 4, we discuss some extensions of the model.

2. The Model

2.1. Basic Setup

We consider an overlapping generations model in which people live two periods. In the first, they work and contribute to the PAYG pension system. In the second period, they retire and earn their pension. The wage level is identical for all individuals and exogenously given. Therefore we can normalize it to 1. We assume also that there are no disincentive effects of taxation.

The problem under investigation is the political determination of a two-dimensional policy \((\tau, a)\), where \(\tau \in [0, 1]\) is the contribution rate to the PAYG system and \(a \in \{0, 1\}\) denotes a noneconomic, ideological decision which we call, for simplicity, a reform; \(a = 1\) (resp. 0) means that the reform is implemented (resp. not implemented). Note that this reform has nothing to do with the pension system. To give some examples, the government might choose to amend the drug legislation or to introduce, in the case of France, the “pacte civil de solidarité” (PACS).

The (indirect) utility of the young individuals is written as follows:

\[
V^f(\tau, a) = v(\tau) + \theta a,
\]

\[
V^o(\tau, a) = v(\tau) + \theta (1 - a)
\]
where
\[ v(\tau) = u(1 - \tau) + u(\tau (1 + n)). \]

The rate of population growth is denoted by \( n > 0 \), and \( \theta > 0 \) denotes the intensity of preferences for the reform issue. The utility function of the young favorable to the reform is \( V_f \) whereas \( V_o \) is the utility function of the young opposed to it. We say in the following that \( i \in \{ f, o \} \) is the type of a young individual. Workers expect the tax rate decided today to remain effective in their old age, so that the pension expected in a PAYG system, considering that \( n \) will also remain unchanged, is \( p = \tau (1 + n) \). This is a traditional assumption in the literature dealing with the vote over pensions. Although not fully satisfactory, it permits avoidance of complications arising because of the separation between contributions and benefits. We make the usual assumptions that \( u' > 0 \) and \( u'' < 0 \), which imply that \( v \) is a strictly concave function of \( \tau \). Finally, we assume that individuals don’t have the possibility to save through private capital markets. This assumption could be relaxed easily and does not affect the results qualitatively.

Working individuals all have the same preferred tax rate \( \tau^*(n) \). Yet they differ according to their preferences over the reform issue. The number of type \( i \) young individuals is \( N_i^w \), \( i \in \{ f, o \} \), and the total number of workers is \( N_w = N_f^w + N_o^w \). Without loss of generality, we assume that \( N_f^w \), the number of young individuals in favor of the reform, is strictly higher than \( N_o^w \), the number of young individuals opposed to it.

We make the assumption that the retirees do not care about the reform issue. They are only concerned about the level of their pension and want to maximize its size which is equivalent to maximizing the value of the tax rate. Indeed, as emphasized above, the pension given to the retirees is \( p = \tau (1 + n) \) in a PAYG pension system. To be precise, we don’t pretend that the retirees are completely indifferent regarding policies not related to pensions; however, we believe they hold some kind of lexicographic preferences with the pensions being their first preoccupation. On the contrary, for the young individuals, pensions are only one of their concerns. This assumption is not critical in generating the results but allows us to greatly simplify the analysis. Denoting \( N_r \) as the number of retirees, we have \( (1 + n) = N_w^r / N_r \) and we make the natural assumption that no group in society constitutes a majority alone, that is, \( N_f^w < N_r (2 + n) / 2 \).

### 2.2. The Political Game

As is well known in the voting literature since Plott (1967), a Condorcet winner usually fails to exist in a multidimensional setting. In our problem, a Condorcet winner is a policy \( (\tau, a) \) that beats any other feasible policy in a pairwise majority voting comparison and such a solution indeed does not
exist. The immediate implication of this is that there is no (pure strategy) Nash equilibrium in an electoral competition game between two opportunistic parties whose sole objective is to be elected.

Following Barro (1973) or Ferejohn (1986), we choose to model the political process as follows. At a given period of time, an incumbent government must choose a policy \((\tau, a)\). At the end of the period, an election occurs with the incumbent government competing against a challenger. We make the assumption that the government is purely opportunistic; its only objective is to be reelected and it is the case when it receives at least 50% of the votes. Other assumptions concerning the behavior of the government are the following:

**ASSUMPTION 1:** The government selects the policy maximizing the number of votes obtained, whether it is reelected or not.

**ASSUMPTION 2:** When two policies favoring different coalitions of voters give the same number of votes, the government chooses the policy favoring young individuals.

**ASSUMPTION 3:** When different policies satisfy the reelection rules of the same coalition of voters, the government chooses the one that gives the highest utility level to the largest group in this coalition.

Assumption 2 is simply a tie-breaking assumption. Assumptions 1 and 3 always bias the policy choice in the direction of the largest groups in society and allow us to narrow down the set of equilibria.

Politicians are all identical and, importantly, the challenger cannot commit itself to a policy. Whatever the policy announced, once in office he can implement a different one. Therefore, all the messages that the challenger may send are purely “rhetoric.” This is in sharp contrast to the Downsian framework in which parties announce policies and stick to them if elected.

In the political game we consider, there are two types of players, the voters and the government. The voters are called the principals and the government the agent. A (pure) strategy for a voter, either young or old, is to fix a reelection rule, that is, to fix a utility level above which he decides to vote for the government and conversely below which he votes for the challenger.

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2To see this in our framework, consider first the policy \((\tau^*, 1)\). This policy is beaten by \((\tau^* + \varepsilon, 0)\) because a coalition made of the young opposed to the reform and of the old prefers this new policy to the former. A policy \((\tau, 1)\), with \(\tau < \tau^*\), loses against \((\tau + \varepsilon, 1)\) (and there is unanimity in favor of this new proposal). Finally, \((\tau, 1)\), with \(\tau > \tau^*\), is defeated by \((\tau - \varepsilon, 1)\). The analysis is symmetric for a policy involving \(a = 0\).

3Following our assumption that no group constitutes a majority alone, receiving at least 50% of the votes means obtaining the votes of at least two groups of individuals out of the three.

4Of course, there exist other ways of representing the political process; for example, the electoral competition models in Besley and Coate (2000) and Roemer (1998), or the citizen candidacy model developed in Besley and Coate (1997). However, we believe this specification is well suited to the problem under consideration.
Hence the retirees vote for the incumbent government if their pension is above a given level \( \bar{p} \), that is, \( p = \tau (1 + n) \geq \bar{p} \iff \tau \geq \bar{\tau} = \bar{p} / (1 + n) \). Similarly, the types \( f \) and \( o \) young individuals choose to reelect the government if \( V^f(\tau, a) \geq V^f \) and \( V^o(\tau, a) \geq V^o \), respectively, where \((\tau, a)\) is the government's choice. A strategy for a retiree is then a choice of \( \tau \in [0, 1] \) and for a type \( i \) working individual, \( i \in \{f, o\} \), a choice of \( \overline{V}^i \in [v(0), v(\tau^*) + \theta] \). We say that the strategies of at least two groups out of the three are compatible, when there exist policies satisfying simultaneously the reelection rules of these groups, which means that the government can be reelected by them.

In the remainder of this paper, we consider only symmetric strategies, that is, we assume that all individuals belonging to a given group will adopt the same strategy. This enables us to consider only the strategy of a representative individual in a given group and not the strategies of each individual voter.

Given the reelection rules fixed by the voters, the government chooses which pair of policies \((\tau, a)\) to implement. A (pure) strategy for the government is thus a mapping \( s^G : [v(0), v(\tau^*) + \theta]^2 \times [0, 1] \to [0, 1] \times \{0, 1\} \). It selects its strategy such as described in assumptions 1, 2, and 3.

The timing of the game is then as follows. In the first stage, the principals play a noncooperative simultaneous game in order to decide their reelection rule. In the second stage, the government observes these strategies and chooses which policy to implement. Finally, voters observe the policy chosen by the government and vote according to their reelection rule. Thus, we assume that they are able to commit in the first stage to these voting rules.

We have defined a two-stage game with complete information and the natural solution concept for such a game is subgame perfection. To solve it, we proceed backward, determining first the optimal choice of the government and then the equilibria of the first stage game played by the principals. Put differently, when choosing their equilibrium strategies, the voters anticipate the optimal reaction of the government in the second stage.

\[\text{It is worth insisting that the politicians are all identical. It follows that the citizens are indifferent between voting for the incumbent or the challenger in the last stage. For this reason, the assumption of commitment to the reelection rules is reasonable. In a model with differentiated politicians (for example, according to their preferences) the voters would compute the expected payoff obtained with the challenger in office to the utility level obtained with the incumbent government (assuming it would choose the same policy if reelected, i.e., that it follows a stationary strategy). In this case, the assumption of commitment would have to be abandoned.}\]

\[\text{It should be noted that this game is a particular form of the common agency game defined by Bernheim and Whinston (1986) and applied to government policymaking by, among others, Dixit, Grossman, and Helpman (1997). The only difference is that here the groups of individuals do not try to influence the incumbent government with monetary contributions but by promising their votes.}\]
3. Voting Equilibria and Policy Outcomes

In this section we describe all the (pure strategy) subgame perfect equilibria of this game. In this purpose, we first define \( \tilde{\tau} \) as follows:

\[
V_f(\tau^*, 0) = V_f(\tilde{\tau}, 1)
\]
or

\[
V_o(\tau^*, 1) = V_o(\tilde{\tau}, 0),
\]

and \( \tilde{\tau} > \tau^* \). This gives the tax rate (higher than \( \tau^* \)) that leaves a working individual indifferent between having the tax rate \( \tau^* \) and the undesired outcome in the other dimension or the tax rate \( \tilde{\tau} \) and his preferred reform outcome. Because \( \theta \) is assumed to be the same for the two types of working individuals, the value of \( \tilde{\tau} \) is the same for both types. In the remainder of the paper \( \tilde{\tau} \) is assumed to be smaller than 1. This assumption may not be satisfied when the reform decision is much more important than the pension policy in the eyes of the voters. The utility levels of the two young groups are represented in Figure 1.

Let’s define also \( \tau_o \) and \( \tau_f \), which are the tax rates satisfying, respectively, for given \( V^o \) and \( V^f \), \( V^o(\tau_o, 0) \) with \( \tau_o > \tau^* \) and \( V^f(\tau_f, 1) \) with \( \tau_f > \tau^* \).

We now prove the following proposition, which describes the only (pure strategy) equilibrium with the government voted out of office.

![Figure 1: Utility levels of the young](image-url)
PROPOSITION 1: There is only one (pure strategy) equilibrium with the government not reelected and it occurs iff $N_f^o \geq N_r$. The equilibrium strategies are $V^f = V^f(\tau^*, 1)$, $V^o = V^o(\tau^*, 0)$, $\tau \geq \overline{\tau}$ and the government chooses $(\tau^*, 1)$.

Proof: See Appendix. ■

We have identified the only equilibrium in which the government is not reelected. In this equilibrium, both young types ask for their maximal utility level; the government adopts the preferred tax rate of the young and chooses to implement the reform. The retirees want so high a tax rate that it is not in the interest of the type $o$ to deviate from their equilibrium strategy.

We are now able to describe all the possible (pure strategy) equilibria with reelection of this game. In the following proposition, we first describe equilibria involving the choice of $\tau = \tau^*$ by the government.

PROPOSITION 2: When $N_r \leq N_f^o$, there exist equilibria with reelection in which the choice of the government is $(\tau^*, 1)$. When $N_r > N_f^o$, this equilibrium outcome is not sustainable.

Proof: See Appendix. ■

In this proposition, we show that the preferred tax rate of the young is sustainable at equilibrium. We describe in the Appendix all the equilibria giving rise to the outcome $\tau = \tau^*$ and $a = 1$.

When both young groups opposed and favorable to the reform outnumber the old, not only is $\tau^*$ a possible equilibrium outcome but this is true even when the old ask for $\overline{\tau} > \tau^*$. This comes from the fact that the government receives more votes favoring the coalition made of both groups in the young population than favoring a coalition made of the old and either of the two young groups. The only possibility for the old to undo the young coalition is to ask for a tax rate between $\tau^*$ and $\overline{\tau}$, trying to induce the young opposed to the reform to deviate. However, the type $f$ are able to prevent such a deviation by adopting a strategy $V^f \in \{ V^f(\overline{\tau}, 1), V^f(\tau, 1) \}$. Indeed, if type $o$ deviate to $V^o(\overline{\tau}, 0)$, for example, the government will not choose the policy $(\overline{\tau}, 0)$ but will choose the policel $(\tau, 1)$, and the type $o$ would get a utility level of $V^o(\overline{\tau}, 1)$, which is lower than $V^o(\tau^*, 1)$, the utility level they obtain if they stick to the strategy $V^o = V^o(\tau^*, 1)$.

When $N_f^o \geq N_r > N_o^o$, equilibria with $\overline{\tau} \in \{ \tau^*, \overline{\tau} \}$ are no longer possible. If the type $f$ choose a strategy $V^f \in \{ V^f(\tau^*, 1), V^f(\overline{\tau}, 1) \}$, the choice of the government is not $(\tau^*, 1)$ anymore but $(\overline{\tau}, 1)$. The government prefers to

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7We restrict our attention to equilibria with $\tau \geq \tau^*$. Indeed, equilibria with $\tau < \tau^*$ are clearly impossible. If the government chooses $\tau < \tau^*$, it could receive the same number of votes by choosing $\tau^*$ ($\tau < \tau^*$ is Pareto-dominated and the three types of individuals would obtain a strictly greater utility level with $\tau^*$). Under assumption 3, $\tau < \tau^*$ cannot be an equilibrium choice for the government.
implement \((\bar{\tau}, 1)\), getting \(N^f_w + N_r\) votes, rather than \((\tau^*, 1)\), this policy ensuring the support of \(N^f_w + N^o_w\) individuals. However, equilibria with \(\bar{\tau} \geq \tilde{\tau}\) still exist. In this case, the tax rate asked by the retirees is so high that the type \(o\) workers are not tempted to make an alliance with them. Knowing that the government prefers to satisfy a coalition of the types \(f\) and \(o\) workers rather than a coalition of the type \(o\) workers and the retirees, there is no way for the retirees to change the outcome, that is, to obtain a tax rate larger than \(\tau^*\).

Consequently, the retirees do not want to deviate and the situation described in the proposition is an equilibrium.

When \(N_r > N^f_w > N^o_w\) the power of the old is much larger. An equilibrium with \(\tau = \tau^*\) is not possible anymore and we will see in the next proposition that equilibria with a larger tax rate exist. The idea is simple. When the retirees constitute the largest group, the government necessarily satisfies the coalition to which the retirees belong.

**Proposition 3:** (i) Equilibria with reelection in which the choice of the government is \((\tau', 1)\) with \(\tau' \in ]\tau^*, \tilde{\tau}\) occur only when \(N^f_w \geq N_r > N^o_w\) and are supported by the strategies \(\bar{V}^f = V^f(\tau', 1), \bar{V}^o = V^o(\tau', 0), \) and \(\bar{\tau} = \tau'\).

(ii) Equilibria with reelection in which the choice of the government is \((\tilde{\tau}, 1)\) occur only when \(N_r > N^f_w\) and are supported by the strategies \(\bar{V}^f = V^f(\tilde{\tau}, 1), \bar{V}^o \in [V^o(\tilde{\tau}, 0), V^o(\tau^*, 0)]\) and \(\bar{\tau} = \tilde{\tau}\).

(iii) Equilibria with reelection in which the choice of the government is \((\tau'', 1)\) with \(\tau'' \in ]\tilde{\tau}, 1]\) occur only when \(N_r > N^f_w\) and are supported by the strategies \(\bar{V}^f = V^f(\tau'', 1), \bar{V}^o = V^o(\tau'', 0), \) and \(\bar{\tau} = \tau''\).

**Proof:** See Appendix.

This proposition establishes the existence of equilibria with \(\tau \in ]\tau^*, 1]\).

Note first that, when \(N^f_w > N^o_w \geq N_r\), such equilibria do not exist. Combining the results of propositions 2 and 3 we can conclude that, in this case, the only possible equilibrium outcome is \((\tau^*, 1)\). For a young population sufficiently large and properly distributed between the two groups that compose it, the outcome of this game is the majoritarian outcome along the tax dimension. Old people are not enough to prevent such a collective decision.

On the contrary, when \(N_r > N^o_w\), \(\tau > \tau^*\) can be part of an equilibrium. For intermediate values of \(N_r (N^f_w \geq N_r > N^o_w )\), the retirees “drive” the outcome as soon as \(\bar{\tau} < \tilde{\tau}\). If they ask too high a tax rate, namely, \(\bar{\tau} \geq \tilde{\tau}\), they are not numerous enough to obtain satisfaction. We know from proposition 2 that the equilibrium tax rate is \(\tau^*\) in this case and they end up with a pension equal to \(\tau^*(1 + n)\). If \(N_r > N^f_w\), there are no equilibria with \(\tau \in ]\tau^*, \tilde{\tau}\) because, by deviating, the retirees are able to obtain a strictly larger tax rate. Indeed, if they deviate, the government is not reelected. Because they constitute the largest group in society and the government maximizes the number of votes obtained, they obtain the tax rate they wish. On the other hand, equilibria
with $\tau \in [\tilde{\tau}, 1]$ exist. In those equilibria, the retirees do not want to deviate because the government can be reelected by the coalition of the workers.

It is worth insisting on this result which is the main point of this paper. When the old are the largest group, there are no equilibria with $\tau = \tau^*$ selected by the government. A tax rate that comprises between $\tilde{\tau}$ and 1 is implemented. It illustrates in a striking way that the outcome of the voting process cannot be analyzed from a demographic perspective in isolation.

Note that in these equilibria with $\tau > \tau^*$, the young could all be better off by switching to a lower tax rate and the maximum feasible utility would be attained at $\tau = \tau^*$. However, in the noncooperative game we consider, they cannot ensure the outcome $\tau^*$. When $N^f_w \geq N^r > N^o_w$, this outcome could be achieved with type $f$ playing $V^f = V^f(\tau^*, 1)$ and type $o$ playing $V^o = V^o(\tau^*, 1)$. But, without commitment, if $\tilde{\tau} < \tau$, type $o$ have an interest to deviate and join the retirees to form a coalition. The only way for the type $f$ to prevent such a deviation is to make an alliance with the retirees. This situation is clearly of a prisoner’s dilemma nature. When $N^r > N^f_w$, they cannot obtain $\tau^*$ even with some commitment. Indeed, the government prefers to satisfy a coalition of the retirees and one young group rather than the two young groups.

We have seen in the previous propositions that a lot of equilibrium outcomes are possible. In particular, one cannot rule out $\tau^*$ as an equilibrium outcome when the retirees are more numerous than the type $o$ workers and less numerous than the type $f$ workers. Moreover, when the retirees constitute the largest group, their political power may be very important as a tax rate as large as 1 can be implemented. One can show that an equilibrium refinement such as the iterative deletion of weakly dominated strategies allows us to obtain clear-cut results.\(^8\) We obtain the following. When the number of retirees is between the number of type $o$ and $f$ workers, only equilibria with a tax rate strictly larger than $\tau^*$ and strictly lower than $\tilde{\tau}$ are possible. No equilibrium leading to the choice of $\tau^*$ by the government survives the deletion of dominated strategies. When the retirees are the largest group, the equilibrium tax rate is $\tilde{\tau}$. Equilibria with a larger tax rate are eliminated. This leads us to the conclusion that the political power of the retirees is larger than in the one-dimensional pension model but it is not unlimited in the sense that the equilibrium tax rate is strictly lower than 1.

Until now, we have described equilibria in which the government decides to adopt the reform. We conclude this section by showing that there are no equilibria with the reform not undertaken.

**PROPOSITION 4:** There are no equilibria with reelection in which the choice of the government is $(\tau, 0)$, $\forall \, \tau \in [0, 1]$. 

\(^8\)The proofs are available upon request.
**Proof:** Consider a potential equilibrium outcome \((\tau, 0)\) with \(\tau \in [0, 1]\). We show that the type \(f\) are always able to improve their welfare relative to this situation. The maximal utility level that the type \(f\) achieve when \(a = 0\) is \(V_f^f(\tau^*, 0)\). If \(N_{fw} \geq N_r\) and \(\tau \geq \tilde{\tau}\), they can move to \(\overline{V}_f^f = V_f^f(\tau^*, 1)\) and obtain the policy \((\tau^*, 1)\), whether the government is reelected or not. If \(\tau < \tilde{\tau}\), they can achieve at least \(V_f^f(\tau, 1)\) by deviating to \(\overline{V}_f^f = V_f^f(\tau, 1)\) if \(N_r > N_{fr}\). Finally, when \(N_r > N_{fw}\), necessarily the equilibrium tax rate is \(\tau = \tau\) and the type \(f\) can always obtain the utility level \(V_f^f(\tau, 1)\) by deviating to \(\overline{V}_f^f = V_f^f(\tau, 1)\), inducing the government to implement \((\tau, 1)\).

This proposition follows simply from our assumptions that the government maximizes the number of votes obtained and that the young in favor of \(a = 1\) outnumber the other young.

### 4. Extensions

We relax in this section two important assumptions of the model. The first one is the fact that the retirees are only interested in the level of their pension (in other words, they are single-minded). The second one concerns the discrete feature of the reform dimension. It is important to know to what extent the results obtained so far are driven by these assumptions. In the following subsections, we ask the following question: is it possible to obtain an equilibrium tax rate larger than \(\tau^*\) when the retirees are not single-minded or when the reform dimension is continuous?

#### 4.1. Partisan Retirees

We have made the assumption that the retirees were indifferent concerning the noneconomic dimension. We show in the following example that this is not an essential assumption. The results obtained in the previous section do not rely on the single-mindedness of the retirees but on the multidimensionality of the political process. The example is illustrated in Figure 2.

We denote \(V_r^f(\tau, a) = u(\tau(1+n)) + \theta a\) and \(V_r^o(\tau, a) = u(\tau(1+n)) + \theta(1-a)\), the utility functions of types \(f\) and \(o\) retirees, respectively. Using natural modifications of the notations, we argue that \((\tau', 1)\) is an equilibrium outcome supported by the strategies \(\overline{V}_r^f = V_r^f(\tau', 1), \overline{V}_r^o = V_r^o(\tau, 0), \overline{V}_w^f = V_w^f(\tau', 1),\) and \(\overline{V}_w^o = V_w^o(\tau, 0)\) when \(N_{fw} \geq N_r\) and \(N_{fr} > N_{wo}\). The government then selects the policy \((\tau', 1)\). Choosing \((\tau', 1)\) leads to its reelection by \(N_{fw} + N_{fr}\) individuals which is larger than \(N_r + N_{wo}\), the number of votes it obtains if it chooses the policy \((\tau, 0)\).

We verify that none of the principals wants to deviate. If type \(o\), whether workers or retirees, deviate, it does not change anything in the outcome. In particular, the type \(o\) workers could deviate to \(\overline{V}_w^o = V_w^o(\tau^*, 1)\) in order to
obtain the policy \((\tau^*, 1)\). However, because of our assumption that \(N^f_r > N^o_w\), the government prefers to adopt the policy \((\tau', 1)\) rather than \((\tau^*, 1)\), getting \(N^f_w + N^f_r\) votes instead of \(N^f_w + N^o_w\) votes. If type \(f\) workers ask for a higher level of utility, the government switches to the policy \((\tau', 0)\) and is reelected by all the retirees and the type \(o\) workers. Finally, if the type \(f\) retirees ask for a higher level of utility, the government cannot be reelected. It then selects a policy \((\tau, 1), \tau \in [\tau^*, \tau']\), giving satisfaction to the largest group, that is, the type \(f\) workers.

4.2. Second Dimension Continuous

We show in the following example that a tax rate larger than \(\tau^*\) is also possible when the ideological dimension is continuous: \(a \in [0, 1]\), in the case \(N^f_w \geq N^r_r > N^o_w\). Consider the strategies \(\bar{V}^f = \bar{V}^o = v(\bar{\tau}) + \theta > v(\tau^*) + \theta/2\), where \(\bar{\tau} > \tau^*\). It is easy to verify that none of the principals wants to deviate.
5. Conclusion

This paper has proposed an explanation of the political power of some minorities, here the retirees, based on the multidimensional nature of the political process. Starting from the traditional (unidimensional) voting model on social security, we show that adding a second dimension in the political debate leads to a higher equilibrium tax rate, making the retirees better off. This analysis therefore sheds light on the weakness of the median voter framework and the importance of considering the interactions between the multiple issues bundled in the political process.

The basic idea developed is the following. The workers are unanimous with respect to the pension policy but divided along the other dimension. As a consequence, the government cannot please both groups of workers simultaneously. The retirees are then able to exploit strategically this division by making an alliance with the worse-off workers, defending their preferred reform policy in exchange for a higher tax rate.

Our model implies that, even when the young would like to abandon the PAYG system, the retirees are able to obtain a positive pension level, thereby protecting part of the pension they were promised. In Casamatta, Cremer and Pestieau (2001), we just assume it. Even though these two models are quite different, in particular because we do not have income heterogeneity here, they can be considered as complementary.

In the last section we relax some important assumptions of the model, the single-mindedness of the retirees and the discrete nature of the reform policy, and we argue, using some examples, that the results obtained are qualitatively unchanged. One other important assumption is that individuals live only two periods. This implies that a tax rate less than the preferred tax rate of the workers is Pareto-dominated. It follows that only tax rates higher than this latter may occur at equilibrium, which allows us to conclude that the political power of the retirees unambiguously increases when moving from the one-dimensional to the two-dimensional model. It is therefore desirable to investigate a more general model with individuals living three periods. Another extension of interest would be to explore the dynamic implications of this model, giving a more satisfactory representation of the situation currently faced by the pension systems in most developed countries.

Appendix

Proof of Proposition 1: For the government not to be reelected, we must necessarily have $V_f > V_f(\bar{\tau}, 1)$, $V_o > V_o(\bar{\tau}, 0)$, and $\tau > \max\{\tau^o, \tau^f\}$.

Consider first the case where $N_r > N_w^f$. Following assumption 3, the government selects the policy $(1, 1)$. Then type $o$ clearly prefer to deviate to $V^o = V^o(\bar{\tau}, 0)$. Indeed, the government, by choosing the policy $(\bar{\tau}, 0)$, is then reelected by a coalition of type $o$ young individuals and the
retirees; type \( o \) are strictly better off with \((\tilde{\tau}, 0)\) than with \((1, 1)\). Therefore, we cannot have an equilibrium with \( N_r > N_w^f \) and the government not reelected.

Now let’s turn to the case \( N_w^f \geq N_r \). We first verify that the situation described in the proposition is an equilibrium. We know from assumption 1 that the government chooses \((\tau^*, 1)\). Type \( f \) obtain their ideal policy and don’t want to deviate. If the type \( o \) deviate to \( V^o = V^o(\tau, 0) \) with \( \tau < \tilde{\tau} \), the government is still voted out of office and choose \((\tau^*, 1)\). If the type \( o \) deviate to \( V^o = V^o(\tau, 0) \) with \( \tau \geq \tilde{\tau} \), the government is reelected by the coalition of the workers and chooses \((\tau^*, 1)\). Therefore, type \( o \) are indifferent between deviating or not. Retirees are also indifferent. If they deviate to \( \tau^o = V^o(\tau, 0) \) with \( \tau < \tilde{\tau} \), the government still adopts \((\tau^*, 1)\). Hence, this is an equilibrium configuration. We then verify that there is no other equilibrium configuration. If \( V_f < V^o \leq V^o(\tau^*, 0) \), the retirees should deviate to \( \tau = \tau^f \) because they would obtain \( \tau^f > \tau^* \) instead of \( \tau^* \).

Similarly, If \( V^o < V^f \leq V^f(\tau^*, 1) \) or \( V^o = V^f < V^f(\tau^*, 1) \), the retirees should deviate to \( \tau = \tau^o \).

**Proof of Proposition 2:** We first argue that in an equilibrium with \((\tau^*, 1)\) chosen by the government, necessarily \( V^f \in \{ V^f(\tilde{\tau}, 1), V^f(\tau^*, 1) \} \). If the type \( f \) set \( V^f \leq V^f(\tilde{\tau}, 1) \) and the government decides to implement \((\tau^*, 1)\), the type \( o \) should deviate to \( V^o = V^o(\tau^*, 0) \) in order to obtain the policy \((\tau^*, 0)\). We then consider the different possible cases:

**Case 1:**
\[ V^f \in \{ V^f(\tilde{\tau}, 1), V^f(\tau^*, 1) \} \]

In this case the equilibrium strategy of the type \( o \) is necessarily \( \overline{V}^o = V^o(\tau^*, 1) \). If they set \( \overline{V}^o < V^o(\tau^*, 1) \), there always exists a strategy for the retirees, \( \tilde{\tau} > \tau^* \), such that the government chooses a tax rate higher than \( \tau^* \). If the type \( o \) adopt the strategy \( \overline{V}^o > V^o(\tau^*, 1) \), the government can be reelected with the policy \((\tau^*, 1)\) only if \( \tilde{\tau} = \tau^* \) or \( \tilde{\tau} > \max\{\tau^o, \tau^f\} \). Clearly, this is not the optimal strategy for the retirees who can obtain at least \( \tau^f \) when deviating to \( \tau = \tau^f \).

\[ N_w^f > N_w^o \geq N_r \]

When \( \tilde{\tau} = \tau^* \), \( \overline{V}^f \in \{ V^f(\tilde{\tau}, 1), V^f(\tau^*, 1) \} \) and \( \overline{V}^o = V^o(\tau^*, 1) \), we are at equilibrium. The government chooses \((\tau^*, 1)\) and is reelected with unanimity. Type \( f \) do not want to deviate. Because they obtain their ideal policy, they weakly prefer this strategy to any other strategy. If the retirees deviate, it does not change anything in the outcome. The government gets the largest possible support, \( N_w^f + N_w^o \), with the policy \((\tau^*, 1)\). Therefore they are indifferent. Type \( o \) are also indifferent.
Whatever the deviation they consider, the government still implements $(\tau^*, 1)$.

The strategies $\tau \in]\tau^*, \tilde{\tau}[,$ $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ and $\overline{V}^o = V^o(\tau^*, 1)$ are not equilibrium strategies. Indeed, with these strategies, the government chooses the policy $(\tau^*, 1)$. The strategy of the type $o$ is not optimal. They should deviate, for example, to $\overline{V}^o = V^o(\tilde{\tau}, 0)$. The government would adopt the policy $(\tilde{\tau}, 0)$ and would be reelected by the type $o$ and the retirees. However, when $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ this is an equilibrium. With $\overline{V}^f$ contained in this interval and as soon as $N^f_w + N_r > N^o_w + N_r$, the optimal reaction of the government is to adopt the policy $(\tau^f, 1)$ if the type $o$ deviate to $\overline{V}^o > V^o(\tau^*, 1)$.

When $\overline{\tau} \geq \tilde{\tau}$, the type $o$ do not want to deviate and $\overline{\tau} \geq \tilde{\tau}$, $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ and $\overline{V}^o = V^o(\tau^*, 1)$ is an equilibrium.

$$N^f_w \geq N_r > N^o_w.$$ 

The strategies $\tau = \tau^*$, $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ and $\overline{V}^o = V^o(\tau^*, 1)$ are not part of an equilibrium. The retirees should deviate, for example, to $\overline{\tau} = \tau^f$. The government can then be reelected by the coalition of the workers, setting the policy $(\tau^*, 1)$, or the coalition of the type $f$ and the retirees, setting the policy $(\tau^f, 1)$. When $N^f_w + N_r > N^f_w + N^o_w$, it chooses the second alternative and the retirees are strictly better off.

As soon as $N_r > N^o_w$ there is no equilibrium with $\overline{\tau} \in]\tau^*, \tilde{\tau}[,$ and the policy $(\tau^*, 1)$ is adopted. To see this, consider the equilibrium with the strategies $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ $\overline{V}^o = V^o(\tau^*, 1)$, $\overline{\tau} \in]\tau^*, \tilde{\tau}[,$ depicted above. Confronted by these strategies, the government now chooses $(\tau^*, 1)$ and the type $f$ would be better off moving to $\overline{V}^f > V^f(\tilde{\tau}, 1)$.

Finally, $\overline{\tau} \geq \tilde{\tau}$, $\overline{V}^f \in]V^f(\tilde{\tau}, 1), V^f(\tau^*, 1)[,$ and $\overline{V}^o = V^o(\tau^*, 1)$ is not an equilibrium because the retirees would like to deviate, for example, to $\overline{\tau} = \tau^f$.

$$N_r > N^f_w > N^o_w.$$ 

Following the same arguments, we can conclude that there is no equilibrium with $(\tau^*, 1)$ chosen by the government in this case.

Case 2:

$$\overline{V}^f = V^f(\tau^*, 1).$$

When $\overline{V}^f = V^f(\tau^*, 1)$, the strategy of the type $o$ is necessarily $\overline{V}^o \leq V^o(\tau^*, 1)$ or $\overline{V}^o = V^o(\tau^*, 0)$. If they adopt the strategy $\overline{V}^o \in]V^o(\tau^*, 1), V^o(\tau^*, 0)[,$ the government can be reelected with the policy $(\tau^*, 1)$ only if $\overline{\tau} = \tau^*$. Clearly, this is not the optimal strategy for the retirees who can obtain at least $\tau^o$ when deviating to $\overline{\tau} = \tau^o$. 

\(N_w^{f} \geq N_r.\)

(1) \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o = V^o(\tau^*, 0), \bar{\tau} = \tau^*.\)

Given that \(\bar{V}^o = V^o(\tau^*, 0)\) and \(\bar{V}^f = V^f(\tau^*, 1)\), if the old deviate, the government is not reelected anymore, in which case we know that it selects \((\tau^*, 1)\); retirees are then indifferent between deviating or not. Knowing that \(\bar{\tau} = \tau^*\) and \(\bar{V}^f = V^f(\tau^*, 1)\), type \(o\) individuals are indifferent between deviating or not because it does not change the outcome.

(2) \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o \leq V^o(\tau^*, 1), \bar{\tau} = \tau^*.\)

For given strategies of types \(o\) and \(f\), the retirees are indifferent between deviating or not because it would not change the outcome. Suppose, for example, that they try to make an alliance with the type \(o\) and deviate to \(\tau = \tau^o\). As \(N_w^f + N_o^w > N_o^w + N_r\), the government still prefers to implement \((\tau^*, 1)\) rather than \((\tau^o, 0)\). Given the other players’ strategies, the type \(o\) could be tempted to deviate to \(V^o > V^o(\tau^*, 1)\). However, the policy \((\tau^*, 1)\) ensures a support of \(N_w^f + N_r\) to the government which is larger than \(N_o^w + N_r\), the number of votes the government obtains if it selects \((\tau, 0)\) with \(\tau > \tau^*.\) Consequently, type \(o\) do not want to deviate either.

(3) \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o = V^o(\tau^*, 0)\) or \(\bar{V}^o \leq V^o(\tau^*, 1)\) and \(\bar{\tau} \in]\tau^*, \bar{\tau}[.\)

These are not equilibrium strategies. When \(\bar{V}^o = V^o(\tau^*, 0)\), the government is not reelected and we are looking for equilibria with reelection. When \(\bar{V}^o \leq V^o(\tau^*, 1)\), the type \(o\) would clearly prefer the strategy \(\bar{V}^o = V^o(\bar{\tau}, 0)\), in which case the government would have no other possibility than choosing \((\tau, 0)\), instead of \((\tau^*, 1)\).

(4) \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o \leq V^o(\tau^*, 1), \) or \(\bar{V}^o = V^o(\tau^*, 0), \bar{\tau} \in [\bar{\tau}, 1]\.\)

Now the retirees ask for a tax rate larger than \(\bar{\tau}.\) When \(\bar{V}^o \leq V^o(\tau^*, 1)\), there is thus no chance that the type \(o\) deviate in order to form a coalition with them and these strategies constitute an equilibrium. As in the previous case, the government is not reelected when \(\bar{V}^o = V^o(\tau^*, 0)\) and it cannot be an equilibrium with reelection.

\(N_r > N_w^{f}.\)

The situations described in (1) and (2) are not equilibria now. Indeed, when \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o = V^o(\tau^*, 0),\) and \(\bar{\tau} = \tau^*,\) the retirees should deviate to a higher tax rate. The government is then not reelected. Following assumptions 1 and 3, it would set a tax rate equal to 1. If \(\bar{V}^f = V^f(\tau^*, 1), \bar{V}^o \leq V^o(\tau^*, 1),\) and \(\bar{\tau} = \tau^*,\) the retirees want to deviate, for example, to \(\bar{\tau} = \tau^o.\) The government can implement
(τ*, 1), obtaining $N_f^w + N_o^w$ votes, or (τ*, 0), obtaining $N_r + N_o^w$ votes. When $N_r > N_f^w$, it prefers the latter policy.

We show that the strategies in case (3) above is not an equilibrium configuration either. When $V_o = V_o(τ^*, 0)$, the government is not re-elected and when $V_o ≤ V_o(τ^*, 1)$, it is re-elected by the coalition of the type o and the retirees by choosing $(τ^*, 0)$. These strategies cannot therefore be part of an equilibrium with the policy outcome $(τ^*, 1)$.

Finally, if $V_f = V_f(τ^*, 1)$, $V_o ≤ V_o(τ^*, 1)$, and $τ ∈ [τ_o, 1]$, two cases are possible. Either $τ > τ_o$ and the retirees want to deviate to $τ = τ_o$ or $τ ≤ τ_o$ and the government chooses $(τ^*, 0)$. When $V_o = V_o(τ^*, 0)$, the government is not re-elected and selects the tax rate 1. ■

Proof of Proposition 3:

Equilibria with $(τ', 1), τ' ∈ ]τ^*, τ_o]$.

For $(τ', 1)$ to be an equilibrium with reelection, it must be supported by the type f and at least one of the other groups, the type o and the retirees.

Assume that type f and o support this equilibrium, that is, $V_f(τ', 1) ≥ V_f'$ and $V_o(τ', 1) ≥ V_o'$. When $N_f^w ≥ N_r$, this cannot be an equilibrium because the type f should deviate, for example, to $V_f' = V_f(τ^*, 1)$. Indeed, in this case, the government will always select $(τ^*, 1)$, whatever the retirees do. When $N_r > N_f^w$, the retirees can always obtain a better policy than $τ'$, for example, by fixing $τ = \max\{τ^*, τ'\}$. So, at equilibrium, $(τ', 1)$ cannot be supported by the coalition of the workers, whatever the retirees do. This reasoning holds more generally for any $τ' > τ^*$. Therefore, we must now look only at the case where a coalition of type f and of the retirees supports the equilibrium outcome.

Now assume that $V_f(τ', 1) ≥ V_f'$, $τ ≤ τ'$, and $V_o > V_o(τ', 1)$. Remembering Assumption 3, we can restrict our attention to the case $V_f(τ', 1) = V_f'$ and $τ = τ'$ (if $V_f' < τ$, the type f have a strict interest to deviate when $N_r > N_f^w$ and the retirees have a strict interest to deviate when $N_r ≤ N_f^w$). Several possibilities must then be distinguished.

1. $V_o ≤ V_o(τ^*, 1)$.

When $N_o^w ≥ N_r$, two cases must be considered. If $V_o = V_o(τ^*, 1)$, the government chooses the policy $(τ^*, 1)$. If $V_o < V_o(τ^*, 1)$, either the government can only be re-elected by the coalition of the workers and chooses $(τ^*, 1)$, or it is re-elected with unanimity choosing $(τ, 1)$ and the type o deviate to $V_o = V_o(τ^*, 1)$, for example, inducing the government to choose $(τ^*, 1)$. Consequently, these strategies cannot constitute an equilibrium with $(τ', 1), τ' > τ^*$. When $N_f^w ≥ N_r > N_o^w$, the government
chooses \((\tau', 1)\) and the type \(f\) end up strictly better off by moving to \(V^f = V^f(\tau^*, 1)\). When \(N_r > N_w^f\), the retirees deviate to \(\tau \in ]\tau', \tau^*.\)

- \(V^o(\tau^*, 1) < V^o < V^o(\tau', 0)\).

Whatever the relative size of each group of principals, the retirees prefer moving to \(\bar{\tau} = \tau^o\). Indeed, the government has then no other choice than setting \((\bar{\tau}, 0)\).

- \(V^o(\tau^*, 1) < V^o < V^o(\tau', 0)\).

When \(N^o_w \geq N_r\), the type \(o\) improve their welfare if they deviate to \(V^o = V^o(\tau^*, 1)\). Indeed, the government chooses \((\tau^*, 1)\) instead of \((\tau', 1)\). When \(N^f_w \geq N_r > N^o_w\), the type \(o\) are indifferent between deviating or not because it does not change the outcome. The type \(f\) do not gain anything by asking for a lower level of utility. If they fix a higher level \(\bar{V}^f > V^f(\tau', 1)\), the government implements \((\tau', 0)\) and they are strictly worse off. Hence, they do not want to deviate. If the retirees deviate to a higher \(\bar{\tau}\), the government is not reelected anymore. Under our assumptions that the government maximizes the number of votes obtained and favor, when indifferent, the largest group, it chooses \((\tau^*, 1)\). Retirees have thus no interest in deviating. \(V^f = V^f(\tau^*, 1)\) and \(V^o = V^o(\tau^*, 0)\) are subgame perfect Nash equilibria when \(N^f_w \geq N^o_w\). This is not an equilibrium anymore when \(N_r > N^f_w\) for the reason that the retirees want to deviate. Indeed, if the retirees deviate, the government is not reelected but the largest group is now the group of the old.

- \(V^o(\tau^*, 1) < V^o < V^o(\tau', 0)\).

Whenever they are the largest group in society, the type \(f\) or the retirees want to deviate because they know that the government will favor them in such cases.

**Equilibria with** \((\tau'', 1), \tau'' \geq \bar{\tau}\).

We argue that \(\bar{V}^f = V^f(\tau'', 1), \bar{V}^o = V^o(\tau'', 0)\) and \(\bar{\tau} = \tau''\) is not an equilibrium when \(N^o_w \geq N_r > N^o_w\) as the type \(f\) want to deviate, for example, to \(\bar{V}^f = V^f(\tau^*, 1)\). On the contrary it is an equilibrium when \(N_r > N^f_w\) because the retirees do not deviate. If they deviate, the government is still reelected by the coalition of the workers. Note also that, when \(N_r > N^f_w\), \(\bar{V}^f = V^f(\bar{\tau}, 1), \bar{V}^o \in ]V^o(\bar{\tau}, 0), V^o(\tau^*, 0)\) and \(\bar{\tau} = \bar{\tau}\) is an equilibrium. Indeed, the type \(o\) are indifferent between deviating or not because it does not affect the outcome. If the type \(f\) ask for a higher \(\bar{V}^f\), the government is not reelected and then chooses \(\tau = 1\) (Assumption 3). If the retirees ask for a higher \(\bar{\tau}\), the government can be reelected by setting \((\tau^*, 0)\).
We finally show that there are no other equilibria with $\tau'' > \tilde{\tau}$. Following the previous discussion, such an equilibrium possibly occurs only when $\bar{V}^f = V^f(\tau'', 1)$ and $\bar{\tau} = \tau''$. Consider then the following cases regarding type $o$ strategy:

- $\bar{V}^o < V^o(\tau'', 0)$

  If $N_w^f \geq N_r$, either the type $f$ strictly prefer to deviate or the government chooses $\tau^*$. If $N_r > N_w^f$, the retirees deviate, for example, to $\bar{\tau} = \tau^o$.

- $\bar{V}^o > V^o(\tau'', 0)$

  The retirees never want to deviate. If $N_w^o \geq N_r$, the government chooses $\tau^*$. If $N_r > N_w^o$, the type $f$ have a strict interest to deviate.

References


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