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The Political Economy of Social Security

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Abstract
We consider a two-period overlapping generations model in which individual voters differ by age and by productivity. In such a setting, a redistributive pay-as-you-go system is politically sustainable, even when the interest rate is higher than the rate of population growth. The workers with medium wages (not those with the lowest wages) and the retirees form a majority which votes for a positive level of social security. This level depends on the difference between the rates of population growth and interest as well as on the redistributiveness of the benefit rule.

Keywords: Social security; majority voting

JEL classification: H55; O41; O9

I. Introduction

The determination of social security benefits and contribution rates is often studied in majority voting models. This approach was initiated by Browning (1975), who considers a society where people differ only according to age. Within that framework, the decisive voter is the median age individual. The voting equilibrium then leads to an excessive social security budget.

It is increasingly recognized that the political forces involved in the debate on social security cannot be represented along a single age axis. There are other dimensions, particularly that of income and, more specifically, income heterogeneity within generations. The income dimension is likely to be important when the social security system redistributes not only from younger to older generations, as the pay-as-you-go (PAYG) system often does, but also from high- to low-wage workers. In other words, introducing differential

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earning capacities is particularly important when the benefit rule is redistributive rather than actuarially fair.

The voting outcome also depends on the alternatives available for financing old-age consumption. Consider the hypothetical case where private savings are not available. Then, all individuals vote for a positive tax rate: the old because it determines their benefit level and the young to ensure themselves some consumption during retirement. However, when private savings are available and when their rate of return is higher than that offered by the social security system, individuals will vote against social security. This can occur for at least three reasons. First, if the social security system is of the PAYG type, and the rate of interest is higher than the rate of population growth, private saving is more attractive, at least for a young worker. Second, if the payroll tax implies some deadweight loss, the same bias may occur. Third, if the social security system is redistributive, those who pay for redistribution, namely workers with earnings above the average level, will prefer private saving. Conversely, workers with lower than average earning capacities benefit from redistribution. Consequently, they may vote in favour of social security, even when there are some distortions and when the rate of interest is higher than the rate of population growth.

In this paper, we adopt a steady-state setting with given rates of interest and population growth. We also assume that the benefit rule is given. Some countries, labelled Bismarckian, such as Germany or France, offer replacement ratios that are stable across income levels, whereas others, labelled Beveridgean, such as Canada or the Netherlands, tend to have replacement ratios that fall as income increases. Table 1 illustrates this distinction for nine countries.

The assumption that the benefit rule is given implies that we concentrate

<table>
<thead>
<tr>
<th>Half of average</th>
<th>Average income</th>
<th>Twice average</th>
<th>Regime*</th>
<th>Spending as % of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>76</td>
<td>44</td>
<td>25</td>
<td>BE</td>
</tr>
<tr>
<td>France</td>
<td>84</td>
<td>84</td>
<td>73</td>
<td>BI</td>
</tr>
<tr>
<td>Germany</td>
<td>76</td>
<td>72</td>
<td>75</td>
<td>BI</td>
</tr>
<tr>
<td>Italy</td>
<td>103 (4\times)</td>
<td>90</td>
<td>84 (3\times)</td>
<td>BI</td>
</tr>
<tr>
<td>Japan</td>
<td>77</td>
<td>56</td>
<td>43</td>
<td>MI</td>
</tr>
<tr>
<td>Netherlands</td>
<td>73</td>
<td>43</td>
<td>25</td>
<td>BE</td>
</tr>
<tr>
<td>New Zealand</td>
<td>75</td>
<td>38</td>
<td>19</td>
<td>BE</td>
</tr>
<tr>
<td>UK</td>
<td>72</td>
<td>50</td>
<td>35</td>
<td>MI</td>
</tr>
<tr>
<td>USA</td>
<td>65</td>
<td>55</td>
<td>32</td>
<td>MI</td>
</tr>
</tbody>
</table>

*BI: Bismarckian; BE: Beveridgean; MI: mixed.
Source: Johnson (1998).

on one specific aspect of the problem. In other words, our model is intended as a building block in a more ambitious set-up, encompassing a broader range of decision variables. This is a meaningful approach, for instance, if the decision process is inherently sequential. It may be noted that while the redistributive character of a system (i.e., its more or less Bismarckian or Beveridgean design) and its size are both essential features of a retirement system, they are not exactly symmetric. Its redistributive character is, to a large extent, an integral part of the very definition of the system itself. Regardless of whether systems are Bismarckian or Beveridgean, they imply specific institutional and administrative arrangements which cannot be overturned in the short run. By now, the Bismarckian system is solidly anchored in the traditions of countries like Germany and France. On the other hand, the tradition is Beveridgean in the UK.

In the concluding section, we briefly discuss the choice of the benefit rule at an earlier, "constitutional", stage. Decisions at this stage can be made either by a welfare-maximizing authority or through a voting procedure. In either case, decisions in the first stage will be contingent on the induced outcome in the second stage. Consequently, characterization of the outcome for any given benefit rule is a necessary step in the analysis. A noteworthy observation that can be derived from analysing constitutional decisions is that, even from a pure Rawlsian viewpoint, it may be optimal to adopt a benefit rule that is not "too redistributive". Interestingly, the less redistributive than otherwise optimal benefit rule is not (or not only) adopted to mitigate labour market distortions but also to induce a majority to opt for generous retirement benefits.

Following Browning (1975), most of the subsequent literature assumes that individuals differ only in age and focuses on simple majority voting and the median voter outcome. Several variations on the Browning model have been considered; see Myles (1995) for a survey. While these studies have produced different specific results, the fundamental conclusion remains the same: majority voting tends to yield overspending on social security. Among the most representative variations, let us mention Hu (1982) who considers uncertainty of benefit receipts, Broadway and Wildasin (1989) who introduce an explicit capital market, and Veall (1986) who assumes intergenerational altruism.

A more drastic departure from Browning's setting is provided by Tabellini (2000). He assumes that individuals are altruistic (children towards parents and parents towards children) and introduces differences in income as a second source of heterogeneity, along with the traditional age differences. Another specific feature of his model is that there is no commitment to preserve past decisions in the future. The main result is that in such a setting, a coalition of the young poor and the retired may sustain a positive tax rate.

In our paper, there are also two sources of heterogeneity. However, there is
no altruism and we return to the conventional commitment assumption. Individual voters differ not only according to age but also according to productivity. In our setting, medium-wage workers, rather than those with the lowest wage, join the retirees to form a majority and vote for a positive level of social security. Furthermore, this level is often in excess of the one which maximizes lifetime welfare. The majority equilibrium level is also shown to depend on the difference between the rates of population growth and interest, as well as on the redistributiveness of the benefit rule.

II. The Model

Consider a small open one-sector economy with given interest rate, \( r_t \). At each period of time \( t \), two generations overlap: \( L_t \) workers and \( L_{t-1} \) retirees, with \( L_t = L_{t-1}(1 + n_t) \), where \( n_t \) is the rate of population growth. Individuals differ in two ways: the generation they belong to and their wage \( w \), a continuous variable with support \([w_-, w_+]\), mean \( \bar{w} \) and median \( w_m < \bar{w} \). Individual labour supply is given and normalized to 1.

The pension benefits that an individual earning \( w \) expects to receive is \( p_{t+1}(w) \). We assume that \( p_{t+1}(w) \) consists of two parts: a contributory part which is directly related to individual earning, \( w \), and a non-contributory part which depends on average earnings, \( \bar{w} \). With a pay-as-you-go (PAYG) scheme, the average return of the social security system is given by the population growth rate. These properties yield the following expression for \( p_{t+1}(w) \):

\[
p_{t+1}(w) = (1 + n_{t+1})\tau_{t+1}(\alpha w + (1 - \alpha)\bar{w}).
\]  

The parameter \( \alpha \) is the Bismarckian factor, that is the fraction of pension benefits that is related to contributions; we assume \( 0 \leq \alpha \leq 1 \). When \( \alpha = 1 \), the pension scheme is purely Bismarckian or contributory; when \( \alpha = 0 \), pension benefits are uniform and the scheme is Beveridgean. Moreover, throughout the paper, we assume dynamic efficiency: \( r_t \geq n_t > 0, \forall t \).

We first analyse the optimal saving decision of a working individual born in period \( t \) with earning \( w \). He is subject to a payroll tax \( \tau_t \) and expects the future tax rate to be \( \tau_{t+1} \) when old. He can then allocate his disposable income between consumption \( c_t \) and saving \( s_t \). When he retires, his consumption \( d_{t+1} \) is equal to the gross return of his saving, \((1 + r_{t+1})s_t\), and a pension \( p_{t+1} \). Formally, he solves the following programme:

\[
\max_{s \geq 0} U_t = u(c_t) + \beta u(d_{t+1})
\]

subject to:

\[ w(1 - \tau_t) = c_t + s_t \]  

(3)  

and  

\[ d_{t+1} = (1 + r_{t+1})s_t + p_{t+1}(w). \]  

(4)  

In (2), \( u(.) \) is strictly concave and \( \beta \leq 1 \) is a factor of time preference. Let \( \sigma \) denote the elasticity of substitution between \( c_t \) and \( d_{t+1} \). We assume throughout the paper that \( \sigma \) is constant and that \( \sigma < 1 \), which means that there is not much substitution in consumption, a widely accepted assumption. The coefficient of relative risk aversion, \( R_r(x) = -xu''(x)/u'(x) \) is thus also constant and equal to \( \varepsilon = 1/\sigma \). We restrict savings to be non-negative.\(^1\)  

The first-order condition associated with an interior solution of \( s_t \) is the following:  

\[ -u'(c_t) + \beta u'(d_{t+1})(1 + r_{t+1}) = 0. \]  

(5)  

Denoting \( s_t^A \geq 0 \) the optimal value of \( s_t \), we can define the (indirect) utility function of an individual with income \( w \) as:  

\[ V_t(\tau_t, \tau_{t+1}, w) = u(w(1 - \tau_t) - s_t^A) + \beta u((1 + r_{t+1})s_t^A + p_{t+1}(w)). \]  

(6)  

We now determine the steady-state majority voting equilibrium tax rate. We consider \( \alpha \) as given; the problem is therefore unidimensional and, under the condition that preferences are single-peaked, the median voter theorem, which ensures the existence of a Condorcet winner, applies. Individuals vote for \( \tau \) believing that the value of \( \tau \) chosen by the majority will hold for ever.\(^2\) Formally, they expect that \( \tau_{t+1} = \tau_t, \forall t \). We can thus remove all the time subscripts in the subsequent equations.  

In the following, we first derive the preferred tax rate of the retirees and the workers, respectively. Then, to identify the Condorcet winner, we order these preferred alternatives. The Condorcet winner is the tax rate such that half of the population prefers a higher and half of the population a lower tax rate. For the time being, there is no tax distortion. Next, we derive the comparative statics of the majority voting solution with respect to some parameters of the model, namely \( \alpha \) and \( n \). We then compare the PAYG majority voting equilibrium tax rate with a (collective) fully funded (FF)  

---  

\(^1\)Allowing negative savings would generate extreme solutions. Young people in favour of the PAYG pension system would vote for a tax rate of 1 and would rely entirely on borrowing to finance their present consumption.  

\(^2\)Or that it will hold at least for the next two periods; the tax rate in later periods is of no relevance for these individuals.  

solution, granted that both systems adopt the same benefit rule. Finally, we solve the problem when taxation generates (quadratic) distortions.

III. Preferred Tax Rates of the Different Agents

The Retirees

Each retiree has some non-negative private savings, $s$, with return $r$. Such private saving is the result of a past decision and the retirees have no control over it. The only variable they can affect is the tax rate which determines their pension level. As there is no altruism, their preferred tax rate, $\tau^R$, is the one which maximizes their consumption:

$$d = (1 + r)s + (1 + n)\tau(\alpha w + (1 - \alpha)\bar{w}).$$  \hspace{2cm} (7)

The solution is given by $\tau^R = 1$ for all retirees.\(^3\) In words, retirees want the tax to be as high as possible. This is because the current tax affects their pension level, but has no impact on their contributions (which were paid in the previous period).

The Workers

A worker with earning $w$ chooses $\tau^A(w)$ which maximizes:

$$\nu(\tau, w) = u(w(1 - \tau) - s^A) + \beta u((1 + r)s^A + (1 + n)\tau(\alpha w + (1 - \alpha)\bar{w})), \hspace{2cm} (8)$$

where $s^A \geq 0$ is the optimal level of private saving and $\nu(\tau, w) = V(\tau, \tau, w)$.

Note that a worker will always be in favour of a zero tax if:

$$1 + r > (\alpha + (1 - \alpha)\bar{w}/w)(1 + n).$$  \hspace{2cm} (9)

This means that the return from private saving is higher than the return from PAYG social security. Without tax distortions, these returns are independent of the tax rate. Consequently, a given individual will prefer either a zero tax rate and positive private saving if (9) holds or a strictly positive tax rate and no private saving in the opposite case.

In other words, an individual prefers private saving if his wage is higher than $\hat{w}$ defined as:

\(^3\)This result is rather extreme, but introducing tax distortions or some degree of altruism from the old to the young would yield a value of $\tau^R$ smaller than 1.
\[
\hat{w} = \frac{1 - \alpha}{(1 + r)/(1 + n) - \alpha} \bar{w} \leq \bar{w}.
\]

(10)

It can easily be checked that \(\hat{w} = \bar{w}\) if \(n = r\), \(\partial \hat{w}/\partial n > 0\), and \(\partial \hat{w}/\partial \alpha < 0\). The solution to the worker's problem is characterized in

**Proposition 1.** The preferred tax rates of the workers have the following properties:

(i) \(\tau^A(w) = 0\) if \(w > \hat{w}\) and \(\tau^A(w) > 0\) if \(w \leq \hat{w}\);

(ii) \(\frac{\partial \tau^A(w)}{\partial w} > 0\) if \(w < \hat{w}\);

(iii) \(\max \tau^A(w) = \tau^A(\hat{w}) < \tau^R = 1\).

**Proof:** See Appendix.

The first point of the proposition recalls that only workers with a sufficiently low wage level want a positive tax rate. The second point says that preferred tax rates are increasing with income. This result arises because the intertemporal elasticity of substitution, \(\sigma\), is smaller than one. Intuitively, the relationship between low intertemporal substitution and the property that the preferred tax rate increases with income is easily understood. Consider, for example, the extreme case where there is no substitution at all: individuals want to equalize their two-period consumptions. Because the rate of return of PAYG social security is decreasing with income, the high-wage workers would like to transfer a greater proportion of their income from the first period to the second period as compared to those with a low wage.\(^4\) Finally, point (iii) states that the maximal preferred tax rate of the workers is lower than 1, the preferred tax rate of the retirees. This is simply due to the fact that individuals want to consume when young.\(^5\)

**IV. The Majority Voting Solution**

Because their utility is increasing with the value of the tax rate, preferences of the retirees are obviously single-peaked. The objective function of the

\(^4\)The property that preferred tax rates are increasing with income and may seem surprising. Within our framework, it results from the (realistic) assumption that the intertemporal elasticity of substitution is low. However, the assumption of a strictly proportional tax is crucial. If we had instead assumed that the poorest individuals are exempted from taxation, they would have a high preferred tax rate, thereby joining the retirees to sustain a generous social security system.

\(^5\)Only for a linear utility function (which corresponds to an infinite intertemporal elasticity of substitution), would a corner solution with \(\tau^A = 1\) be obtained.
workers can easily be shown to be strictly concave, which implies that it is single-peaked; see the proof of Proposition 1. The workers are divided into two classes, those who prefer a zero tax rate and positive savings and those who prefer a positive tax rate and no savings. The utility of individuals in the first group decreases with the tax rate. For the second group, the preferred value of the tax rate is given by an interior solution.

Summing up, a fraction $1/(2 + n)$ of citizens, the retirees, is in favour of $\tau^R = 1$. Further from Proposition 1, all workers with earnings above $\hat{w}$ are in favour of a zero tax. The preferred tax rate for the workers with earnings below $\hat{w}$ increases with $w$. The profile of preferred tax rates is represented in Figures 1 and 2 for the cases $r = n$ and $r > n$, respectively. We can now determine the decisive voter and thus report the majority voting equilibrium tax rate in the following proposition.

**Proposition 2.** If $\int_{\hat{w}}^{w^*} f(w) dw < n/(2 + n)$, the majority voting equilibrium tax rate, $\tau^*$, is 0. If $\int_{\hat{w}}^{w^*} f(w) dw \geq n/(2 + n)$, the majority voting equilibrium tax rate is the rate preferred by the workers with earnings $\hat{w}$ defined as follows:

$$\int_{\hat{w}}^{w^*} f(w) dw = \frac{n}{2(1 + n)}. \quad (11)$$

**Proof:** The Condorcet winner is the tax rate such that one-half of the total population prefers a higher tax rate and the other half a lower tax rate. The total number of individuals who prefer a tax rate higher than 0 is $L + L(1 + n) \int_{\hat{w}}^{w^*} f(w) dw$, where $L$ is the number of old individuals. The total population is $L + L(1 + n) = L(2 + n)$. Consequently, if $L + L(1 + n) \int_{\hat{w}}^{w^*} f(w) dw < L(2 + n)/2 \iff \int_{\hat{w}}^{w^*} f(w) dw < n/(2 + n)$, less than one-half of the total population prefers a strictly positive tax rate and the Condorcet winner is 0. On the other hand, when $\int_{\hat{w}}^{w^*} f(w) dw \geq n/(2 + n)$, the individuals who prefer a strictly positive tax rate constitute more than one-half of the total population and the Condorcet winner is strictly positive. It is the tax rate preferred by the individual with income $\hat{w}$ such that those in the population who prefer a higher tax rate constitute half of the total population:

$$L + L(1 + n) \int_{\hat{w}}^{w^*} f(w) dw = \frac{L(2 + n)}{2}. \quad (12)$$

Straightforward manipulations lead to (11). ■
The proposition first states that the majority voting equilibrium tax rate is zero when the number of working individuals in favour of a positive tax rate is too low. This may occur in particular when \( r \) is large relative to \( n \). In this case, the redistributive effect of PAYG social security is dominated by the high return of private saving and even the poorest workers may prefer to save privately. The second part of the proposition states that when the Condorcet winner is strictly positive, the majority coalition consists of the retirees and of the workers with medium wages. This result is reminiscent of Epple and Romano's (1996) "ends against the middle" equilibrium in which there is a coalition made up of the tails of the income distribution; see also Casamatta, Cremen and Pestieau (2000). This property clearly hinges on our assumption on the elasticity of substitution. With \( \sigma > 1 \), preferred tax rates would be decreasing with income and the majority coalition would be composed of the retirees and the poorest working individuals.

V. Comparative Statics

The illustrative figures in Table 1 suggest that the size and the redistributive character of a system are inversely related. Put differently, the most generous systems also appear to be those which redistribute the least. To see whether this stylized fact is consistent with our model, let us now study the impact of the contributive part on the equilibrium tax rate. Consider the case where the Condorcet winner is strictly positive.

Differentiating (11) with respect to \( \alpha \) yields:

\[
\frac{\partial \tilde{w}}{\partial \alpha} = \frac{\partial \tilde{w} f(\tilde{w})}{\partial \alpha f(\tilde{w})} \leq 0. \tag{13}
\]

\[\text{Fig. 1. Preferred tax rates when } r = n\]

Furthermore, the first-order condition for $\tau^A$, implies:

$$
\frac{\partial \tau^A}{\partial \alpha}(\tilde{w}(\alpha), \alpha) = \frac{\beta(1 + n)(\tilde{w} - \tilde{w})u'(d)(1 - R_d(d))}{-D_t}.
$$

(14)

Keeping in mind that $\tilde{w} < \tilde{w}$ and that $R_d(.) > 1$, this expression is positive. Finally, recalling that $\tau^*(\alpha) = \tau^A(\tilde{w}(\alpha), \alpha)$ yields:

$$
\frac{\partial \tau^*}{\partial \alpha} = \frac{\partial \tau^A}{\partial \alpha}(\tilde{w}(\alpha), \alpha) + \frac{\partial \tau^A}{\partial \tilde{w}}(\tilde{w}(\alpha), \alpha) \frac{\partial \tilde{w}}{\partial \alpha}.
$$

(15)

Clearly the sign of $\partial \tau^*/\partial \alpha$ is positive when $r = n$; in that case the identity of the decisive voter does not depend on $\alpha$ and the second term vanishes. The first, positive, term then determines the sign of the expression and we get the expected positive relation between $\alpha$ and $\tau^*$. When $r < n$, however, the sign of (14) is ambiguous. There are now two opposite effects. When $\alpha$ rises, the preferred tax rate of the decisive voter rises. However, when $\alpha$ changes, the identity of the decisive voter changes as well. When $\alpha$ rises, PAYG public pension becomes less attractive to low-wage workers (who support this system). Some of them (the more productive) "switch" to the private sector and the new decisive voter is poorer than before. As preferred tax rates are increasing with income, the tax rate chosen by this new decisive voter is lower than before.

We should note, however, that when $r > n > 0$, the majority voting equilibrium tax rate jumps discontinuously to 0 for $\alpha$ high enough. This

\[\text{Fig. 2. Preferred tax rates when } r > n\]

comes from the fact that, when \( \alpha \) is sufficiently close to 1, the PAYG system brings about so little redistribution that even the poorest among the workers do not find it attractive any more and vote for the abandonment of the system.

Following the same approach, we can also study the comparative statics of \( \partial \tau^*/\partial n \). The result is also ambiguous: the negative "direct" effect induced by the decrease in the rate of return of the PAYG system has to be balanced against an ambiguous "indirect" effect which arises because the identity of the decisive voter changes. Consequently, a decline in fertility may indeed result in an increase in the majority voting equilibrium tax.\(^6\)

VI. Pay-as-you-go versus Fully Funded

So far, we have concentrated on voting with regard to PAYG systems. Let us now compare the majority voting equilibrium under PAYG and the equilibrium that arises with an "equivalent" fully funded (FF) scheme. We are not concerned with the transition from one to the other. We simply assume that voting takes place in two alternative steady states. To make the comparison fair we assume \( r = n > 0 \), so that the average return is the same under both systems. Furthermore, we consider identical benefit rules to ensure that the two schemes achieve the same degree of redistribution. In other words, we do not compare a redistributive PAYG system to a totally individualized FF system, as is often done in the literature on the privatization of social security.

The major difference between the two systems is that under an FF scheme, the retirees have no stake in the vote. All the decisions which are relevant for them have been taken in the past: private saving, \( s \), and collective saving through pension funds, if any. Recall that the interest rate, \( r \), and the Bismarckian factor, \( \alpha \), are also given.

Consequently, only the active population matters for determination of the voting equilibrium. Moreover, preferred tax rates of workers continue to be characterized by Proposition 1 and the decisive voter has earnings \( \tilde{w} \) such that:

\[
\int_{\tilde{W}}^{\tilde{w}} f(w) \, dw = \frac{1}{2},
\]

\(^6\)For a "large" decline in fertility, however, the tax rate falls to zero, as long as the condition \( w_- > \tilde{w}/(1 + r) \) is satisfied. The explanation is simple: for \( n \) close to 0, \( \tilde{w} \) is close to \( (1 - \alpha)\tilde{w}/(1 + r - \alpha) \). Moreover, we know that \( \tilde{w} \) declines with \( \alpha \). Hence the maximal value of \( \tilde{w} \) when \( n \) tends to 0 is \( \tilde{w}/(1 + r) \). Therefore, if \( w_- > \tilde{w}/(1 + r) \), we are sure that the poorest individual does not support the PAYG pension system, so that a majority votes for abandonment of the system.
In other words, the young population is divided into two groups: those with earnings between \( \hat{w} \) and \( \bar{w} \) and who want a tax rate higher than \( \tau^A(\hat{w}) \), and those with earnings below \( \hat{w} \) or above \( \bar{w} \) who want a lower tax rate. Note that \( \omega_m < \bar{w} \) ensures that majority voting yields a strictly positive tax rate.

Comparing (16) and (11), while recalling that \( r = n \) implies \( \hat{w} = \bar{w} \), it follows that \( \hat{w} < \bar{w} \). Consequently, the decisive voter under the FF scheme has a lower wage than the decisive voter under the PAYG system. This leads to Proposition 3.

**Proposition 3.** The majority voting equilibrium size of social security is larger under a PAYG than under an FF scheme.

This result can be explained as follows. With a PAYG system, the retirees favour a payroll tax rate which is as large as possible. They do not have a majority, and the median voter is a worker who backs social security because of its redistributiveness. Nevertheless, it is clear that the vote of the retirees tends to increase the size of the social security system. With an FF system, on the other hand, the retirees do not have any leverage on the workers’ tax payments.

In a sense, the voting under PAYG amounts to giving a larger weight to the older than to the younger generation. The current young will be old in the next period and take this into account when voting. Consequently, they assign some weight to old-age consumption—as, of course, do the currently old. First-period consumption, on the other hand, affects only the currently young.

What about social welfare? In the setting with heterogeneous individuals, the comparison is not straightforward and depends on which welfare function is used. With a Rawlsian criterion, FF always dominates PAYG, as it gives the decisive vote to workers with lower earnings (recall that, \( \hat{w} < \bar{w} \)). With a general utilitarian criterion, the comparison is ambiguous.

**VII. Extension: Distortionary Payroll Taxation**

Up to now the tax system was assumed to imply no efficiency loss. Let us now briefly examine the implications of distortionary taxation. For simplicity, we use a quadratic loss function so that the steady-state relationship between contributions and pensions is now given by:

\[
p(w) = (1 + n)\tau(\alpha w + (1 - \gamma\tau)(1 - \alpha)\bar{w}),
\]

where \( \gamma > 0 \) is the distortion factor. Note that the distortion applies only to

---

7Welfare depends on (ex ante) lifetime utilities.
the non-contributory part of social security. In other words, voters see through the budgetary veil and realize that the fraction $\alpha$ of their tax payment is given back to them with a return of $n$.\(^8\)

**The Retirees**

It is straightforward to show that the preferred tax rate of a retiree with (past) wage $w$ is given by:

$$
\tau^R(w) = \min \left\{ \frac{1}{2\gamma} \left( 1 + \frac{\alpha}{1 - \alpha} \frac{w}{w} \right), 1 \right\}.
$$

The retiree chooses the tax rate that maximizes his pension under the constraint that $\tau \leq 1$. Note that $\tau^R(w)$ is increasing in $w$. This is because tax distortions are related to the Beveridgean part of pensions. Consequently, poor retirees suffer more from a tax increase than the rich.

**The Workers**

The problem of the workers is drastically modified when tax distortions are introduced. The rate of return of the PAYG system now depends on the value of the tax rate. Consequently, it is now possible that a given individual prefers the PAYG system to private saving for some values of $\tau$ and that his preferences change in favour of private saving for some other values. This individual will then choose a positive tax rate, such that the rates of return of public pension and private saving are equalized, and will supplement these mandatory public savings with private savings. Consequently, the possibility of a mixed choice—public pension supplemented with private savings—can no longer be ruled out.

Formally, the preferred tax rate, $\tau^A(w)$, and the savings, $s^A(w)$, of a worker are the solutions to the following problem:

\(^8\)This specification is a reduced form of a more general (and complicated) model where labour supply is endogenous. Indeed the first-order condition for an interior value of $\tau$ would be:

$$
-yu'(c) + \beta(1 + n)(\alpha y + (1 - \alpha)\bar{y})u'(d)
$$

$$
+ \beta \left( \tau(1 + n)(1 - \alpha) \int_{w_-}^{w_+} w \frac{\partial l^A}{\partial \tau f(w)} dw \right) u'(d) = 0,
$$

where $\bar{y} = \int_{w_-}^{w_+} yf(w) dw$, $y = wl^A(w)$ and $l^A(w)$ is the optimal labour supply of an individual with productivity $w$. It appears clear that tax distortions are associated with the third term of this first-order condition.
\[
\max_{\tau, s} U = u(c) + \beta u(d)
\]  
subject to:
\[
w(1 - \tau) = c + s
\]
\[
d = (1 + r)s + (1 + n)(\tau \alpha w + (1 - \alpha)(1 - \gamma \tau)\bar{w})
\]  
and
\[
\tau \geq 0, \quad s \geq 0.
\]

The next proposition states that low-wage individuals want a positive tax rate but no private savings, middle-wage individuals will make a mixed choice, with both PAYG social security and private saving, and higher-wage individuals will prefer to rely only on private saving.

**Proposition 4.** (i) \(\max \tau^A(w) < \min \tau^R(w)\).

Moreover, there exists a value of \(w\), \(w' < \hat{w}\) (defined in (10)), such that:

(ii) if \(w \leq w'\), \(\tau^A(w) \geq 0\), \(s^A(w) = 0\);

(iii) if \(w' < w < \hat{w}\), \(\tau^A(w) > 0\) and \(s^A(w) > 0\);

(iv) if \(w = \hat{w}\), \(\tau^A(w) = 0\) and \(s^A(w) > 0\);

(v) if \(w < w'\), \(\partial \tau^A(w)/\partial w > 0\) and if \(w' < w < \hat{w}\), \(\partial \tau^A(w)/\partial w < 0\).

**Proof:** See Appendix.

The first point in Proposition 4 says that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The underlying idea is the same as in the case without distortions: workers do not want the tax to be too high because it decreases the first-period consumption. The other points imply that \(\tau^A(w)\) is first increasing and then decreasing with \(w\); it is equal to zero when \(w = \hat{w}\). For workers with \(w < \hat{w}\), social security is attractive up to a certain point; its relative return is now equal to \((\alpha + (1 - \alpha)(1 - \gamma \tau)\bar{w}/w)\), which can be lower than \(1 + r\) for some \(\tau\). For workers with wage close to \(w_\gamma\), there is no saving and the preferred tax rate increases with \(w\). Figure 3 shows these results for the case where \(r = n\), so that \(\hat{w} = \bar{w}\).

**The Majority Voting Solution**

It can easily be shown that the objective function of each worker is concave relative to its arguments, \(\tau\) and \(s\). Therefore, preferences over \(\tau\) are single-peaked and the median voter theorem applies. In the next proposition, we
characterize the Condorcet winner. For this purpose, we define \( w^* \) as the income level satisfying \( \tau^A(w_-) = \tau^A(w^*) \).

**Proposition 5.** When the non-contributory part of social security implies quadratic distortions, the majority voting equilibrium tax rate is characterized as follows:

(i) If \( \int_{w_-}^{w^*} f(w) \, dw < n/2(1 + n) \), the majority voting equilibrium tax rate, \( \tau^* \), is 0.

(ii) If \( \int_{w_-}^{w^*} f(w) \, dw \leq n/2(1 + n) \leq \int_{w_-}^{\bar{w}} f(w) \, dw \), the majority voting equilibrium tax rate is the rate preferred by the workers with wage \( w^p \) defined as follows:

\[
\int_{w_-}^{w^p} f(w) \, dw = \frac{n}{2(1 + n)}. \tag{22}
\]

(iii) If \( \int_{w_-}^{w^*} f(w) \, dw > n/2(1 + n) \), the majority voting equilibrium tax rate is the rate preferred by the workers with wage \( w^1 \) or \( w^2 \) satisfying:

\[
\int_{w^1}^{w^2} f(w) \, dw = \frac{n}{2(1 + n)} \quad \text{and} \quad \tau^A(w^1) = \tau^A(w^2). \tag{23}
\]

The proof goes along the same lines as that of Proposition 2, so we do not develop the formal argument here. The idea is to find a sufficient number of individuals in the working population to form a majority coalition with the
retirees and sustain a positive equilibrium tax rate. Under the condition in (i), the number of young who favour a positive tax rate is not sufficient to form a majority coalition with the old, and the PAYG pension system is not sustainable in the steady state. In (ii), the number of young individuals who favour the PAYG scheme is sufficient but there are not enough young with income between \( w^1 \) and \( w^2 \) to form a majority coalition with the old. Therefore, some young people with income \( w > w^2 \) also belong to that majority. Finally, in (iii), there are enough people in the interval \([w^1, w^2]\) and workers with income \( w > w^2 \) belong to the minority in favour of a tax rate lower than \( \tau^d(w^1) = \tau^d(w^2) \). It is then clear that the set of workers who join the retirees to form a majority in favour of a positive tax rate is different from what it was without distortion. In particular, when \( \int_{w_{\hat{w}}}^{\hat{w}} f(w) dw \) is not close to \( n/2(1 + n) \), those with earnings equal or just below \( \hat{w} \) do not belong to that majority.

VIII. Concluding Comments

Throughout the paper, the more or less Bismarckian feature of the social security system was regarded as given. A natural extension of our analysis would be to study the determination of \( \alpha \). A possible approach is to assume that the benefit formula is determined by a welfare-maximizing authority at a first, constitutional stage. This approach is in line with the sequential nature of the decision process alluded to in the Introduction. It has been adopted by Casamatta et al. (2000) but within a different context, namely social insurance. In that paper the choice of the Bismarckian parameter, \( \alpha \), is made at the constitutional level, on the expectation that the payroll tax is determined later through majority voting. One of the main results is that a positive Bismarckian parameter can be desirable, even though the constitutional planner, with full control of \( \alpha \) and \( \tau \), would set \( \alpha \) equal to zero. This result arises because the redistributive character of the system has an impact on its political support. Specifically, when \( \alpha \) is positive, the decisive voter chooses a tax rate that better fits the preferences of the low-wage individuals. A Rawlsian constitutionalist can then favour such a positive \( \alpha \).

Alternatively, one could consider a setting where both \( \alpha \) and \( \tau \) are chosen by majority voting. In this two-dimensional collective choice problem, a Condorcet winner does not generally exist. Therefore, the political process has to be given more structure. Some examples, again in the context of social insurance, can be found in Casamatta (1999), who studies sequential voting procedures, or in De Donder and Hendricks (1998), who study (two-party) electoral competition.

Another extension would be to study the properties of our setting out of the steady state. In particular, it is interesting to examine the transition following the occurrence of a shock such as, for example, a sudden drop in

fertility or in productivity. In such a setting, the issue of voting is more complex and depends to a large extent on how expectations are formed. Moreover, the degree of redistributiveness can be very important when considering social security reforms. It is often observed that the resistance to change by vested interests is related to entitlements founded on the contributory part of social security. In other words, the choice of $\alpha$ has an effect not only on political support in the steady state, but also on political resistance out of the steady state when reforms are contemplated. But this is clearly another story.

Appendix

Proof of Proposition 1

First, observe that some straightforward algebra is sufficient to show that $v(\tau, w)$ is a concave function of $\tau$. To prove (i), we differentiate $v(\tau, w)$ at $\tau = 0$:

$$\frac{\partial v}{\partial \tau} \bigg|_{\tau = 0} = -wu'(c) + \beta(1 + n(\alpha w + (1 - \alpha)\bar{w}))u'(d). \quad (A1)$$

At $\tau = 0$, everyone saves. Hence, substituting (5) in the previous expression, we obtain:

$$\frac{\partial v}{\partial \tau} \bigg|_{\tau = 0} = u'(c)\left(\frac{1 + n}{1 + r} (\alpha w + (1 - \alpha)\bar{w}) - w\right). \quad (A2)$$

This expression is greater than 0 iff $w \leq \bar{w}$. Therefore, only these individuals will have a strictly positive preferred tax rate.

To prove (ii), let us write the first-order condition for $\tau$ for an individual with income $w < \hat{w}$

$$-wu'(c) + \beta(1 + n(\alpha w + (1 - \alpha)\bar{w}))u'(d) = 0. \quad (A3)$$

Differentiating this expression with respect to $w$, we obtain:

$$\frac{\partial \tau^*}{\partial w} = -\frac{\beta(1 + n)(1 - \alpha)(\bar{w}/w)u'(d)(1 - e)}{D_\tau}, \quad (A4)$$

where $D_\tau < 0$ is the second order derivative of $v(\tau, s)$ with respect to $\tau$. Clearly, under our assumption that $\sigma < 1$, the preferred tax rate of individuals with income below $\hat{w}$.

To prove (iii), we note that the first-order condition $\partial v/\partial \tau = 0$ for $w < \hat{w}$ implies that workers with less than a break-even level of earnings oppose too high a tax rate because it decreases their first-period consumption. Indeed, when $\tau \to 1$, $u'(c) \to +\infty$ whereas the limit of $u'(d)$ is finite, which implies that the first-order condition cannot be satisfied.
Proof of Proposition 4

The first-order conditions associated with the programme of a working individual are:

\[-wu'(c) + \beta(1 + n)(aw + (1 - \alpha)\bar{w}(1 - \gamma \tau))u'(d) + \lambda_{r} = 0 \quad (A5)\]

and

\[-u'(c) + \beta(1 + r)u'(d) + \lambda_{s} = 0, \quad (A6)\]

where \(\lambda_{r}\) and \(\lambda_{s}\) are the Lagrange multipliers associated with the non-negativity constraints on \(\tau\) and \(s\), respectively.

To solve the programme, we have to consider different cases, depending on which constraint binds.

Case 1: \(\lambda_{r} = \lambda_{s} = 0\).

Here, none of the non-negativity constraints bind, which means that we have an interior solution for both \(\tau\) and \(s\). Substituting (A6) into (A5), we obtain:

\[\tau^{A}(w) = \frac{(1 + n)(aw + (1 - \alpha)\bar{w}) - w(1 + r)}{2\gamma(1 - \alpha)\bar{w}(1 + n)}. \quad (A7)\]

A necessary and sufficient condition for \(\tau^{A}(w)\) to be positive is that

\[w < \hat{w} = (1 + n)(1 - \alpha)\bar{w}/((1 + r) - \alpha(1 + n)).\]

All the people with earnings above this threshold will have a preferred tax rate of 0 and will choose to rely exclusively on private savings. For people with a wage lower than \(\hat{w}\) who save privately, we have:

\[\frac{\partial \tau^{A}}{\partial w} = \frac{\alpha(1 + n) - (1 + r)}{2\gamma(1 - \alpha)\bar{w}(1 + n)} < 0. \quad (A8)\]

Consequently, preferred tax rates are decreasing with income.

Using (A6), we obtain:

\[\frac{\partial s^{A}(w)}{\partial w} = \frac{(-1 - \tau^{A}) + w[\partial \tau^{A} / \partial w]u''(c)}{u''(c) + (\beta(1 + r))^{2}u''(d)} \]

\[= \frac{\beta(1 + r)[(\partial \tau^{A} / \partial w)(1 + n)(aw + (1 - \alpha)\bar{w}) + \alpha(1 + n)\tau^{A})u''(d)}{u''(c) + (\beta(1 + r))^{2}u''(d)} \]

\[= \frac{\alpha((1 + n)(aw + (1 - \alpha)\bar{w}) - w(1 + r))}{\gamma(1 - \alpha)\bar{w}}. \quad (A9)\]

This expression is positive for individuals with a wage below \(\hat{w}\). The function \(s^{A}(w)\) is thus increasing. Moreover, from (A6), it is negative for \(w\) sufficiently close to 0.
and positive for $w$ sufficiently close to $\hat{w}$. Therefore, there exists a value of $w, w'$, such that each individual with income above this value has strictly positive savings and for those with income below $w'$, the constraint $s \geq 0$ is binding.

To sum up, we have found that all the people with earnings between $w'$ and $\hat{w}$ have an interior solution for both $\tau$ and $s$. These solutions are given by (A7) and (A6), respectively. The other individuals have either a positive preferred tax rate with no savings or positive savings and a zero preferred tax rate. We now analyse these two cases.

Case 2: $\lambda_\tau = 0, \lambda_s > 0$.

As already shown, individuals with earnings less than $w'$ will choose a strictly positive tax rate and no savings. The value of the optimal tax rate is given by the condition (A5).

Case 3: $\lambda_\tau > 0, \lambda_s = 0$.

The richer individuals (those with earnings above $\hat{w}$) will rely exclusively on savings, the value of which is given by (A6).

In order to determine the majority voting solution, we must know how the preferred tax rates vary with income for individuals below $w'$. We obtain from (A5) that:

$$\frac{\partial \tau^A}{\partial w} = \frac{(1 - \varepsilon)\beta\alpha(1 + n)u'(d) - u'(c)) - \alpha(1 - \alpha)\beta\gamma\tau^2(1 + n)^2\bar{w}u''(d)}{-D_\tau}. \quad (A10)$$

For those people who do not save privately, we know from (A6) that $u'(c) > \beta(1 + r)u'(d)$ which implies that $u'(c) > \beta\alpha(1 + n)u'(d)$. Hence, when $\varepsilon > 1$, this expression is positive, so that preferred tax rates are increasing with income.

Finally, we show that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The maximal preferred tax rate of the workers is:

$$\tau^A(w') = \frac{(1 + n)(\alpha w' + (1 - \alpha)\bar{w}) - w'(1 + r)}{2\gamma(1 - \alpha)\bar{w}(1 + n)}, \quad (A11)$$

and, assuming an interior solution, the minimal preferred tax rate of the retirees is:

$$\tau^R(w_-) = \frac{1}{2\gamma} \left(1 + \frac{\alpha}{1 - \alpha} \frac{w_-}{\bar{w}}\right). \quad (A12)$$

Therefore, the condition for the maximal preferred tax rate of the workers to be higher than the minimal preferred tax rate of the retirees is

$$\tau^A(w') > \tau^R(w_-) \Leftrightarrow w'(\alpha(1 + n) - (1 + r)) - \alpha w_-(1 + n) > 0.$$ 

Knowing that $\alpha(1 + n) < 1 + r$, this is impossible. Hence the result.
References


