Optimal bequests taxation in the steady state

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Abstract

We consider an infinite-horizon economy populated by two types of individuals, some individuals being more productive than others. Individuals live one period and are altruistic toward their children. We first determine the second-best steady state allocation and then study the optimal bequest and labor income tax functions. We consider independent income and bequests tax functions. We first show that the second-best is not implementable with such tax schedules. We then exhibit a condition under which high bequests should be taxed (and low bequests subsidized). A numerical example suggests that this condition is likely to be met when individuals are sufficiently altruistic. Under moderate altruism, no taxation of bequests is desirable. Finally high bequests should be subsidized when individuals are poorly altruistic.

Keywords: bequests, taxation, steady state.


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1. Introduction

Despite representing a negligible part of fiscal revenues,\(^1\) the taxation of bequests has always been subject to a heated controversy. Its opponents raise the concern that it is unbearable to tax individuals at death. Moreover, taxing bequests discourage labor supply, savings and destroy small businesses. Supporters of this tax argue that it allows to achieve equality of opportunity and has low efficiency costs.

The theoretical studies so far on the optimal taxation of bequests have not been totally convincing in evaluating these arguments. The first difficulty is that there is no clear empirical evidence about the bequest motive, the optimal level of bequests taxation being crucially dependent on this motive. Bequests could be driven by pure altruism (Barro (1974)). They can also be accidental due to the absence of perfect annuity markets. It could also be true that individuals derive utility from the mere fact of giving (Andreoni (1989)). Finally, it could be the outcome of a game between parents and children: parents exchange the promise of bequests with services provided by their children (Bernheim, Shleifer, and Summers (1985)). Presumably the decision to give results from a combination of these different motives and furthermore differs from an individual to another.

To illustrate the importance of the bequest motive, consider accidental bequests. In that case, taxation is not distortive and as a consequence bequests should be taxed heavily (Blumkin and Sadka (2004), Cremer, Gahvari, and Pestieau (2009)). In the pure altruism model on the other hand, taxation discourages individuals from leaving bequests and as such create distortions. We focus in this paper on this setting and ask what is the optimal bequest tax when individuals value the welfare of their offspring.

Earlier arguments in the literature build on the work by Chamley (1986) and Judd (1985) on capital income taxation, who show that capital should not be taxed in the long run. In the representative agent framework, bequests and savings are equivalent, implying that the Chamley-Judd result extends to the taxation of bequests (see Cremer and Pestieau (2006)).

This result is valid when there is no heterogeneity among agents, meaning that only the efficiency role of taxes is taken into account. With individuals differing in productivity within each generation, the standard equity-efficiency trade-off appears. The first attempt to analyze

\(^1\)In 1997, tax revenues from transfer taxes represented 0.44% of fiscal revenues on average in all OECD countries, the maximum being 1.66% in Japan (Gale and Slemrod (2001)).
this trade-off is due to Kaplow (2001). His analysis generated two main insights. First, when bequests are interpreted as a particular form of consumption, the Atkinson-Stiglitz theorem applies (Atkinson and Stiglitz (1976)): When consumption and leisure are separable, there is no need to tax consumption. When applied to bequests, this suggests that they should not be taxed or subsidized. Second, the specificity of bequests is that they generate a positive externality from the donor to the recipient. As a consequence, bequests should be subsidized at the margin, acting like a Pigovian correction. Subsequent work has mainly developed these arguments (Kopczuk (2001), Cremer and Pestieau (2001), Farhi and Werning (2010)). These studies put two main restrictions on the model of the economy: either perfect correlation between the parents and the children is assumed (Kopczuk (2001)) or a two-period model is considered (Cremer and Pestieau (2001), Farhi and Werning (2010)).

In this paper we remove these restrictions. We consider an infinite horizon model and assume no correlation between the productivities of the parents and the children. Individuals live one period and have each one child. These individuals are altruistic and differ according to their productivity.

We consider the steady state of this economy and first describe the second-best optimum. The optimal allocation is such that the child of a highly productive individuals should be better-off than the one of a low productivity individual with the same productivity level. This allows to relax the incentive constraint of the parents. This result has been put forward by Farhi and Werning (2010).

We then analyze the implementation of this optimum through separate labor income and bequests nonlinear tax functions. We show that these tax instruments do not allow to implement the optimum. As for the design of the bequests tax, we show that it may be optimal to redistribute from high to low bequests. Numerical examples suggest that it is more likely when individuals are very altruistic. For lower values of altruism however, it is possible that the redistribution of bequests takes place the other way around.

2. The economy

Individuals live one period and differ by the level of their productivity (type), which is assumed to be private information. In each period, there are $N^L$ individuals with productivity $\omega^L$ and $N^H$ individuals with productivity $\omega^H$, $\omega^L < \omega^H$. 

We assume a utility function additively separable between leisure and consumption. Note that the allocation received by a given individual may depend on his own productivity but also on the productivity of his ancestors. We will show later on that there is no loss of generality in considering only the productivities of the individual and his parent for characterizing the steady state optimum (see lemma 1). With this restriction, the preferences of an individual with productivity $i$ living in period $t$ with a parent of productivity $j$ are given by:

$$V_{ij}^t \equiv U(c_{ij}^t, l_{ij}^t) + \gamma \sum_k p_{kj} V_{ij}^{k+1}, \quad (2.1)$$

where $U(c_i, l_i) = u(c_i) - v(l_i)$, $c_i$ is consumption at date $t$ and $l_i$ labor supply. The probability that a child is of type $k$ when his parent is of type $j$ is $p_{kj}$. We assume no correlation between the types of the parents and the children, so that: $p_{HH} = p_{HL} \equiv p_H$ and $p_{LH} = p_{LL} \equiv p_L$. The parameter $\gamma \ (\leq 1)$ represents the degree of altruism. It is assumed to be identical for all individuals.

3. Optimal steady state allocation

We consider the steady state of this economy: the distribution of consumptions and labor supplies is the same in every period. As mentioned before, we can restrict ourselves without loss of generality to allocations that depend on the productivity of a given individual as well as the type of his parent but not on the productivities of other members of the dynasty (see lemma 1 below).

The social planner maximizes the utilitarian welfare of a representative generation. His program is the following:

$$\max N_L p_L U(c_{LL}, y_{LL}/\omega_L) + N_H p_L U(c_{LH}, y_{LH}/\omega_L) \quad \text{+} \quad N_L p_H U(c_{HL}, y_{HL}/\omega_H) + N_H p_H U(c_{HH}, y_{HH}/\omega_H)$$

subject to

$$N_L p_L (y_{LL} - c_{LL}) + N_H p_L (y_{LH} - c_{LH}) + N_L p_H (y_{HL} - c_{HL}) + N_H p_H (y_{HH} - c_{HH}) \geq 0,$$

and

$$U(c_{HH}, y_{HH}/\omega_H) + \gamma (p_L V_{LH} + p_H V_{HH}) \geq U(c_{LH}, y_{LH}/\omega_H) + \gamma (p_L V_{LL} + p_H V_{HL})$$

and

$$U(c_{HL}, y_{HL}/\omega_H) + \gamma (p_L V_{LH} + p_H V_{HH}) \geq U(c_{LL}, y_{LL}/\omega_H) + \gamma (p_L V_{LL} + p_H V_{HL}),$$
where \( y = \omega l \) is the production of a type \( j \) individual. The second constraint is the resource constraint in each period: total consumption should exceed total production. Bequests do not appear in this equation because inheritances received and bequests left exactly cancel out in the aggregate. The second group of constraints represent incentive constraints: a type \( j \) individual should not want to pretend that he is of type \( i \). As usual in the optimal taxation literature (Stiglitz (1987)), only the constraints from the high to the low types bind at the optimum. This is checked in proposition 1.

From (2.1), we have:

\[
\begin{align*}
V^{LL} &= U(c^{LL}, y^{LL}/\omega^L) + \gamma(p^L V^{LL} + p^H V^{HL}) \\
V^{LH} &= U(c^{LH}, y^{LH}/\omega^L) + \gamma(p^L V^{LH} + p^H V^{HH}) \\
V^{HL} &= U(c^{HL}, y^{HL}/\omega^L) + \gamma(p^L V^{HL} + p^H V^{HH}) \\
V^{HH} &= U(c^{HH}, y^{HH}/\omega^H) + \gamma(p^L V^{HH} + p^H V^{HH}),
\end{align*}
\]

implying:

\[
\begin{align*}
V^{LH} - V^{LL} &= U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) \\
V^{HH} - V^{HL} &= U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H).
\end{align*}
\]

The incentive constraints can thus be re-written in the following way:

\[
\begin{align*}
U(c^{HH}, y^{HH}/\omega^H) + \gamma(p^L U(c^{LH}, y^{LH}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \\ \geq U(c^{LH}, y^{LH}/\omega^L) + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \quad (3.1) \\
U(c^{HL}, y^{HL}/\omega^H) + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) \\ \geq U(c^{LL}, y^{LL}/\omega^L) + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)). \quad (3.2)
\end{align*}
\]

Denoting \( \mu_1 \) and \( \mu_2 \) the Lagrange multipliers associated to the resource and incentive constraints respectively, the first-order conditions with respect to consumption and income are respectively:

\[
\begin{align*}
\frac{\partial L}{\partial c_{LL}} &= (N^L p^L - \lambda_1 \gamma p^L - \lambda_2 - \lambda_2 \gamma p^L) u'(c_{LL}^{opt}) - \mu N^L p^L = 0 \quad (3.3a) \\
\frac{\partial L}{\partial c_{LH}} &= (N^H p^L + \lambda_1 \gamma p^L - \lambda_1 + \lambda_2 \gamma p^L) u'(c_{opt}^{HL}) - \mu N^H p^L = 0 \quad (3.3b) \\
\frac{\partial L}{\partial c_{HL}} &= (N^L p^H - \lambda_1 \gamma p^H + \lambda_2 - \lambda_2 \gamma p^H) u'(c_{opt}^{HL}) - \mu N^L p^H = 0 \quad (3.3c) \\
\frac{\partial L}{\partial c_{HH}} &= (N^H p^H + \lambda_1 + \lambda_1 \gamma p^H + \lambda_2 \gamma p^H) u'(c_{opt}^{HH}) - \mu N^H p^H = 0 \quad (3.3d)
\end{align*}
\]
\[
\frac{\partial L}{\partial y^L} = (N^Lp^L - \lambda_1 \gamma p^L - \lambda_2 \gamma p^L)(-\frac{1}{\omega^L} v'(\frac{y_{opt}^L}{\omega^L})) - \lambda_2 (-\frac{1}{\omega^H} v'(\frac{y_{opt}^L}{\omega^H})) + \mu N^L p^L = 0 \tag{3.4a}
\]

\[
\frac{\partial L}{\partial y^H} = (N^H p^L + \lambda_1 \gamma p^L + \lambda_2 \gamma p^L)(-\frac{1}{\omega^L} v'(\frac{y_{opt}^L}{\omega^L})) - \lambda_1 (-\frac{1}{\omega^H} v'(\frac{y_{opt}^L}{\omega^H})) + \mu N^H p^L = 0 \tag{3.4b}
\]

\[
\frac{\partial L}{\partial y^{HL}} = (N^L p^H - \lambda_1 \gamma p^H + \lambda_2 - \lambda_2 \gamma p^H)(-\frac{1}{\omega^L} v'(\frac{y_{opt}^H}{\omega^H})) + \mu N^L p^H = 0 \tag{3.4c}
\]

\[
\frac{\partial L}{\partial y^{HH}} = (N^H p^H + \lambda_1 + \lambda_1 \gamma p^H + \lambda_2 \gamma p^H)(-\frac{1}{\omega^H} v'(\frac{y_{opt}^H}{\omega^H})) + \mu N^H p^H = 0. \tag{3.4d}
\]

The last two conditions, combined with (3.3c) and (3.3d), imply that the marginal utility of consumption of the \(H\) types should be equal to their marginal disutility of work. In other words, there should be no-distortion-at-the-top, a standard property in the optimal taxation literature (Stiglitz (1987)).

We describe in the next proposition some properties of the second-best allocation.

**Proposition 1.** (i) \(c_{opt}^{HL} > c_{opt}^{LH} \forall \gamma \in [0,1]; c_{opt}^{HL} = c_{opt}^{LH}\) when \(\gamma = 1\).

(ii) \(c_{opt}^{HH} > c_{opt}^{HL} > c_{opt}^{LH} > c_{opt}^{LL}, y_{opt}^{HH} > y_{opt}^{HL}, y_{opt}^{LL} > y_{opt}^{LH}, \forall \gamma \in (0,1] \); When \(\gamma = 0, c_{opt}^{HH} = c_{opt}^{HL} = c_{opt}^{LL}, y_{opt}^{HH} = y_{opt}^{HL} = y_{opt}^{LL}\).

(iii) Incentive constraints from the high to the low types, (3.1) and (3.2), are binding at the second-best optimum, whereas incentive constraints from the low to the high types are not.

**Proof.** See appendix.

This proposition allows to rank the second-best allocations, as represented on the figure below. It is shown that the children of highly productive individuals should consume more and work less than the children of individuals with a low productivity. They thus obtain a higher utility level. This is explained by the incentive constraints (3.1) and (3.2): in order to provide better incentives to the \(H\) types not to mimic \(L\) types, the planner promises a higher utility level to their children.

To conclude this section, we prove in the appendix the following lemma.

**Lemma 1.** There is no loss of generality in considering allocations that depend on the type of a given individual and of his parent only for characterizing the second-best optimum.
Figure 3.1: Second-best allocation
4. The optimal redistribution of bequests with independent tax schedules

We now ask which allocation can be implemented through a tax schedule. The observable variables being the bequests and labor incomes, we consider taxes that depend on these two variables. With a joint tax schedule, that is if we allow the tax on labor income to depend on the level of bequests (and vice versa), it can be shown easily that the second-best optimum can be implemented. There is moreover some indeterminacy in the absolute level of the implementing taxes (see Farhi and Werning (2010)). We rule out this possibility and consider separate (nonlinear) tax functions on labor income and bequests. This means that the government is precluded from using the available information about inheritances received when it determines the tax on labor income. Conversely, it cannot make the tax on bequests dependent on the level of labor income. Under this assumption, we ask whether the second-best can be implemented. Moreover, we are interested in determining if bequests should be taxed or subsidized, a long-standing controversy in the literature.

We first argue that, at the steady state optimum, there can be only two levels of bequests: the high productivity individuals should bequeath \( b^H \) and the low productivity ones \( b^L \). There are indeed four possible levels of consumptions and labor supplies: \((c^{LL},y^{LL}), (c^{LH},y^{LH}), (c^{HL},y^{HL})\) and \((c^{HH},y^{HH})\). Then consider two individuals \( HH \) and \( HL \). They have the same continuation value, whatever the bequests they make. This continuation value is \( p^L V^{LH} + p^H V^{HH} \). It then cannot be the case that \( b^{HH} \neq b^{HL} \), as one of the two individuals (the one with the higher bequest) should select the bequest level intended for the other individual. The same reasoning can be made for type \( L \) individuals. Taking this into account, the government chooses the policy \((y^{kj},T^{kj}_y,T^{kj}_b)\) that solves the following program:

\[
\max N^L p^L U(y^{LL} - T^{LL}_y - T^{LL}_b, y^{LL}/\omega^L) + N^H p^H U(y^{LH} - T^{LH}_y + b^H - T^{LH}_b - b^L, y^{LH}/\omega^L) \\
+ N^L p^H U(y^{HL} - T^{HL}_y + b^L - T^{HL}_b - b^H, y^{HL}/\omega^H) + N^H p^H U(y^{HH} - T^{HH}_y - T^{HH}_b, y^{HH}/\omega^H)
\]

\[
st \\
N^L p^{L} T^{LL}_y + N^H p^L T^{LH}_y + N^L p^H T^{HL}_y + N^H p^H T^{HH}_y \geq 0 \\
N^L T^{L}_b + N^H T^{H}_b \geq 0
\]

and

\[
U(y^{kj} - T^{kj}_y + b^j - T^{kj}_b - b^k, y^{kj}/\omega^k)
\]
\[ +\gamma(p_H U(y^{H_k} - T_y^{H_k} + b^k - T_b^k - b^H, y^{H_k}/\omega^H) + p_L U(y^{L_k} - T_y^{L_k} + b^k - T_b^k - b^L, y^{L_k}/\omega^L)) \]
\[ \geq U(y^{k'j'} - T_y^{k'j'} + b^j - T_b^j - b^{k'}, y^{k'j'}/\omega^k) \]
\[ +\gamma(p_H U(y^{H_{k'}} - T_y^{H_{k'}} + b^{k'} - T_b^{k'} - b^H, y^{H_{k'}}/\omega^H) + p_L U(y^{L_{k'}} - T_y^{L_{k'}} + b^{k'} - T_b^{k'} - b^L, y^{L_{k'}}/\omega^L)) \]
\[ U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^{kj}, y^{kj}/\omega^k) \geq U(y^{k'j'} - T_y^{k'j'} + b^j - T_b^j - b^{k'}, y^{k'j'}/\omega^k) \] (4.6)
\[ U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^{kj}, y^{kj}/\omega^k) \]
\[ +\gamma(p_H U(y^{H_{k'}} - T_y^{H_{k'}} + b^{k'} - T_b^{k'} - b^H, y^{H_{k'}}/\omega^H) + p_L U(y^{L_{k'}} - T_y^{L_{k'}} + b^{k'} - T_b^{k'} - b^L, y^{L_{k'}}/\omega^L)) \]
\[ \geq U(y^{kj} - T_y^{kj} + b^j - T_b^j - b^{kj}, y^{kj}/\omega^k) \]
\[ +\gamma(p_H U(y^{H_{k'}} - T_y^{H_{k'}} + b^{k'} - T_b^{k'} - b^H, y^{H_{k'}}/\omega^H) + p_L U(y^{L_{k'}} - T_y^{L_{k'}} + b^{k'} - T_b^{k'} - b^L, y^{L_{k'}}/\omega^L)) \] (4.7)

The first two constraints are the balanced-budget conditions on the tax schedules. The second group of constraints represents incentive constraints. They can be split into three sub-groups. The first constraints prevent individuals from making a joint deviation: type \( k \) individuals should select the pre- and post- tax labor incomes intended for them as well as the appropriate level of bequests \( (b^k) \). The other constraints are meant to prevent unilateral deviations. Constraints in the second sub-group impose that individuals should not select the income tax schedule intended for other individuals while constraints in the last sub-group require that individuals select the “right” level of bequests.\(^2\)

Observe that in government’s program, only the difference \( b^H - b^L \) matters so that \( b^L \) can be normalized to 0. Defining \( \Omega^k \) as follows:
\[ \Omega^k \equiv p_L U(c^{L_k}, y^{L_k}/\omega^L) + p_H U(c^{H_k}, y^{H_k}/\omega^H), \]
we can thus write the constraints preventing joint deviations as follows:
\[ u(c^{HH}) - v(y^{HH}/\omega^H) \geq u(c^{HL} + T_b^L + b^H - T_b^H) - v(y^{HL}/\omega^H) \] (4.8a)
\[ u(c^{HL}) - v(y^{HL}/\omega^H) \geq u(c^{HH} - T_b^L - b^H + T_b^H) - v(y^{HH}/\omega^H) \] (4.8b)
\[ u(c^{HH}) - v(y^{HH}/\omega^H) + \gamma \Omega^H \geq u(c^{LH}) - v(y^{LH}/\omega^H) + \gamma \Omega^L \] (4.8c)
\[ u(c^{LH}) - v(y^{LH}/\omega^L) + \gamma \Omega^L \geq u(c^{HH}) - v(y^{HH}/\omega^H) + \gamma \Omega^H \] (4.8d)

\(^2\)Recall that, while bequests are observable, the government cannot use this information when designing the income tax schedule. Therefore an individual who has received high inheritances for example could well select the income tax schedule intended for low inheritances individuals. If we allowed the government to use all the relevant information at its disposal, this latter would propose bundles \( (y^{k'j'}, T_y^{k'j'}, T_b^{k'j}, b^{k'}) \) and \( (y^{k'j'}, T_y^{k'j'}, T_b^{k'j}, b^{k'}) \) to an individual who has received inheritances \( b^j \). The program of the government would then be identical to the second-best problem, implying that the optimal tax implements the second-best allocation.
Finally, the constraints ensuring that individuals select the appropriate level of bequests are:

\[
\begin{align*}
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) + \gamma \Omega^H & \geq u(c^{LL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{LL}}{\omega}T\right) + \gamma \Omega^L \quad (4.8e) \\
    u(c^{LL}) - v\left(\frac{y^{LL}}{\omega}T\right) + \gamma \Omega^L & \geq u(c^{HH} - T_b^L - b^H + T_b^H) - v\left(\frac{y^{HH}}{\omega}T\right) + \gamma \Omega^H \quad (4.8f) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) + \gamma \Omega^H & \geq u(c^{LL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{LL}}{\omega}T\right) + \gamma \Omega^L \quad (4.8g) \\
    u(c^{LH}) - v\left(\frac{y^{LH}}{\omega}T\right) + \gamma \Omega^L & \geq u(c^{HL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{HL}}{\omega}T\right) + \gamma \Omega^H \quad (4.8h) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) + \gamma \Omega^H & \geq u(c^{LL}) - v\left(\frac{y^{LL}}{\omega}T\right) + \gamma \Omega^L \quad (4.8i) \\
    u(c^{LH}) - v\left(\frac{y^{LH}}{\omega}T\right) + \gamma \Omega^L & \geq u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) + \gamma \Omega^H \quad (4.8j) \\
    u(c^{LH}) - v\left(\frac{y^{LH}}{\omega}T\right) & \geq u(c^{LL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{LL}}{\omega}T\right) \quad (4.8k) \\
    u(c^{LL}) - v\left(\frac{y^{LL}}{\omega}T\right) & \geq u(c^{LH} - T_b^L - b^H + T_b^H) - v\left(\frac{y^{LH}}{\omega}T\right). \quad (4.8l)
\end{align*}
\]

The constraints on the income tax schedule are the following:

\[
\begin{align*}
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) & \geq u(c^{HL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{HL}}{\omega}T\right) \quad (4.9a) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) & \geq u(c^{HH} - T_b^L - b^H + T_b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9b) \\
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) & \geq u(c^{LL} - b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9c) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) & \geq u(c^{HH} + b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9d) \\
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) & \geq u(c^{LL} + T_b^L - T_b^H) - v\left(\frac{y^{LL}}{\omega}T\right) \quad (4.9e) \\
    u(c^{LL}) - v\left(\frac{y^{LL}}{\omega}T\right) & \geq u(c^{HH} - T_b^L + T_b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9f) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) & \geq u(c^{LL} - 2b^H + T_b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9g) \\
    u(c^{LH}) - v\left(\frac{y^{LH}}{\omega}T\right) & \geq u(c^{HL} + T_b^L + 2b^H - T_b^H) - v\left(\frac{y^{HL}}{\omega}T\right) \quad (4.9h) \\
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) & \geq u(c^{LL} - b^H) - v\left(\frac{y^{LL}}{\omega}T\right) \quad (4.9i) \\
    u(c^{HL}) - v\left(\frac{y^{HL}}{\omega}T\right) & \geq u(c^{HH} + b^H) - v\left(\frac{y^{HH}}{\omega}T\right) \quad (4.9j) \\
    u(c^{HH}) - v\left(\frac{y^{HH}}{\omega}T\right) & \geq u(c^{LL} + T_b^L + b^H - T_b^H) - v\left(\frac{y^{LL}}{\omega}T\right) \quad (4.9k) \\
    u(c^{LL}) - v\left(\frac{y^{LL}}{\omega}T\right) & \geq u(c^{LH} - T_b^L - b^H + T_b^H) - v\left(\frac{y^{LH}}{\omega}T\right). \quad (4.9l)
\end{align*}
\]

Finally, the constraints ensuring that individuals select the appropriate level of bequests are:
We solve the government problem in two steps. We first assume that there is no taxation of bequests \((T_H^L = T_H^H = 0)\) and determine the optimal tax schedule. We then introduce (redistributive) taxes on bequests. This allows to conclude about the desirability of taxing bequests and in which direction the redistribution of bequests should take place.

The results that follow hold when the ordering of allocations satisfies the following condition:

Condition \(o\): at the optimum with taxes, the ordering of consumption levels satisfies \(c_{HH} \geq c_{HL} \geq c_{LL}, c_{HH} \geq c_{LH} \geq c_{LL}\).

Before presenting the main results of the paper, we prove the following lemma.

**Lemma 2.** Under condition \(o\),

(i) The constraints (4.9c), (4.9i), (4.9d), (4.9j), (4.9h), (4.10c) and (4.10d) cannot be binding at the optimum with taxes when \(T_L^L = T_H^H = 0\).

(ii) When (4.9e) binds, constraints (4.9f), (4.10a) and (4.10b) cannot be binding at the optimum with taxes when \(T_L^L = T_H^H = 0\).

**Proof.** See appendix.

Following this lemma, the program of the government when \(T_L^L\) and \(T_H^H\) are close to 0 can thus be written:

\[
\begin{align*}
\max & \, N_L p_L U(c_{LL}, y_{LL}/\omega^L) + N_H p_L U(c_{HH}, y_{HH}/\omega^H) \\
\text{st} & \, N_L p_L (y_{LL} - c_{LL}) + N_H p_L (y_{HH} - c_{HH}) + N_L p_H (y_{HL} - c_{HL}) + N_H p_H (y_{HH} - c_{HH}) \geq 0 \\
& \, N_L T_L^L + N_H T_H^H \geq 0 \\
& \, U(c_{kj}, y_{kj}/\omega^L) + \gamma \Omega^k \geq U(c_{kj}^j, b^j - T_b^L - b^j + T_b^H, y_{kj}^j/\omega^k) + \gamma \Omega^k,
\end{align*}
\]
We now evaluate the impact of introducing a small tax on the high bequests $T_b^H$, this tax being redistributed to individuals with low inheritances ($T_b^L = -(N^H/N^L)T_b^H$):

\[
\frac{\partial L}{\partial T_b^H} \bigg|_{T_b^H=0} = (1 + \frac{N^H}{N^L}) \left( \lambda_5 u'(c^{LL} + b^H) - \lambda_6 u'(c^{HH} - b^H) - \lambda_7 u'(c^{HL} - b^H) + \lambda_8 u'(c^{HL} + b^H) + \lambda_{11} u'(c^{LL} + b^H) - \lambda_{12} u'(c^{HH} - b^H) + \beta_1 u'(c^{HL} + b^H) - \beta_2 u'(c^{HH} - b^H) + \beta_3 u'(c^{LL}) - \beta_6 u'(c^{HH}) + \beta_{11} u'(c^{LL} + b^H) - \beta_{12} u'(c^{HL} - b^H) \right) = 0.
\]

Using the first-order condition on $b^H$, we obtain:

\[
\frac{\partial L}{\partial T_b^H} \bigg|_{T_b^H=0} = (1 + \frac{N^H}{N^L})(\beta_3 u'(c^{LL}) - \beta_6 u'(c^{HH}) - \gamma_1 u'(c^{HH} + b^H) - \gamma_2 u'(c^{HL} + b^H)).
\]
Following lemma 2, (4.9f), (4.10a) and (4.10b) cannot be binding when (4.9e) is. This means that \( \beta_5 > 0 \) and \( \beta_6 = \gamma_1 = \gamma_2 = 0 \). It thus appears from the previous expression that high bequests should be taxed in this case, the proceeds being redistributed to individuals having received low bequests. On the other hand, when (4.9e) does not bind \( (\beta_5 = 0) \), it may be possible that \( \beta_6, \gamma_1 \) or \( \gamma_2 \) are positive. In such a case, either no taxation of bequests is desirable or high bequests should be subsidized. These results are summarized in the following proposition.

**Proposition 2.** High bequests should be taxed (and low bequests subsidized) if and only if (4.9e) binds at the optimum with taxes when \( T^L_b \) and \( T^H_b \) are set to 0. When it does not bind, either no taxation of bequests is desirable or high bequests should be subsidized.

This proposition puts forward the crucial role played by constraint (4.9e). We are however not able to say if this constraint binds or not, and if it is the case, when it happens. In order to get more insights, we study in the next section a numerical example.

Before turning to this example, we show in the following proposition, which proof is in the appendix, that the second-best allocation cannot be implemented with taxes.

**Proposition 3.** The second-best allocation can never be implemented with independent tax schedules on bequests and labor income.

The proof consists in showing that constraint (4.8e) is always binding at the second-best allocation. This prevents this allocation from being implemented with independent tax schedules.

5. Numerical example

We consider a isolelastic utility function, \( u(x) = x^{1-\varepsilon}/(1-\varepsilon) \), with \( \varepsilon = 4 \). The disutility of labor is given by \( v(l) = l^4/4 \). We assume an identical number of low and high productivity individuals, with population in each group normalized to 1: \( N^L = N^H = 1 \). In the absence of correlation, this implies that \( p^L = p^H = 1/2 \). Finally productivities are \( \omega^L = 1 \) and \( \omega^H = 2 \).

This simulations allows to determine which constraints are binding at the optimum with taxes when \( T^L_b \) and \( T^H_b \) are set to 0 and thus the sign of the optimal bequest tax.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Binding constraints</th>
<th>Opt. bequest tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(4.9e), (4.9b), (4.8e)</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>0.98</td>
<td>(4.9e), (4.9b), (4.9i), (4.8e)</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>0.9</td>
<td>(4.9b), (4.9i), (4.8e), (4.8i)</td>
<td>= 0</td>
</tr>
<tr>
<td>0.5</td>
<td>(4.9b), (4.9i), (4.8e), (4.10a), (4.10b)</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>0.02</td>
<td>(4.9b), (4.9i), (4.8e), (4.10a), (4.10b)</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>
The results presented in this table suggest that the taxation of high bequests is most likely for high values of $\gamma$. For intermediate values, there should neither taxation nor subsidization. High bequests should be subsidized when the degree of altruism is low.

6. Tax implementation: marginal taxes on bequests

Given tax functions $T^b(.)$ and $T^y(.)$ on bequests and labor income respectively, individual $kj$ solves:

$$\max_{c^{kj}, l^{kj}, b^k} V^{kj} = u(c^{kj}) - v(l^{kj}) + \gamma \sum_j p^{jk}(u(c^{jk}) - v(l^{jk}))$$
$$\text{st } c^{kj} = y^{kj} - T^y(y^{kj}) + b^j - T^b(b^j) - b^k.$$  

Assuming the bequests tax schedule is everywhere differentiable, the first-order conditions with respect to bequests write:

$$-u'(c^{kj}) + \gamma \sum_j p^{jk}(1 - T^b(b^k))u'(c^{jk}) = 0.$$  

Therefore:

$$1 - T^b(b^k) = \frac{u'(c^{kj})}{\gamma \sum_j p^{jk}u'(c^{jk})}.$$  

We develop this expression for high bequests individuals:

$$1 - T^b(b^H) = \frac{u'(c^{HH})}{\gamma(p^H u'(c^{HH}) + p^L u'(c^{HL}))} = \frac{u'(c^{HL})}{\gamma(p^H u'(c^{HL}) + p^L u'(c^{LL}))}.$$  

The two last expressions are in general not equal. The optimal bequest tax schedule is thus not differentiable. If we however interpret $1 - u'(c^{kj})/\gamma \sum_j p^{jk}u'(c^{jk})$ as the marginal tax, it should be noted that it is greater than 0 when $\gamma$ is large enough. The marginal tax is thus positive when individuals are sufficiently altruistic. The reason is that purely altruistic individuals would like to equalize the consumption of their offspring, whether they be high or low skills. In order to create a wedge between these consumptions levels, it is optimal to tax bequests at the margin. When $\gamma$ is low enough, it however may be optimal to subsidize bequests at the margin. When individuals do not value enough the welfare of their children, the government should give incentives to increase their bequests, acting like a Pigovian correction.

7. Conclusion

Our analysis has contributed to shed light on the optimal level of bequest taxation. We have shown that, depending on the level of altruism, high bequests should be taxed or subsidized.
Numerical examples suggest that the former case is more likely when individuals are altruistic enough. We would like in future research to assess the theoretical validity of this numerical insight, by studying the comparative statics of the optimal bequest tax with respect to the altruism parameter.

We also have assumed in this work that the productivities of the parents and the children were uncorrelated. Of particular interest would be to generalize the model to the case where these productivities are correlated (though not perfectly). This would allow us to study how the optimal taxes vary with the degree of correlation.
Appendix

A. Proof of proposition 1

(i) We evaluate $\frac{\partial L}{\partial c^H}$ at the point $c^H = c^L$:

$$\frac{\partial L}{\partial c^H} \bigg|_{c^H = c^L} = N^L p^H u'(c^L) - \mu N^L p^L - \lambda_1 y^H u'(c^L) + \lambda_2 y^L u'(c^L) - \lambda_2 y^H u'(c^L).$$

Noting that $N^L p^H = N^H p^L$ and using (3.3b), we obtain:

$$\frac{\partial L}{\partial c^H} \bigg|_{c^H = c^L} = -\lambda_1 y^H u'(c^L) + \lambda_1 y^L u'(c^L) - \lambda_2 y^H u'(c^L) + \lambda_2 y^L u'(c^L) - \lambda_2 y^H u'(c^L) = -\lambda_1 y^H u'(c^L) + \lambda_1 y^L u'(c^L) - \lambda_2 y^H u'(c^L) + \lambda_2 y^L u'(c^L) \geq 0.$$

Therefore $c^H > c^L \forall \gamma \in [0, 1)$ and $c^H = c^L$ if $\gamma = 1$.

(ii) Suppose that $c^{HH} < c^{HL}$. Dividing (3.3c) and (3.3d) by $N^L p^H$ and $N^H p^L$ respectively and comparing these two expressions, one must have

$$\frac{\lambda_1}{N^H p^H} + \frac{\lambda_1}{N^H} + \frac{\lambda_2}{N^H} < \frac{\lambda_2}{N^L p^L} - \frac{\lambda_1}{N^H} - \frac{\lambda_2}{N^L},$$

which implies in particular $\lambda_1/N^H < \lambda_2/N^L$. Using this relationship and dividing (3.3a) and (3.3b) by $N^L p^L$ and $N^H p^L$ respectively, it appears that $c^{LL} < c^L$.

When $c^{HH} < c^{HL}$, we can deduce from (3.4c) and (3.4d) that $y^{HH} > y^{HL}$. This means that $(y^{HL}, c^{HL})$ is on a higher indifference curve than $(y^{HH}, c^{HH})$. From the incentive constraints (3.1) and (3.2), it must then be that $(y^{LL}, c^{LL})$ is on a higher indifference curve of the $H$ types than $(y^{HH}, c^{HH})$. As $c^{LL} < c^{HL}$, this is only possible when $y^{LL} < y^{HL}$. However when $\lambda_2/N^L > \lambda_1/N^H$, the inspection of (3.4a) and (3.4b) makes clear that $y^{LL} < y^{HL}$, a contradiction.

We thus have shown $c^{HH} > c^{HL}$ and $y^{HH} < y^{HL}$ when $\gamma > 0$. Similar arguments lead to the conclusion that $c^{HL} > c^{LL}$ and $y^{HL} < y^{LL}$. When $\gamma = 0$, it can be checked that the first-order conditions on the second-best optimum are satisfied when $(y^{LL}, c^{LL}) = (y^{HH}, c^{HH})$, $(y^{HH}, c^{HH}) = (y^{HL}, c^{HL})$ and $\lambda_1/N^H = \lambda_2/N^L$.

(iii) It can easily be shown that the first-best implies complete equalization of consumptions and incomes. At this allocation, at least one of the two incentive constraints (3.1) and (3.2) is necessarily binding. We prove that in fact both constraints are. Suppose that (3.2) is not $(\lambda_2 = 0)$; (3.3a) and (3.3c) then write:

$$\frac{\partial L}{\partial c^L} = (N^L p^L - \lambda_1 y^L) u'(c^{opt}^{LL}) - \mu N^L p^L = 0$$

$$\frac{\partial L}{\partial c^H} = (N^H p^H - \lambda_1 y^H) u'(c^{opt}^{HL}) - \mu N^H p^H = 0.$$
Dividing these two conditions by $N^L p^L$ and $N^L p^H$ respectively, it appears that they are identical and thus that $c^{LL} = c^{HL}$. This is in contradiction with (ii).

A similar reasoning can be made to show that $\lambda_2 \neq 0$.

Knowing that (3.1) and (3.2) are binding, standard argument can be used to prove that constraints from the low to the high types cannot be binding.
B. Proof of lemma 1

We consider allocations that depend also on the type of the grandparent. The program of the social planner is then:

\[
\max N^{LLL} U(c^{LLL}, y^{LLL} / \omega^L) + N^{LHH} U(c^{LHH}, y^{LHH} / \omega^L) + N^{LHL} U(c^{LHL}, y^{LHL} / \omega^L) \\
+ N^{LHH} U(c^{LHH}, y^{LHH} / \omega^L) + N^{HLL} U(c^{HLL}, y^{HLL} / \omega^H) + N^{HHL} U(c^{HHL}, y^{HHL} / \omega^H) \\
+ N^{HLL} U(c^{HLL}, y^{HLL} / \omega^H) + N^{HHH} U(c^{HHH}, y^{HHH} / \omega^H)
\]

st

\[
N^{LLL}(y^{LLL} - c^{LLL}) + N^{LHH}(y^{LHH} - c^{LHH}) + N^{LHL}(y^{LHL} - c^{LHL}) + N^{LHH}(y^{LHH} - c^{LHH}) \\
+ N^{HLL}(y^{HLL} - c^{HLL}) + N^{HHH}(y^{HHH} - c^{HHH}) \geq 0
\]

and

\[
U(c^{HHH}, y^{HHH} / \omega^H) + \gamma(p^L U(c^{LHH}, y^{LHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H)) \\
\geq U(c^{LHH}, y^{LHH} / \omega^H) + \gamma(p^L U(c^{LHH}, y^{LHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H)) \\
U(c^{HHH}, y^{HHH} / \omega^H) + \gamma(p^L U(c^{HHH}, y^{HHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H)) \\
\geq U(c^{LHH}, y^{LHH} / \omega^H) + \gamma(p^L U(c^{LHH}, y^{LHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H)) \\
U(c^{HHH}, y^{HHH} / \omega^H) + \gamma(p^L U(c^{HHH}, y^{HHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H)) \\
\geq U(c^{LHH}, y^{LHH} / \omega^H) + \gamma(p^L U(c^{LHH}, y^{LHH} / \omega^L) + p^H U(c^{HHH}, y^{HHH} / \omega^H))
\]

The first-order conditions with respect to \(c^{HHH}\) and \(c^{HHL}\) are:

\[
N^{HHH} u'(c^{HHH}) - \mu N^{HHH} + \lambda^{HHH} (1 + \gamma p^H) u'(c^{HHH}) + \lambda^{HHL} \gamma p^H u'(c^{HHH}) = 0 \\
N^{HHL} u'(c^{HHL}) - \mu N^{HHL} + \lambda^{HHL} u'(c^{HHL}) + (\lambda^{HHL} + \lambda^{HLL}) \gamma p^H u'(c^{HHL}) = 0
\]

Summing these two conditions, we recover condition (3.3d) when \(\lambda^{HHH} + \lambda^{HHL} = \lambda_1, \lambda^{HHL} + \lambda^{HLL} = \lambda_2\) and \(c^{HHH} = c^{HHL}\). Therefore \(c^{HHH} = c^{HHL} = c^{HH}\). This result can be demonstrated for the other variables in the same way.

We have considered in this proof histories of length 3. It can be shown using the same technique that this result holds true for any history of types.
C. Proof of lemma 2

(i) 1) At the optimum with taxes, (4.9e) must be satisfied. This implies readily:

\[ U(c^L, y^L) - U(c^L, y^L) \geq U(c^L, y^L) \geq U(c^L, y^L) - U(c^L, y^L) \]

Therefore as soon as (4.9l) is satisfied, (4.9c) is also satisfied.

We now turn to (4.9i). When \( u(c^{HH}) - v(y^{HH}/\omega^H) \geq u(c^{LL}) - v(y^{LL}/\omega^H) \), (4.9b) implies (4.9i).

Finally, combining (4.9l) and (4.9i), (4.9g) must also be satisfied.

2) (4.9k) and (4.9a) imply readily (4.9d) and (4.9j) (graphical argument).

Then, combining (4.9d) with (4.9a), (4.9i), (4.9h) must also be satisfied.

3) We show that constraints (4.10c) and (4.10d) do not bind at the optimum with taxes.

We rewrite condition (4.10d) as follows:

\[ U(c^{LL}, y^{LL}/\omega^L) - U(c^{LL} - b^H, y^{LL}/\omega^L) \]

\[ + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) - \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \geq 0. \]

From (4.9b), \( U(c^{HL}, y^{HL}/\omega^H) \geq U(c^{HH} - b^H, y^{HH}/\omega^H) \), and therefore \( U(c^{HH}, y^{HH}/\omega^H) \geq U(c^{HH} - b^H, y^{HH}/\omega^H) - U(c^{HH}, y^{HH}/\omega^H) = u(c^{HH} - b^H) - u(c^{HH}) \).

Recalling that \( c^{HH} > c^{LL} \) under condition \( o \), the concavity of the utility function implies \( u(c^{HH} - b^H) - u(c^{HH}) > u(c^{LL} - b^H) - u(c^{LL}) \). From (4.9l), \( U(c^{LL}, y^{LL}/\omega^L) \geq U(c^{LL} - b^H, y^{LL}/\omega^L) \) and therefore \( U(c^{LL}, y^{LL}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) \geq U(c^{LL} - b^H, y^{LL}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) = u(c^{LL} - b^H) - u(c^{LL}) \). Recalling that \( c^{LL} \geq c^{LL} \) under condition \( o \), the concavity of the utility function implies \( u(c^{LL} - b^H) - u(c^{LL}) \geq u(c^{LL} - b^H) - u(c^{LL}) \).

It follows that

\[ \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) - \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \]

\[ \geq \gamma(u(c^{LL} - b^H) - u(c^{LL})), \]

and thus

\[ U(c^{LL}, y^{LL}/\omega^L) - U(c^{LL} - b^H, y^{LL}/\omega^L) \]

\[ + \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HL}, y^{HL}/\omega^H)) - \gamma(p^L U(c^{LL}, y^{LL}/\omega^L) + p^H U(c^{HH}, y^{HH}/\omega^H)) \]

\[ \geq (1 - \gamma)(u(c^{LL} - b^H)) \geq 0. \]
We now turn to condition (4.10c), that can be rewritten as follows:

\[
U(c^{LL}, y^{LL}/\omega^L) - U(c^{HL}, y^{HL}/\omega^L) + \gamma(p^L(U(c^{LL}, y^{LL}/\omega^L) - U(c^{HL}, y^{HL}/\omega^L)) + p^H(U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H))) \\
\leq 0.
\]

From (4.9f), \(U(c^{LL}, y^{LL}/\omega^L) \geq U(c^{LH} - b^H, y^{LH}/\omega^L)\) and thus \(U(c^{LH}, y^{LH}/\omega^L) - U(c^{LL}, y^{LL}/\omega^L) \leq u(c^{LH}) - u(c^{LH} - b^H)\). From (4.9b), \(U(c^{HL}, y^{HL}/\omega^H) \geq U(c^{HH} - y^H, y^{HH}/\omega^H)\) and thus \(U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H) \leq u(c^{HH}) - u(c^{HH} - b^H)\). Under condition \(o\), \(c^{HH} \geq c^{LH}\).

Therefore \(u(c^{HH}) - u(c^{HH} - b^H) \leq u(c^{LH}) - u(c^{LH} - b^H)\). All this implies:

\[
U(c^{LH} - b^H, y^{LH}/\omega^L) - U(c^{LH}, y^{LH}/\omega^L) + \gamma(p^L(U(c^{LH}, y^{LH}/\omega^L) - U(c^{HL}, y^{HL}/\omega^L)) + p^H(U(c^{HH}, y^{HH}/\omega^H) - U(c^{HL}, y^{HL}/\omega^H))) \\
\leq (1 - \gamma)(u(c^{LH} - b^H) - u(c^{LH})) \leq 0.
\]

(ii) (4.9f) is obviously not binding when (4.9e) is. We then show that (4.10a) and (4.10b) are not binding at the optimum with taxes when (4.9e) is binding.

Let us consider first constraint (4.10b). From (4.8i), we have:

\[
U(c^{HL}, y^{HL}/\omega^H) + \gamma(p^L(U(c^{LH}, y^{LH}/\omega^L) + p^H(U(c^{HH}, y^{HH}/\omega^H)) - \gamma(p^L(U(c^{LH}, y^{LH}/\omega^L) + p^H(U(c^{HL}, y^{HL}/\omega^L))) \\
\geq U(c^{LL}, y^{LL}/\omega^L).
\]

Therefore (4.10b) will be satisfied if:

\[
U(c^{LL}, y^{LL}/\omega^H) \geq U(c^{HL} + b^H, y^{HL}/\omega^H).
\]

When (4.9e) binds, we have \(U(c^{LL}, y^{LL}/\omega^H) = U(c^{HH}, y^{HH}/\omega^H)\). Using (4.9a), (4.10b) must be satisfied.

We then show that (4.10a) is satisfied when (4.10b) is. These two conditions can indeed be rewritten respectively:

\[
u(c^{HH}) - u(c^{HH} + b^H) \geq \gamma(\Omega^L - \Omega^H)\]

\[
u(c^{HL}) - u(c^{HL} + b^H) \geq \gamma(\Omega^L - \Omega^H).
\]

As \(c^{HH} \geq c^{HL}\) under condition \(o\), the concavity of the utility function implies that \(u(c^{HH}) - u(c^{HH} + b^H) \geq u(c^{HL}) - u(c^{HL} + b^H)\) and therefore that (4.10a) is satisfied when (4.10b) is.
D. Proof of proposition 3

We have shown previously that (3.1) is binding at the second-best allocation:

\[ U(c^{HH}, y^{HH}/\omega^{H}) + \gamma \Omega^{H} = U(c^{LH}, y^{LH}/\omega^{H}) + \gamma \Omega^{L}. \]

It follows that (4.8e) can be satisfied iff:

\[ U(c^{LH}, y^{LH}/\omega^{H}) \geq U(c^{LL} + b^{H}, y^{LL}/\omega^{H}). \]

This condition is satisfied iff \( b^{H} \leq b_{1} \), with \( b_{1} \) implicitly defined by

\[ U(c^{LH}, y^{LH}/\omega^{H}) = U(c^{LL} + b_{1}, y^{LL}/\omega^{H}) \]  \hspace{1cm} (D.1)

We then argue that this condition is not compatible with (4.9l). This latter constraint indeed imposes that \( b^{H} \geq b_{2} \), where \( b_{2} \) is implicitly defined by:

\[ U(c^{LL}, y^{LL}/\omega^{L}) = U(c^{LH} - b_{2}, y^{LH}/\omega^{L}). \]

A graphical inspection makes clear that \( b_{2} > b_{1} \) and thus that the two conditions are not compatible. Formally this can be shown by differentiating (D.1):

\[ \frac{db_{1}}{d\omega^{H}} = \frac{y^{LH}v'(y^{LH}/\omega^{H}) - y^{LL}v'(y^{LL}/\omega^{H})}{(\omega^{H})^{2}u'(c^{LL} + b^{H})}. \]

This expression is, recalling that \( v \) is convex and that \( y^{LL} > y^{LH} \), negative. Noting that \( b_{1} = b_{2} \) when \( \omega^{L} = \omega^{H} \), this implies that \( b_{1} < b_{2} \). qed.
References


