Choosing the legal retirement age in presence of unemployment

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Abstract

A general conclusion of the theoretical literature on pensions is, confronted to an increased longevity, to encourage continued activity. This literature however assumes a perfect labour market. The central question addressed in this article is whether it is still desirable to increase the retirement age when individuals face the risk of being unemployed.

In this purpose, we study the design of the Pay-As-You-Go pension system, focusing on the determination of the retirement age, in a model where people differ according to age only and face in every period a given probability of becoming unemployed.

We first determine the optimal pension system, which consists in a payroll tax rate, a pension benefit level and a retirement age and study its comparative statics with respect to a change of the unemployment rate and the length of life. Our main findings are the following. First, it is optimal to postpone retirement when life expectancy increases. Second, when unemployment benefits are low the optimal retirement age may decrease with the unemployment rate.

We then characterize the issue-by-issue voting equilibrium and compare it to the optimal pension scheme. It is shown that the median voter in general chooses a retirement age lower than the optimal one as well as a higher payroll tax rate.

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**Résumé**

La littérature théorique sur les retraites conclut que, dans une situation où la longévité augmente, les individus devraient travailler plus longtemps. Dans cette article, nous cherchons à déterminer si cette conclusion est toujours valide lorsque les individus font face au risque de chômage. Dans cette optique, nous étudions le choix du système de retraite par répartition, en mettant l’accent sur l’âge de départ en retraite, dans un modèle où les individus diffèrent par leur âge seulement et font face au risque de chômage. Nous déterminons dans un premier temps le système de retraite optimal, ce système étant caractérisé par le taux de cotisation, le niveau de prestation et l’âge de la retraite et nous étudions la statique comparative par rapport au taux de chômage et la durée de vie. Nos résultats principaux sont les suivants. Tout d’abord, il est optimal de différer l’âge de la retraite lorsque la durée de vie augmente. Ensuite, nous trouvons que l’âge de la retraite optimal peut diminuer lorsque le risque de chômage augmente. Nous caractérisons enfin l’équilibre de vote et le comparons avec le système optimal. Nous montrons que l’électeur médian choisit en général un âge de la retraite inférieur à l’âge optimal ainsi qu’un taux de cotisation plus important.
1 Introduction

Both the political debate and the scientific literature aim at finding sustainable reforms to solve the financial unbalance of public Pay-As-You-Go (PAYG) pensions. Three policies are usually pointed out: raising contributions, cutting benefits and raising the retirement age. Among those, a rise in the retirement age has received a special attention. A general conclusion of the literature (Cremer and Pestieau (2003)) is to advocate policies that encourage continued activity as a way to improve financial problems of pension systems.

This literature rests on the assumption that labour markets are competitive and can completely absorb increases in the labour stock. However, if we depart from the assumption of perfectly competitive labour markets and consider unemployment, the issue of the retirement age cannot set aside labour market features.

A popular argument stresses the fact that early retirement schemes were introduced to make the old leave the labor force, hence providing more jobs for the young. Symmetrically, the reluctance to increase the retirement age is often explained by its detrimental effect on the employment opportunities of the young. This mechanism is studied in details in Gruber, Milligan, and Wise (2009).

There is however no clear empirical evidence on the link between the rate of unemployment and the retirement age. Keuschnigg and Keuschnigg (2004) find a negative general equilibrium effect on unemployment of an increase in $R$. But Diamond (2005), using data from Gruber and Wise (1998), finds no relationship between the unemployment rate for males and the log of the tax force. Taking this absence of clear evidence into account, we assume that the unemployment rate does not depend on the age of retirement.

Another crucial issue pointed out in the political debate is that raising the legal retirement age is not feasible if there is no demand for older labour force. This paper sheds light on this argument since the central question we are interested in is weather it is still desirable to increase the retirement age when workers face the risk of being unemployed. Our framework also allows us to study the impact of the unemployment rate on the chosen retirement age.

In this purpose we study the design of the PAYG system, in particular concerning the legal retirement age,\footnote{In our setting, the legal retirement age, the effective retirement age an the age at which retirement benefits can be claimed are the same.} when there is unemployment. More specifically, we are interested in the choice of the retirement age (mandatory and common to all) when the level of
unemployment rate is taken into account. Are people willing to trade the insurance of avoided unemployment against a smaller pension associated to anticipated retirement?

We build up a continuous overlapping-generations (OLG) model in which agents inelastically supply labour in each period but express preferences over how many periods they want to work, given that labour gets costlier for older people. In each period, they face the risk of being unemployed. Redistribution occurs towards the retirees and the unemployed, and benefits are financed through a payroll tax on labour.

In this setting, our objective is twofold. We first clarify how the individually optimal policy concerning the pension system and unemployment subsidies depends on the demography and the labour market characteristics. We then characterize the political choice over this policy and compare it to the optimal policy.

Our results are the following.

As we do not endogenise labour supply within periods (on the intensive margin), we consider a setting in which unemployment benefits are exogenous. Indeed, with optimal unemployment benefits, there would be perfect unemployment insurance at the optimum, which is an artifact of the model. We first show that people are willing to work longer when life expectancy increases, in order to achieve a satisfactory consumption level during retirement. At the beginning of the life cycle, they also want to increase the payroll tax rate and decrease pension benefits. This result can be explained by the negative income effect identified by Conde-Ruiz, Galasso, and Profeta (2005) and Galasso (2008). Following an increase in the life length or in the unemployment rate, the level of pension benefits has to be reduced if one keeps the other parameters of the PAYG system unaltered. The optimal way to counter this income loss consists in delaying retirement.

We then study the optimal choice when confronted to a rise in the probability of unemployment and show that the change in the optimal policy is ambiguous: a higher unemployment risk does not necessarily lead to earlier desired retirement age, the crucial element being the amount of unemployment benefit received in case of unemployment. More precisely, when the unemployment compensation is high enough and the disutility of foregone consumption is not too strong, the marginal gain in terms of increased pension benefits offsets the marginal cost of postponed retirement.

The second part of the paper is devoted to the analysis of the political equilibrium. We consider issue-by-issue voting: individuals vote independently on two elements of
the pension system, the payroll tax rate and the legal retirement age.\textsuperscript{2} We show that a corner equilibrium with a maximum tax rate (equal to 1) and the age of retirement being equal to the age of the median voter always exists. Interior equilibria may also exist, depending on the parameters of the model. In these equilibria, the retirement age is larger and the payroll tax rate is lower than at the corner equilibrium. When comparing the political equilibrium with the optimal pension scheme, we find that \textit{the median voter has some incentives to retire earlier and to choose a higher tax rate than what optimality commands.}

\textit{Related literature}

A few other papers (Demmel and Keuschnigg (2000), Corneo and Marquardt (2000), Keuschnigg and Keuschnigg (2004)) merge the labour market and pension reform issues but they aim at proving that wide public pension systems are harmful for unemployment. They focus on the consequences of high social security contributions on labour market equilibrium rather than on the impact of existing unemployment for pension reform: in a union wage-setting model, Corneo and Marquardt (2000) show that contributions to the retirement system have a negative effect on growth and that a shift from a PAYG to a funded system entails a Pareto improvement. Keuschnigg and Keuschnigg (2004) calibrate an OLG model with search unemployment to Austrian data and study the impact of a rise in the retirement age on unemployment: they show that this policy is welfare improving, that it boosts growth and lowers unemployment.

Our paper is also related to the literature on voting on pensions. Casamatta, Cremer, and Pestieau (2005) study the issue-by-issue voting equilibrium on the contribution rate and on the tax on the retirement benefit, in a setting where the retirement decision is endogenous.\textsuperscript{3} Their focus is different from ours in that the retirement decision is individual (there is no global determination of a mandatory retirement age); they are more interested in understanding the role of the implicit tax on continued activity for individuals with different productivity. Lacomba and Lagos (2007) focus on the political determination

\textsuperscript{2}It would be interesting to consider the joint political equilibrium on the legal retirement age and the level of unemployment compensation. We do not proceed along these lines here for two reasons: for the sake of uniformity, we want to stick to the setting developed in the first part of the paper which considers exogenous unemployment benefits. Moreover, we argue that the political debate over the retirement system focuses mainly on the elements of the system itself, i.e. the amount of pensions, the contribution rate and the retirement age.

\textsuperscript{3}Other papers use this equilibrium concept to determine pension policies; see, e.g., Conde-Ruiz and Galasso (2004) and Galasso (2008).
of the retirement age in a model with savings. They study the role of the redistribution degree of the pension system for the political outcome and show that more redistribution induces people to retire later. There are many differences with our model: they do not consider unemployment, they do not say anything about the effect of an increase in the length of life and they assume that pensions can only be reformed by changing the retirement age. We show that changes in the retirement age are linked to reforms on the other dimensions, the contribution rate and the level of pensions.

The paper is organized as follows. Section 2 presents the main features of the model. Section 3 studies the life cycle utility maximization problem, investigates the role of unemployment and performs a sensitivity analysis on the length of life. Section 4 focuses on the determination of the political equilibrium and its comparison with the optimal policy.

2 The model

2.1 Production

We set up a continuous time model. At each time $t$, an homogeneous good is produced in the economy with a single input technology: labour $L$ is used in the production function $f_t(L_t)$, which has non-increasing returns to scale. We assume that the labour market does not clear, so that there is unemployment and denote $\pi_t$ the unemployment rate at date $t$. We take the market imperfection as given.

The cost of labour is $w_t$ and assumed not to depend on $\pi_t$, which is consistent with the minimum wage model of unemployment.

2.2 Consumption

A new generation is born at each point in time $t$, with a certain and finite time horizon $T$. Life can be divided into two broad periods: until $R$, which corresponds to the retirement age, the individual is active. After $R$, she retires.

2.2.1 The young

At $t$, each individual born between $t - R$ and $t$ inelastically supplies one unit of labour to the firm. With probability $\pi_t$, exogenous to the individual, she is employed and receives the wage $w_t(1 - \tau_t)$ where $\tau$ is the proportional tax on labour income. With probability
1 − \pi_t she does not find a job and receives an unemployment benefit \( b_t^U \). Note that history does not play any role in the model: at \( t + dt \) individuals face the same probability \( \pi_{t+dt} \) to be employed, irrespective of their employment status at time \( t \).

### 2.2.2 The old

We assume that the retirement age \( R \) is imposed and common to everybody. Therefore, at \( t \), each individual born between \( t − T \) and \( t − R \) is a retiree. She receives a retirement benefit \( b_t^R \). The pension system is Pay-As-You-Go (PAYG), financed with the payroll tax on labour at time \( t \).

### 2.3 Population dynamics

We denote \( C_t \) the number of individuals born at time \( t \):

\[
C_t = C_0 e^{nt},
\]

where \( n \) is the rate of population growth.

In the following, we consider a stationary population, i.e. we set \( n = 0 \). If we denote \( N_t \) the total population, \( N_t^R \) the number of retirees, and \( N_t^A \) the number of young people living at time \( t \), we therefore have that \( N_t = C_0 T, N_t^A = C_0 R \) and \( N_t^R = C_0(T − R) \).

In other words, with a stationary population, at each \( t \) the population is the same so that all subscripts \( t \) can be removed.

### 3 Maximization of life-cycle utility

#### 3.1 Preferences

Individuals are identical in all respects, except for age. The utility of an individual of age \( a \in [0, T] \) is given by

\[
\begin{align*}
\text{if } a < R & \quad U = \int_a^R \left( (1 − \pi)\left[ u(c) − v(s) \right] + \pi u(b^U) \right) ds + \int_R^T u(b^R) ds \\
& \quad = (1 − \pi)(R − a)u(c) + \pi(R − a)u(b^U) − (1 − \pi) \int_a^R v(s) ds + (T − R)u(b^R) \\
\text{if } a \geq R & \quad U = (T − a)u(b^R)
\end{align*}
\]

(1)
where $c$ is consumption, which is equal to the net wage $w(1 – \tau)$. We assume that $u(.)$ is increasing and concave and that $v(.)$, which denotes the marginal disutility of labour, is non-decreasing with age. Furthermore, the unemployment probability, $\pi$, is assumed not to depend on the retirement age, $R$.

The life-cycle utility is obtained when we set $a = 0$.

### 3.2 The resource constraint

The payroll tax on labour is used to finance both the unemployment and the retirement benefits from the same budget. Given the stationarity assumptions about the population, the government budget constraint (GBC) does not depend on $t$. It is given by:

$$\tau w(1 – \pi)R – \pi R b^U – (T – R)b^R = 0. \quad (2)$$

### 3.3 Optimal policies

We are now interested in the optimal values of the fiscal instruments for an individual at the beginning of his life, i.e. we want to maximize his life-cycle utility. We solve the life-cycle maximization problem when unemployment benefits are given. The reason for doing so is twofold. First, as shown at the end of this section, the optimum policy mix with endogenous benefits involves perfect insurance. This is due to the fact that individuals do not choose their labour supply within time periods. With endogenous labour supply, perfect insurance would not occur as nobody would have an incentive to continue working. This result is thus totally dependent on the simplified nature of our model and would disappear in a more elaborated model. Second, we consider in the next section the political equilibrium with exogenous benefits. Considering exogenous benefits in the life-cycle maximization problem then allows to make relevant comparisons between the two settings.

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4 People do not have the possibility to save or borrow in our model. The only possibility to “transfer” wealth across time is through the pension system. This is of course an important limitation of our approach. With a credit market, the preferences of a given individual on the pension system would depend on accumulated savings and thus on the previous retirement age. See Lacomba and Lagos (2007).

5 The assumptions that both the wage level and the unemployment rate are independent of the retirement age may seem incompatible at first glance. A raise in the retirement age increases the supply of labour and thus, given that labour demand is unchanged, should reflect in a higher unemployment rate. We can however show that these two assumptions are satisfied simultaneously in a slightly enriched model with two types of labour (high skilled and low skilled) and a CES production function. The labour market for the skilled being perfectly competitive, an increase in labour supply leads to an increase in the quantity of skilled labour employed at equilibrium. This in turn leads to an increase in the demand for low skilled workers that, under the CES specification, just offsets the initial increase in labour supply.
The life-cycle program writes

\[
\max_{\tau, b_R, R} (1 - \pi)Ru(c) + \pi Ru(b^U) - (1 - \pi) \int_0^R v(a)da + (T - R)u(b^R)
\]

s.t. (2).

Denoting \( L \) the Lagrangian function and \( \lambda \) the Lagrange multiplier, the first-order conditions (FOC) are given by:

\[
\begin{align*}
\frac{\partial L}{\partial \tau} &= -u'(c) + \lambda = 0 \\
\frac{\partial L}{\partial b_R} &= u'(b^R) - \lambda = 0 \\
\frac{\partial L}{\partial R} &= (1 - \pi)u(c) + \pi u(b^U) - (1 - \pi)v(R) - u(b^R) + \lambda(\tau w(1 - \pi) - \pi b^U + b^R) = 0 \\
\frac{\partial L}{\partial \lambda} &= \tau w(1 - \pi)R - \pi Rb^U - (T - R)b^R = 0
\end{align*}
\]

which can be simplified as:

\[
\begin{align*}
\frac{c}{b^R} &= b^R, \quad \text{(3)} \\
\frac{T}{R}R\left[\frac{b^R u'(b^R)}{u(b^U)} - \frac{u(b^R)}{u(b^R)}\right] &= v(R) - \pi v(R), \quad \text{(4)} \\
\frac{b^R}{T - R} &= \left(\tau w(1 - \pi) - \pi b^U\right). \quad \text{(5)}
\end{align*}
\]

The first condition means that individuals at the beginning of the life cycle want to smooth consumption between the working and retirement period. The second condition corresponds to the first-order condition on the retirement age and states that individuals choose the retirement age so as to equalise the marginal benefit of working one more year with the marginal disutility of labour.

When there is no unemployment, the marginal utility of postponing retirement is given by \(1\) and the marginal disutility of labour is \(3\). With unemployment, we also have \(2\) and \(4\): when retirement is postponed for one year, the individual gets consumption \(c\) with probability \((1 - \pi)\) and unemployment benefits \(b^U\) with probability \(\pi\). He also loses pension benefits for that year. The total effect on utility is thus \((1 - \pi)u(c) + \pi u(b^U) - u(b^R)\). Under consumption smoothing \((c = b^R)\), this becomes \(\pi(u(b^U) - u(b^R))\), which corresponds to \(2\): \(4\) simply means the disutility \(v(R)\) is less important because it only occurs when people are at work.
The first order conditions can be rewritten as

\begin{align*}
    c &= b^R \\
    \pi(u(b^R) - u(b^U)) + \frac{T}{R}b^R u'(b^R) &= (1 - \pi)v(R) \\
    b^R &= \frac{R}{T - R}(\tau w(1 - \pi) - \pi b^U).
\end{align*}

**Endogenous unemployment benefits**

If unemployment benefits were also endogenous and the optimisation programme solved with respect to \(b^U\) as well, the first order conditions would then be written as

\begin{align*}
    c &= b^R = b^U \\
    wu'(b^R) &= v(R) \\
    b^R &= \frac{R}{T - R}w(\tau - \pi).
\end{align*}

The main difference with the previous case is related to the first-order condition with respect to \(R\): we can notice that, when \(b^R = b^U\), the solutions to the problems with endogenous and exogenous unemployment benefits are the same. Besides, with endogenous unemployment, the optimal value of the retirement age depends on unemployment only through the GBC whereas the first-order condition with respect to \(R\) is more complex in the case of exogenous unemployment insurance. In particular, it does now depend on \(\pi\), which was not the case with endogenous insurance. This has significant implications for the comparative statics.

### 3.3.1 Sensitivity of the retirement age to an increase in the life length

We investigate in the next proposition how preferences at the beginning of the life cycle are affected by a change in the life length, when unemployment benefits are exogenous.

**Proposition 1.** When the length of life increases, agents at the beginning of the life-cycle want to postpone retirement, increase the payroll tax rate and decrease pension benefits.

**Proof.** See appendix A. 

The intuition for this result is the following. Condition (6) tells us that \(b^R\) and \(\tau\) necessarily move in opposite direction. Assume then that, consequently to an increase in the length of life \(T\), \(b^R\) increases and \(\tau\) decreases. Inspecting (8) leads us to the conclusion...
that $R$ should increase (otherwise $b^R$ could not increase). This contradicts equation (7): if $b^R$ and $R$ increase, this condition cannot be satisfied anymore, as the left-hand side and the right-hand side move in opposite directions. Therefore, $b^R$ must decrease and $\tau$ must increase following an increase in the length of life. Understanding this, it is clear that $R$ should be increased so as to satisfy (7).

Later on in the paper, we will focus on the particular case in which the disutility of working one more year does not depend on age ($v(R) = v$). In this case, condition (7) implies that the level of pension benefits is unaffected by a change in the length of life, the reason being that the right-hand side is fixed. Then (6) implies that $c$ (and therefore $\tau$) is also unaffected. Following an increase in the length of life, the only way to satisfy the GBC (11) is thus to increase $R$. In the particular case where $v(R)$ is constant, individuals at the beginning of the life cycle react by adjusting the age of retirement only, keeping the payroll tax rate and the level of pension benefits unchanged.

### 3.3.2 Comparison of the optimal retirement age with and without unemployment

In this section we seek to better understand how the optimal retirement age varies with the unemployment rate.

In appendix A, we prove the following

**Proposition 2.** When the unemployment rate increases, the change in the retirement age, the payroll tax rate and pension benefits is ambiguous. In particular, the retirement age may increase or decrease following an increase in the unemployment probability.

The expression derived in appendix A shows that the effect of $\pi$ on the pension policy cannot be unambiguously signed anymore. To illustrate this point, we reproduce below equation (23) in the appendix:

$$
\frac{dR}{d\pi} = \frac{w}{M} \left[ -(u(b^R) - u(b^U) - v(R))(T - \pi R) - (\tau w + b^U) Ru'(b^R)\left(\frac{T}{R}(1 - R_w(b^R)) - \pi\right) \right].
$$

Consider a isoelastic utility function, $u(x) = x^{1-\varepsilon}/(1 - \varepsilon)$, with $\varepsilon > 1$. As $u(b^U) \to -\infty$ when $b^U \to 0$, it is clear that $dR/d\pi$ is negative for $b^U$ sufficiently small and positive when $b^U$ and $b^R$ are close. This contrasts with the endogenous insurance case in which the retirement age always increases with the unemployment rate (see proposition 3 below). The reason for this difference can be found by inspecting (7). The marginal benefit of postponing retirement is composed of two terms, $\pi(u(b^U) - u(b^R))$ and $(T/R)b^R u'(b^R)$.
The first of these two terms expresses the fact that with probability $\pi$, the individual is unemployed in the marginal working year and thus earns $b^U$ instead of $b^R$. When the unemployment compensation is small and the disutility of living one year without resources is large (which is true as long as $\varepsilon \geq 1$), the marginal cost of postponing retirement is large and cannot be offset by the marginal gain in terms of increased pension benefits, $(T/R)b^R u'(b^R)$.

It can be shown that the comparative statics exercise provides similar results when unemployment benefits are endogenous as far as an increase in the life length is concerned and the reason is also similar. Conversely, the results are different in the endogenous and exogenous cases when we consider an increase in the unemployment rate: in the endogenous case, we can unambiguously state the following

**Proposition 3.** Assume that unemployment benefits are optimally chosen. When the unemployment rate increases, agents at the beginning of the life cycle want to postpone retirement, increase the payroll tax and decrease pension benefits.

**Proof.** See appendix B. ■

The fact that individuals prefer to delay retirement when the unemployment probability increases is somewhat surprising. One could think that, confronted with a higher risk of being unemployed, individuals would react by anticipating retirement. This result is driven by the fact that there is perfect insurance against unemployment in the first-best. As individuals get the same consumption whether working or unemployed, the only adverse effect of increasing the unemployment rate comes from the macro budget constraint (2). For a given tax rate, the level of unemployment and social security benefits is lowered. Individuals respond optimally to this negative income shock by delaying retirement (see Galasso (2008)).

### 4 Issue-by-issue voting equilibrium

We determine in this section the political equilibrium when individuals vote on one issue at a time.\(^6\) The two issues determined by the vote are the payroll tax rate and the retirement age. We take the unemployment benefit as given, the pension benefit level being residually determined by the government budget constraint.

\(^6\)This equilibrium concept has been proposed by Shepsle (1979).
4.1 Voting on $\tau$ for a given $R$

4.1.1 Individually optimal tax rates

We have to distinguish two cases. If, at the time of the vote, the individual is a retiree, he will vote for the tax rate that maximizes his pension level, namely $\tau = 1$. Therefore we have:

$$\text{If } a \geq R, \tau^* = 1.$$ 

We now consider the case of a worker. The government budget constraint (2) can be rewritten:

$$b^R(\tau, R) = \frac{\tau w(1 - \pi)R - \pi R b^U}{(T - R)}. \quad (12)$$

Substituting this expression into the life-cycle utility of an aged $a$ worker (see (1)), the program of this latter is:

$$\max_{\tau \in ([\pi b^R]/w(1-\pi), 1]} \int_a^R \left( (1 - \pi)[u(c) - v(s)] + \pi u(b^U) \right) ds + \int_T^R u(b^R(\tau, R)) da.$$ 

The lower bound on $\tau$, $\tau_{\min} \equiv (\pi b^U)/(w(1 - \pi))$, comes from the fact that, even if the pension level is set to 0, some taxes have to be collected so as to finance unemployment benefits. After some manipulations, we obtain the following first-order condition:

$$\frac{u'(c)}{u'(b^R)} = \frac{R}{R - a}. \quad (13)$$

This condition tells us that $\forall a > 0, c < b^R \ (c = b^R \text{ when } a = 0)$. At the beginning of the life-cycle, people want to smooth consumption perfectly. As they get older, the return from the PAYG system increases and this leads them to consume more during retirement than during the working period. Note that this result would not hold with a perfect credit market in which agents would be able to borrow as much as they want at the market interest rate. In such a case, they would choose the pension system that is the most generous possible (which corresponds to setting the payroll tax rate equal to 1) and would use borrowing to smooth consumption over time.

4.1.2 Majority voting tax rate

By differentiating the utility function (1), we obtain that the slope of an indifference curve in the $(\tau, b^R)$ plane is:

$$\frac{db^R}{d\tau} = \frac{(1 - \pi)w(R - a)u'(c)}{(T - R)u'(b^R)}.$$
The slope of indifference curves decreases with age. This implies both that the optimal payroll tax rates increase with age and that the single-crossing condition holds. This latter property in turn implies that a Condorcet winner exists (Gans and Smart (1996)). The majority voting solution is the preferred tax of the individual with median age: \( a = T/2 \). Using (12) and (13), the majority voting tax rate, \( \tau_{mv} \), is implicitly defined by the following condition:

\[
(R - \frac{T}{2})u'(w(1 - \tau_{mv})) - Ru'((R/(T - R))(\tau_{mv} w(1 - \pi) - \pi b^U)) = 0. \tag{14}
\]

4.2 Majority voting retirement age

In general, the optimal choice of the retirement age by any individual depends on the status quo retirement age through its effect on savings. Agents adjust their optimal savings depending on the retirement age. If this latter is low for example, the number of contributors is small relatively to the beneficiaries and the pension level is accordingly low. In such a situation, individuals may want to make important savings in order to finance their old age consumption. When, on the other hand, pensions are generous, individuals do not have to save a lot. Therefore the level of savings depend on the current retirement age and in turn they affect the newly chosen retirement age: the optimal retirement choice of a given individual at a given date depends on his accumulated wealth. We abstract from these difficulties in this model by considering that individuals do not have the possibility to save.

We first argue that an individual has no interest in choosing a retirement age strictly lower than his own age. In such a situation, he would benefit from increasing the retirement age, as it would increase the number of contributors to the pension system and decrease the number of beneficiaries. An individual aged \( a \) thus solves

\[
\max_{R \in [a, T]} V(R) = \int_a^R ((1 - \pi)[u(c) - v(s)] + \pi u(b^U))ds + (T - R)u(b^R(\tau, R)).
\]

The first-order conditions for an interior solution is

\[
\frac{\partial V}{\partial R} = (1 - \pi)u(c) + \pi u(b^U) - u(b^R) - (1 - \pi)v(R) + \frac{T}{R}b^R u'(b^R) = 0. \tag{15}
\]

This condition does not depend on age \( a \). Consequently, when they choose an interior solution, individuals all choose the same value for the retirement age. In other words,

\[\text{Increasing the retirement age also increases the number of unemployed and thus total unemployment payments. One can show however that the gain in alleged pension payments outweighs this cost.}\]
there is no conflict over this decision. The majority voting retirement age is thus the preferred retirement age of the individual with median age, $a = T/2$.\(^8\)

### 4.3 Political equilibrium

An interior equilibrium is characterized by equations (14) and (15) that we rewrite below:

\[
(R^e - \frac{T}{2})u'(c) - R^e u'(b^R) = 0 \tag{16}
\]

\[
(1 - \pi)u(c) + \pi u(b^U) - u(b^R) - (1 - \pi)v(R^e) + \frac{T}{R^e} b^R u'(b^R) = 0, \tag{17}
\]

where

\[
c = w(1 - \tau^e)
\]

\[
b^R = \frac{R^e}{T - R^e} (\tau^e w(1 - \pi) - \pi b^U).
\]

One should note that these conditions are the same as (6) - (8), except for the first. The decisive voter in the political equilibrium is indeed the voter of median age, $T/2$, whereas it is a newborn individual at the optimum. The main difference is that this latter wants to smooth consumption perfectly across time whereas the median voter prefers to consume more during retirement ($c < b^R$).\(^9\)

#### 4.3.1 Existence

We argue in the following proposition that, with an isoelastic utility function, a stable corner equilibrium always exists whereas the existence of a stable interior equilibrium is not guaranteed.

**Proposition 4.** Assume that the utility function is isoelastic: $u(x) = x^{1-\varepsilon}/(1 - \varepsilon)$. The political equilibrium such that $\tau^e = 1$ and $R^e = T/2$ always exists. There might also exist interior equilibria, depending on the parameters of the model. In these equilibria, $\tau^e < 1$ and $R^e > T/2$.

**Proof.** See appendix C.

Issue-by-issue voting equilibria are represented on the figures below.

\(^8\)To be precise, if the median age individual wants a retirement age larger than his own age, all individuals younger than him as well as part of the older ones have the same optimal retirement age. When the preferred retirement age of the median individual is equal to his own age, all younger (resp. older) individuals have a lower (resp. higher) optimal retirement age.

\(^9\)This feature of the model may appear surprising as individuals should “realistically” consume more during working life. It comes from our simplifying assumptions that individuals do not discount the future and have the same preferences during the working and retirement periods. These assumptions are made mainly for simplification and do not affect our results.
In order to get a better sense of the conditions ensuring the existence of interior equilibria, we solve the model with a logarithmic utility function, \( u(x) = \ln x \), and a constant disutility of work, \( v(R) = v \). Condition (17) becomes in this case:

\[
(1 - \pi) \ln c + \pi \ln b^U - \ln b^R - (1 - \pi)v + \frac{T}{R^e} = 0. \tag{18}
\]

From (16),

\[
\frac{c}{b^R} = \frac{R^e - T/2}{R^e}.
\]

Substituting into (18), we have:

\[
\phi(R^e) \equiv \ln \frac{R^e - T/2}{R^e} + \pi \ln \frac{b^U}{c} - (1 - \pi)v + \frac{T}{R^e} = 0. \tag{19}
\]

After some manipulations, we obtain

\[
c = \frac{R - T/2}{(T/2)(1 + \pi) - \pi R}(w(1 - \pi) - \pi b^U).
\]

Consumption is thus decreasing with the unemployment compensation \( b^U \). Using (19), this implies that \( \phi(\cdot) \) is increasing with \( b^U \). When \( b^U \to 0 \), we readily deduce from (19) that \( \phi(R) \to -\infty \) and thus that there is no interior equilibrium. The intuition is straightforward. If the individual decides to postpone retirement for one year, he takes the risk of being unemployed with probability \( \pi \) during this year, being left with almost
no resources when the unemployment compensation is low. Starting from any candidate equilibrium, he has thus an incentive to reduce the retirement age so as to avoid this risk.

Under perfect insurance \((b^U = c)\), the equilibrium condition writes:

\[
\ln \frac{R^e - T/2}{R^e} - (1 - \pi)v + \frac{T}{R^e} = 0.
\]

When \(R \to T/2\), \(\phi(R) \to -\infty\). Moreover,

\[
\phi(T) = \ln 1/2 - (1 - \pi)v + 2.
\]

To have an interior equilibrium, it must be that \(\phi(T) > 0\) (it can be checked that \(\phi\) is increasing), that is:

\[
\ln 1/2 - (1 - \pi)v + 1 > 0
\]

\[
\Leftrightarrow \pi > 1 - \frac{\ln 1/2 + 1}{v}.
\]

Therefore, an interior equilibrium with perfect insurance exists whenever the unemployment probability is sufficiently large. The intuition for this result is that a large unemployment probability makes unlikely that the individual will work in the additional year if retirement is postponed. The expected labour disutility, \((1 - \pi)v\), is accordingly small. To decide whether to delay retirement or not, the individual compares the loss in consumption for that year \(u(c) - u(b^R) = \ln(c/b^R)\) (instead of being on retirement, he either works or is unemployed, enjoying the same level of consumption in these two states under perfect insurance) with the rise in pension benefits obtained when retirement is delayed. We thus have shown that, when the unemployment probability is large enough, there exist values of the retirement age such that the gain in pension benefits outweighs the sum of the consumption loss and the expected labour disutility.

To sum up, in the log utility case, unemployment benefits should be high enough and the probability of unemployment large enough for an interior equilibrium to exist.

### 4.3.2 Comparative statics of the equilibrium retirement age with respect to the unemployment rate

As emphasized previously the program of the median voter and of the social planner are very close. In particular, the first-order conditions on the retirement age, (7) and (17), are the same. This implies that the comparative statics with respect to the unemployment rate have the same qualitative features: one can show that the equilibrium retirement age may decrease or increase with the unemployment rate, depending on the value of the unemployment compensation \(b^U\).
4.3.3 Comparison between the majority voting and the optimal solutions

In this last section, we aim at comparing the pension policy chosen optimally at the beginning of the life cycle and at the political equilibrium. We argue in the next proposition that the comparison depends critically on whether the optimal retirement age is lower or larger than the median age in the population.

Proposition 5. The retirement age chosen at the political equilibrium is lower than the optimal retirement age if and only if this latter is larger than the median age, $T/2$.

Proof. When See appendix D. 

This proposition is driven by the fact that the median voter, being closer to retirement than the newborn individual, chooses a larger tax rate. This in turn leads him to choose a lower retirement age. This conclusion is not valid in the case where the optimal retirement age is lower than $T/2$ for the simple reason that the median voter never has an interest in choosing a retirement age lower than his own age, namely $T/2$.

5 Conclusion

This paper contributes to the literature on the design of the PAYG pension scheme, focusing on the impact of unemployment. We start from the analysis of individually optimal preferences and then move to the discussion of the political decision making, based on the issue-by-issue equilibrium concept. Our results shed light on the interaction between unemployment insurance and the optimal design of the pension system. In particular, we have shown that the optimal retirement age always increases with life expectancy. Moreover, the optimal adjustment of the pension system in a context of increasing unemployment depends dramatically on the size of the unemployment compensation. We have also emphasized the tendency of the political process to make people retire too early.

Two important assumptions are made that should be relaxed in future research. First, we consider that agents do not have the possibility to save or borrow. This of course makes our analysis simpler. However, the savings and retirement decisions are intimately related. Taking savings into account when analyzing retirement policy thus appears to

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This comes from the fact that the curves $R^{mv}(\tau)$ and $R^{opt}(\tau)$ (that coincide when $R \in (T/2, T)$) are decreasing. Recall that an increase in $R$ induces a loss $(1 - \pi)u(c) + \pi u(b^R) - u(b^R)$ and a gain $(T/R)b^R u'(b^R)$. Increasing $\tau$ makes both this gain and this loss bigger. It can however be shown that the aggregate effect is negative. To compensate this drop in the marginal benefit, individuals choose to reduce $R$.
be a desirable continuation of our work. Second, we have assumed exogenous labour supply within time periods. This allowed us to have a straightforward characterization of the optimal unemployment insurance scheme. A realistic approach of the labour market however makes desirable to endogenize labour supply.

Finally, further work, both empirical and theoretical, should be devoted to understanding the impact of the collective retirement decision on the unemployment rate and its implications for the design of pension systems.
Appendix

A Comparative statics with exogenous unemployment benefits

The differentiation of the system (6)-(8) with respect to $T$ can be written in matrix form:

$Mx = b$ where $M$ is

$$
\begin{pmatrix}
-w & -1 & 0 \\
0 & T/R(u'(b^R) + b^R u''(b^R)) - \pi u'(b^R) & -(1 - \pi) v'(R) - (T/R^2) b^R u'(b^R) \\
w(1 - \pi) R & -(T - R) & (T/R) b^R
\end{pmatrix},
$$

$x$ is the vector

$$
\begin{bmatrix}
\frac{d\tau}{dT} \\
\frac{db^R}{dT} \\
\frac{dR}{dT}
\end{bmatrix},
$$

and $b$ is the vector

$$
\begin{bmatrix}
0 \\
-(1/R) b^R u'(b^R)
\end{bmatrix}.
$$

We then get

$$
\begin{align*}
\frac{d\tau}{dT} &= \frac{1}{|M|} [b^R(1 - \pi) v'(R)] \geq 0 \\
\frac{db^R}{dT} &= \frac{1}{|M|} [-w b^R (1 - \pi) v'(R)] \leq 0 \\
\frac{dR}{dT} &= \frac{1}{|M|} \left[-w \frac{T}{R} (b^R)^2 u''(b^R)\right] > 0,
\end{align*}
$$

where

$$|M| = -w \frac{T^2}{R^2} (b^R)^2 u''(b^R) + w(1 - \pi) v'(R)(T - \pi R) > 0.$$

When $v'(R) = 0$, these expressions simplify to:

$$
\begin{align*}
\frac{d\tau}{dT} &= 0 \\
\frac{db^R}{dT} &= 0 \\
\frac{dR}{dT} &= \frac{R}{T},
\end{align*}
$$

We next turn to the comparative statics of the optimal policy with respect to $\pi$, the unemployment probability. The vector $b$ is modified to:

$$
\begin{bmatrix}
0 \\
u(b^R) - u(b^U) - v(R) \\
(\tau w + b^U) R
\end{bmatrix}.$$
This leads to:

\[
\frac{d\tau}{d\pi} = \frac{1}{|M|} \left[ \frac{T}{R} b^R (u(b^R) - u(b^U) - v(R)) + (\tau w + b^U) R \left( \frac{T}{R^2} b^R u'(b^R) + (1 - \pi) v'(R) \right) \right]
\]

(21)

\[
\frac{db^R}{d\pi} = -\frac{w}{|M|} \left[ \frac{T}{R} b^R (u(b^R) - u(b^U) - v(R)) + (\tau w + b^U) R \left( \frac{T}{R^2} b^R u'(b^R) + (1 - \pi) v'(R) \right) \right]
\]

(22)

\[
\frac{dR}{d\pi} = \frac{w}{|M|} \left[ -(u(b^R) - u(b^U) - v(R))(T - \pi R) - (\tau w + b^U) Ru'(b^R) \left( \frac{T}{R} (1 - R\tau(b^R)) \right) \right]
\]

(23)

B Proof of proposition 3

If we differentiate the first-order conditions (9)-(11) with respect to \(\pi\) (after substitution of the first condition into the third) we get:

\[
-w \frac{d\tau}{d\pi} - \frac{db^R}{d\pi} = 0
\]

\[
v'(R) \frac{dR}{d\pi} - u''(b^R) \frac{db^R}{d\pi} w = 0
\]

\[
\frac{dR}{d\pi} + \frac{T}{1 - \pi} \frac{d\tau}{d\pi} - \frac{T(1 - \tau)}{(1 - \pi)^2} = 0
\]

We can rewrite these equations in matrix form \(M x = b\) where \(M\) is the matrix

\[
\begin{pmatrix}
w & 1 & 0 \\
0 & u''(b^R) w & -v'(R) \\
T/(1 - \pi) & 0 & 1
\end{pmatrix},
\]

\(x\) the vector

\[
\begin{pmatrix}
\frac{d\tau}{d\pi} \\
\frac{db^R}{d\pi} \\
\frac{dR}{d\pi}
\end{pmatrix},
\]

and \(b\) is the vector

\[
\begin{pmatrix}
0 \\
0 \\
(T(1 - \tau))/(1 - \pi)^2
\end{pmatrix}.
\]

Applying Cramer’s rule, we have

\[
\frac{d\tau}{d\pi} = \frac{1}{|M|} \left[ -v'(R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] \leq 0
\]

\[
\frac{db^R}{d\pi} = \frac{w}{|M|} \left[ v'(R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] \geq 0
\]

\[
\frac{dR}{d\pi} = \frac{1}{|M|} \left[ w^2 u''(b^R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] > 0.
\]
C Proof of proposition 4

Conditions (14) and (15) define two reactions curves $\tau_{mv}(R)$ and $R_{mv}(\tau)$ respectively in the $(R,\tau)$ space. A political equilibrium is characterized by the intersection of these curves.

As stated before, the optimal tax rate of the median voter is 1 when $R = T/2$. Therefore $\tau_{mv}(R)$ passes through the point $(T/2,1)$. Moreover, $\lim_{R \to T} \tau_{mv}(R) = \tau_{\text{min}}$: when $R \to T$, the only way to satisfy (14) is to have $\tau \to \tau_{\text{min}}$; otherwise, $bR \to +\infty$ and the first-order condition (14) cannot be satisfied.

We now study the reaction curve $R_{mv}(\tau)$, implicitly defined in (15). As a first step, we characterize this curve without imposing the constraint $R_{mv} \geq T/2$ (recall that the median voter’s preferred retirement age is larger than his own age). With an isoelastic utility function, the term $-u(b^R) + (T/R)b^R u'(b^R)$ in (15) becomes $u(b^R)((1-\varepsilon)(T/R) - 1)$. When $R \to T$, $b^R \to +\infty$, unless $\tau \to \tau_{\text{min}}$, which implies that $u(b^R)((1-\varepsilon)(T/R) - 1) \to \infty$.

As all other terms in the first-order condition take finite values, the only way to satisfy this condition is to have $\tau \to \tau_{\text{min}}$. Consequently, $\lim_{\tau \to \tau_{\text{min}}} R_{mv}(\tau) = T$.

Next, we construct the curve $\tau_{opt}(R)$, that corresponds to the optimal tax rate chosen at the beginning of the life cycle, for a given $R$. This curve is implicitly defined in (6). It is straightforward to show that this curve is decreasing and passes through the points $(0,1)$ and $(T,\tau_{\text{min}})$. The optimal pension policy at the beginning of the life cycle with exogenous unemployment benefits is obtained at the intersection point of $\tau_{opt}(R)$ and $R_{opt}(\tau)$, this latter curve being implicitly defined in (7). Note that $R_{opt}(\tau)$ coincides with $R_{mv}(\tau)$ when the median age individual chooses an interior retirement age. It can be shown that the optimization program at the beginning of the life cycle is convex. Therefore, the curve $R_{opt}(\tau)$ crosses $\tau_{opt}(R)$ once and only once (at the optimum). Moreover, $R_{opt}(\tau)$ can be shown to be decreasing. These two properties imply that $R_{opt}(\tau)$ crosses the vertical axis $R = T/2$ at $\tau = R_{opt}^{-1}(T/2)$. For $\tau \leq R_{opt}^{-1}(T/2)$, $R_{mv}(\tau)$ coincides with $R_{opt}(\tau)$ (and it is strictly larger than $T/2$). For $\tau \leq R_{opt}^{-1}(T/2)$, $R_{mv}(\tau) = T/2$. Reminding that $\tau_{mv}(T/2) = 1$, we have shown that $\tau = 1$ and $R = T/2$ is an equilibrium.

Other equilibria possibly exist. It depends whether the curves $\tau_{mv}(R)$ and $R_{mv}(\tau)$ cross in the region $R \in (T/2,T)$. Numerical examples show that both cases are possible: for some parameters values, interior equilibria do exist whereas they don’t for other values.
D Proof of proposition 5

When $R^{\text{opt}} < T/2$, it is obviously lower than $R^e$, this latter being always greater than $T/2$. We now study the case $R^{\text{opt}} > T/2$. As emphasized in the proof of proposition 4, the curves $R^{\text{opt}}(\tau)$ and $R^{\text{mv}}(\tau)$ coincide when $R \in (T/2, T)$. On the other hand, both $\tau^{\text{opt}}(R)$ and $\tau^{\text{mv}}(R)$ converge to $\tau_{\min}$ when $R \to T$. Moreover, $\tau^{\text{mv}}(R)$ is everywhere larger than $\tau^{\text{opt}}(R)$. This implies that the intersection point of $R^{\text{opt}}(\tau)$ and $\tau^{\text{opt}}(R)$ (that corresponds to the optimal policy) is necessarily to the right of the intersection point of $R^{\text{mv}}(\tau)$ and $\tau^{\text{mv}}(R)$ (this point corresponding to the political equilibrium).
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