Inefficient public provision in a repeated elections model

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INEFFICIENT PUBLIC PROVISION IN A REPEATED ELECTIONS MODEL

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Abstract
We consider a dynamic setting with no policy commitment. Two parties that compete for election must choose the level of provision of a public good as well as the tax payment needed to finance it. The cost of producing the good may be high or low and this information is not known to the voters. We show that there exists an equilibrium in which the party that does not want much of the public good uses the inefficient (high cost) technology even though the efficient one is available. Using the low cost technology would, by informing the voters about the cost parameter, force it to produce an excessively high level of the good in the future. Interestingly, this equilibrium is not symmetric, suggesting that a party with a strong taste for the public good is less likely to adopt a wasteful policy.

1. Introduction
The central question addressed in this paper is whether democracies produce efficient policies.

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According to the Chicago school of political economy, which ideas are summarized in Wittman (1989), the answer to this question is in the affirmative. The argument is simple and powerful: a politician adopting an inefficient policy would be voted out of office and replaced by a challenger.

While this argument is compelling, it misses an important point: the voters may not be perfectly informed about some characteristics of the policy and/or of the politicians. This view is associated with the Virginia school of political economy. Coate and Morris (1995) develop in an important paper a model along these lines. They show that a “bad” incumbent politician may adopt an inefficient policy because it consists in a disguised transfer to a special interest. Hidden transfers to special interest groups, although inefficient, are preferable for politicians who care about their reputation; a direct cash transfer to the special interest would indeed inform the citizens about the type of the incumbent politician.

Another issue that has been neglected until recently is the issue of policy commitment, as emphasized by Acemoglu (2003). When politicians are not able to commit to future policies, they may have some incentives to adopt inefficient policies today.

The conclusion that economic choices may be inefficient is in line with recent empirical evidence that points to the fact that “Governments differ dramatically in quality” (Treisman 2002).¹ So there is a widespread agreement that democratic countries may fail in providing efficiently some goods to the citizens. Investigating the causes of such failures and determining the circumstances in which they will arise remains an important issue. Our paper contributes to a growing theoretical literature which aim is to answering these questions.

We propose a model characterized both by asymmetric information between the politicians and the voters and absence of policy commitment. The economy is the same as in Schultz (1996). A public good is produced at a constant marginal cost that can be high or low and is only observed by the politician in power (not by the voters). The provision of this good is financed with a uniform tax. The voters/citizens differ only according to their valuation for the public good.

Schultz considers a standard static electoral competition model in which two parties compete for election (based on the majority rule) and are able to commit to the policy announced during the campaign: if elected, they implement the platform announced during the campaign. Those parties are assumed to be policy motivated. They care about winning office but also about which policy is implemented.

¹Quality is a broader concept than efficiency, it may include aspects such as corruption, the degree of inequality or political freedom. Treisman adopts a narrow view of quality, defined as “the extent to which the government provides public goods and services that the public demands at minimum cost in taxation and regulatory burden” which corresponds to the efficiency criterion used in this paper.
He obtains that the electoral equilibrium may be pooling: the parties' platforms, that consist of quantities of the public good, are the same whether the cost is low or high. The basic intuition is that parties are unable to credibly convey the information to the voters. At equilibrium, the two parties propose the ideal policy of the (uninformed) median voter and are elected with probability $1/2$. Because a small deviation from the equilibrium does not modify the beliefs of the voters, the deviating party loses with certainty. It follows that no party should deviate. The author deduces from this result that the equilibrium is \textit{ex ante} inefficient: the expected utility of all individuals could be increased before the true value of the cost is known to the voters. However, the equilibrium is efficient \textit{ex post}, in the sense that the elected party chooses a value of the tax rate that makes the budget constraint binding at the true cost value.

We depart from Schultz by considering a repeated elections model, based on Duggan (2000) and Banks and Duggan (2006), in the tradition of political agency models developed by Barro (1973) and Ferejohn (1986). In this model, elections are \textit{infinitely repeated} and there is no political campaigning at all, so that \textit{there is no possible policy commitment}. In each period, a party has to choose a policy. At the beginning of the following period, this incumbent competes in an election against the challenger. The elected party is the one that receives the most votes (majority rule). The voters' strategy is assumed to be \textit{retrospective}: they vote for the incumbent if and only if the utility generated by the last policy choice meets a given utility level.

We consider two parties with policy preferences, these preferences being known to the voters. Party $A$ is not favorable to a large provision of the public good whereas party $B$ has a strong taste for it. The only source of asymmetric information is thus the marginal cost of production of the public good.\footnote{This is a notable difference with the work of Coate and Morris in which the voters are imperfectly informed about both some characteristics of the policy and of the politicians. These authors argue that such a double uncertainty is necessary to generate some political inefficiency. Our study clearly contradicts their claim.}

We show that, as soon as the preferences of the parties are sufficiently polarized and the discount factor is high enough, there exists a perfect Bayesian equilibrium in which $A$ adopts a pooling strategy: he implements the same policy whether the cost is high or low. In other words, $A$ offers the same quantity of public good at the same tax level, whatever the cost. When the cost is actually low, such a policy entails a waste of resources: more of the good could be produced at the same tax level. The underlying idea is that if producing efficiently, party $A$ would inform the voters that the cost is low. It would then be induced to produce, in all future periods, a quantity of the good which is too high from its point of view. As soon as this party prefers to remain in power, it should follow this inefficient pooling strategy. In some sense, this
mechanism is thus a political ratchet effect.\textsuperscript{3} We obtain the interesting additional result that this equilibrium is not symmetric: party B should not pretend that the cost is high when it is low. This follows from the fact that when the cost is low, party B benefits from a higher production at a lower cost. These results suggest that parties with a strong taste for the public good are less likely to adopt a wasteful behavior than others.

1.1. Related Literature

Schultz (2002) also considers a setting of asymmetric information coupled with the absence of policy commitment. However his main concern is different from ours: he argues that the nature of asymmetric information (whether related to the parties’ preferences or to the state of the economy) determines the political equilibrium and the departure from the median voter’s preferred policy. We rather focus on the question of inefficiency itself, as determined jointly by asymmetric information and the absence of commitment.

There are surprisingly few papers that address the question of inefficient policy-making. Apart from the above cited work by Coate and Morris (1995) and Acemoglu (2003), the following papers are relevant for our purpose. In Acemoglu and Robinson (2001), the political power of a group depends on its size. Adopting an inefficient policy (like a price distortion) can then be an effective way of expanding the size of this group in order to guarantee its future political power. The typical example of such a mechanism can be found in the distortive price support to farmers. Robinson and Verdier (2002) argue that employment in the public sector is an inefficient way of redistributing income. However, it is politically attractive as it is credible (income transfers are not), excludable (public goods are not) and reversible (public investments are not). The articles by Besley and Coate (1998) and Robinson and Torvik (2005a, b) are more closely related to our analysis. Both emphasize that the inability for the politicians to commit about future policies can be a source of inefficiency. In Besley and Coate, the incumbent politician may refrain from implementing a potentially Pareto improving public project because it modifies the preferences of the voters and thus distorts the political equilibrium in the next period. The argument developed by Robinson and Torvik (2005b) is that the implementation of an inefficient project ties the voters to the incumbent politician. If the former want to benefit from this project, they have to re-elect the incumbent as the challenger would not refinance it. Therefore implementing this project, even if economically inefficient, is politically attractive as it increases the re-election probability of the incumbent. It is worth noticing that in this study the absence of commitment is sufficient

\textsuperscript{3}This term was first used to designate the tendency of firms in a centrally planned economy to underproduce in order to avoid demanding schemes in the future (see Freixas, Guesnerie, and Tirole 1985).
to generate a political inefficiency. This is due to the fact that the policy considered has long lasting effects; if the voters want to enjoy the benefits from this policy in the future they have to re-elect the incumbent. In our model, the policy considered lasts only one period. We show that in this framework, the absence of policy commitment is not sufficient for the politicians to behave inefficiently and it must be coupled with asymmetric information.

To summarize, there are two strands of the literature on political inefficiencies, both building on dynamic models with no policy commitment. In the first one, the inefficiency comes from the signaling property of the political action. The paper by Coate and Morris (1995) and ours belong to this class. In the other strand of the literature, that encompasses the papers by Besley and Coate (1998), Acemoglu and Robinson (2001), Robinson and Verdier (2002), and Robinson and Torvik (2005a, b), asymmetric information is not the reason why elected representatives choose inefficient policies. The explanation lies instead in the fact that the political action today has impact on the political equilibrium tomorrow, this impact not being channeled by asymmetric information.

We are not aware of papers developing theories of inefficient behavior by elected politicians, that consider a static framework and/or policy commitment. We conjecture that (i) dynamics and (ii) absence of commitment are both necessary to explain such a phenomenon. A formal demonstration of this conjecture is however outside the scope of this paper.

The model is presented in the next section. Sections 3 and 4 are devoted to the analysis of equilibria.

2. The Model

2.1. Description of the Economy

The economic problem is exactly the same as in Schultz (1996). A quantity $x$ of a public good is produced.\footnote{As it should become clear in the following of the section, the public good nature of government provision is inessential for the derivation of the results. What matters is that this provision be uniform across all citizens.} The cost of production of $x$ units of the good is $c x$ where $c$ can take on one of two values: $c^l$ or $c^h$ with $0 < c^l < c^h$. The government finances the provision of the public good through a uniform tax $\tau$.

The citizens are identical in all respects but their valuation for the public good, $\theta$. The preferences of a type $\theta$ individual are described by the following function:

$$v(x, \tau; \theta) = \theta u(x) - \tau,$$

where $u$ is increasing and strictly concave; $\theta$ is distributed on $[\theta_\bar{\theta}, \bar{\theta}]$ according to a density function $f$, the median of this distribution being denoted $\theta_m$.
We impose that the budget be balanced each period (no debt is allowed). Normalizing total population to 1, the government budget constraint is
\[
\begin{align*}
  cx & \leq \int_{\theta_m}^{\theta_m} \tau f(\theta) d\theta \\
  \iff cx & \leq \tau.
\end{align*}
\]
When \(cx < \tau\), there is obviously a waste of resources: with the tax receipts \(\tau\), a quantity of the good higher than \(cx\) could be provided to the citizens.

The optimal policy of individual \(j\) in state \(s \in \{l, h\}\) will be denoted \((x^s_j, \tau^s_j)\). This is given by the conditions
\[
\begin{align*}
  \theta_j u'(x^s_j) &= c^s \quad (2) \\
  \tau^s_j &= c^s x^s_j. \quad (3)
\end{align*}
\]
Equation (2) comes from the first-order conditions on \(x\) and \(\tau\). Equation (3) means that individual \(j\)'s optimal policy requires a binding budget constraint. Differentiating these equations, we obtain
\[
\frac{dx^s_j}{dc^s} = \frac{1}{\theta_j u''(x^s_j)} < 0 \quad (4)
\]
and
\[
\begin{align*}
  \frac{d\tau^s_j}{dc^s} &= x^s_j + c^s \frac{dx^s_j}{dc^s} \\
  &= x^s_j + \theta_j u'(x^s_j) \frac{1}{\theta_j u''(x^s_j)} \\
  &< 0 \text{ iff } E(x^s_j) = -x^s_j \frac{u''(x^s_j)}{u'(x^s_j)} < 1.
\end{align*}
\]
We will assume in the following that \(E(x)\), the elasticity of the marginal utility of consumption, is always lower than 1. This implies that the optimal tax level of any individual increases when the marginal cost decreases. We further make the simplifying assumption that this coefficient is constant, considering an isoelastic utility function:
\[
u(x) = \frac{x^{1-\varepsilon}}{1-\varepsilon},
\]
where \(E(x) = \varepsilon \leq 1\).

2.2. The Political Game

There are two parties/politicians \(A\) and \(B\) with policy preferences \(\theta_A\) and \(\theta_B\) such that \(\theta_A < \theta_m < \theta_B\). Party \(A\) represents individuals with a moderate taste for
the public good whereas party B’s constituency favors a high provision of this
good. The players of this game are thus the two parties and the voters. There
is asymmetric information among these players. The marginal cost $c$ is constant
across periods.\footnote{Our results and the main messages of the model do not change if we assume that the cost is variable but there is some persistence across periods. However, we make the assumption that $c$ is constant for simplicity.} It is revealed to a party as soon as it takes office\footnote{In the equilibria we consider, the party initially in power remains in power forever. Therefore, the challenger never observes the true value of the marginal cost. Observe that our results would remain valid if we had assumed that the challenger was informed about the value of the marginal cost but was not able to communicate it to the voters. The case in which the challenger is informed and communication with the electorate is possible is more complicated and lies outside the scope of this paper.} whereas voters are initially uninformed. Their prior belief is that the cost is high with probability $\mu_0$. They may or may not learn the true value of the cost later on, depending on the equilibrium path of play.

2.2.1. Timing

Building on Duggan (2000) and Banks and Duggan (2006), we study a repeated
elections model with infinite horizon. At the beginning of each period $t$, an election
takes place in which the voters decide whether to re-elect the incumbent party
or to appoint the challenger. The elected politician is the one who receives
most votes (majority rule).\footnote{We assume that when the two parties receive the same number of votes, the incumbent politician is re-elected for sure. In the equilibria described below, the incumbent and the challenger will always be tying. This comes however from our assumption that there is a continuum of types. With a finite number of types, such a case would occur only when the proportion of individuals having a valuation for the public good lower or equal to the median is exactly 1/2.} He then chooses a policy for the current period $(x_t, \tau_t)$. In the following period $t + 1$, the same sequence of events occurs again.

The history at date $t$, $h_t$, describes the publicly observed events in the first
$t$ periods, namely the party in power and the policy chosen in each period.

One important feature of this model is that there is no campaign stage
and thus no policy announcements. The parties are not able to commit before
the election to the policy they would implement if elected. This contrasts with
most of the election models developed insofar, that belongs to the Downsian
tradition. The model we build upon (Schultz 1996) is not an exception.

Another important difference between Schultz and our analysis is that
in our study the policy space is composed of both the public good level and
the tax needed to finance it. In Schultz, electoral competition only takes
place on the quantity of the public good provided by the state; the tax rate
is adjusted ex post (after the election) so as to satisfy the government budget
constraint. In such a setting, the public good allocation is necessarily (ex post)
Pareto efficient. What would happen if one were to consider a model with commitment, as in Schultz, but with both the public good level and the tax rate announced during the campaign stage? We argue that the outcome would also be Pareto efficient. The argument is straightforward. Suppose that both parties propose wasteful policies, that is they pretend that the cost is high when it is in fact low. Clearly the two parties should deviate towards a policy that is feasible only in the low cost state and that makes half the total population better off. Doing so, their probability of being elected would jump from 0 to 1. As a consequence the no commitment assumption is necessary to obtain our inefficiency result. We will see later that it is not sufficient alone and that it must coupled with asymmetric information for that result to hold (see Proposition 1).

2.2.2. Payoffs

The voters: Type \( \theta \) voter’s payoff from the sequence \( \{x_t, \tau_t\} \) of policy outcomes is the discounted sum of per period utility levels:

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} v(x_t, \tau_t; \theta),
\]

where \( \delta < 1 \) is the time discount factor.

The parties: Party \( i \)'s (\( i = \{A, B\} \)) payoff from the sequence \( \{x_t, \tau_t\} \) of outcomes is:

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} [v(x_t, \tau_t; \theta) + \beta \omega_i(I_t)],
\]

where \( \omega_i(I_t) \) is the indicator function taking the value 1 if and only if \( I_t = i \), \( i = A, B \). The parties have therefore some preferences on the policy implemented in each period, but they also value the fact of holding office, deriving a utility level \( \beta \) in such a case.

\( ^8 \)In Schultz’s paper as well as in ours, the efficiency criterion is constrained Pareto optimality, i.e., given the instruments available to the policymaker. In particular, we do not consider side transfers between the players. This means that a public good allocation will not, except in the particular case where it corresponds to the optimal choice of the mean valuation type, satisfy the Bowen–Lindahl–Samuelson rule. In order to be optimal, it must simply lie on the boundary of the budget set. Any allocation in the interior of the budget set entails a waste of resources and is thus Pareto dominated.

\( ^9 \)We have established that the commitment assumption in the static game generates an efficient policy choice. It is still possible that an inefficient choice emerges in a repeated elections context in which at each period the two parties compete for election and are able to commit to the policy implemented if elected (see Duggan and Fey 2006 for a formal treatment of this game in the complete information case).

\( ^{10} \)The reader may wonder if an inefficient policy could arise as an equilibrium of a static game without commitment. It is easily shown that it is not possible: once elected, the politician in power has no incentive to waste resources, he simply chooses the optimal policy from his point of view.
2.2.3. Strategies

The parties: A (pure) strategy of party $i$ specifies the policy chosen $p_{i,t} = (x_{i,t}, \tau_{i,t})$ if elected in period $t$. It is a function of history at date $t-1$, $h_{t-1}$, and of the state of the world $s \in \{l, h\}$.

The voters: A (pure) strategy of voter $j$ specifies, for every possible history at date $t-1$, $h_{t-1}$, the action chosen in period $t$, $a_{j,t} \in \{I, C\}$, where $a_{j,t} = I$ (resp. $C$) means that this individual votes for the incumbent (resp. challenger). We consider voting strategies that are retrospective: each voter decides to vote for the incumbent if and only if the utility generated by its last policy choice is at least equal to a given threshold level $\tilde{V}$. Formally,

$$a_{j,t} = I \text{ iff } v(x_{t-1}, \tau_{t-1}; \theta_j) \geq \tilde{V}_{j,t}. \quad (5)$$

In the equilibria studied in the following sections, $\tilde{V}$ is assumed to be equal to the expected continuation value of electing the challenger.

2.2.4. Beliefs

The party in power is perfectly informed about the state of the world. Initial beliefs of the voters are given by $\mu_0$ (probability that the cost is high). These beliefs are updated at the end of each period following the policy choice of the incumbent politician: $\mu_t = \mu(h_{t-1})$. As usual, these beliefs are updated according to Bayes rule whenever possible, that is following an equilibrium play.

Out of equilibrium beliefs are assumed to be the following. When the voters observe a policy choice on or below the high cost frontier, that is a choice which is feasible in both states of the world, they do not modify their beliefs. Conversely, a policy choice outside this frontier is necessarily informative because it is only feasible in the low cost state. Whenever they observe such a play, the voters conclude that the cost of production of the public good is low.

3. Political Equilibria

As one can guess, there are a lot of possible equilibria in this repeated game setting. We will restrict our attention to strategies that are stationary, except possibly with respect to the beliefs of the voters. In other words we will consider Markov perfect equilibria with the beliefs of the voters as the state variable. This means that the re-election rule of the voters and the policy choice of the incumbent will remain unchanged as soon as the beliefs are not modified. Additionally, we will consider equilibria such that the party initially in power remains in office forever and always chooses the same policy $(x^e_i, \tau^e_i)$ on the equilibrium path. In these equilibria, called Equilibria with Policy Persistence (EPP), the following lemma holds.
LEMMA 1: Median decisiveness

In any EPP, the incumbent politician is re-elected if and only if the median type individual votes for him.

Proof: See Appendix.

The median type individual will be called the median voter in the remainder of the paper. This lemma is very useful as it allows us, on the voters’ side, to consider only the strategy of the median voter. In other words, the game can be reduced to a game between three players: party A, party B and the median voter.

3.1. Complete Information

Before presenting equilibria with asymmetric information, we show that the policy chosen under complete information is efficient.

PROPOSITION 1: Assume that the voters are perfectly informed about the value of the marginal cost. Then the policy choice in any given period is Pareto-optimal.

Proof: Suppose that in a given period, the incumbent chooses a Pareto dominated policy. If this policy does not satisfy the re-election rule of the median voter, the incumbent can still implement a policy that yields a higher utility for himself whether it leads to re-election or not. If the policy considered satisfies the re-election threshold, there exist policies that still meet this threshold and generate a higher utility for the incumbent.

This proposition implies that asymmetric information is a necessary condition in our setting to have some political inefficiencies. This contrasts with papers such as Robinson and Torvik (2005a, b) in which the absence of commitment alone is sufficient for such a result to hold. As explained in the introduction this is due to the long lasting nature of the policy they consider: in their model it takes two periods for a project to yield some benefits. If the voters want to enjoy these benefits they have to re-elect the incumbent politician. The mechanism we identify is quite different. It relies on the interaction between the absence of policy commitment and asymmetric information.

3.2. Inefficient Equilibrium

We now turn to the main result of the paper, that is the possibility of obtaining an equilibrium in which a politician chooses a Pareto dominated policy.
3.2.1. Intuitive Description of the Equilibrium

Before formally demonstrating it, we provide the reader with an intuitive explanation of our main result. The equilibrium we identify works as follows.

Initially, party A is in power. It adopts a pooling strategy, which implies that the policy chosen is not informative to the voters, it is the same whether the cost is high or low.

The (uninformed) median voter is indifferent between this policy and the one that B would implement if elected. This latter policy is separating (and hence would reveal the true value of the cost to the voters) and consists in B proposing its optimal policy in both states of the world. The median voter being indifferent, he chooses to re-elect the incumbent politician following our assumptions on the voting behavior. It is worth emphasizing that he has no incentive whatsoever to experiment and elect the challenger, even though he would learn the true value of the cost by electing party B: his expected utility is the same whether he votes for the incumbent or the challenger.

In the second period, the party in office is A. Because its first period policy choice was not informative, the beliefs of the voters are unchanged. As we consider Markov strategies with beliefs as the state variable, the policy choice by A in the second period remains the same and the argument developed previously still applies. The process then repeats forever: A always remains in power and adopts the same policy.

We have argued that the strategy of the median voter is optimal. We also have to check that the strategies of the parties are optimal. Under this strategy, A is re-elected in all periods. Is it optimal for A? Could not it prefer to deviate to a preferred policy, even though this entails foregoing the benefits of re-election? We consider a discount factor sufficiently large so that this is not the case, that is such that A wants primarily to be re-elected. Then the question is why A shouldn’t want to deviate in the low cost state and propose a more efficient policy. The answer is simply that it would reveal to the voters that the cost is low. These latter would then require that A provides them with a utility level as least as high as the one proposed by B in the low cost state. We give conditions on the preferences of the parties such that A is not willing to do so: A prefers to pretend that the cost is high and implement a policy which is more in line with its preferences.

So far, we have thus shown informally that a situation in which party A plays a pooling strategy, and thus wastes resources in the low cost state, can be an equilibrium. The last thing we need to check is whether this equilibrium is subgame perfect. In other words, we have to make sure that the strategies of the parties and the voters are optimal on paths out the equilibrium. In this purpose, we have to verify first that B should not deviate if elected and

\[\beta\] These benefits are twofold. First, they consist in the exogenous benefit $\beta$ of being in office. Second, being in office allows the party to choose the policy. If the party were to abandon power, it would have to leave this choice to the challenger and would have to bear a less preferred policy.
second that A’s strategy would be optimal if this party were to be re-elected after B, that is with the voters being perfectly informed (recall that B’s strategy is revealing).

The important point is that A’s strategy is modified when the voters become informed about the cost. In such a case, A proposes the best possible policy from its point of view under the constraint that it makes the median voter indifferent between this policy and the one proposed by B in each state of the world. This policy is separating and it ensures re-election for A in both states of the world. It is thus optimal as soon as the discount factor is sufficiently large, which implies that A values re-election a lot. The last question is whether B’s strategy is optimal. Observing that B’s policy guarantees re-election (the median voter is indifferent between A and B’s policies) and moreover that it corresponds to the optimal unconstrained choice for B, it is obvious that B should not deviate.

3.2.2. Formal Argument

Let \((\tilde{x}_A, \tilde{\tau}_A)\) be the policy such that
\[
\theta_m u(\tilde{x}_A) - \tilde{\tau}_A = \mu_0 v(x^h_B, \tau^h_B; \theta_m) + (1 - \mu_0) v(x^l_B, \tau^l_B; \theta_m)
\]
and \((\tilde{x}^s_A, \tilde{\tau}^s_A), s = h, l,\) the policies such that
\[
v(x^s_B, \tau^s_B; \theta_m) = v(\tilde{x}^s_A, \tilde{\tau}^s_A; \theta_m)
\]
\[
\tilde{\tau}^s_A = c^s \tilde{x}^s_A.
\]

It should be noted that these policies do not exist for all possible values of the parameters of the model, \(\theta_m, \theta_B, c^h, c^l, \) and \(\mu_0). However, one can always find some values for these parameters such that the policies defined above exist. In particular, for \((\tilde{x}_A, \tilde{\tau}_A)\) to exist, \(\mu_0,\) the initial probability that the cost is high, must be large enough.

PROPOSITION 2: Provided that the following necessary and sufficient conditions are satisfied,
\[
v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) \geq v(\tilde{x}^l_A, \tilde{\tau}^l_A; \theta_A)
\]
\[
\frac{\partial x}{\partial \tau} \bigg|_{\tau = v(\tilde{x}^h_A, \tilde{\tau}^h_A; \theta_A)} \geq c^h
\]
and
\[
\delta \geq \max \left\{ \frac{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(x^l_A, \tau^l_A; \theta_A) + \beta}, \frac{v(x^l_A, \tau^l_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^l_A, \tau^l_A; \theta_A) - v(x^h_A, \tau^h_A; \theta_A) + \beta} \right\},
\]
the strategies below constitute a perfect Bayesian equilibrium:
If \( \mu_t = \mu_0 \),

\[
p_{c,t}^e(c^h) = p_{A,t}^e(c^l) = (\tilde{x}_A, \tilde{\tau}_A)
\]

\[
p_{c,t}^e(c^h) = (x_B^h, \tau_B^h); p_{A,t}^e(c^l) = (x_B^l, \tau_B^l)
\]

\[
\tilde{V}_t = \mu_0 v(x_B^h, \tau_B^h; \theta_m) + (1 - \mu_0) v(x_B^l, \tau_B^l; \theta_m) = v(\tilde{x}_A, \tilde{\tau}_A; \theta_m).
\]

If \( \mu_t \neq \mu_0 \),

\[
p_{A,t}^e(c^h) = (\tilde{x}_A, \tilde{\tau}_A); p_{A,t}^e(c^l) = (\tilde{x}_A, \tilde{\tau}_A)
\]

\[
p_{B,t}^e(c^h) = (x_B^h, \tau_B^h); p_{B,t}^e(c^l) = (x_B^l, \tau_B^l)
\]

\[
\tilde{V}_t = \mu_t v(x_B^h, \tau_B^h; \theta_m) + (1 - \mu_t) v(x_B^l, \tau_B^l; \theta_m)
\]

\[
= \mu_t v(\tilde{x}_A, \tilde{\tau}_A; \theta_m) + (1 - \mu_t) v(\tilde{x}_A, \tilde{\tau}_A; \theta_m).
\]

**Proof:** Suppose first that \( A \) is in power at the beginning of period \( t \).

(1) The beliefs of the voters are the initial beliefs.

The equilibrium play of \( A \) is pooling at \((\tilde{x}_A, \tilde{\tau}_A)\). The re-election rule of the (uninformed) median voter, \( \tilde{V} = v(\tilde{x}_A, \tilde{\tau}_A; \theta_m) \), is satisfied so that \( A \) is re-elected. The conditions ensuring that \( A \) does not want to give up re-election in states \( h \) and \( l \), respectively, are

\[
v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) + \beta \geq (1 - \delta) \left[ v(\tilde{x}_A^h, \tilde{\tau}_A^h; \theta_A) + \beta \right] + \delta v(\tilde{x}_B^h, \tilde{\tau}_B^h; \theta_A)
\]

\[
\Leftrightarrow \delta \geq \frac{v(\tilde{x}_A^h, \tilde{\tau}_A^h; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(\tilde{x}_A^h, \tilde{\tau}_A^h; \theta_A) - v(\tilde{x}_B^h, \tilde{\tau}_B^h; \theta_A) + \beta} \quad (10)
\]

and

\[
v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) + \beta \geq (1 - \delta) \left[ v(\tilde{x}_A^l, \tilde{\tau}_A^l; \theta_A) + \beta \right] + \delta v(\tilde{x}_B^l, \tilde{\tau}_B^l; \theta_A)
\]

\[
\Leftrightarrow \delta \geq \frac{v(\tilde{x}_A^l, \tilde{\tau}_A^l; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(\tilde{x}_A^l, \tilde{\tau}_A^l; \theta_A) - v(\tilde{x}_B^l, \tilde{\tau}_B^l; \theta_A) + \beta}. \quad (11)
\]

A deviation by \( A \) on or below the high cost frontier is not informative to the voters (following our specification of out-of-equilibrium beliefs). Therefore, if \( A \) deviates “to the left” (lower \( \tau \)), he is not re-elected anymore and accordingly obtains a lower payoff. If he deviates “to the right”, he is still re-elected but with a less desirable policy if

\[
\partial x/\partial \tau|_{v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)} \geq c^h.
\]

One can easily verify that this condition is implied by (8), so that this deviation is not profitable.

A deviation above the high cost frontier (when feasible) informs the voters that the cost is low. To be re-elected, \( A \) must provide the median voter with a utility level at least as high as \( \tilde{V} = v(x_B^l, \tau_B^l; \theta_m) = v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) \).
When condition (7) is satisfied, \( A \) does not want to make such a deviation.

(2) When the voters are informed, the pooling strategy is not optimal: \( A \) should deviate in both states of the world. In state \( l \), he is not re-elected if he sticks to the pooling strategy. In state \( h \), he is re-elected but could obtain a higher utility level.

The separating strategy described in the proposition is optimal. Whether the cost is low or high, party \( A \) is voted out of office if deviating “to the left” whereas, under condition (8), a deviation “to the right” allows party \( A \) to be re-elected but yields a lower utility level. Conditions ensuring that \( A \) prefers to be re-elected are

\[
\delta \geq \frac{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}^h_A, \tilde{\tau}^h_A; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}^h_A, \tilde{\tau}^h_A; \theta_A) + \beta}
\]

and

\[
\delta \geq \frac{v(x^l_A, \tau^l_A; \theta_A) - v(\tilde{x}^l_A, \tilde{\tau}^l_A; \theta_A)}{v(x^l_A, \tau^l_A; \theta_A) - v(\tilde{x}^l_A, \tilde{\tau}^l_A; \theta_A) + \beta}.
\]

From (8), \( v(\tilde{x}^h_A, \tilde{\tau}^h_A; \theta_A) > v(x^h_A, \tilde{\tau}^h_A; \theta_A) \). Therefore (10) implies (12). Moreover from (7), (13) implies (11). Consequently, the conditions ensuring that \( A \) always prefers to be re-elected are those in (9).

Suppose now that \( B \) is in power at the beginning of period \( t \). The strategy described in the proposition is clearly optimal as \( B \) obtains his optimal policy in both states of the world and is always re-elected.

We finally have to show that condition (7) is possible. The following lemma, proved in the Appendix, will be useful:

**LEMMA 2:** For a coefficient of relative risk aversion \( \varepsilon \) sufficiently close to 1, \( \tilde{\tau}^A_m \) is decreasing with \( c^s \).

Let us denote \( \tilde{A}^h \) (resp. \( \tilde{A}^l \) and \( \tilde{A} \)) the point \((\tilde{x}^h_A, \tilde{\tau}^h_A)\) (resp. \((\tilde{x}^l_A, \tilde{\tau}^l_A) \) and \((\tilde{x}_A, \tilde{\tau}_A)\)). Let us also denote \( I^h_m \) (resp. \( I^l_m \) and \( I_m \)) the indifference curve for \( m \) through \( \tilde{A}^h \) (resp. \( \tilde{A}^l \) and \( \tilde{A} \)).

Lemma 2 implies that \( \tilde{A}^l \) lies to the right of \( \tilde{A}^h \). Therefore \( I^l_m \) lies above \( I^h_m \). By construction, \( I_m \) lies between \( I^h_m \) and \( I^l_m \) so that \( \tilde{A} \) must be to the right of \( \tilde{A}^h \). Lemma 2 also implies that it is possible that \( \tilde{\tau}^A_m < \tilde{\tau}^A \). As indifference curves are increasing, this is a necessary condition for the indifference curve for \( A \) through \( \tilde{A} \) to be above the indifference curve for \( A \) through \( \tilde{A}^l \), which is equivalent to condition (7). This condition is more likely to hold the steeper the indifference curves for \( A \) (i.e., the smaller \( \theta_A \)) and the bigger \( \mu_0 \).

This equilibrium is represented on Figure 1.

The steeper indifference curve passing through \((\tilde{x}_A, \tilde{\tau}_A)\) is an indifference curve of politician \( A \). The indifference curves tangent to the budget frontier
at \((x_B^h, \tau_B^h)\) and \((x_B^l, \tau_B^l)\) are indifference curves of politician \(B\). The other indifference curves represented on this graph are those of the median voter.

The strategy of party \(B\) consists in proposing his ideal policy \((x_B^i, \tau_B^i)\) in state \(s = h, l\). It is separating. Consequently, if \(B\) is in power at some point in time, voters learn the value of the cost. This is not true for \(A\) whose policy choice depends on the voters’ beliefs. When these latter are not informed (they hold the initial beliefs), \(A\) adopts a pooling strategy: he proposes the same policy \((\tilde{x}_A, \tilde{\tau}_A)\) in both states of the world. This implies that the policy choice is inefficient when the cost of production is low as more of the public good could be produced with the same tax receipts.

The logic underlying this result is that if \(A\) adopts an efficient policy when the cost is low, he reveals this information to the voters. In such a case, the strategy of \(B\) implies that he would choose the policy \((x_B^h, \tau_B^h)\) if elected. In order to give the median voter a utility level at least equal to \(v(x_B^l, \tau_B^l; \theta_m)\), \(A\) must select a policy “to the right” of \((\tilde{x}_A, \tilde{\tau}_A)\). However, under condition (7) he is not willing to do so. In other words, even though the good is produced more efficiently, the increase in production that \(A\) must ensure in order to be re-elected is excessively high given his preferences for the good. Condition (9) precisely guarantees that \(A\) wants to be re-elected. It should be noted that this may be the case even when the benefit of holding office is 0. The explanation is straightforward: if \(A\) is voted out of office, he will never be re-elected again. Therefore the policy choice will be the optimal policy of \(B\) forever. Even though there is a short run gain for \(A\) due to the implementation of his ideal policy, there is a long run opportunity cost. When the preference for the present is not too strong, \(A\) prefers to remain in office.
We have just argued that A does not want to deviate to a more efficient policy in the low cost state and prefers to stick to the pooling strategy (as long as the voters are uninformed). For this strategy to be an equilibrium, it must be also the case that there are no other pooling strategies that make A better off. Two possible deviations on the high cost budget frontier should be considered. On the one hand, A could deviate to the left and forego the benefits of re-election. Under (9), he does not want to do so. On the other hand, A could deviate to the right and still be re-elected. However, condition (8) tells us that A is worse-off after this deviation.

What about subgame perfection? Should B be in power at any point in time, it is re-elected (the median voter is indifferent between A and B’s policies) and adopts its most preferred unconstrained policy \((x^B_s, \tau^B_s)\) in state \(s\). It thus should obviously not deviate. Should A be in power with the voters informed, it is also re-elected with a policy, \((x^A_s, \tau^A_s)\) in state \(s\), that leaves the median voter indifferent between voting for A and B. As soon as A wants to be re-elected, which is ensured by (9), it has no better option than this.

We have thus shown that politician A who does not want much of the public good may adopt a Pareto dominated policy at the political equilibrium. Is the converse true? Is it also possible that the party B displaying a high taste for the public good also adopts an inefficient behavior? Section 4 addresses this question. Before moving to this section, we consider in the next subsection other possible equilibria.

### 3.3. Other Equilibria

As one can imagine, there is large number of equilibria in this infinitely repeated game and we will not attempt to describe all of them. We may however be interested in knowing if there exist equilibria in which both parties adopt a revealing strategy, implying an efficient level of the publicly provided good. If we consider equilibria in which B proposed his optimal policy, as in Proposition 2, the answer is no under the conditions stated in this proposition. If A were to play a separating strategy, the policy adopted by A is state \(s\) should necessarily be \((\tilde{x}^A_s, \tilde{\tau}^A_s)\). Otherwise, it could not be re-elected (and, by condition (9), we assume a discount factor sufficiently large such that A wants to be re-elected). But then, condition (7) implies that A should deviate in the low cost state: he should pretend that the cost is high whenever it is low.

Therefore, the **equilibrium in which party B proposes its optimal policy in both states of the world and A plays a separating strategy does not exist under the conditions stated in Proposition 2.** One could however envisage situations in which B plays a different strategy. For example one could consider an equilibrium in which A proposes its optimal policy contingent on the state of the world and B plays a separating strategy such that the median voter is indifferent between A and B’s policies in both states of the world. One can prove that such an equilibrium exists as soon as the valuation for the public good of
party $B$ is sufficiently high enough compared to the valuation of the median voter.

Therefore a fully revealing (and thus efficient) equilibrium exists when the preferences of $B$ and the median voter are sufficiently far apart. What happens if the preferences of the median voter and politician $B$ are very aligned? When the valuation of $A$ is sufficiently low and the voters puts a large probability on the cost being high, the only possible equilibrium involves a pooling behavior by $A$. Under these conditions, the policy proposed by $B$ should be close to the optimal policy of the median voter (it is necessarily between the optimal policy of $B$ and the median voter). Consequently, $A$ should not deviate from the pooling strategy as doing so he would be forced to provide the preferred public good level of the median voter in the low cost state, which is excessive from his point of view.\textsuperscript{12} There thus exist circumstances in which the only possible equilibria are inefficient.

As a concluding comment, observe that there are also cases in which there is no equilibrium with policy persistence, either separating or not. This occurs when the preferences of $B$ and the median voter are close enough and the valuation of $A$ low enough. Moreover, the voters should place a non negligible probability on the cost being low. In these circumstances, if $A$ plays pooling, he is not re-elected as the median voter strictly prefers the policy proposed by $B$. On the other hand, a separating strategy by $A$ is not optimal for the reasons by now familiar that $A$ should misreport the value of the marginal cost.

4. Not All Parties Behave Inefficiently

We show in the next proposition that the equilibrium symmetric to the one in Proposition 2, in which $A$ always proposes his optimal public good level and $B$ plays pooling, does not exist.

**PROPOSITION 3:** Strategies symmetric to the ones presented in Proposition 2 do not constitute an equilibrium.

**Proof:** See Appendix.

Such an equilibrium would require that party $A$ chose his preferred policy in the two states of nature. On the other side, party $B$ would pick up a pooling policy until voters are informed about the cost. Symmetrically to

\textsuperscript{12}Note incidentally that, for the same reasons, an equilibrium in which both parties implement the optimal policy of the median voter, which would be a kind of median voter theorem result, does not exit when $A$ has a very low valuation for the public good. This latter would have an incentive to deviate from this separating strategy so as to avoid too large a provision of the public good.
what happened with Proposition 2, we would have the following equilibrium strategies:

If \( \mu_t = \mu_0 \),

\[
p_{B,t}^p(c^h) = p_{B,t}^p(c^l) = (\hat{x}_B, \hat{\tau}_B) \]
\[
p_{A,t}^p(c^h) = (x^h_A, \tau^h_A); p_{A,t}^p(c^l) = (x^l_A, \tau^l_A) \]
\[
\hat{V}_t = \mu_0 v(x^h_A, \tau^h_A; \theta_m) + (1 - \mu_0) v(x^l_A, \tau^l_A; \theta_m) = v(\hat{x}_B, \hat{\tau}_B; \theta_m). 
\]

If \( \mu_t \neq \mu_0 \),

\[
p_{B,t}^p(c^h) = (\hat{x}^h_B, \hat{\tau}^h_B); p_{B,t}^p(c^l) = (\hat{x}^l_B, \hat{\tau}^l_B) \]
\[
p_{A,t}^p(c^h) = (x^h_A, \tau^h_A); p_{A,t}^p(c^l) = (x^l_A, \tau^l_A) \]
\[
\hat{V}_t = \mu_t v(x^h_A, \tau^h_A; \theta_m) + (1 - \mu_t) v(x^l_A, \tau^l_A; \theta_m) 
= \mu_t v(\hat{x}^h_B, \hat{\tau}^h_B; \theta_m) + (1 - \mu_t) v(\hat{x}^l_B, \hat{\tau}^l_B; \theta_m). 
\]

These strategies are represented on Figure A1 in the Appendix.

We show that it is never optimal for \( B \) to offer the pooling policy when the cost is low so that the previous strategies cannot be sustained in equilibrium. The idea underlying this result is that when \( A \) plays his optimal policy in each state of the world, \( B \) should deviate from the pooling strategy in the low cost state, proposing a policy that entails more public good provision at a lower cost and leaving the median voter indifferent between this policy and \( A \)'s optimal policy, therefore guaranteeing re-election for \( B \).

This result means that there is a fundamental asymmetry between the parties in the sense that the party with a low taste for the public good is more likely to select inefficient policies. However, this does not mean that there is no equilibrium in which the party with a strong taste for the public good wastes resources. The existence of an equilibrium with \( B \) adopting a pooling strategy is still a possibility.

5. Concluding Comments

We have shown that a party in power may prefer to supply a low quantity of a public good when more could be produced at the same tax level. This is clearly a wasteful behavior. The reason for such a behavior is that if the median voter were informed that the cost of production is low, he would ask for a larger provision of the good, against the interests of the party with a low taste for the public good. In other words, our analysis suggests that a party opposed to an excessive state intervention has very weak incentives to promote an efficient provision of goods by the public sector. Observing that the state performs well in providing such goods, the voters would urge the party in power to develop public provision.

This also suggests that when a party with a strong taste for the public good is in power, the policy adopted is less likely to involve a waste of resources.
Proposition 3 proves that this intuition is correct. However, it relies very much on a technological asymmetry of the model: because the budget is balanced each period, the set of feasible policies expands when the cost decreases. This implies that a party can pretend that the cost is high when it is low but not the converse. Allowing some debt would restore some symmetry. However, as soon as debt is observable, it still does not allow a party to pretend that the cost is lower than it really is.

Our argument is built on the combination of three assumptions: asymmetric information, absence of commitment and repeated elections. All these three assumptions are needed for our results. As argued before, other papers developed theories of inefficient political decision-making in a dynamic setting without commitment but with complete information (Besley and Coate 1998, Acemoglu and Robinson 2001, Robinson and Torvik 2005a, b). In our setting, asymmetric information is necessary to obtain inefficiency. This difference stems from the fact that in all these papers, the choice made by a politician in a given period has an impact on the political equilibrium in subsequent periods, which is not channeled through asymmetric information. Because the politicians are unable to commit to the future policies, they may have incentives to distort policies in the current period so as to generate more favorable outcomes in the future. As a conclusion, asymmetric information is not in general necessary to have political inefficiencies. On the other hand, we are not aware of elections models that yields inefficiencies in a static framework (with or without policy commitment) or in a dynamic framework with policy commitment. This suggests that the assumptions of no commitment and of a dynamic political process are necessary for the political equilibrium to be inefficient. While this conclusion is true in our framework, its generalization lies outside the scope of this paper and should be the object of future research.

Our paper is very close in spirit to Coate and Morris (1995) who consider a dynamic model with no commitment and asymmetric information. The mechanism through which politicians deliver an inefficient policy is however different. In their model, politicians, by implementing the efficient action, reveal information about their type. A “bad” politician who cares about some special interest and not only about the general public incurs the risk of not being re-elected if detected by the voters. He may then prefer to use disguised transfers to special interests even though it results in a Pareto dominated policy. In our model, information with respect to the type of the politicians is public and the “risk” that politicians face when disclosing information to the electorate is to be forced to implement a policy that they dislike. Therefore, their argument is based on reputational concerns whereas our can be interpreted as a ratchet effect.

Formally, an important difference between the two models is that they need a double uncertainty, both on the type of the politicians and of the policies to be selected by them, whereas we only have asymmetric information with respect to the type of the policy. The “price” of this simplification is that
we need to consider and infinitely repeated elections model when they only need two periods.

Lastly, a few words concerning the nature of the political inefficiency are in order. A possible interpretation of this inefficiency is that the politician in power collects more money than needed and “burns” the unused amount. While this is of course an unrealistic feature of the model, it captures the incentives faced by elected policy makers. A proper framework should allow the politicians to “steal” public money and use it for their own personal consumption, which is not the case in our modeling. We guess that introducing such a possibility would in fact reinforce our main result as it represents an additional motive for the politician to pretend that the technology is inefficient.

Appendix

**Proof of Lemma 1:** This lemma follows directly from the single crossing property of individuals’ preferences. From (1), the slope of an indifference curve

\[
\frac{dx}{d\tau} = \frac{1}{\theta u'(x)}
\]

is decreasing with \(\theta\). This implies that the indifference curves cross only once.

In any given period \(t\), the continuation value from electing party \(i\) for a type \(\theta\) individual is

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} v(x_i^e, \tau_e^i; \theta) = v(x_i^e, \tau_e^i; \theta).
\]

Let us denote \(-i\) the alternative party to party \(i\). If \(v(x_i^e, \tau_e^i; \theta) > v(x_e^{-i}, \tau_e^{-i}; \theta)\), individual \(\theta\), who behaves as pivotal, should vote for party \(i\). In case of equality, consistency with the retrospective voting behavior described in (5) requires that this individual votes for the incumbent.

Consider now the case \(x_A^e < x_B^e\). If \(v(x_A^e, \tau_A^e; \theta_m) > v(x_B^e, \tau_B^e; \theta_m)\), the median type votes for \(A\). The single crossing property implies that \(v(x_A^e, \tau_A^e; \theta) > v(x_B^e, \tau_B^e; \theta)\) \(\forall \theta < \theta_m\) and therefore that these individuals also vote for \(A\). Therefore if \(A\) is the incumbent, he is re-elected. If \(B\) is the incumbent he is voted out of office. When \(v(x_A^e, \tau_A^e; \theta_m) = v(x_B^e, \tau_B^e; \theta_m)\), each party receives half of the votes and the incumbent is re-elected by assumption (see footnote 7). If \(v(x_A^e, \tau_A^e; \theta_m) < v(x_B^e, \tau_B^e; \theta_m)\), the median type votes for \(B\). The single crossing property implies that \(v(x_B^e, \tau_B^e; \theta) > v(x_A^e, \tau_A^e; \theta)\) \(\forall \theta > \theta_m\) and therefore that these individuals also vote for \(B\). Therefore if \(B\) is the incumbent, he is re-elected. If \(A\) is the incumbent, he is voted out of office. When \(v(x_A^e, \tau_A^e; \theta_m) = v(x_B^e, \tau_B^e; \theta_m)\), each party receives half of the votes and the incumbent is re-elected by assumption.

The analysis in the case \(x_A^e > x_B^e\) is similar. When \(x_A^e = x_B^e\) and \(\tau_A^e \neq \tau_B^e\), all the citizens vote for the party proposing the lower tax rate. Finally,
when \( x_A^e = x_B^e \) and \( \tau_A^e = \tau_B^e \), they are all indifferent between the two parties. ■

**Proof of Lemma 2:** We remove in this proof the superscript denoting the state of the world. From (6), \( \tilde{x}_A \) is implicitly defined by the following equation:

\[
\theta_m u(\tilde{x}_A) - c \tilde{x}_A - \theta_m u(x_B) + cx_B = 0, \tag{A1}
\]

where \( x_B \) satisfies the first-order condition \( c = \theta_B u'(x_B) \).

Differentiating equation (A1) with respect to \( c \), we obtain

\[
\frac{d\tilde{x}_A}{dc} = \frac{\tilde{x}_A + \theta_m \frac{dx_B}{dc} u'(x_B) - x_B - c \frac{dx_B}{dc} \theta_m u'(\tilde{x}_A) - c}{\theta_m u'(\tilde{x}_A) - c}.
\]

Differentiating \( \tilde{\tau}_A = c \tilde{x}_A \) and recalling that \( u(x) = x^{1-\varepsilon}/(1-\varepsilon) \), we get

\[
\frac{d\tilde{\tau}_A}{dc} = \tilde{x}_A + c \frac{d\tilde{x}_A}{dc} = \frac{\tilde{x}_A \theta_m u'(\tilde{x}_A) - x_B \theta_B u'(x_B) - \frac{x_B}{\varepsilon} u'(x_B) (\theta_m - \theta_B)}{\theta_m u'(\tilde{x}_A) - c}.
\]

The denominator is positive. We thus have to check that the numerator, denoted \( N \), is negative.

\[
N = x_B u'(x_B) \left[ \theta_m \frac{\tilde{x}_A u'(\tilde{x}_A)}{x_B u'(x_B)} - \theta_B - \frac{\theta_m - \theta_B}{\varepsilon} \right].
\]

Observing that

\[
\frac{\tilde{x}_A u'(\tilde{x}_A)}{x_B u'(x_B)} = \frac{u(\tilde{x}_A)}{u(x_B)},
\]

we have that \( N < 0 \) if and only if

\[
\frac{u(\tilde{x}_A)}{u(x_B)} < \frac{\theta_B (\varepsilon - 1) + \theta_m}{\varepsilon \theta_m}.
\]

From (A1),

\[
\frac{u(\tilde{x}_A)}{u(x_B)} = \frac{\theta_B}{\theta_m} (1 - \varepsilon) \left( \frac{\tilde{x}_A}{x_B} - 1 \right) + 1.
\]

Therefore the previous inequality is satisfied if and only if

\[
\frac{\tilde{x}_A}{x_B} < \frac{\theta_m - \theta_B}{\varepsilon \theta_B} + 1.
\]
From the strict concavity of $u$, $u(x_B) - u(\tilde{x}_A) < u'(\tilde{x}_A)(x_B - \tilde{x}_A)$. Rearranging (A1), we obtain

$$u(x_B) - u(\tilde{x}_A) = \frac{c(x_B - \tilde{x}_A)}{\theta_m} < u'(\tilde{x}_A)(x_B - \tilde{x}_A)$$

$$\iff u'(\tilde{x}_A) > \frac{\theta_B}{\theta_m}$$

$$\iff \frac{\tilde{x}_A}{x_B} < \left(\frac{\theta_m}{\theta_B}\right)^{1/\epsilon}.$$ 

Observing that

$$\lim_{\epsilon \to 1} \frac{\theta_m - \theta_B}{\epsilon \theta_B} + 1 = \lim_{\epsilon \to 1} \left(\frac{\theta_m}{\theta_B}\right)^{1/\epsilon} = \frac{\theta_m}{\theta_B},$$

and since $(\theta_m - \theta_B)/\epsilon \theta_B$ is continuous in $\epsilon$, we have that $\tilde{x}_A/x_B < (\theta_m - \theta_B)/\epsilon \theta_B + 1$ for $\epsilon$ sufficiently close to 1 and therefore that $d\tilde{x}_A/dc < 0$. ■

Proof of Proposition 3: In order to prove Proposition 3, we use the following lemma:

**LEMMA 3:** $\tilde{x}_B^s$ decreases when $c^s$ increases, where $\tilde{x}_B^s$ is implicitly defined by the equation

$$\theta_m u(\tilde{x}_B^s) - c^s \tilde{x}_B^s - \theta_m u(x_A^s) + c^s x_A^s = 0,$$  \hspace{1cm} (A2)

where $c^s = \theta_A u'(x_A^s)$.

**Proof:** Differentiating (A2) and dropping superscripts, we have

$$\frac{d\tilde{x}_B}{dc} = \frac{\tilde{x}_B - x_A - \frac{x_A}{\epsilon} \left(\frac{\theta_m}{\theta_A} - 1\right)}{\theta_m u'(\tilde{x}_B) - c}.$$  

The denominator being negative, we want to show that the numerator $N$ is positive. After some manipulations we obtain

$$N > 0 \iff \frac{\tilde{x}_B}{x_A} > \frac{\theta_m - \theta_A}{\epsilon \theta_A} + 1.$$  

From the concavity of $u$,

$$\frac{\tilde{x}_B}{x_A} > \left(\frac{\theta_m}{\theta_A}\right)^{1/\epsilon}.$$  

To achieve the proof, we only need to show that

$$\left(\frac{\theta_m}{\theta_A}\right)^{1/\epsilon} > \frac{\theta_m - \theta_A}{\epsilon \theta_A} + 1, \forall \epsilon < 1.$$
Observing that these two functions are decreasing with $\varepsilon$ and equal when $\varepsilon = 1$, this will be the case if the slope of $(\theta_m/\theta_A)^{1/\varepsilon}$ is lower than the slope of $(\theta_m - \theta_A)/\varepsilon\theta_A + 1$, that is if

$$-\frac{1}{\varepsilon^2} \ln \left(\frac{\theta_m}{\theta_A}\right) \left(\frac{\theta_m}{\theta_A}\right)^{1/\varepsilon} < -\frac{1}{\varepsilon^2} \left(\frac{\theta_m}{\theta_A} - 1\right)$$

$$\Leftrightarrow \ln \left(\frac{\theta_m}{\theta_A}\right) \left(\frac{\theta_m}{\theta_A}\right)^{1/\varepsilon} > \frac{\theta_m}{\theta_A} - 1.$$

As $(\theta_m/\theta_A)^{1/\varepsilon} > \theta_m/\theta_A, \forall \varepsilon < 1$, it is sufficient to show that

$$\ln \left(\frac{\theta_m}{\theta_A}\right) \frac{\theta_m}{\theta_A} > \frac{\theta_m}{\theta_A} - 1.$$

One can easily verify that $x \ln x > x - 1, \forall x > 0$. Hence the result.

We are now ready to prove Proposition 3.

Let us denote $\tilde{B}'$ and $\tilde{B}$ the points $(\tilde{\tau}_B, \tilde{x}_B)$ and $(\tilde{\tau}_B, \tilde{x}_B)$ respectively. Let us also denote $\tilde{I}_m$ and $\tilde{I}_B$ the indifference curves for $m$ and $B$ through $\tilde{B}$. Finally, denote $I_B'$ the indifference curve for $B$ through $\tilde{B}'$. These indifference curves are represented on Figure A1.

We want to prove that $B$ always prefers the policy $\tilde{B}'$ to the policy $\tilde{B}$. $B$ may prefer $\tilde{B}$ to $\tilde{B}'$ either because $\tilde{x}_B'$ is too low or too large. We know from Lemma 3 that $\tilde{x}_B' > \tilde{x}_B$ and thus that $\tilde{x}_B'$ is not too low. We now show that it is

Figure A1: The symmetric configuration with B wasting resources is not an equilibrium.
not too large either. Consider $\tilde{I}_B$, the indifference curve for $B$ through $\tilde{B}$. We know from the fact that $\theta_m \leq \theta_B$ that $\tilde{I}_B$ is flatter than $\tilde{I}_m$ at $\tilde{B}$. Moreover, $\tilde{I}_B$ and $\tilde{I}_m$ cross only once, precisely at $\tilde{B}$. This implies that $\tilde{I}_B$ crosses the low cost frontier to the right of the point where $\tilde{I}_m$ crosses the same frontier, which occurs to the right of $\tilde{B}^l$. Consequently, $\tilde{I}_B$ crosses the low cost frontier to the right of $\tilde{B}^l$ so that $\tilde{I}_B$ lies below $I^l_B$. Hence the result.

References


