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Estimation and Welfare Analysis with Large Demand Systems

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We develop an approach for estimating individual or household level preferences for a large set of quality-differentiated goods and for constructing Hicksian welfare measures within the demand system framework. Our approach uses a maximum simulated likelihood procedure to recover estimates of the structural parameters and a multistage, Monte Carlo Markov chain algorithm for constructing Hicksian consumer surplus estimates. We illustrate our approach with a recreation dataset consisting of day trips to 62 Mid-Atlantic beaches.

KEY WORDS: Beach recreation; Demand system models; Random parameters; Simulation; Welfare analysis.

1. INTRODUCTION

In this article we develop a demand system approach for estimating preferences for a large set of quality-differentiated goods at the individual or household level. We apply the approach to an outdoor recreation dataset consisting of day trips to 62 beaches in the Mid-Atlantic region and analyze the welfare effects of changes in beach characteristics and availability. Interest in the value of beach recreation opportunities arises from policymakers' need to assess the merits of beach nourishment programs and to measure the damages resulting from acute environmental accidents that impact beach availability. The Hicksian consumer surplus estimates reported in this article address these issues and represent the first welfare measures derived from a theoretically consistent demand system model that accounts for interior and corner solutions and accommodates a large set of quality-differentiated goods.

Because of the computational difficulties associated with estimating and generating welfare measures from demand system models, nearly all empirical strategies for modeling consumer choice for many goods have relied on the discrete choice random utility maximization (RUM) model developed by McFadden (1974). A large and growing body of empirical research has shown that the discrete choice RUM framework is attractive for modeling extensive margin choices made on single choice occasions, but how the framework can be modified or augmented to represent consumer choices made over longer time horizons when realized demands are a mixture of interior and corner solutions remains an unresolved modeling issue. At present there are several discrete choice RUM-based approaches for modeling consumer choices in these situations (e.g., Morey, Rowe, and Watson 1993; Feather, Hellerstein, and Tomasi 1995; Hausman, Leonard, and McFadden 1995; Parsons and Kealy 1995), all of which have strengths and weaknesses. Some common features of these RUM-based modeling strategies are the assumptions that the time horizon of choice can be decomposed into separable choice occasions, that the objects of choice on each choice occasion are quality-adjustedperfect substitutes, and, with rare exception, that income effects are absent (for a more extensive review and comparison of these approaches, see Parsons, Jakus, and Tomasi 1999). In part because of the restrictiveness of these assumptions, Phaneuf, Kling, and Herriges (2000; hereafter PKH) and Phaneuf (1999) reconsidered modeling consumer choice in these situations within a unified demand system framework that consistently accounts for interior and corner solutions. Their empirical applications, however, considered only a small number of quality-differentiated goods, and, thus, the relevance of the demand system framework for policy applications with many goods remained uncertain.

We demonstrate in this article that if preferences are additively separable the demand system framework can be estimated and used to generate Hicksian welfare measures for applications with many goods. Additive separability implies strong restrictions for consumer behavior, but in our view these restrictions have similarities with the assumptions embedded in the discrete choice RUM models that have traditionally been used for this class of problems. Although discrete choice RUM models can generate substitution patterns that are more flexible than our additively separable demand system model, our model has the advantage of allowing for diminishing marginal utility in consumption for a given commodity and, more generally, combining the intensive and extensive margins of consumer choice for all quality-differentiated goods in a coordinated and behaviorally consistent framework.

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In addition to permitting the construction of welfare measures for a large set of quality-differentiated goods, our empirical models incorporate several innovations over existing demand system recreation applications. Our specifications allow a subset of the parameters entering the direct utility function to vary randomly across individuals in the population. From an econometric perspective introducing random parameters is attractive because it facilitates a relatively flexible specification for the unobserved heterogeneity without substantially expanding the number of estimable parameters. In addition, our empirical application incorporates an approach to welfare measurement suggested by von Haefen (2003) that conditions on an individual's observed choice. In contrast to traditional approaches to welfare measurement from RUM models (e.g., Small and Rosen 1981), we construct welfare measures in this article by simulating the unobserved heterogeneity entering preferences such that our model predicts observed behavior perfectly at baseline conditions. The structure of the model is then used to predict how individuals respond to price, quality, and income changes. To implement our conditional approach, we develop a sequential Monte Carlo procedure that employs an adaptive Metropolis–Hastings algorithm. Whereas conditioning on an individual's observed choice adds more complexity to our welfare calculation procedure, it substantially reduces the number of simulations necessary to generate precise welfare estimates as well as the computational time involved.

Although our modeling approach can be applied to disaggregate consumption data for any number of quality-differentiated goods (e.g., transportation mode, brand level demand for commodities such as cereal or coffee) and policy applications (welfare measurement, exact price index construction, valuing new goods), we illustrate its usefulness by assessing the welfare implications arising from changes in the availability and quality of outdoor recreation sites. Using a detailed dataset consisting of 540 Delaware residents' beach recreation activities, our empirical application examines the demand for day trips to 62 ocean beaches in the Mid-Atlantic region. Owing to infrequent but acute oil and toxic spills, state officials in New Jersey, Delaware, Maryland, and Virginia occasionally close beaches for health and safety reasons. In addition, beach erosion resulting from rising sea levels, development, and natural causes has led state officials to initiate beach nourishment programs throughout the region. Because beach recreators are among the individuals most affected by beach closures and erosion, our welfare scenarios can inform state officials of the potential economic losses arising from these impacts.

The remaining structure of the article is as follows. The next section gives a general overview of the issues involved in demand system estimation and welfare calculation with large choice sets. Section 3 follows with a discussion of the empirical specifications and estimation strategies we employ in this article, and Section 4 discusses our strategy for constructing welfare measures. Section 5 discusses the Mid-Atlantic beach recreation dataset we use in our application, and Section 6 summarizes our estimation results. Section 7 discusses our welfare scenarios and results, and Section 8 concludes.

2. GENERAL OVERVIEW

In principle, there are two generic strategies for developing demand system models that consistently account for both interior and corner solutions and can be applied to problems with many goods. The first, developed independently by Hanemann (1978) and Wales and Woodland (1983) and referred to as the Kuhn–Tucker framework, exploits the Kuhn–Tucker conditions that implicitly define the consumer's optimal consumption bundle. Alternatively, Lee and Pitt (1986) developed a demand system framework that relies on the concepts of notional demand and virtual price functions (Neary and Roberts 1980). Although these approaches are dual, we focus on the Kuhn–Tucker framework in this section and in our subsequent empirical work. Much of our general discussion that follows, however, transfers to the dual approach in a straightforward manner.

As discussed in, for example, PKH, the Kuhn–Tucker framework begins with a specification of consumer preferences represented by a continuously differentiable, strictly increasing, and strictly quasi-concave direct utility function, $U(\mathbf{x}, \mathbf{Q}, z, \boldsymbol{\beta}, \boldsymbol{\varepsilon})$, where $\mathbf{x}$ is an M-dimensional vector of consumption levels for the quality-differentiated goods that are consumed in nonnegative quantities, $\mathbf{Q}$ denotes an $M \times K$ matrix of commodity-specific quality attributes of the goods in $\mathbf{x}$ (i.e., $\mathbf{Q} = [\mathbf{q}_1, \ldots, \mathbf{q}_M]^{\mathrm{T}}$, where $\mathbf{q}_i$, $i = 1, \ldots, M$, is a $K \times 1$ vector of attributes for good i), z is an essential Hicksian composite commodity representing spending on all other goods, $\boldsymbol{\beta}$ is a vector of structural parameters entering preferences, and $\boldsymbol{\varepsilon}$ is a vector or matrix of unobserved heterogeneity. Because $\boldsymbol{\varepsilon}$ is interpreted as components of the utility function known by the individual but unobserved and random from the analyst's perspective, the structure of preferences is consistent with McFadden's random utility maximization (RUM) hypothesis (see McFadden 2001 for a recent discussion).

The consumer maximizes utility subject to a linear budget constraint and M nonnegativity constraints:

$$\max_{x,z} U(\mathbf{x}, \mathbf{Q}, z, \boldsymbol{\beta}, \boldsymbol{\varepsilon}) \quad \text{s.t.} \quad \mathbf{p}^{\mathrm{T}}\mathbf{x} + z = y, \mathbf{x} \geq \mathbf{0}, \tag{1}$$

where $\mathbf{p}$ is an M-dimensional vector of prices, y is income, and the price of the Hicksian composite commodity is normalized to 1 with no loss in generality. In addition to the constraints in (1), the Kuhn–Tucker conditions that implicitly define the optimal solution to the consumer's problem can be written as

$$\partial U/\partial x_j \leq (\partial U/\partial z)p_j, \qquad j = 1, \ldots, M, \tag{2}$$

$$x_j\big(\partial U/\partial x_j - (\partial U/\partial z)p_j\big) = 0, \qquad j = 1, \ldots, M. \tag{3}$$

Estimation of the structural parameters entering the preference specification within the Kuhn–Tucker framework exploits (2). These weak inequalities and an individual's observed choices place restrictions on the support of the unobserved heterogeneity's distribution. Assuming the errors representing unobserved heterogeneity are drawn from some known family of distributions with parameter vector $\boldsymbol{\Sigma}$, these restrictions permit recovery of estimates for $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ within the maximum likelihood framework.

From an econometric perspective the Kuhn–Tucker model can be interpreted as an endogenous regime-switching model

where regimes are defined as combinations of interior and corner solutions for the M goods and determined by (2). When dealing with applications involving many goods, two related issues must be resolved in order to estimate Kuhn–Tucker endogenous regime-switching models. The analyst must choose a flexible yet parsimoniously parameterized direct utility function. This requires restricting the dimension of $\boldsymbol{\beta}$ to be sufficiently low. Moreover, the analyst must specify a distribution for the unobserved heterogeneity that has an estimable parameter vector $\boldsymbol{\Sigma}$ that is of relatively low dimension and that allows calculation of the multiple-dimensional integrals that correspond to the probabilities of observing each of the 2^M possible regimes. If these issues are adequately addressed, the Kuhn–Tucker demand system framework represents a viable approach for estimating consumer preferences for many quality-differentiated goods.

Welfare measurement from demand system models raises a separate and in many ways more complicated set of issues. The Hicksian consumer surplus, CS^{H}, associated with a price and quality change from $(\mathbf{p}^0, \mathbf{Q}^0)$ to $(\mathbf{p}^1, \mathbf{Q}^1)$ is implicitly defined as

$$\max_{\omega\in\Omega} V_\omega(\mathbf{p}^0, \mathbf{Q}^0, y, \boldsymbol{\beta}, \boldsymbol{\varepsilon}) = \max_{\omega\in\Omega} V_\omega(\mathbf{p}^1, \mathbf{Q}^1, y - CS^{\mathrm{H}}, \boldsymbol{\beta}, \boldsymbol{\varepsilon}), \quad (4)$$

where ω indexes each of the 2^M separate regimes and $V_\omega(\cdot)$ represents the corresponding conditional indirect utility function. Unless preferences are quasilinear or homothetic in income, no closed-form solution exists for CS^{H}, and iterative techniques such as numerical bisection are required to solve for CS^{H}. However, as discussed by PKH, procedures such as numerical bisection require that the analyst solve the consumer's problem at each iteration conditional on an arbitrary set of $(\mathbf{p}, \mathbf{Q}, y, \boldsymbol{\varepsilon})$ values. PKH proposed a strategy for accomplishing this task that analytically calculates each of the possible 2^M conditional indirect utility functions and ascertains which is the maximum. Although this strategy is computationally feasible for small M, it quickly becomes intractable as M grows large. For example, in our subsequent empirical application where M equals 62, the number of possible regimes is 4.6117×10^{18}.

An additional complication with constructing welfare estimates is that the analyst does not observe $\boldsymbol{\varepsilon}$. This limitation suggests that the analyst cannot determine the individual's Hicksian consumer surplus precisely and can at best construct an estimate of the welfare measure's central tendency over the support of $\boldsymbol{\varepsilon}$ such as its expectation, $E(CS^{\mathrm{H}})$. As described in PKH constructing $E(CS^{\mathrm{H}})$ requires the use of Monte Carlo techniques that involve simulating several realizations of $\boldsymbol{\varepsilon}$ from its estimated distribution, solving for CS^{H} conditional on each simulated $\boldsymbol{\varepsilon}$, and averaging the simulated values of CS^{H}. Increasing the number of simulations improves the precision of the estimate but also increases the computational time involved.

One final notable difficulty arises because these welfare estimates are functions of estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ that are random variables from the analyst's perspective. Quantifying the implications of uncertainty about the parameters' true values by constructing standard errors for the welfare estimates requires replication of the entire simulation routine for several alternative parameter estimates.

The preceding discussion suggests the significant computation challenges arising with welfare estimation from demand system applications with large choice sets. For welfare measurement to be viable with demand system models, the analyst must be able to quickly solve for the utility the individual obtains conditional on $(\mathbf{p}, \mathbf{Q}, y, \boldsymbol{\varepsilon})$. As discussed in the Introduction, the difficulties associated with this task as well as estimating demand system models have led researchers in the outdoor recreation literature to largely abandon demand system models and instead rely on the discrete choice RUM framework. More recently, researchers in the field of industrial organization have also adopted the discrete choice framework for modeling this class of problems due to its computational advantages (e.g., Berry, Levinsohn, and Pakes 1995; Nevo 2001). In the next section we develop econometrically tractable preference specifications that can be used to model the demand for a large set of quality-differentiated goods and to construct Hicksian welfare measures.

3. PREFERENCE SPECIFICATIONS AND ESTIMATION STRATEGIES

In this article our approach to modeling consumer choice within the Kuhn–Tucker demand system framework relies on the assumption that consumer preferences are additively separable in each element of $\mathbf{x}$ and z:

$$U(\mathbf{x}, z) = \sum_j^M u_j(x_j) + u_z(z), \quad (5)$$

where we have suppressed all arguments entering preferences except $\mathbf{x}$ and z in (5). This specification of preferences has similarities with the utility structures employed by Hanemann (1984) and Chiang and Lee (1992) to model mixed discrete/continuous choices. Their preference specifications can be nested within the following general form:

$$U(\mathbf{x}, z) = u\left(\sum_j^M x_j, z\right). \quad (6)$$

Note that the structure in (6) assumes that the elements of $\mathbf{x}$ enter preferences additively through a separable subfunction and, therefore, are perfect (quality-adjusted) substitutes, although the elements of $\mathbf{x}$ collectively need not be additively separable from z. By contrast, (5) implies that strictly increasing, strictly concave functions of each element of $\mathbf{x}$ and z enter preferences additively. Hanemann's and Chiang and Lee's approach to structuring preferences implies that the individual consumes at most one element of $\mathbf{x}$, whereas our preference structure gives rise to multiple elements of $\mathbf{x}$ being consumed.

Although PKH also assumed preferences are additively separable in their empirical work, we recognize that it is a strong preference restriction; it rules out a priori inferior goods and implies that all goods are Hicksian substitutes (see Pollak and Wales 1992 for a discussion). Additive separability, moreover, implies that the marginal utility for each good is independent of the level of all other goods. Thus, although the preference function is consistent with the intuitively appealing notion of diminishing marginal utility of consumption for a given good, it does not allow marginal utility to decrease (or increase) with

increases in consumption of other goods. This in turn limits the richness of substitution patterns that can be captured with this preference structure.

For perspective, it is instructive to compare these implications with those that arise from the separability assumptions embedded in discrete choice RUM models. These models decompose the time horizon of choice into separable choice occasions on which the individual makes discrete choices. Consumer behavior across each of the choice occasions is uncoordinated and, thus, diminishing marginal utility of consumption for a good is absent from these models. Although discrete choice RUM models can generate fairly rich substitution patterns among goods, they assume that on a given choice occasion all goods are quality-adjusted perfect substitutes and income effects are absent (for a notable exception, see Herriges and Kling 1999).

The additively separable empirical demand system specifications estimated in this article can be nested within the following general structure:

$$U(\mathbf{x}, \mathbf{Q}, z, \boldsymbol{\beta}, \boldsymbol{\varepsilon}) = \sum_j^M \frac{1}{\rho_j} \Psi(\mathbf{s}, \mathbf{d}_j, \boldsymbol{\varepsilon}) \big(\phi(\mathbf{q}_j) x_j + \theta_j\big)^{\rho_j} + \frac{1}{\rho_z} z^{\rho_z},$$
$$\ln \Psi(\mathbf{s}, \mathbf{d}_j, \boldsymbol{\varepsilon}) = (\boldsymbol{\delta} + \boldsymbol{\varepsilon}_\delta)^{\mathrm{T}} \mathbf{s} + (\boldsymbol{\zeta} + \boldsymbol{\varepsilon}_\zeta)^{\mathrm{T}} \mathbf{d}_j + \boldsymbol{\varepsilon}_j,$$
$$\ln \phi(\mathbf{q}_j) = \boldsymbol{\gamma}^{\mathrm{T}} \mathbf{q}_j, \tag{7}$$

where $\mathbf{s}$ and $\mathbf{d}_j$ are vectors of individual specific demographic variables and site-specific dummy variables, respectively; $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_M]^{\mathrm{T}} > \mathbf{0}$; $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_M, \rho_z] < \mathbf{1}$; $\boldsymbol{\delta}$, $\boldsymbol{\zeta}$, and $\boldsymbol{\gamma}$ are estimable parameters; $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$ represent unobserved heterogeneity that varies randomly across individuals in the population; and $(\varepsilon_1, \ldots, \varepsilon_M)$ represent unobserved heterogeneity that varies randomly across individuals and goods.

Our preference structure is a close relative of the linear expenditure system employed by PKH but differs in three important respects. First, our specification can be interpreted as a more general specification because, in the limit as all elements of $\boldsymbol{\rho}$ approach 0, our specification nests the linear expenditure system, that is,

$$\lim_{\rho \to 0} \left[\sum_j^M \frac{1}{\rho_j} \Psi(\mathbf{s}, \mathbf{d}_j, \boldsymbol{\varepsilon}) \big(\phi(\mathbf{q}_j) x_j + \theta_j\big)^{\rho_j} + \frac{1}{\rho_z} z^{\rho_z} \right]$$
$$= \sum_j^M \Psi(\mathbf{s}, \mathbf{d}_j, \boldsymbol{\varepsilon}) \ln\big(\phi(\mathbf{q}_j) x_j + \theta_j\big) + \ln z. \tag{8}$$

Second, PKH assumed, using our notation in (7), that a good's quality attributes enter preferences through Ψ instead of ϕ. This approach to introducing quality implies that weak complementarity (i.e., $\partial U / \partial \mathbf{q}_j = 0$ if $x_j = 0, \forall j$; see Mäler 1974; Bradford and Hildebrandt 1977 for discussions) is not, in general, satisfied unless $\theta_j = 0$ when $\rho_j \neq 0$ and $\theta_j = 1$ for the limiting case when $\rho_j = 0, \forall j$. As a result a potentially large component of the total value associated with a quality change will be independent of the consumer's use of a particular commodity, a conceptually troubling implication for many applications. Because our specification introduces quality through the simple repackaging ϕ parameters (Griliches 1964), weak complementarity is satisfied for all parameter values; thus, only use-related values will arise from our model. Finally, our specification allows the parameters for the demographic and site-specific dummy variables entering the Ψ parameters to vary randomly across individuals in the population. As discussed later, this feature of our model allows us to introduce a more flexible structure for unobserved heterogeneity.

Maximizing the utility function in (7) with respect to $\mathbf{p}^{\mathrm{T}}\mathbf{x} + z = y$ and the nonnegativity constraints implies a set of first-order conditions that, with some manipulation, can be written as

$$\varepsilon_j \leq -(\boldsymbol{\delta} + \boldsymbol{\varepsilon}_\delta)^{\mathrm{T}} \mathbf{s} - (\boldsymbol{\zeta} + \boldsymbol{\varepsilon}_\zeta)^{\mathrm{T}} \mathbf{d}_j$$
$$+ \ln \frac{p_j}{\phi(\mathbf{q}_j)} + (\rho_z - 1) \ln(y - \mathbf{p}^{\mathrm{T}}\mathbf{x})$$
$$+ (1 - \rho_j) \ln\big(\phi(\mathbf{q}_j) x_j + \theta_j\big) \qquad \forall j. \tag{9}$$

These weak inequalities, along with assumptions for the distributions of $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta, \varepsilon_1, \ldots, \varepsilon_M)$, permit estimation of the structural parameters using maximum likelihood techniques. In our application we assume that each ε_j is an independent and identically distributed draw from the Type I extreme-value distribution with common scale parameter μ. Defining the right side of (9) as $g_j(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$, the likelihood of observing a particular vector of choices $\mathbf{x}$ conditional on $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$ can be written as

$$l(\mathbf{x} | \boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta) = |\mathbf{J}| \prod_j \Big[\big(\exp(-g_j(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)/\mu)/\mu\big)^{\mathbb{1}_{x_j > 0}}$$
$$\times \exp\big(-\exp(-g_j(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)/\mu)\big) \Big], \tag{10}$$

where $\mathbf{J}$ is the Jacobian of transformation. The unconditional likelihood of observing $\mathbf{x}$ is

$$l(\mathbf{x}) = \int l(\mathbf{x} | \boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta) f(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)\, d\boldsymbol{\varepsilon}_\delta\, d\boldsymbol{\varepsilon}_\zeta, \tag{11}$$

where the integral is over the full support of $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$. For our application we assume that $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$ are mean-zero random draws from the normal distribution with unequal vectors of scale parameters $(\boldsymbol{\sigma}_\delta, \boldsymbol{\sigma}_\zeta)$ and no correlations. Given these assumptions and the structure of our conditional likelihood function, (11) has no closed-form solution and cannot be evaluated using numerical integration techniques unless the dimension of $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$ is small. In this article, we follow common empirical practice in the discrete choice literature (e.g., Train 2003) and use simulation to evaluate (11). Our estimation strategy, therefore, falls under the rubric of maximum simulated likelihood estimation (Gourieroux and Monfort 1996). Although estimation of our random parameter model is more computationally difficult than the fixed parameter model with generalized extreme value unobserved heterogeneity employed by PKH, it is more flexible. In addition to allowing for heteroscedasticity and correlations in the unobserved heterogeneity across groups of goods as PKH's specification does, our specification also allows for heteroscedasticity across individuals.

4. WELFARE CALCULATION

4.1 Solving the Consumer's Problem

An essential component of constructing Hicksian welfare measures involves evaluating the utility an individual achieves conditional on $(\mathbf{p}, \mathbf{Q}, y, \boldsymbol{\varepsilon})$. When the dimension of the choice set is large, PKH's strategy of analytically constructing all 2^M

possible conditional indirect utility functions and determining which is the maximum is not feasible. In this article, we pursue an alternative, computationally tractable strategy that numerically solves the Kuhn–Tucker conditions for the optimal consumption levels. These optimal values are then inserted into the direct utility function to ascertain the individual's utility conditional on $(\mathbf{p}, \mathbf{Q}, y, \boldsymbol{\varepsilon})$.

Given our additive separability assumption, solving the consumer's problem is greatly simplified. In particular, the Kuhn–Tucker conditions take the general form:

$$\frac{\partial u_j(x_j)}{\partial x_j} \leq \frac{\partial u_z(z)}{\partial z} p_j \qquad \forall j, \tag{12}$$

$$x_j \left[\frac{\partial u_j(x_j)}{\partial x_j} - \frac{\partial u_z(z)}{\partial z} p_j \right] = 0 \qquad \forall j, \tag{13}$$

$$x_j \geq 0 \qquad \forall j, \tag{14}$$

$$z = y - \sum_j p_j x_j. \tag{15}$$

Note that additive separability implies that only x_j and z enter the jth inequalities in (12)–(14). This structure suggests that if the analyst knew the optimal value for z, he or she could use (12)–(14) to solve for each x_j. Therefore, solving the consumer's problem reduces to solving for the optimal value of z. Building on this insight, we developed the following numerical bisection algorithm to solve the consumer's problem:

1. At iteration i set $z_a^i = (z_l^{i-1} + z_u^{i-1})/2$. To initialize the algorithm, set $z_l^0 = 0$ and $z_u^0 = y$.
2. Conditional on z_a^i, solve for $\mathbf{x}^i$ using (12)–(14).
3. Use (15) and $\mathbf{x}^i$ to construct $\tilde{z}^i$.
4. If $\tilde{z}^i > z_a^i$ set $z_l^i = z_a^i$ and $z_u^i = z_u^{i-1}$. Otherwise, set $z_l^i = z_l^{i-1}$ and $z_u^i = z_a^i$.
5. Iterate until $\text{abs}(z_l^i - z_u^i) \leq c$, where c is arbitrarily small.

The ability of our algorithm to solve the consumer's problem relies on the strict concavity of our utility function (note that monotonic transformations of the utility function will not affect the consistency of our algorithm). By totally differentiating (12), one can show that strict concavity implies $\partial x_j / \partial z \geq 0 \ \forall j$. This inequality, in conjunction with the fact that our model has a unique solution, $(\mathbf{x}^*, z^*)$, suggests that if z_a^i is less than or equal to z^*, the implied solutions for the quality differentiated goods, $\mathbf{x}^i$, will be less than or equal to their optimal solutions, and, by implication, $\tilde{z}^i \geq z_a^i$. Conversely, if $z_a^i \geq z^*$, each element of $\mathbf{x}^i$ will be greater than or equal to its optimal solution, and, by implication, $\tilde{z}^i \leq z_a^i$. These relationships suggest that updating the upper and lower bounds using the criteria stipulated in step 4 and iterating will solve for $(\mathbf{x}^*, z^*)$ to any level of precision. Plugging these optimal solutions into (7) allows the analyst to evaluate the consumer's utility conditional on $(\mathbf{p}, \mathbf{Q}, y, \boldsymbol{\varepsilon})$. Nesting this algorithm for solving the consumer's problem within a numerical bisection routine that iteratively solves for the income compensation that equates utility before and after the price and quality change will allow the analyst to construct the Hicksian consumer surplus, CS^{H}.

Additional insight into how our algorithm for solving the consumer's problem works is provided graphically in Figure 1.

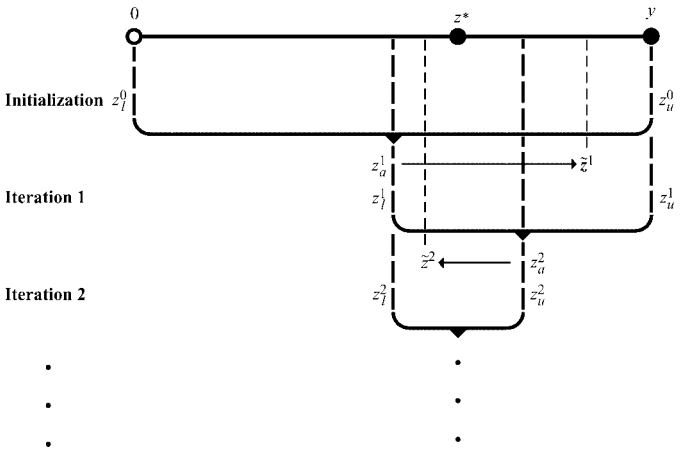

Figure 1. Solving the Consumer's Problem.

The real line from $(0, y]$ represents the feasible set of values that z^* can take. For illustration assume that, unknown to the analyst, z^* is located as shown in Figure 1. To initialize our numerical bisection algorithm, we set $z_l^0 = 0$ and $z_h^0 = y$. At the first iteration we insert the average of z_l^0 and z_h^0 (i.e., z_a^1) into (12)–(14) to solve for the implied demands, $\mathbf{x}^1$. Using the budget constraint, we solve for $\tilde{z}^1$. Because z_a^1 is less than z^*, the curvature properties of (12) imply that $\tilde{z}^1$ will be greater than z_a^1, and, thus, z_a^1 can be used to update the lower bound ($z_l^1 = z_a^1$). Keeping the upper bound estimate the same ($z_u^1 = z_u^0$), this routine can be iterated a second time where it is found that $z_a^2 > \tilde{z}^2$. As a result the upper bound estimate is updated in the second iteration with z_a^2 whereas the lower bound remains unchanged (i.e., $z_u^2 = z_a^2$ and $z_l^2 = z_l^1$). This numerical bisection procedure can be used to solve for $(\mathbf{x}^*, z^*)$ to any level of precision by replicating these basic steps.

4.2 Efficient Simulation of the Unobserved Heterogeneity

The algorithm described in the previous section permits construction of welfare measures conditional on a set of unobserved heterogeneity values. As discussed in Section 2, a precise estimate of the individual's Hicksian consumer surplus is not possible because the elements of $\boldsymbol{\varepsilon}$ are not observed. However, using simulation techniques and the distribution of the unobserved heterogeneity, the analyst can construct estimates of CS^{H} such as its expectation, $E(CS^{\mathrm{H}})$. The approach taken by PKH to simulating the unobserved heterogeneity employs the full distributional support of $\boldsymbol{\varepsilon}$. In this article, we build on an approach suggested by von Haefen (2003) and instead simulate the unobserved heterogeneity from the region of the unobserved heterogeneity's support that is consistent with the individual's observed choice. In other words, we simulate $\boldsymbol{\varepsilon}$ such that at baseline conditions our model perfectly predicts the observed choices we find in our data, and we use the model's structure of substitution to predict how individuals respond to price, quality, and income changes. This approach contrasts with PKH's more traditional approach that uses the structure of the model to predict both what individuals do at baseline conditions as well as how they respond to price, quality, and income changes. For the purposes of our application in this

article, incorporating observed choice is appealing because, although it requires the use of more complicated simulation techniques, it greatly reduces the number of simulations required to produce a precise estimate of $E(CS^{\mathrm{H}})$ as well as the computational time involved. Based on some Monte Carlo experiments with low-dimensional choice sets, we found that, relative to the unconditional approach, the conditional approach to welfare measurement required roughly one-third the simulations and time to produce mean welfare estimates that were insensitive to additional simulations.

We follow von Haefen and use a sequential strategy to simulate the unobserved heterogeneity consistent with the individual's observed choice. Note that our objective is to simulate from the distribution of $\boldsymbol{\varepsilon}$ conditional on $\mathbf{x}$, $f(\boldsymbol{\varepsilon}|\mathbf{x})$. This distribution can be decomposed as follows:

$$f(\boldsymbol{\varepsilon}|\mathbf{x}) = f_1(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta|\mathbf{x}) f_2(\varepsilon_1, \ldots, \varepsilon_M|\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta, \mathbf{x}). \tag{16}$$

Equation (16) states that the joint conditional distribution for the unobserved heterogeneity can be decomposed into the marginal distribution for the random parameters conditional on $\mathbf{x}$ multiplied by the conditional distribution for the site-specific unobserved heterogeneity conditional on $\mathbf{x}$ and the random parameters. We use an adaptive Metropolis–Hastings algorithm (Chib and Greenberg 1995) tailored to our problem to simulate from $f_1(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta|\mathbf{x})$. The steps of the algorithm are as follows:

1. At iteration i simulate a candidate vector of unobserved heterogeneity, $(\tilde{\boldsymbol{\varepsilon}}_\delta^i, \tilde{\boldsymbol{\varepsilon}}_\zeta^i)$, from the normal distribution with location parameters $(\boldsymbol{\varepsilon}_\delta^{i-1}, \boldsymbol{\varepsilon}_\zeta^{i-1})$ and scale parameters $(r^{i-1}\boldsymbol{\sigma}_\delta, r^{i-1}\boldsymbol{\sigma}_\zeta)$ where r^{i-1} is a constant. To initialize the process, set each element of $(\boldsymbol{\varepsilon}_\delta^0, \boldsymbol{\varepsilon}_\zeta^0)$ equal to 0 and r^0 equal to .1.
2. Construct the following statistic:

$$\chi^i = \frac{N(\tilde{\boldsymbol{\varepsilon}}_\delta^i/\boldsymbol{\sigma}_\delta, \tilde{\boldsymbol{\varepsilon}}_\zeta^i/\boldsymbol{\sigma}_\zeta) l(\mathbf{x}|\tilde{\boldsymbol{\varepsilon}}_\delta^i, \tilde{\boldsymbol{\varepsilon}}_\zeta^i)}{N(\boldsymbol{\varepsilon}_\delta^{i-1}/\boldsymbol{\sigma}_\delta, \boldsymbol{\varepsilon}_\zeta^{i-1}/\boldsymbol{\sigma}_\zeta) l(\mathbf{x}|\boldsymbol{\varepsilon}_\delta^{i-1}, \boldsymbol{\varepsilon}_\zeta^{i-1})}, \tag{17}$$

   where $N(\cdot)$ is the probability density function for the normal distribution and $l(\cdot)$ is defined in (10). If $\chi^i \geq U$ where U is a uniform random draw, accept the candidate random parameters, that is, $(\boldsymbol{\varepsilon}_\delta^i, \boldsymbol{\varepsilon}_\zeta^i) = (\tilde{\boldsymbol{\varepsilon}}_\delta^i, \tilde{\boldsymbol{\varepsilon}}_\zeta^i)$. Otherwise, set $(\boldsymbol{\varepsilon}_\delta^i, \boldsymbol{\varepsilon}_\zeta^i) = (\boldsymbol{\varepsilon}_\delta^{i-1}, \boldsymbol{\varepsilon}_\zeta^{i-1})$.
3. Gelman, Carlin, Stern, and Rubin (1995) argued that the Metropolis–Hastings algorithm for the normal distribution is most efficient if the acceptance rate of candidate parameters is between .23 and .44. Accordingly, we employ the following updating rule for r^i. If the proportion of accepted candidate parameters is less than .3, set $r^i = 1.1r^{i-1}$. Otherwise, set $r^i = .9r^{i-1}$.
4. Iterate.

After a burn-in period, this Monte Carlo Markov chain simulator generates draws from $f_1(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta|\mathbf{x})$ that can be used to construct $E(CS^{\mathrm{H}})$.

After values of $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$ are simulated, drawing from $f_2(\varepsilon_1, \ldots, \varepsilon_M|\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta, \mathbf{x})$ is far simpler. If good j is consumed in a strictly positive quantity, the structure of our model, the simulated random parameters, and the individual's observed choice imply that $\varepsilon_j^i = g_j(\boldsymbol{\varepsilon}_\delta^i, \boldsymbol{\varepsilon}_\zeta^i)$, where $g_j(\cdot)$ is the right side of (9). Otherwise, ε_j^i can be simulated from the truncated Type I extreme value distribution via

$$\varepsilon_j^i = -\ln\big(-\ln\big(\exp\big(-\exp(-g_j(\boldsymbol{\varepsilon}_\delta^i, \boldsymbol{\varepsilon}_\zeta^i)/\mu)\big)U\big)\big)\mu, \tag{18}$$

where U again is a uniform random draw.

4.3 Summary

Before proceeding to the data summary and empirical results, we summarize the key components of our welfare measurement algorithm:

1. For simulation i use the sequential procedure described in the previous section to simulate $\boldsymbol{\varepsilon}$ consistent with the individual's observed choice. Because simulating from $f_1(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta|\mathbf{x})$ requires the use of an adaptive Metropolis–Hastings algorithm, discard the first T simulations.
2. For every jth simulation after the first T, use a numerical bisection routine to solve for the simulated Hicksian consumer surplus associated with a price and quality change. j is generally set to some value greater than 1 to reduce the impact of serial correlation in the Markov chain sequence for $(\boldsymbol{\varepsilon}_\delta, \boldsymbol{\varepsilon}_\zeta)$.

2a. At each step of the numerical bisection routine that solves for the Hicksian consumer surplus associated with a given simulation, use the numerical bisection routine described in Section 4.1 to solve the consumer's problem. Inserting these optimal values into (7) permits the analyst to determine the utility the individual achieves.

3. Average each of the simulated Hicksian consumer surplus values to construct an estimate of $E(CS^{\mathrm{H}})$, the individual's expected Hicksian consumer surplus.

Although our algorithm for estimating welfare measures has multiple layers and numerous details, our experience has been that it is surprisingly easy and fast to use in an applied setting. One of the algorithm's most appealing attributes is that each of its steps can be executed simultaneously for every observation in the sample using vector and matrix notation. This feature implies that coding and executing the algorithm in a matrix programming language reduces the computational burden significantly.

5. APPLICATION

We apply our demand system framework to a random sample of Delaware residents' recreational day trips to Mid-Atlantic ocean beaches in 1997. From a policy perspective understanding beach recreation demand is important for at least two reasons. Oil and toxic spills in coastal waters often result in beach closings. Under the Oil Pollution Act of 1990 and other Natural Resource Damage Assessment (NRDA) statutes, the public's lost economic benefits arising from these spills are compensable. Deacon and Kolstad (2000) discussed issues associated with estimating these losses and argued that revealed preference recreation demand models are a preferred method for ascertaining beach users' resource values. Furthermore, understanding how characteristics such as beach width influence the demand for beach recreation can help inform the ongoing debate over beach nourishment as a strategy for combating

coastal erosion. Beach nourishment, or pumping imported sand onto an eroding beach, is a technically feasible but costly engineering approach for maintaining coastal areas threatened by erosion from rising sea levels, development, and natural causes. At present, substantial state and federal resources have been earmarked for these activities in response to a perceived need to protect tourism and recreation-related infrastructure. The literature reviews presented in Massey (2001) and Parsons and Massey (2003) suggest that environmental economists' understanding of the recreational values associated with policies that impact beach erosion and availability are in many ways incomplete and outdated.

To assess the recreational values of beach amenities and to gauge residents' willingness to pay for beach quality improvements such as nourishment, researchers at the University of Delaware collected data on visits by Delaware residents to 62 ocean beaches in New Jersey, Delaware, Maryland, and Virginia during 1997. Figure 2 shows a map of the region and several of the major beaches included in the dataset. Massey (2001) discussed the data collection effort in detail. A mail survey of 1,000 Delaware residents selected via stratified random sampling resulted in 540 completed responses. Although respondents provided information on the number of day trips, multiday trips, and side trips made to each of the 62 beaches, we follow conventional empirical practice in the recreation literature and consider only day trips in our analysis. Excluding multiday and side trips from our analysis implies that our aggregate welfare estimates are incomplete measures of the total value of beach recreation. The exclusion does, however, allow us to avoid the difficult modeling issues arising when on-site time at a beach is a separate argument of the individual's utility function and when recreation trips involve multiple activities and jointly produce multiple service flows. For further discussion of these and other important issues pertaining to recreation demand modeling, see Phaneuf and Smith (2003).

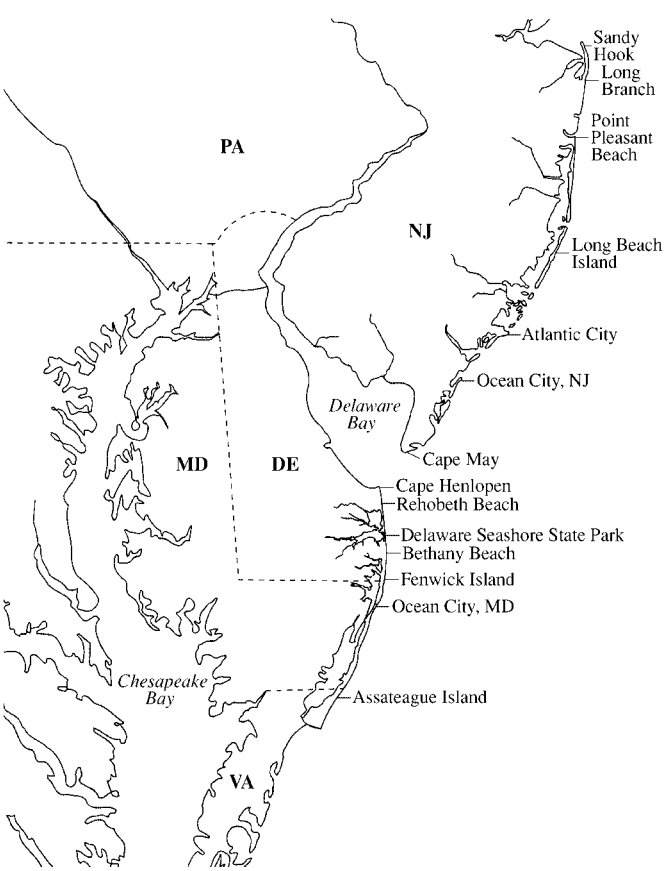

Figure 2. Mid-Atlantic Application Region.

The survey collected sufficient information to compute round trip travel costs to each of the beaches. For every individual in the sample, a beach's price is assumed to consist of transit costs (valued at \$.35 per mile), beach fees, highway tolls, parking fees, a ferry toll on trips from southern Delaware to New Jersey beaches, and the opportunity cost of travel time valued at the individual's average wage rate. Distances and travel times to each of the 62 sites from each of 540 residents' homes were calculated using PC Miler. The wage rate was estimated as the individual's annual income divided by 2,040 (i.e., the typical number of hours worked in a year).

Household characteristic and demographic data were also collected and are used to parameterize our utility function. Household-specific variables and recreational summary statistics are listed and described in the top part of Table 1. Note that respondents took on average 9.77 trips and visited 2.77 different beaches during the season. The maximum number of beaches visited by a respondent was 19, and 165 people surveyed (30% of the sample) did not visit any beaches during the season.

In addition to the behavioral data, auxiliary information on the characteristics of the 62 beaches was also gathered. Fourteen variables are used to differentiate the beaches included in the study. These variables are listed and described in the lower part of Table 1. Of interest for policy purposes are the indicator variables for wide and narrow beach. Of the 62 beaches, 25% are wide (i.e., greater than 200 feet in width) and 14% are narrow (less than 75 feet in width). The impacts of the wide and narrow dummy variables on the demand for beach visits allow us to gauge the effects of beach width on recreation demand and to assess the welfare implications of policies designed to alter beach width such as beach nourishment.

6. PARAMETER ESTIMATES

Although we estimated several demand system specifications consistent with the generic structure described in Section 3, we only present a representative set of our findings in this section. Table 2 contains estimates for two fixed parameter models (i.e., $\boldsymbol{\varepsilon}_\delta = \boldsymbol{\varepsilon}_\varsigma = \mathbf{0}$) nested in (7): a translated constant elasticity of substitution (CES) specification (column 2) resulting from the restrictions $\rho_z = \rho_j = \rho,\ \forall j$ and $\theta_j = \theta,\ \forall j$, and a second specification (column 3) resulting from the restrictions $\rho_j = 0,\ \forall j$ and $\theta_j = \theta,\ \forall j$. Table 3 contains estimates for more general random parameter versions of these specifications that allow the parameters in the Ψ index to vary randomly across the population. Although the parameter estimates in Table 2 are estimated with conventional maximum likelihood techniques, the parameter estimates in Table 3 are obtained via maximum simulated likelihood procedures. To improve simulation efficiency, we follow common empirical practice in the discrete choice literature and employ Halton draws rather than random draws in the calculation of our simulated

Table 1. Household and Beach Characteristics Variables

Variable	*Description*	*Summary*[a]
Household-specific variables		
ln(age)	Natural log of respondent age	3.82 (.33)
Kids under 10	Respondent has kids under 10 (0/1)	.26
Kids 10–16	Respondent has kids between 10 and 16 (0/1)	.20
Vacation property in DE	Respondent owns vacation home in DE (0/1)	.03
Retired	Respondent is retired (0/1)	.24
Student	Respondent is student (0/1)	.05
Income	Household annual income	49,944 (30,295)
Trips	Total visits for day trips to all sites	9.77 (14.06)
Sites visited	Number of beaches visited during 1997	2.70 (3.19)
Site characteristics[b]		
Beach length	Length of beach in miles	.62 (.87)
Boardwalk	Boardwalk with shops and attractions (0/1)	.40
Amusements	Amusement park near beach (0/1)	.13
Private/limited access	Access limited (0/1)	.25
Park	State or federal park or wildlife refuge (0/1)	.09
Wide beach	Beach is more than 200 feet wide (0/1)	.25
Narrow beach	Beach is less than 75 feet wide (0/1)	.14
Atlantic City	Atlantic City indicator (0/1)	.016
Surfing	Recognized as good surfing location (0/1)	.35
High rise	Highly developed beach front (0/1)	.24
Park within	Part of the beach is a park area (0/1)	.14
Facility	Bathrooms available (0/1)	.48
Parking	Public parking available (0/1)	.45
New Jersey beach	New Jersey beach indicator (0/1)	.74
Price	Person-specific money and time cost of travel	$118[c]

[a]Summary statistics for household variables are means (standard errors) over the 540 individuals. Summary statistics for site variables are means (standard errors) over the 62 sites.

[b]We thank Tony Pratt and Michael Powell of the Department of Natural Resources and Environmental Control and Steve Hafner of the Coastal Research Center at Richard Stockton College of New Jersey for their help in compiling and verifying the characteristics data.

[c]This statistic is the mean (standard error) of each individual's mean trip price. Each individual in the sample has a unique price associated with visiting each of the 62 sites. Because prices are functions of distance, there is substantial variability in site prices both for and across individuals.

*Table 2. Kuhn–Tucker Fixed Parameter Estimates**

Parameter	*#1 Translated CES*	*#2*
Log-likelihood	−7,837.40	−7,676.21
Ψ_i index parameters		
Constant	−3.8169 (−3.480)	2.5506 (2.090)
ln(age)	−.5822 (−2.142)	−.5356 (−2.137)
Kids under 10	.0593 (.463)	.0686 (.597)
Kids 10–16	.1996 (1.376)	.1960 (1.465)
Vacation property in DE	.7924 (3.696)	.9318 (4.700)
Retired	.3611 (1.787)	.0566 (.301)
Student	.4328 (2.140)	.2906 (1.549)
Park	−.0589 (−.744)	−.0671 (−.861)
New Jersey beach	−1.4401 (−8.900)	−1.2213 (−8.788)
Translating parameter		
θ	3.5132 (9.693)	4.9187 (10.963)
Simple repackaging quality index parameters		
Beach length (miles)	.0726 (1.843)	.0764 (2.029)
Boardwalk	−.2199 (−2.670)	−.2020 (−2.575)
Amusements	.8568 (9.178)	.7551 (8.641)
Private/limited access	−.6099 (−5.739)	−.5750 (−5.815)
Wide beach	−.2286 (−2.999)	−.1901 (−2.658)
Narrow beach	−.4088 (−3.141)	−.3729 (−3.021)
Atlantic City	.7657 (7.100)	.6994 (7.208)
Surfing	.1805 (2.235)	.1554 (2.000)
High rise	.1206 (1.415)	.1331 (1.667)
Park within	.5775 (6.220)	.5256 (5.943)
Facility	−.0347 (−.582)	−.0275 (−.440)
Parking	.4996 (5.126)	.4667 (4.706)
Rho parameters		
$\ln \rho$	−1.6185 (−6.189)	—
$\ln \rho_z$	—	−.3063 (−3.104)
Type I extreme value scale parameter		
μ	.8759 (19.241)	.8080 (22.865)

* *t*-statistics based on robust standard errors in parentheses.

*Table 3. Kuhn–Tucker Random Parameter Estimates**

Parameter	#1 Translated CES		#2	
Log-likelihood	−6,825.98		−6,777.70	
Ψ_i *index parameters*	*Mean*	*SD*	*Mean*	*SD*
Constant	−2.5630 (−5.457)	—	3.1698 (4.453)	—
ln(age)	−.6421 (−5.800)	.2396 (16.975)	−.6478 (−5.242)	.1994 (22.493)
Kids under 10	.0805 (1.137)	.0107 (.142)	.0588 (.818)	.0306 (.609)
Kids 10–16	.2688 (3.873)	.1403 (2.734)	.1189 (1.535)	.0564 (1.082)
Vacation property in DE	.6985 (5.528)	.3650 (6.264)	.9131 (11.170)	.0668 (4.119)
Retired	.2993 (3.725)	.1034 (4.976)	−.0031 (−.034)	.1534 (4.100)
Student	.2605 (2.664)	.0746 (1.210)	.3026 (3.505)	.1210 (4.542)
Park	−.0758 (−1.247)	.3578 (7.462)	−.0870 (−1.469)	.3416 (6.606)
New Jersey beach	−.9097 (−9.058)	.9890 (16.686)	−.9747 (−8.317)	.9177 (9.550)
Translating parameter				
θ	6.0661 (14.353)		7.4352 (15.237)	
Simple repackaging quality index parameters				
Beach length (miles)	.0813 (3.648)		.0795 (3.409)	
Boardwalk	−.0366 (−.687)		−.0540 (−.999)	
Amusements	.5880 (9.146)		.6007 (9.348)	
Private/limited access	−.3743 (−6.794)		−.3880 (−6.753)	
Wide beach	−.1575 (−3.195)		−.1564 (−3.072)	
Narrow beach	−.2343 (−3.244)		−.2428 (−3.233)	
Atlantic City	.5115 (5.859)		.5419 (5.979)	
Surfing	.0983 (1.946)		.1034 (1.991)	
High rise	−.0404 (−.758)		−.0247 (−.452)	
Park within	.3718 (5.663)		.3876 (5.787)	
Facility	−.0086 (−.268)		−.0123 (−.340)	
Parking	.2396 (3.737)		.2625 (4.011)	
Rho Parameters				
$\ln \rho$	−1.7876 (−7.798)		—	
$\ln \rho_z$	—		−.4200 (−6.831)	
Type I extreme value scale parameter				
μ	.3817 (22.421)		.4013 (23.432)	

* t-statistics based on robust standard errors in parentheses. Simulated probabilities computed using 500 Halton draws.

probabilities. Train (2003) demonstrated that Halton draws outperform random draws in terms of the number of simulations needed to achieve an arbitrary level of precision for many random parameter models.

In general, we find statistically significant, plausibly signed, and robust coefficient estimates for the four models. For example, we find that age negatively impacts trips to all destinations, whereas ownership of vacation property in Delaware is positively related to increased beach visitation. Students as well as respondents with children also tend to take more trips. In addition, our results suggest that several site characteristics are significant determinants of choice. Of particular interest for policy purposes are the signs on the narrow and wide beach dummy variables. We find for all four models negative and statistically significant coefficient estimates for both of these variables, suggesting that respondents prefer beaches of moderate width *ceteris paribus*. This finding is consistent with the empirical findings of Parsons, Massey, and Tomasi (1999) and suggests that individuals dislike narrow beaches with limited available recreation area and wide beaches that require long walks to the waterfront.

For both specifications the fixed parameter models can be interpreted as restricted versions of the random parameter models. As a result we compare their log-likelihood values to gauge the improvement in statistical fit arising from the inclusion of random parameters. Although formal statistical testing of the fixed versus random parameter specifications is confounded by the fact that the fixed parameter models constrain the random coefficient standard errors to their lower bound values (0), we find in both cases substantial increases in log-likelihood values with the addition of random parameters. These empirical results suggest the importance in this application of allowing for additional unobserved heterogeneity in the characterization of preferences. In addition, formal statistical tests of the translated CES versus second specification are confounded because the models are not nested. However, because both models are restricted versions of the same more general specification, we use Pollak and Wales' (1991) likelihood dominance criteria to evaluate their comparative fits. For both fixed and random parameter specifications, the log-likelihood values suggest that the second specification fits the data better, implying that on statistical grounds our second random parameter specification provides the better characterization of preferences in this application.

7. WELFARE ANALYSIS

7.1 Individual Welfare Estimates

The parameter estimates allow welfare analysis for our beach application employing the methods described in Section 4. We analyze three scenarios designed to provide different types of valuation information for beach recreation and to provide a demonstration of the feasibility of our algorithm for welfare measurement. Two pertain to the recreation value lost when beaches close, and the third addresses the recreational losses

associated with beach erosion. The specific scenarios analyzed include:

- closing of Rehoboth Beach
- closing of northern Delaware beaches
- lost beach width at all Delaware, Maryland, and Virginia (DE/MD/VA) developed beaches

These three scenarios have policy relevance for the Mid-Atlantic region. Oil tankers enter the Delaware Bay regularly and pass near the most frequently visited ocean beaches in the state. The possibility of a spill and consequent closure of beaches, especially along the northernmost beaches from Cape Henlopen State Park to the Delaware Seashore State Park (see Fig. 2), is widely recognized. For example, had the oil spill by the *Presidente Rivera* in 1989 in the Delaware Bay occurred farther south at the mouth of the bay, one or more ocean beaches in Delaware may have been closed. In our analysis the first two scenarios simulate the welfare loss that might be associated with such a spill. We consider the closure of Rehoboth Beach, located along the northern Delaware coast, because it is the most visited beach in the state. We also consider the closure of all beaches from Cape Henlopen State Park to the Delaware Seashore State Park. These are 7 of the 62 beaches in our analysis and comprise the 11 northernmost miles of Delaware's 25 miles of ocean beaches. In our judgment these beaches are most likely to experience the effects of a spill.

Our third scenario pertains to beach erosion on the DE/MD/VA beaches, a major policy issue in the region. For more than 20 years, the three states have pumped sand onto their beaches to maintain beach width in support of recreation uses. Using our estimated model, we simulate the welfare loss associated with all developed (i.e., nonpark) beaches in the DE/MD/VA area eroding to a width of 75 feet or less. This will affect most of the popular beaches in the area. Although more severe erosion is possible, the scenario we consider is both plausible and within the range of our data. The losses arising from this scenario approximate the recreation gains associated with a nourishment project in the region, assuming that full beach migration is not a policy option. In our scenario all natural (park) beaches are assumed to maintain their current width. These beaches tend to migrate inland naturally and maintain width.

Point estimates and standard errors for the three welfare scenarios (1997 dollars per respondent per season) are presented in Table 4. Columns 2 and 3 present the translated CES and second specification estimates, respectively. To evaluate whether our demand system models generate qualitatively different policy inference from the discrete choice RUM-based strategies that dominate current empirical practice, column 4 presents welfare estimates from repeated discrete choice RUM models (e.g., Morey et. al. 1993) that are generated by the conditional welfare measurement procedure outlined in von Haefen (2003). The repeated discrete choice models assume (1) the recreation season can be decomposed into 75 separable choice occasions; (2) each individual on each choice occasion makes a discrete choice among the 62 beaches and an option not to recreate; (3) preferences for a beach (and the outside alternative) are a linear additive index of its price, attributes (a constant and demographic variables), and an i.i.d. Type I extreme value random draw; and (4) a subset of the parameters entering the indexes is statistically independent and normally distributed across the population. A table with parameter estimates for the specifications we consider in this paper is available from the authors upon request.

For each scenario fixed and random parameter model estimates are presented together for comparison purposes. In all cases the estimates have plausible magnitudes and standard errors that suggest statistically significant differences from 0. Additionally, a few general patterns emerge. Perhaps the most striking is that the inclusion of random coefficients decreases the magnitude of the estimated welfare effects for all models and all scenarios. This is consistent with some empirical findings in other random parameter applications (e.g., Petrin 2002; Phaneuf, Kling, and Herriges 1998) and suggests that the additional unobserved heterogeneity accounted for by random parameter models allows for a greater degree of substitutability among goods and, as a result, smaller (absolute) economic values. Next, in spite of the statistical dominance of the second specification over the translated CES specification, both demand system specifications provide qualitatively

Table 4. Mean Seasonal Welfare Estimates (1997 dollars)[a]

Scenario	*#1 Translated CES*	*#2*	*Repeated discrete choice*
Closing of Rehoboth Beach			
Fixed coefficients	−$73.36 (7.48)	−$82.93 (4.17)	−$59.18 (.81)
Random coefficients[b,c]	−$49.77 (3.57)	−$57.32 (2.59)	−$26.61 (.74)
Closing of Northern Delaware beaches			
Fixed coefficients	−$152.47 (13.47)	−$151.24 (7.04)	−$134.64 (1.83)
Random coefficients[b,c]	−$102.99 (7.55)	−$106.24 (4.45)	−$74.18 (1.61)
Lost beach width at all DE/MD/VA developed beaches			
Fixed coefficients	−$57.28 (14.63)	−$53.92 (14.88)	−$55.11 (4.80)
Random coefficients[b,c]	−$33.75 (8.61)	−$35.16 (8.54)	−$23.95 (2.60)

[a] Robust standard errors based on 200 Krinsky and Robb (1986) simulations in parentheses. All estimates employ the sampling weights implied by the county-stratified sampling design.

[b] The fixed coefficient Kuhn–Tucker welfare estimates were constructed with 25 simulations per observation. For the random coefficient Kuhn–Tucker welfare estimates, a total of 2,000 simulations were generated. The first 1,000 simulations were discarded as burn-in and every 10th simulation thereafter was used to construct the estimates.

[c] The fixed coefficient repeated discrete choice welfare estimates were constructed with 2,500 simulations per observation. For the random coefficient discrete choice welfare estimates, a total of 13,500 simulations were generated. The first 1,000 simulations were discarded as burn-in and every fifth simulation thereafter was used to construct the estimates.

similar welfare measures for both the fixed and the random parameter versions. Finally, our results suggest that the repeated discrete choice model generally produces welfare estimates that are smaller in absolute value. This result may in part reflect the greater cross-site substitution patterns that can be characterized within the discrete choice framework (particularly in the random parameter case) relative to our additively separable demand system model. However, this finding should be corroborated with results from other datasets and policy applications before general conclusions are drawn with confidence.

7.2 Aggregate Welfare Estimates

The per person welfare measures can be used to construct aggregate estimates of the welfare impacts of the policies considered. The survey was designed to allow inference for the 576,246 Delaware residents 16 years and older in 1997. Because the mean estimates in Table 4 incorporate the sampling weights implied by the stratified random sampling design, multiplying the per person estimates by the population provides an estimate of the population welfare impact. These aggregate losses can be quite large for the scenarios considered. For example, our preferred demand system model (specification 2 with random parameters) suggests an aggregate welfare loss of $33.1 million for Delaware residents for the closure of Rehoboth Beach for one season, and losses of $61.3 million for the closure of all northern beaches. The erosion losses are also substantial, estimated at $20.1 million per season for lost beach width.

Two caveats should be attached to these population welfare estimates. We have assumed that nonresponse to the survey instrument was randomly assigned throughout the Delaware population. If in fact beach visitors were more likely to respond to the survey than nonvisitors, the aggregate estimates are likely biased upward. In addition, these aggregate welfare measures do not reflect the losses accruing to multiday and side trip users as well as nonresidents of Delaware. For a full accounting of the total welfare impact of the proposed policies to all beach users, the welfare impacts to these other groups should be added to the estimates presented here.

8. CONCLUSION

Our general conclusion from the research presented in this article is that the demand system framework represents a viable strategy for modeling consumer choice and generating Hicksian welfare measures for individual or household level applications with many quality-differentiated goods. Using Monte Carlo estimation and welfare construction procedures, we present empirical results from a beach recreation application that demonstrate how this can be accomplished. Our methodological approach and empirical findings suggest that the demand system framework, which fully integrates the intensive and extensive margins of consumer choice within a behaviorally consistent framework, represents a genuine alternative to the RUM-based strategies that dominate current empirical practice.

Numerous extensions to our approach are possible, and we discuss two in closing. Relaxing the assumption that preferences are additively separable represents a significant area for future work. From a computational perspective relaxing additive separability without jeopardizing the demand system framework's tractability in estimation and welfare construction for applications with many goods represents a formidable task. However, our sense is that progress in this area is possible. In addition, our representation of the consumer's problem assumes that all Delaware residents consider all 62 beaches in the Mid-Atlantic region. Introspection, however, suggests that it is doubtful that all individuals seriously consider all beaches. In the marketing literature discrete choice statistical models have been developed that build on Manski's (1977) original formulation of the individual's choice process and formally model the individual's first-stage formulation of his or her "consideration set" as well as his or her second-stage conditional choice (e.g., Ben-Akiva and Boccara 1995). Augmenting the demand system framework developed in this article with a model of the individual's consideration set formulation process would represent an innovation that may more accurately represent how individuals make choices.

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