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Multiscale characterization of materials with distributed pores and inclusions and application to crack formation in an aluminum alloy

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Abstract

The paper reports a study on the mechanical behavior of materials containing pores and inclusions distributed over a wide range of length scales. Utilizing a wavelet-based multiscale process such microstructures are characterized and their effects on material properties is studied. In order to present the process in a semi-analytical fashion, the variance of the strain field for an approximated one-dimensional deformation problem is examined in detail. It is shown that with respect to crack initiation, there is a strong interplay between the distribution of pores and inclusions. Furthermore, their interaction with boundaries proves to be paramount. The process is applied to a particular cast aluminum alloy where pores are, in general, about two orders of magnitude larger than the silicon particles (inclusions). Results agree well with recent experimental reports on crack initiation where the interplay of pores, inclusions, and boundaries is observed, yet not explained on a fundamental basis. The present work extends recent efforts on porous materials [Frantziskonis G. Wavelet-based analysis of multiscale phenomena—application to material porosity and identification of dominant scales. Prob Eng Mech (this issue). PII: S0266-8920(2)00032-2] to include the interaction of pores at certain scales with inclusions at other scales as well as the interaction of both with boundaries. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Multiscale; Porosity; Inclusions; Wavelets; Compound matrix

1. Introduction

In materials processing, design, and applications to structures and structural components, it is critical to comprehend the effect of microstructures on properties. Even though a large number of such studies have been, and are continuously being, reported in the literature, very few deal with the fact that microstructures of (most) engineering materials appear at a wide hierarchy of spatial scales. It is difficulties in dealing with such multiscale problems that has hindered such studies. For several materials, important microstructures are in the form of inclusions as well as pores appearing at a hierarchy of scales. Of particular interest to this contribution are cast aluminum alloys, which have great potential as replacements for ferrous castings and fabrications in the automobile and rail industries where energy efficiency and low cost component production are crucial; also, the application of aluminum castings in aerospace structures can result in significant cost reductions in aircraft manufacturing. Detailed information on the present use, future potential, and studies on properties of such materials can be found mostly in the materials science and engineering literature, which can be traced through [2,3]. A complete understanding of the properties of such materials is still lacking, mostly from difficulties in characterizing the microstructures as they appear over a wide range of spatial scales. Here we examine two particular microstructures, i.e. pores and inclusions, known to be related to mechanical properties in aluminum castings such as A356 [2], and attempt to characterize them using a wavelet-based multiscale approach. We concentrate on crack initiation under imposed external mechanical load, and for illustrating the wavelet-based multiscale process we study a 1D problem, which, of course, involves certain simplifying assumptions. The 1D results set the foundation for 2D as well as 3D studies, which are presently undergoing.

In a related work [1], a wavelet-based multiscale material characterization framework was proposed and examined with respect to porous media. The work presented herein is a natural, yet by no means straightforward, extension of Ref. [1], thus, at the expense of frequent cross-referencing it, we do not repeat some of the relevant information appearing therein. Thus, a brief review of multiscale approaches in material characterization and a brief introduction to wavelet analysis is not repeated herein. The same holds for some
relevant mathematical derivations detailed in Ref. [1]. With this in mind, the rest of the paper is organized as follows. Section 2 sets up the basis for characterizing the multiscale microstructures through their spatial statistics, and Section 3 presents experimental information obtained from actual micrographs of an A356 alloy. Section 4 presents the study of the variance of the strain field for a uniaxial load problem, and identifies the role of each microstructure. Section 5 shows the importance of the interaction of microstructures with boundaries through an analytical example. Section 6 compares all the semi-analytical results to actual experiments on A356 and is followed by conclusions.

2. Statistical properties of a medium with pores and inclusions

Let us consider a material containing both pores and inclusions embedded in a matrix. We consider each phase, i.e. matrix, inclusions and pores, to be homogeneous (no microstructure). Let $F$ be a specific local property, e.g. Young’s modulus, yield stress, failure stress, etc. Within the material, $F = F(x)$ is a function of the position vector $x$ ($x$ is a scalar in 1D) and has a value equal to $m$, $i$, $p$ denoting the value of $F$ for spatial positions within the matrix, inclusions, pores, respectively. For random spatial distribution of the pores and of the inclusions, $F$ is a random function of space. Pioneer works [4,5] for porous media set up an elegant framework for describing the geometry. The material is considered as a composite one, in the terminology of Ref. [6], consisting of a matrix with embedded pores and matrix with embedded inclusions. For convenience, we shall refer to the former as medium-P and the latter as medium-I. Each component of the composite medium is built by placing a set of given arbitrary pores, inclusions, respectively, at random in space, while keeping the porosity, inclusion density, constant. Thus, each component forms a ‘perfectly random medium’ in the terminology of Debye et al. [4] or a medium constructed according to a Boolean scheme in the terminology of Matheron [5].

Let $\varphi_p$ denote the porosity and $\varphi_I$ the volume density of inclusions. The following relations hold, where subscripts P, I refer to medium-P, medium-I, respectively. Eqs. (1)–(4) can be directly derived from works on porous media [4,5] some of which can also be found in Ref. [1]. For the expected values, we have

$$\langle F_p(x) \rangle = p\varphi_p + m(1 - \varphi_p),$$

(1)

$$\langle F_I(x) \rangle = i\varphi_I + m(1 - \varphi_I)$$

while the variances are given by

$$\text{Var}[F_p(x)] = (m - p)^2 \varphi_p(1 - \varphi_p),$$

(2)

$$\text{Var}[F_I(x)] = (i - m)^2 \varphi_I(1 - \varphi_I)$$

Each of the components is considered stationary (and isotropic, even though this is irrelevant for the 1D case), thus the autocorrelation $\rho(r)$ is a function of the distance $r$ between two points in space. Each component of the composite medium is two-phase with exponential autocorrelation [1,4,5], thus for the autocorrelations $\rho_{F_p}$, $\rho_{F_I}$, of the matrix phase in both components, P, I, we have

$$\rho_{F_p}(r) = \exp(-rl_p), \quad \rho_{F_I}(r) = \exp(-rl_I)$$

(3)

where $l_p$, $l_I$ denote the autocorrelation distances of the matrix material for the medium-P and medium-I, respectively. The autocorrelation function of the matrix phase for each component is expressed as

$$l_p = \frac{4\varphi_p(1 - \varphi_p)}{s_p}, \quad l_I = \frac{4\varphi_I(1 - \varphi_I)}{s_I}$$

(4)

where $s_p, s_I$ denote the specific area of pores and inclusions, respectively, i.e. it is the ensemble average of the density of matrix-pores and matrix-inclusions interface elements, respectively. An illustrative example for, say, medium-I, can be created by placing at random disjoint spherical particles of radii $R$ of probability density distribution $f(R)$. We have, from the definition of $s_I$ through straightforward geometrical calculations [7], for $i > m$

$$s_I = \frac{3\varphi_I}{R} \int_0^\infty \frac{R^2 dR}{\int_0^\infty R^2 f(R) dR}$$

(5)

Eq. (5), for the special case of spheres of constant radius $R$, thus $f(R) = \delta(R)$, $\delta(\cdot)$ denoting the delta function, yields

$$s_I = \frac{3\varphi_I}{R}$$

(6)

and thus, from Eq. (4), it follows that

$$l_I = \frac{4}{3} (1 - \varphi_I) R$$

(7)

For lognormal distribution of the inclusions we have [7]

$$s_I = \frac{3\varphi_I}{\langle R \rangle} \left( \frac{\langle R \rangle^2}{\langle R^2 \rangle} \right), \quad l_I = \frac{4}{3} (1 - \varphi_I) \langle R \rangle \left( \frac{\langle R \rangle^2}{\langle R^2 \rangle} \right)^2$$

(8)

for the specific area and correlation distance, respectively.

In the case $i < m$ $\varphi$ has to be replaced by $1 - \varphi_I$ in Eq. (5) and in the ensuing relations.

For the composite medium, i.e. medium-P plus medium-I, we have [6]

$$\langle F \rangle = \langle F_p \rangle + \langle F_I \rangle$$

(9)

for the expected value of $F$ and

$$\text{Var}[F] = \text{Var}[F_p] + \text{Var}[F_I]$$

(10)

for its variance. The composite medium’s autocorrelation reads

$$\rho(r) = \lambda_p \rho_p(r) + \lambda_I \rho_I(r)$$

(11)
and its correlation distance \( l \) is expressed as
\[
I = \lambda_p l_p + \lambda_l l_l
\]  
(12)
where
\[
\lambda_p = \frac{\text{Var}[F_p]}{\text{Var}[F_p] + \text{Var}[F_l]},
\]
\[
\lambda_l = \frac{\text{Var}[F_l]}{\text{Var}[F_p] + \text{Var}[F_l]}
\]  
(13)
Given the above relations, we next investigate the statistical properties of an actual material with embedded pores and inclusions, i.e. a cast aluminum alloy, A356.2.

3. Application to an aluminum alloy

The mechanical properties of aluminum castings have been related to microstructural details such as pores and inclusions, cf. the extensive literature reviewed in Ref. [2]. The material chosen for the present application is a cast and heat-treated alloy, A356.2 (Al–7Si–0.3Mg), which can be produced at low cost and thus has potential for a much wider range of applications than those at present. Important microstructures of A356 at length scales in the range of 1 \( \mu m \) to a few millimeters are the silicon particles, the pores, and oxides. These microstructures have been found to correlate far better with mechanical properties than microstructures appearing at smaller length scales such as precipitates, intermetallic particles, etc. We concentrate at those scales, yet the process can be extended to smaller (and larger if appropriate) scales. As mentioned in Ref. [2] the role of oxides on crack initiation is similar to that of pores in this material, thus oxides may be included in the description of porosity. The size and distribution of the pores/oxides and of the silicon particles (inclusions) depend strongly on the processing; the cooling rate is one of the major processing parameters in this regard. Thus, as shown in Refs. [2,3,8] the pores as well as the inclusions in the material can vary in size and spatial distribution.

In the experiments reported in Ref. [8] and utilized also in Ref. [2] a mold with water-cooling at the bottom was used in order to obtain a variation of inclusions and pores with distance from the bottom in a cast ingot of A356.2 [8]. Thus, by considering specimens at various distances from the chill, various microstructures can be examined with respect to their contribution to mechanical properties of the material. In general, as the cooling rate decreases (distance from the chill increases) the size of pores increases with the porosity remaining approximately constant. The volume density of the inclusions also remains the same, yet their spatial distribution changes; the so-called secondary dendrite arm spacing (SDAS), increases as the cooling rate decreases and this indicates an increase in the spatial correlations of the matrix. Instead of SDAS, the pore size can be used to classify the crack initiation sites since SDAS and pores size are related [3]. For compactness we concentrate only on one type of microstructures, i.e. those observed at a distance of 140 mm from the chill; some images of such microstructures are shown below.

Fig. 1 shows a typical micrograph of the microstructure at 25 \( \times \) optical microscope magnification, for A356.2 with a cooling rate of 0.3 K s \(^{-1}\) (140 mm from the chill). At this cooling rate, the pores as well as the SDAS are rather large in size. The pores can be clearly seen, while the rest of the material appears homogeneous at this scale. It is essential to note that Fig. 1 is at a particular spatial position where a few pores can be seen. Images of the same size at other spatial positions may show less number of pores or no pores at all. This relates to the spatial distribution of the pores and is addressed further later.

At smaller scales different structures are observable. Fig. 2 shows images at various magnifications. Here, analysis at corresponding scales is applicable, as described below.

An effective way of studying (up to second order) spatial correlations is through the variance of increments of function \( F \), defined as
\[
V_F = \langle (F(x_2) - F(x_1))^2 \rangle
\]  
(14)
where subscripts 1, 2 imply two different positions in the \( x \) domain. Using Eq. (14) instead of the correlation function, which is related to \( V_p \), is advantageous when the variance of the process is unknown. For stationary \( F \), the following holds
\[
\frac{V_F(r)}{\text{Var}[F]} = 1 - \rho_p(r)
\]  
(15)
Thus, from the images shown in Fig. 2, with fixed values of gray scale in the matrix and in the inclusions, we evaluate the left-hand-side of Eq. (15). As expected, the images at higher magnifications may not be large enough for convergence of Eq. (15) (assuming ergodicity). Fig. 3 shows the variance of increments \( V_F(r)/\text{Var}[F] \) as a function of distance \( r \) between two points in space, obtained from the images of Fig. 2. Clearly, the variance of increments reaches a plateau for small magnifications, implying stationarity, but shows an erratic
behavior for large ones. This may be due to (a) the images at large magnifications are not large enough for convergence; (b) the structure is not statistically stationary. The latter case is not pursued further at this point, yet if the structure shows a self-affine character with an upper and a lower cut-off, the convergence at low magnifications could be explained [9]. We leave this issue for future study and in the following consider that the structure of the inclusions is statistically stationary.

The curves in Fig. 3 obtained from the 50× and 100× images fit an exponential curve of the type \(1 - \exp(-r/l_0)\) well; a best fit process has yielded \(l_0 = 8.5 \mu m\). From the images at large magnifications, it is noticeable that the spatial statistics of the medium are mostly due to small clusters of inclusions rather than from individual ones. In a sense, the medium can be considered as built by placing clusters of inclusions at random. The same seems to hold for the pores, yet at the scale where they appear, it would be necessary to obtain very large images in order to see the distribution of pores. This is practically inconvenient and rather difficult with the typical equipment as that used for obtaining the images of Fig. 1. A way to obtain large images is based on advanced image processing, e.g. as reported in Ref. [10]. However, since in the present study we consider a range in the statistical properties of the pores, so we can study their interplay with inclusions, this is rather unnecessary.

4. Study of the variance of the strain field—1D approximation of a deformation problem

Given the geometrical description of the pores and the inclusions in 2 or 3D one can in principle study deformation problems and the effect of the pores as well as the inclusions. For example, simulations through finite element (FE) techniques can yield the statistics of the strain field, which is important for crack initiation studies. Yet, even with modern computers capable of handling a large number of FE, because of the multiscale nature of the problem it becomes practically impossible to simulate a reasonable size of the material. For the aluminum alloy described above, for example, the pores are of size often up to 500 μm and the inclusions 3–6 μm in diameter. These two orders of magnitude range of scales make difficult the simulation of problems containing an appropriate number of inclusions and pores. This has been realized recently [10] thus images of microstructures through advanced image processing, have been used for simulating material behavior hierarchically using FE, from large scales to small ones. Typically this technique uses actual microstructures observed experimentally. A disadvantage of this method is that the constitutive response of the material at increasing scales is not the result of microstructural analysis at smaller scales but rather assumed on the basis of macroscopic experiments.

Here we present an alternative approach based on wavelet analysis. Using the ability of wavelets to represent a medium at several scales concurrently, the process treats features at diverse scales naturally. For special cases such as isotropic porous media it has been demonstrated analytically in 1, 2 and 3D in Ref. [1] where also some other relevant studies are cited. Furthermore, it has been demonstrated analytically for media showing self-affine properties [9]. In Ref. [1], knowledge of the strain field near critical pores allowed analytical and in multiple dimensions investigation of the problem of failure of a porous brittle material (for isotropic media only). For inclusions, however, a similar treatment is very difficult to achieve. Thus, we make some simplifying assumptions (discussed subsequently and when appropriate) leading to a 1D approximation of a deformation problem, and thus analytically

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**Fig. 2.** Image of a section of the material taken from a distance of 140 mm from the chill at magnifications of 50×, 100×, 500× and 1000×, from left to right and from top to bottom. The size of the image at 50× is 1100 μm × 1100 μm. Images (a)–(c) were obtained using optical microscopy and image (d) using SEM.

**Fig. 3.** Variance of increments obtained from the images shown in Fig. 2.
study crack initiation criteria and interaction of microstructures with boundaries.

In 1D, the governing equation for an elastic deformation problem reads
\[
\frac{\partial}{\partial x} \left( E(x) \frac{\partial u(x)}{\partial x} \right) = 0
\]
where \( u \) denotes displacement and \( E \) the modulus of elasticity. For dead-load boundary conditions, a deterministic external stress, \( s \), is applied, while for the specified displacement case we have
\[
u(0) = 0, \quad u(L) = GL
\]
where \( G \) denotes the mean strain, i.e. end displacement over bar length \( L \). We address the dead-load boundary condition first. The flexibility is \( F(x) = 1/E(x) \). Based on the properties of the composite medium, i.e. medium-P plus medium-I it is easy to verify that stationarity of \( E(x) \) guaranties stationarity of \( F(x) \). Furthermore, the statistical properties of \( E(x) \) uniquely define those of \( F(x) \) and vice versa.

We approximate a problem as 1D by assuming homogenized properties over the direction transverse to the applied load. The problem depends, of course, on several factors such as the distance over which homogenization is performed, weights used in the process, etc. We proceed with this in mind, and the fact that we study a range of properties of relevant variables, e.g. \( F(x) \), makes the details behind the assumption rather mild. For the dead-load boundary condition case, by direct integration of Eq. (16) it follows that
\[
\frac{du}{dx} = sF(x)
\]
Since \( s \) is constant in this case, the strain, \( du/dx \), depends directly on the flexibility \( F(x) \). We next study the variance of the strain field for the composite material, i.e. that of medium-P and medium-I. We consider each phase, i.e. matrix, inclusions, and pores, as elastic, thus the variance of the strain provides information on the statistics of criteria for crack initiation for the brittle phase of the material (aluminum rich matrix). This information is crucial for understanding the behavior of such materials [11]. Problems associated with studying the variance of the strain field have to do with the multiscale nature of the microstructure. For this, based on the results presented in Ref. [1], we note that for each of the two components of the material, i.e. medium-P and medium-I, that
\[
E_W(a) \sim \text{Var}[F]
\]
where \( E_W(a) \) denotes the energy of the wavelet transform of the variable \( F \) for the first level of the wavelet decomposition, i.e. the smallest scale \( a \), i.e. \( E_W(a) = E_W[\text{Min}(a)] \). Thus, \( \text{Var}[F] \) can be studied as a function of scale by studying \( E_W(a) \). The process is similar to that appearing in Ref. [1], yet for different variables and for the simplified 1D approximation. It turns out that the interaction of pores at certain scales with inclusions at other ones is entirely different than the interaction of pores studied in Ref. [1].

With respect to the aluminum alloy of which images are shown in Figs. 2 and 3, we consider the following values, in order to study the interaction of pores and inclusions. For the silicon particles (inclusions), \( E_I = 130 \text{ GPa} \), and for the aluminum rich matrix, \( E_M = 39 \text{ GPa} \); these values are the same as those used in Ref. [12]. The flexibilities \( F_I, F_M \) are the inverse of these values while for the pores the flexibility diverges to infinity. However, since the 1D approximation considers properties homogenized in the direction transverse to the applied load, \( F_p \) is finite. For our simulations, it turned out that the results were insensitive to the value of \( F_p \) as far as it was greater than about 30 times that of \( F_M \); it is the geometrical distribution of the pores that is more important. Thus, we proceed with \( F_p = 1 \text{ GPa}^{-1} \).

The central theme of this paper is studying the effects of pores and inclusions on crack initiation as their spatial properties vary, e.g. by varying the solidification process in A356. This is accomplished, for our 1D approximation, as follows. We refer all spatial correlations to those of the matrix; we fix the spatial statistics of the inclusions; we vary the spatial statistics of the pores. With this in mind, and for the flexibility values shown in the previous paragraph, considering \( \varphi_p = 1\% \), it follows from Eqs. (1) and (2) that \( \text{Var}(F_p) = 0.0969^2, \langle F_p \rangle = 0.0354 \). The high coefficient of variation resulting from these values is expected due to the large differences in properties of the matrix and the pores; yet, the rather large correlation distance for the matrix ‘smoothes’ such fluctuations in space. For the inclusions, a value of \( \varphi = 20\% \), yields \( \text{Var}[F_I] = 0.0072^2, \langle F_I \rangle = 0.0221 \).

For medium-I, we consider a correlation distance for the matrix material equal to \( 8.5 \mu m \) (see discussion of Fig. 3). Note that if all inclusions were circular of fixed radius \( R \) and distributed randomly in space, Eq. (7) with \( l = 8.5 \mu m \) and \( \varphi = 0.2 \) yields \( R = 7.97 \mu m \). Yet, as may be evidenced from Fig. 2, it is rather clusters of inclusions that are randomly distributed and this changes the value of \( S_I \) and thus \( l_I \), cf. Eq. (8). For compactness we only present typical results in the following, for which \( l_I = 8.5 \mu m \) and \( l_p \) varies from 0.2 to 10.2 \mu m. For the case of all pores being circular of fixed radius \( R \), instead of Eq. (7) we have, since for flexibilities \( p > m \)
\[
l_p = \frac{1}{2} \varphi_p R
\]
For example, if \( R = 400 \mu m \) and \( \varphi_p = 1\% \) it follows that \( l_p = 5.33 \mu m \). Importantly, this simplifying example shows that the correlation distances for the matrix for medium-I and medium-P are not far apart, even though the actual size of the pores may be two orders of magnitude larger than the size of inclusions. This, together with the fact that the relevant variances are not far apart either, shows that the role of inclusions and the role of pores with respect to crack...
initiation are not far apart. This explains relevant experimental observations as elaborated in the following.

Using Eq. (19) and the range of values indicated above, the variance of strain (19) is studied. For the composite medium with statistical properties given in Eqs. (9)–(13), relevant random fields have been produced [1]. Then the wavelet transform of these fields is performed and thus the variance as a function of scale, based on Eq. (19), is studied. Since the value of the strain relevant to failure strain for the brittle inclusions and relative to the yield strain of the ductile matrix is important for microcrack initiation, we assume for simplicity that the ratio of modulus over stress at failure for the inclusions is the same as the ratio of modulus over yield stress for the matrix; this allows the study of normalized variance of the strain field. With this in mind, 5000 simulations of $F$ each of 256 pixels representing $128 \, \mu m$ were generated numerically, and for each of them the wavelet transform was performed, using the process and wavelets reported in Ref. [1]. Then, Eq. (19) was used to study the variance as a function of scale and results are

![Fig. 4. Animation of $E_W(a)$ for $l_p$ equal to 0.2, 1.2, 2.2…,10.2 \, \mu m$ from left to right and top to bottom. In each plot, the horizontal axis denotes scale; for 256 points, there are 8 scales ($2^8 = 256$). Scale 8 is the finest while scale 1 is the coarsest. 256 points correspond to 128 \, \mu m.](image-url)
presented in Fig. 4. As can be seen, for small values of \( l_P \) the energy \( E_W(a) \), and thus the normalized variance of strain, is at small scales where inclusions are the dominant microstructures. As \( l_P \) increases, the energy gradually transfers to the larger scales where the pores are dominant. The transition is gradual rather than abrupt due to the interaction between the pores and inclusions. The results implied from Fig. 4 agree well with the experimental observations on the effects of microstructure on crack initiation. Yet, in experimental studies ([2,3] and references therein) strong boundary related phenomena are reported. Thus, before we discuss the comparison of the results of this section with experiments, we present an analytical study of boundary effects and present comparison with experiments afterwards.
5. Interaction of microstructures with boundaries

In Refs. [2,3] and in references cited therein it has been reported that pores or inclusions near boundaries are very often dominant for microcrack initiation. There is definitely an interaction of microstructures with boundaries, and it is possible to study this interaction analytically. The foundation for such studies has been set in Ref. [13] and the following extends those studies for the three-phase material. For the 1D problem described by Eqs. (16) and (17), the coefficient of variation of the strain field, \( COV \), for a bar of length \( L \) is expressed as [13]

\[
COV = \frac{\text{Var}[F(x)]}{\langle F(x) \rangle^2} \times \left\{ \frac{1}{L^2} \Delta(L) - \frac{1}{L} \Delta(x) + \Delta(L - x) + 1 \right\}
\]  

where function \( \Delta(x) \) depends on the autocorrelation function of \( F \) and an over dot indicates derivative with respect to the argument. For \( p(r) \) expressed through Eq. (11), a lengthy, yet straightforward derivation yields

\[
\Delta(t) = \alpha_p \left( 2 \hat{f} \left( \frac{t}{l_p} + \exp \left[ -\frac{t}{l_p} \right] \right) \right) + \alpha_t \left( 2 \hat{f} \left( \frac{t}{l_t} + \exp \left[ -\frac{t}{l_t} \right] \right) \right)
\]

and

\[
\Delta(t) = 2 \alpha_p l_p \left( 1 - \exp \left[ -\frac{t}{l_p} \right] \right) + 2 \alpha_t l_t \left( 1 - \exp \left[ -\frac{t}{l_t} \right] \right)
\]

Using the values for the constants as those that produced Fig. 4, the variation of \( COV[du/dx] \) as a function of the spatial variable \( x \) has been studied, using Eqs. (21)–(23). Fig. 5 shows such plots.

Fig. 5 shows that as the size of the pores increase, the extent of boundary interactions increases. Thus, large pores are more prone to be crack initiation sites especially when those pores are positioned near boundaries. The larger the pore, the further their distance from the boundary can be while it remains critical as crack initiation site. This agrees well with the experiments, and detailed comparisons of boundary phenomena as well as crack initiation in the bulk of the material that appear in Section 6.

6. Comparison with relevant experiments

Experiments relevant to the present study, e.g. those reported in Refs. [2,3] and references cited therein, agree well with the present results. Note that the present study refers to monotonic load, while in the relevant experiments used herein for comparison specimens were subjected to cyclic fatigue load. Thus we saliently assume, as is often the case in crack formation studies based on stress/strain concentrations, that the response of the sample to monotonic load will provide accurate information on crack initiation under cyclic load. Furthermore, due to the simplifications implied by the 1D formulation, comparison of the present results with experiments is rather qualitative.

In Refs. [2,3], crack initiation studies on samples of various pore sizes as well as silicon particle distributions, produced by mainly varying the cooling rate, are reported. Crack initiation sites can be classified by either the SDAS (characteristic of the distribution of the inclusions) or, as is more relevant here, by pore size. As the cooling rate decreases, the SDAS as well as the pore size increases. The experimental findings relevant to this study, based on Refs. [2,3] and references cited therein, can be summarized as follows:

(a) for small SDAS (<27 \( \mu \)m, and corresponding pore size of <80 \( \mu \)m) cracks initiated at near surface silicon particles;
(b) for SDAS > 30 \( \mu \)m, cracks initiated from large pores;
(c) when present at or near the surface (boundary), large pores/oxides initiated cracks, regardless of SDAS;
(d) there is a relation between pore initiation size and its distance from the surface—in general, the size of the pore initiation size increases with its distance from the surface (see also Ref. [14], where pore sizes and distance from the surface was controlled).

Here, it may be relevant to quote Ref. [3] in that ‘it is widely believed that most cracks in aluminum castings as A356 initiate from pores at or close to the specimen surface’.

All the experimental findings briefly stated above agree well with the theoretical results. In particular

(a) as shown in Fig. 4, for small pore sizes, the microstructural features dominating crack initiation are the inclusions (silicon particles)—such a ‘critical’ pore size can be identified from Fig. 4, yet, due to the assumptions involved in the 1D approximation it cannot be compared to the \( \sim 80 \mu \)m critical size reported in the experiments;
(b) for pore size larger than the ‘critical’ one, the variance of the strain field dictates crack initiation from the larger pores;
(c) as is evident from Fig. 5, interactions of the pores and of the inclusions with boundaries is crucial—thus, a crack-initiating pore may be smaller than the critical size if positioned near a surface;
(d) Fig. 5 shows clearly that the larger the pore the larger its distance from the surface can be while it remains a crack initiation site.
7. Conclusions

The present semi-analytical work based on wavelet analysis proves to be effective in studying the effect of multiscale microstructure on material properties. For the material with inclusions and pores, the results may seem from a first glance counterintuitive, yet they agree with experimental observations. In particular, for a moderate cooling rate of A356 specimens, the correlation distances for the matrix for medium-I and medium-P are not far apart, even though the actual size of the pores is about two orders of magnitude larger than the size of inclusions. This indeed seems counterintuitive from a first glance, yet a close look at the relevant statistics sheds light. Thus, at a certain cooling rate, the role of the inclusions with respect to crack initiation is statistically similar to that of the pores. For faster cooling rates the dominant microstructures are the inclusions rather than the pores and the opposite holds for slower cooling rates. For all cooling rates, interaction of microstructures with surfaces is paramount. Since all structural components have boundaries, their adverse interaction with the microstructures makes studies of bulk properties, e.g. with respect to crack initiation, of questionable practical use.

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