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The effect of aerodynamic braking on the inertial power requirement of flapping flight: case study of a gull

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Summary

There has been an unresolved question of whether there is any significant degree of aerodynamic braking during wing deceleration in the flapping flight of birds, with direct analogies existing with flapping micro air vehicles. Some authors have assumed a complete conversion of kinetic energy into (useful) aerodynamic work during wing deceleration. Other authors have assumed no aerodynamic braking. The different assumptions have led to predictions of inertial power requirements in birds differing by a factor of 2. Our work is the first to model the aerodynamic braking forces on the wing during wing deceleration. A model has been developed that integrates the aerodynamic forces along the length of the wing and also throughout the wing beat cycle. A ring-billed gull was used in a case study and an adult specimen was used to gather morphometric data including a steady state measurement of the lift coefficient. The model estimates that there is a 50% conversion of kinetic energy into useful aerodynamic work during wing deceleration for minimum power speed. This aerodynamic braking reduces the inertial power requirement from 11.3% to 8.5% of the total power. The analysis shows that energy conversion is sensitive to wing inertia, amplitude of flapping, lift coefficient and wing length. The aerodynamic braking in flapping micro air vehicles can be maximised by maximising flap angle, maximising wing length (for a given inertia), minimising inertia and maximising lift coefficient

Key words: flapping micro air vehicles, bird wing**,** wing inertia, inertial power, aerodynamic braking

Introduction

There are four main contributions to energy demand in level flapping flight of birds and FMAVs (flapping micro air vehicles): induced drag, body drag, wing profile drag and wing inertial drag. The inertial power requirement of birds and FMAVs is significant because the wings need to be accelerated and decelerated twice during the wing beat cycle. In addition, flapping occurs at high frequencies, typically between 3Hz to 30Hz for birds (Pennycuick 1996). The wings of birds and FMAVs have a low inertia in order to minimise the inertial power requirements of flapping flight.

In this paper we develop a theoretical model of inertial power for flapping flight that takes into account conversion of kinetic energy into useful aerodynamic work that occurs during wing deceleration. Previous researchers have been split between those assuming complete conversion of kinetic energy into useful aerodynamic work (Norberg 1990) and those assuming zero conversion (Berg and Rayner 1995). Our study enables the validity of these two approaches to be assessed for the case of the ring-billed gull.

Weis-Fogh studied the hummingbird *Amazilia fumbriata fluviatilis* and estimated that 19% of the kinetic energy was converted into useful aerodynamioc work (Weis-Fogh, 1972). However this was not based on modeling of aerodynamic braking but by estimating wasted energy during the wing beat cycle. In our work we make a direct calculation of aerodynamic braking by calculating the aerodynamic forces along the length of the wing and throughout the wingbeat cycle.

A ring-billed gull was chosen for the case study because it is a common medium-sized bird that is an efficient flyer and convenient in size for a FMAV. The ring-billed gull (*Larus delawarensis*) is perhaps the most common gull in North America, capable of long migratory journeys there, and wandering regularly as far as western Europe, Ireland and Great Britain. A dead adult specimen in excellent condition was obtained in order to get data for this study.

Understanding of flapping flight is important for applications such as flapping micro air vehicles and flapping aquatic vehicles (Techet 2008). Lessons for optimum design of FMAV design are discussed in this paper.

Derivation of wing lift coefficient

In order to model the aerodynamic braking during flapping flight it is necessary to know (or make an estimate of) the lift coefficient of the wings. A steady state lift coefficient for the gull was determined using a terminal velocity experiment with an adult specimen of ring-billed gull. The experimental setup is shown in Fig. 1. The wings were set in a fully deployed geometry and attached rigidly to a slender wooden rod from the shoulder joint to the wrist joint. The wings were positioned to have an angle of attack, $\beta = 10°$. One wing was reversed so that both wings were moving in the same direction equivalent to the down-stroke. A constant torque T_{drive} was applied to the shaft with a dead weight and the terminal velocity ω_T was measured over 20 revolutions.

At steady state velocity there is a torque balance between the aerodynamic resistance and the drive torque, T_{drive}. Aerodynamic force is proportional to the square of the relative air velocity and hence the aerodynamic drag varies along the length of the wing. The aerodynamic resistance of the wing can be calculated by considering an elemental strip of width δr (see Fig. 2) and integrating the drag on the elemental strip along the length of the wing. In order to derive the lift coefficient it was necessary to make an assumption concerning the shape of the deployed gull wings. The assumed shape is shown in Fig. 2 and allows integration of aerodynamic drag along the length of the wing by separately integrating between r_1 and r_2 and between r_2 and 0.

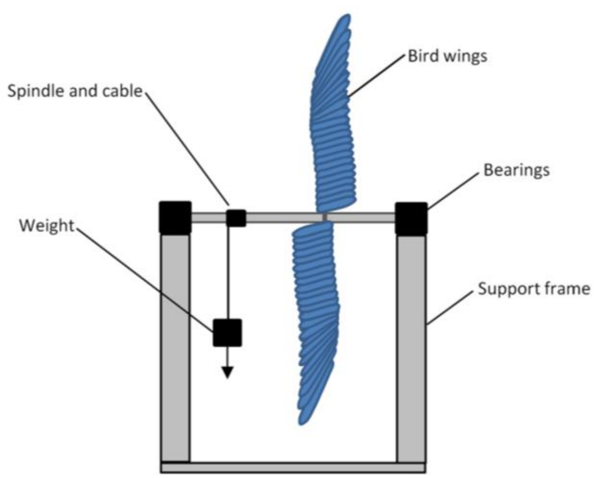


Fig. 1: Experimental setup to measure steady state lift coefficient

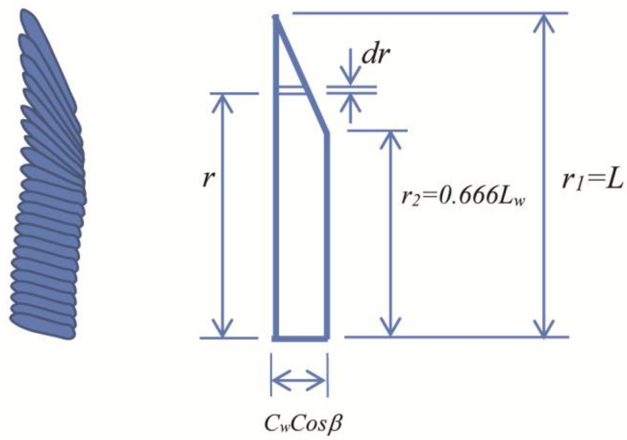


Fig. 2: Simplified representation of fully deployed gull wing

$$T_{drive} = 2\left\{0.5\rho C_L C_w Cos\beta {\omega_T}^2 \int_0^{r_2} r^3\, dr\right\} + 2\left\{0.5\rho C_L C_w Cos\beta {\omega_T}^2 \int_{r_2}^{L_w} \frac{(r_1 - r)}{(r_1 - r_2)} r^3 dr\right\}$$

$$T_{drive} = \rho C_L C_w Cos\beta {\omega_T}^2 \frac{{L_w}^4}{7.7} \tag{1}$$

where ρ is the air density (taken as 1.2 kg/m^3), C_L is the lift coefficient, C_w is the wing chord, β is the angle of attack (β=10°), ω_T is the terminal angular velocity and L_w is the wing length. (Note that the normal velocity on each strip is $\omega_T r$ and that the frontal area of each strip is the width multiplied by dr).

By measuring the terminal angular velocity for a given applied torque, the only unknown is the lift coefficient. The terminal velocity was found to be 1.22 rev/s when the drive torque was 52.3 mNm. The terminal velocity of 1.22 rev/s corresponds to 3.1 Hz flapping frequency (taking into account the total wing excursion angle of 71.7°) which is slightly less than the assumed flapping frequency of the gull of 4.26 Hz (see Table 1). The effect of skin friction was assumed to be very small and was ignored.

This terminal velocity and drive torque gives a lift coefficient of C_L = 0.95. This value of lift coefficient is similar to values found in the literature for other birds for similar angles of attack. Heers (Heers et al. 2011) measured a value of C_L = 1.0 for a partridge at 10° angle of attack. Withers (Withers 1981) measured a value of C_L = 1.15 for a vulture for an angle of attack of 12°. Withers (Withers 1981) measured a value of C_L = 0.88 for a petrel at an angle of attack of 13°.

Variability of lift coefficient

In flapping flight, the lift coefficient is likely to vary during the wing beat cycle because of changing air flows around the wings. For horizontal flight at constant speed, when the wings are moved up and down this changes the direction of air flow over the wings and this changes the angle of attack. The change in angle of attack can be minimised if the bird twists the wings to compensate for the changing air flow vector but it would be very difficult for a bird to maintain a constant angle of attack. Changes in air flow must be particularly significant at wing reversal because this is when acceleration is highest and therefore changes in the angle of attack are highest.

Despite the variability of the lift coefficient in flapping flight, there are reasons why it is justifiable to use a constant value of lift coefficient for the purposes of estimating aerodynamic braking. The majority of energy conversion takes place during the period of highest velocities (since power is proportional to ω^3). Therefore it is most important that the lift coefficient is reasonably accurate during the central part of the wing beat cycle. At this part of the cycle, the lift coefficient is likely to be reasonably constant as the angle of attack is changing the least.

Summary of physical data

The physical data used in this paper is summarised in Table 2. The wing inertia, I was calculated from the equation $I = m_w r_g^2$ where m_w is the wing mass and r_g is the radius of gyration. The radius of gyration was taken as 0.317 L_w because this value of radius of gyration was obtained from experiments by Berg and Rayner for a black-headed gull (*Chroicocephalus ridibundus*) of similar size (b = 0.93m) (Berg and Rayner, 1995). The total wing excursion angle during flapping was calculated from the equation (Scholey 1983):

$$\phi = 1.1048 m_b^{-0.119} \qquad (2)$$

where m_b is the mass of the bird. The flapping frequency was obtained from the equation (Pennycuick 1996):

$$f = {m_b}^{3/8} g^{1/2} b^{-23/24} {S_T}^{-1/3} \rho^{-3/8} \tag{3}$$

where g is the gravitational constant, S_T is the wing area of two wings and ρ is the air density.

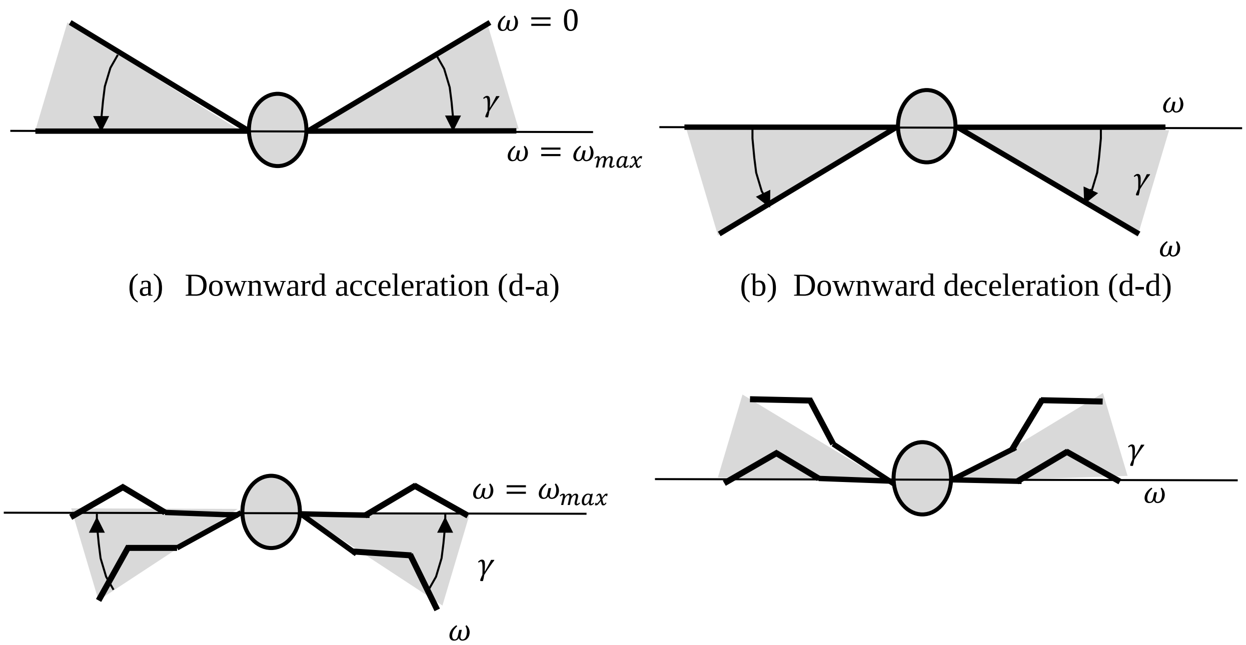
Table 1. Data for the ring-billed gull used in modelling

Wing characteristic	Term	Value	Source
Wing span (m)	b	1.03	Measured
Wing length (m)	L_w	0.48	Measured
Wing chord (m)	C_w	0.115	Measured
Single wing area $\times 10^{-4}$ (m^2)	S	460	Measured
Bird mass (kg)	m_b	0.3515	Measured
Wing mass $\times 10^{-3}$ (kg)	m_w	32	Measured
Lift coefficient (10 deg)	C_L	0.95	Measured
Flapping frequency (Hz)	f	4.26	[Pennycuick 1996]
Total wing flap angle (^{o})	ϕ	71.7	[Scholey 1983]
Wing inertia $\times 10^6$ (kgm^2)	I	740.89	Derived

Power equations for inertial drag

This section develops four separate equations for inertial power for accelerating and decelerating the wings for the down-stroke and upstroke in flapping flight. Fig. 3 shows a schematic of the four main phases of flapping flight. The wings are more extended during the down-stroke in order to maximise lift for flight (see Fig. 3(a) and Fig. 3(b)). The wing is partly folded during the upstroke to reduce inertia and reduce drag (see Fig. 3(c) and Fig. 3(d)). The feathers also have a structure that tends to give a one-way flow of air that further reduces resistance on the upstroke (King and McLellan, 1984). When calculating the inertial component of drag, account must be taken of reduced wing inertia for the upstroke. In this paper we assume the inertia is reduced by 50% in the upstroke. This is the same reduction as that assumed in (Berg and Rayner, 1995).

In the model, the wings are assumed to flap with simple harmonic motion with minimum rotational speed of zero and maximum rotational speed of ω_{max}.



(a) Downward acceleration (d-a)　(b) Downward deceleration (d-d)

(c) Upward acceleration (u-a)　(d) Upward deceleration (u-d)

Fig. 3: Acceleration phases during flapping

Aerodynamic braking

When the bird is flapping the wings, an aerodynamic pressure force is generated as air passes over the wing. This aerodynamic force has the effect of slowing the flapping and thus reducing (or totally eliminating) the need for the bird to use muscle work to decelerate the wings.

For downward flapping, the pressure force acts in the upwards direction, i.e. pushing the bird upwards. In the case of downward deceleration, the aerodynamic work is useful work because it gives lift and thrust. In the case of upward deceleration, the aerodynamic work is partially useful because it gives beneficial forward thrust and a detrimental downward thrust. In both cases, the aerodynamic work reduces the inertial power requirement.

Downward acceleration phase (d-a)

The equation for wing acceleration in the upstroke and down-stroke can be derived by analyzing kinetic energy. The maximum kinetic energy (KE) of a single wing is given by:

$$\mathrm{KE} = \mathrm{I}\omega_{\mathrm{max}}^2/2 \qquad (4)$$

Where I is the wing inertia and ω_{max} is the maximum angular velocity. The bird has to do work to accelerate two wings to the mid position during a quarter wing beat cycle. Therefore the total work done is $I\omega_{max}^2$ and the average power, P, in the quarter cycle is:

$$P = \mathrm{I}\omega_{\mathrm{max}}^2/\mathrm{t} \qquad (5)$$

Where t = (1/f)/4 and where f is the wing beat frequency in hertz. The wings are assumed to go through simple harmonic motion. In this case, the position α of the wing is given by:

$$\alpha = \gamma \sin(2\pi f\mathrm{t}) \qquad (6)$$

where γ is the amplitude of motion (half total excursion angle) and ω_b is the wing beat frequency in r/s.

The wing angular velocity ω is found by differentiating the above equation, i.e.

$$\omega = 2\pi f\gamma \cos(2\pi f\mathrm{t}) \qquad (7)$$

The maximum angular velocity occurs in the mid-stroke when $\cos 2\pi ft = 1$ and is given by:

$$\omega_{max} = 2\pi f\gamma \qquad (8)$$

Therefore the average power required to overcome wing inertia for two wings is approximately given by:

$$P_{iner-d-a} = 16I\pi^2 f^3\gamma^2 \qquad (9)$$

Equation (9) is consistent with the equation given in (Norberg 1990).

Downward deceleration phase (d-d)

Norberg assumes that no power is needed by the bird to slow the wings from ω_{max} to $\omega = 0$ (Norberg 1990). This is based on the assumption that the wings are fully slowed down by the conversion of kinetic energy into aerodynamic work. In contrast to Norberg, Berg and Rayner do not assume that there is any conversion of kinetic energy into useful aerodynamic work (Berg and Rayner, 1995). Hence they predict Equation (9) is valid through the whole downwards cycle.

In our analysis, we make an estimate of the actual conversion of kinetic energy into aerodynamic work. The aerodynamic power at a particular wing position and velocity can be calculated by integrating the aerodynamic power along the length of the wings from $r = 0$ to $r = L_w$ and by integrating the aerodynamic power throughout the wing beat cycle from $\theta = 0$ to $\theta = \pi$.

For two wings the aerodynamic power is given by:

$$P_{aero} = 2\left\{0.5\rho \mathrm{C_L} \mathrm{C_w} Cos\beta \int_0^{2/3L_w} r^3\, dr\right\}\left\{\frac{\omega_{max}^3}{\pi}\int_0^{\pi} sin^3\,\theta\, d\theta\right\} + 2\left\{0.5\rho \mathrm{C_L} C_w Cos\beta \int_{0.666L_w}^{L_w} \frac{(r_1 - r)}{(r_1 - r_2)} r^3 dr\right\}\left\{\frac{\omega_{max}^3}{\pi}\int_0^{\pi} sin^3\,\theta\, d\theta\right\}$$

$$P_{aero} = \rho \mathrm{C_L} C_w Cos\beta\, {\omega_{max}}^3 \frac{L_w^4}{18.1} \qquad (10)$$

where ρ is the air density (taken as 1.2 kg/m^3), $\mathrm{C_L}$ is the lift coefficient, C_w is the wing chord, β is the angle of attack, ω_{max} is the maximum angular velocity, L_w is the wing length and $\theta = 2\pi ft$. Solving Equation (10) and noting that $\omega_{max} = 2f\pi\gamma$ gives:

$$P_{aero-d-d} = 0.0571\, f^3\pi^3\gamma^3 L_w^4 \qquad (11)$$

Therefore taking into account the result in Equation (9) (i.e. the same kinetic energy), the net inertial power for downward deceleration is given by:

$$P_{iner-d-d} = \pi^2 f^3 \gamma^2 (16I - 0.0571\gamma\pi L_w^4) \qquad (12)$$

If $P_{iner-d-d}$ is less than zero (i.e. the aerodynamic work is greater than the inertial work) then the power is assumed to be zero because the analysis is only interested in calculating inertial work. If the aerodynamic work is greater than the inertial work then the bird requires no effort to decelerate the wings.

Upwards acceleration phase (u-a)

Taking into account a 50% reduced inertia due to wing retraction in the upstroke gives the following:

$$P_{iner-u-a} = 8I\pi^2 f^3 \gamma^2 \quad (13)$$

Upwards deceleration phase (u-d)

Assuming a 50% reduction in planform area and assuming the lift coefficient is the same in the upstroke gives the following:

$$P_{iner-u-d} = \pi^2 f^3 \gamma^2 (8I - 0.0286\gamma\pi L_w^4) \quad (14)$$

If $P_{iner-u-d}$ is less than zero (i.e. the aerodynamic work is greater than the inertial work) then the power is assumed to be zero.

Average inertial power

The average inertial power during the whole flapping cycle is given by:

$$P_{iner-av} = [P_{iner-d-a} + P_{iner-d-d} + P_{iner-u-a} + P_{iner-u-d}]/4$$

$$P_{iner-av} = (6I\pi^2 f^3 \gamma^2 + P_{iner-d-d}/4 + P_{iner-u-d}/4) \quad (15)$$

Minimum total power

The total power at minimum power speed is given by (Norberg 1990).

$$P_{total} = 50.2{m_b}^{0.73} \quad (16)$$

where m_b is the total mass of the bird.

Inertial power requirement at minimum power speed

Table 1 shows the inertial power predictions for the gull flying at minimum power speed. The average inertial power requirement through the wing beat cycle is predicted to be 8.5% of the total power requirement. The 8.5% prediction for inertial power is slightly higher than results by (Berg and Rayner, 1995) who calculated an equivalent range of 6.4%-8.3% (when applying 50% conversion of kinetic energy to aerodynamic work). The reason for our slightly higher prediction is due to our analysis using a more recent equation for calculating flapping frequency (Pennycuick 1996). This more recent equation gives very slightly higher frequency for the gull and hence higher inertial energy requirement (note inertial power is related to the cube of the flapping frequency so slight differences make a significant difference in power).

The results show there is 50% conversion of kinetic energy to useful aerodynamic work during wing deceleration caused by the aerodynamic braking. The aerodynamic braking results in a 25% reduction in the inertial power requirement. If there was no aerodynamic braking the inertial power requirement would be 11.3%. This means that the total energy in flight is reduced by around 2.8% due to aerodynamic braking. This is a significant amount of energy and shows that it is desirable to model aerodynamic braking.

Table 2. Inertial power requirement at minimum power speed

Quantity	Phase	Power
$P_{iner\text{-}d\text{-}a}$ *(W)*	Down acceleration	3.54
$P_{iner\text{-}d\text{-}d}$ *(W)*	Down deceleration	1.77
$P_{iner\text{-}u\text{-}a}$ *(W)*	Up acceleration	1.77
$P_{iner\text{-}u\text{-}d}$ *(W)*	Up deceleration	0.88
$P_{iner\text{-}av}$ *(W)*	(Av. inertial power)	1.99
P_{total} *(W)*	(Total power)	23.40
% kinetic to aero energy		50%
% inertia power of total power		8.5%

Sensitivity analysis

Derived equations (10) and (12) in the following section of the research paper show that aerodynamic braking is independent of wing beat frequency. They also show that aerodynamic braking is affected by lift coefficient, wing beat amplitude, wing length and wing inertia. The values of these parameters are subject to uncertainty and also vary from bird to bird. Therefore it is of interest to see how variation in the values affects inertial power predictions.

Lift coefficient

If the lift coefficient varied by +/-10% then the conversion of kinetic energy to aerodynamic work is in the range 45-55%. Even though the calculation of aerodynamic braking is subject to significant uncertainty, at least it gives an estimate. When aerodynamic braking is assumed to be zero or 100%, as happens at present, this leads to an error of up to 50% in the estimation of inertial power. By making an estimate of the aerodynamic braking this immediately leads to an improvement of the inertial power calculation. For example, if our estimate of lift coefficient is in error by 10% then the error in the inertial power prediction is only 5% which is 10 times more accurate.

Wing beat amplitude

If the excursion angle varied by +/- 10% then the conversion of kinetic energy to aerodynamic work is in the range 45-55%. If both lift coefficient and excursion angle had simultaneous variations of +/-10% in the worst case combination then the predicted aerodynamic braking is in in the range 40-60%. Therefore from this analysis we can be confident that energy conversion is taking place.

Wing inertia

Since wing inertia is likely to vary from bird to bird it is of interest to see how energy conversion varies with I. Fig 4 shows how the percentage of energy conversion from kinetic energy to aerodynamic work varies with wing inertia. The graph shows the point where complete conversion occurs. Complete aerodynamic takes place at I = 371 kgm^2 which is half the actual value of wing inertia. At this point, the aerodynamic braking is sufficient to completely decelerate the wings and the bird requires no energy to slow down the wings.

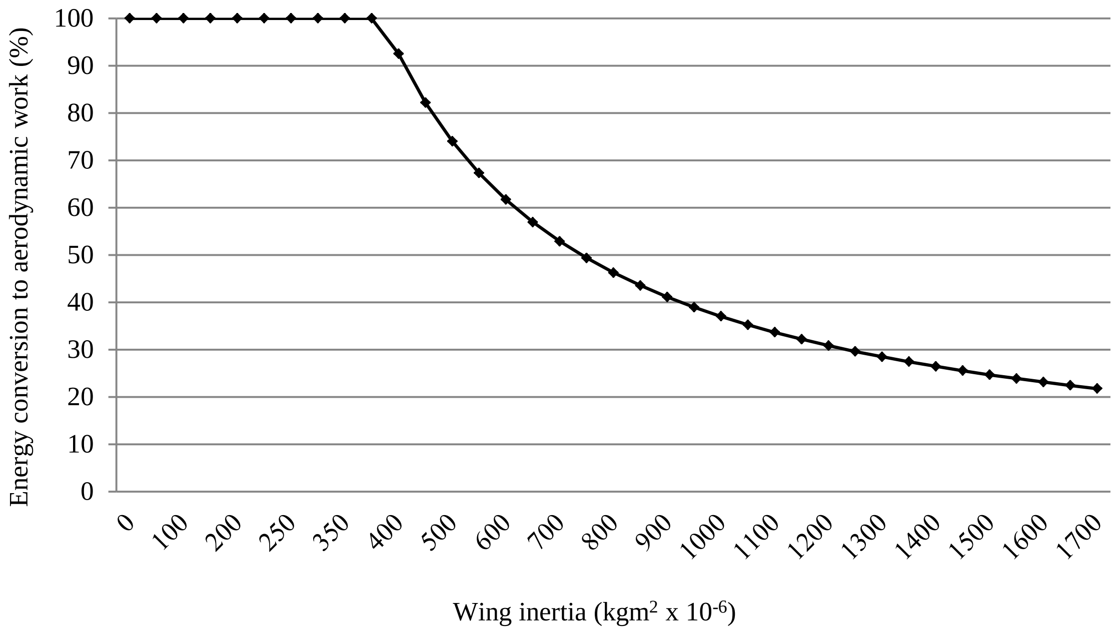


Fig.4 Wing inertia versus energy conversion

Lessons for flapping micro air vehicle (FMAV) design

Wing design

If a flapping micro air vehicle (FMAV) is designed on the scale of a ring-billed gull then there is potential to reduce the inertial power requirement by up to 5.7% if a complete conversion can be achieved of the kinetic energy to aerodynamic work. Currently MAVs are limited to very short flight times because of the high power requirements (Conn et al. 2011) and so every saving is significant. One way to achieve complete conversion is by making the wing inertia very low.

By plotting a graph of percentage of conversion of kinetic energy to aerodynamic work, shown in Fig. 4, a designer can quantify the benefits of reducing wing inertia. The inertial power required to accelerate the wings is proportional to the wing inertia. However the inertial power required to decelerate the wings is more sensitive to the wing inertia. In the case of deceleration there is a double benefit in reducing wing inertia because it reduces inertial power directly by reducing the kinetic energy requirement and reduces it indirectly by increasing aerodynamic braking.

If it is possible to increase wing length whilst maintaining the same inertia then this would also significantly increase aerodynamic braking because braking is proportional L_w^4. Increasing the wing excursion angle and lift coefficient also has potential to increase aerodynamic braking.

Actuator selection

If the wings can be made with low enough inertia that aerodynamic braking can take place then it is still necessary to select an actuator and drive system that is able to allow the aerodynamic braking to take place. To allow aerodynamic braking an actuator must be able to switch off or reduce power during wing deceleration. In addition, there must not be too much compliance in the drive system otherwise complex dynamic effects take place that do not allow intermittent actuation. Ideally the actuator would be like animal muscle and would give a pulsed force just during wing acceleration. Electrically active polymers (EAPs) are a type of human-designed muscle actuator which have the potential to produce such a pulsed actuation. Recently EAP actuators have been demonstrated to pulse at 5Hz (Araromi et al 2011) which is a similar frequency to the flapping frequency of the ring-billed gull.

Discussion

The inertial power requirement of the ring-billed gull has been estimated to be 8.5% of the total power requirement for minimum power speed. This estimation is based on measured morphometric data for a gull and a model based on simple harmonic wing motion. The result is consistent with results by (Berg and Rayner, 1995) when account is taken that they did not model aerodynamic braking.

Our work is the first to model from first principles the conversion of kinetic energy into (useful) aerodynamic work during wing deceleration. The model estimated that 50% of the kinetic energy is converted into useful aerodynamic work for the case of the ring-billed gull. The amount of energy conversion is likely to be subject to significant variation in practice because of variations in wing span, wing excursion angle and lift coefficient. For example, a 10% variation in lift coefficient leads to a 5% change in energy conversion. Our work shows that the assumptions of zero conversion or total conversion are both unsatisfactory for a gull. It would be interesting to see how the degree of energy conversion varies for other birds.

The analysis of aerodynamic braking on the gull has given insight into how to reduce the inertial drag of flapping micro air vehicles. The aerodynamic braking of flapping micro air vehicles can be maximised by maximising flap angle, maximising wing length (for a given inertia), minimising inertia and maximising lift coefficient. If aerodynamic braking is desired then it is important to select an actuator system that allows intermittent actuation of the wing.

Acknowledgments

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Nomenclature

b	Wing span (m)
C_L	Lift coefficient
C_w	Wing chord (m)
f	Flapping frequency (Hz)
g	Gravitational constant (m/s^2)
I	Wing inertia $\times 10^{-6}$ (kgm^2)
L_w	Length of wing (m)
m_b	Bird mass (kg)
m_w	Wing mass $\times 10^{-3}$ (kg)
P	Power (W)
r_g	Radius gyration (m)
S	Single wing area $\times 10^{-4}$ (m^2)
S_T	Total wing area (two wings)

T_{drive}	Drive torque (Nm)
β	Angle of attack (°)
γ	Flapping amplitude (half total excursion) (rad)
θ	Angular position during simple harmonic motion
ϕ	Total excursion angle (rad)
ρ	Air density (kg/m^3)
$\omega_{\max}$	Maximum angular velocity (rad/s)
ω_T	Terminal velocity (rad/s)

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