Green and Ampt Infiltration with Redistribution

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GREEN AND AMPT INFILTRATION WITH REDISTRIBUTION

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ABSTRACT: Distributed, physically based watershed and irrigation advance models require robust infiltration estimation capabilities. The empirical Green and Ampt (GA) equation of infiltration is a popular method for estimating infiltration. The GA parameters have physical basis and considerable prior research has focused on relating these parameters to soil textural classification. However, the original GA method is limited in that it is applicable only for a single ponding period. An explicit Green and Ampt redistribution (GAR) technique is developed herein to estimate interstorm redistribution of soil water and allow multiple ponding simulations using the GA methodology. Soil water redistribution during interponding periods is estimated using the physical constructs of water retention curves without hysteresis. A GA wetting-front suction parameter scaling function is derived for use between arbitrary soil water contents. Simulations on 11 soil textural classifications comparing the GAR method with the numerical solution of Richards equation reveal that the GAR method is a viable technique for estimating infiltration for multiple pondings with acceptable error.

INTRODUCTION

The Green and Ampt (GA) equation of infiltration was first published over 85 years ago (Green and Ampt 1911). The GA method remains popular to this day, for several reasons. First, while the method is empirical it has physical significance (Morrel-Seytoux and Khanji 1974). The concept of the "piston" wetting front and the inclusion of the soil suction head and hydraulic conductivity parameters are the extent of the complexity of the method, which is parsimonious. Second, at \( r = 0 \) and \( t = \infty \) the GA equation correctly models free vertical imbibition and saturated flow, respectively. Third, and perhaps most importantly, the GA parameters have been correlated with some success to soil textural classification (Rawls et al. 1983). Distributed, physically based watershed or irrigation advance models of extensive areas often require infiltration parameter estimates based on surrogate measures of soil texture.

Theoretically, the application of Richards equation for estimating multiponding infiltration rates should produce accurate results. However, there are a number of drawbacks related to the solution of Richards equation. The numerical solution requires an iterative implicit technique with fine spatial discretization of the soil column and is therefore computationally and input-data intensive. Solution method robustness is a serious issue, as a general solution methodology that can accurately change boundary conditions and react to sudden changes in the curvature of the vertical profile of water content is quite cumbersome to implement. Furthermore, the uncertainty in unsaturated soil hydraulic characteristics in all but the most studied locations does not warrant exact solution methodologies.

Smith et al. (1993) added soil water redistribution to the general three-parameter infiltration equation developed by Parlange et al. (1982) to simulate infiltration for multistorm runoff events. The Smith et al. method provides the impetus for the development reported herein. The Smith et al. (1993) method is general in that it provides two parameters that conceptually describe the shape and curvature of the wetting front. The method requires nine input parameters to describe the infiltration characteristics of a given soil. The Smith et al. (1993) method was shown to perform satisfactorily when compared with the numerical solution of Richards equation in a number of test cases with multiple ponding intervals.

The purpose of this paper is to describe a technique to redistribute soil water specifically for the GA infiltration method. The objectives of this development are as follows:

1. Derive soil water redistribution assuming rectangular piston wetting fronts using a minimum number of soil infiltration characteristic parameters
2. Derive a GA wetting-front suction parameter scaling function to allow calculation of the capillary drive between two arbitrary water contents
3. Formulate a two wetting-front GA infiltration scheme
4. Verify the performance of the method against the numerical solution of Richards equation for multiple ponding periods

The motivation for this work comes from the writers' involvement in the development of the two-dimensional, physically based, distributed, watershed rainfall-runoff model CASC2D (Julien et al. 1995). The original formulation of CASC2D included the GA infiltration method. CASC2D simulations with weather radar rainfall estimates (Ogden and Julien 1994) revealed that single-storm rainfall often occurs in pulses, with interpulse "hiatus" periods. The original GA formulation has no facility for redistributing soil water during rainfall hiatus periods, and therefore tends to significantly underestimate infiltration during the second pulse of rainfall. The need for soil water redistribution in the GA framework during rainfall hiatus is pressing for distributed, physically based modeling.

UNSATURATED FLOW DURING REDISTRIBUTION

Simulation of multiple ponding periods that might arise from storms with multiple pulses of rainfall separated by periods of rainfall hiatus requires the capability to compute soil surface moisture content at all times. The original GA infiltration equation for a single ponding event is

\[
f_p = K_s \left( \frac{H_i \Delta \theta}{F} + 1 \right)
\]

where \( f_p \) = potential infiltration rate; \( K_s \) = soil saturated hydraulic conductivity; \( H_i \) = GA wetting-front capillary pressure parameter; \( \Delta \theta \) = soil water deficit given by \( \theta_f - \theta_i \), where \( \theta_f \) = water content of the soil at natural saturation; \( \theta_i \) = initial soil water content; and \( F \) = cumulative infiltrated depth. Note that in (1) if the rainfall rate is less than the potential infiltration rate the actual infiltration rate is equal to the rainfall rate.

Continuous soil water accounting allows simulation of mult-
multiple ponding events. A rainfall hiatus period begins when, after ponding, the rainfall rate \( r \) is less than saturated hydraulic conductivity \( K_s \), all ponded surface water is infiltrated, and soil water redistribution occurs. During rainfall hiatus the water content at the soil surface \( \theta_s \) becomes less than \( \theta_i \). Fig. 1 shows an assumed rectangular soil water profile at some time during the rainfall hiatus after the first ponding. This figure shows that in our formulation, the GA "piston" wetting front is assumed to elongate due to unsaturated flow under the action of capillarity and gravity. This occurs in a fashion that preserves the rectangular shape of the soil water profile while conserving mass. As the depth to the wetting front \( Z \) increases during redistribution, the water content along the entire depth of the profile is assumed to decrease uniformly.

In general, it is possible that \( \theta_i \) is large, and hence, \( K(\theta) \) is of a significant magnitude, representing gravity-driven unsaturated flow throughout the entire soil column. A general analysis therefore requires that this condition be considered. The amount of water contained in the profile at any time during redistribution is equal to \( F_r \), which is equal to \( Z(\theta_i - \theta) \). Defining \( F_r \) as the cumulative infiltrated depth at \( t_o \), the beginning of rainfall hiatus, conservation of mass for the portion of the soil water profile greater than \( \theta_i \) requires (abbreviating \( K(\theta) \) as \( K_o \)):

\[
(\theta_i - \theta) Z = F_r + (r - K_o)(t - t_o)
\]  
(2)

Conservation of mass during the hiatus period may also be expressed in the following differential form, which describes the relation between rainfall influx during hiatus \( r_o \), outflow, profile elongation, and the change in profile water content for a rectangular profile:

\[
(\theta_i - \theta) \frac{dZ}{dt} + Z \frac{d}{dt}(\theta_i - \theta) = r_o - K_o
\]  
(3)

An unsaturated form of Darcy's law can be written as (Smith et al. 1993)

\[
(\theta_i - \theta) \frac{dZ}{dt} = \frac{K(\theta_i, \theta_s)}{Z} + K(\theta_i)
\]  
(4)

The term \( G(\theta_r, \theta_s) \) represents the integral of the capillary drive across the wetting front, and is discussed in the next section. The following differential equation that describes the downward movement of the wetting front during rainfall hiatus is developed by combining (3) and (4)

\[
\frac{d\theta}{dt} = \frac{1}{Z} \left[ r_o - K_i - \left( K(\theta_o) + \frac{K(\theta_i, \theta_o)}{Z} \right) \right]
\]  
(5)

Eq. (5) forms the basis of the Green and Ampt redistribution (GAR) technique. This equation is very similar to (27) in the paper by Smith et al. (1993), with the omission of two parameters Smith et al. employed to define curvature of the water profile. In our algorithm, (5) is integrated explicitly over a small time step \( O(0.00) \) to determine the moisture content at the soil surface during redistribution while preserving a rectangular soil water profile.

**EVALUATION OF CAPILLARY DRIVE TERM**

The solution of (5) requires evaluation of the capillary drive \( G(\theta_r, \theta_s) \). This term represents the integrated capillary head across the wetting front, and can be generally written between \( \theta_i \) and \( \theta_s \) as (Smith et al. 1993)

\[
G(\theta_i, \theta_s) = \frac{S^2(\theta_s, \theta_i)}{2K_s(\theta_s - \theta_i)}
\]  
(6)

where the sorptivity, \( S \), is given by

\[
S^2(\theta_s, \theta_i) = 2(\theta_i - \theta_s) \int_{\theta_i}^{\theta_s} K(\Psi) d\Psi
\]  
(7)

where \( \Psi = \) soil water capillary head; and \( \Psi_i = \) soil water capillary head at \( \theta = \theta_i \).

Combination of (6) and (7) gives the following relation for \( G(\theta_i, \theta_s) \):

\[
G(\theta_i, \theta_s) = \frac{1}{K_s} \int_{\theta_i}^{\theta_s} K(\Psi) d\Psi
\]  
(8)

Integration of (8) requires knowledge of \( K(\Psi) \), or alternatively, both \( K(\theta) \) and \( \Psi(\theta) \). We use the Brooks and Corey (1964) relation for \( K(\theta) \) that requires the soil pore size distribution index \( \lambda \)

\[
K(\theta) = K_o \theta^{b-\alpha} \lambda
\]  
(9)

where \( \Theta = (\theta - \theta_s)/(\theta_i - \theta_s) = \) relative saturation; \( \theta_s = \) residual volumetric soil water content; and \( a = 2 \) and \( b = 3 \) (Brooks and Corey 1964) assuming an isotropic medium.

In addition, we assume the van Genuchten (1980) form of the \( \Psi(\theta) \) relation

\[
\Psi(\theta) = \Psi_s \theta^{-\alpha} \left[ 1 - \theta^{1-\alpha} \right] \lambda
\]  
(10)

In the van Genuchten relation, \( \Psi_s \) is the air entry or "bubbling" pressure, and \( c \) is an empirical constant.

Eq. (8) can be written in an alternative form

\[
G(\theta_i, \theta_s) = \frac{1}{K_s} \int_{\theta_i}^{\theta_s} K(\theta) \frac{d\Psi}{d\theta} d\theta
\]  
(11)

with

\[
\frac{d\Psi}{d\theta} = \frac{\Psi_s \theta^{-\alpha} \left[ 1 - \theta^{1-\alpha} \right] \lambda}{c}
\]  
(12)

Eq. (11) is integrated with the substitution of (9) and (12) with the assumption that \( c = \infty \) under which (10) reverts to the Brooks and Corey (1964) \( \Psi(\theta) \) equation. This method of integration of (11) produces an equation for the capillary drive between water contents \( \theta_i \) and \( \theta_s \),

\[
G(\theta_i, \theta_s) = -\frac{\Psi_s}{\lambda} \left[ \frac{1 - \theta_i^{-1+\alpha}}{3 + 1/\lambda} \right]
\]  
(13)

where
\[ \Theta_e = \frac{\theta_s - \theta_i}{\theta_s - \theta_i} \]

The capillary drive between \( \theta_s \) and an arbitrarily higher water content \( \theta_i \) can similarly be derived as

\[ G(\theta_s, \theta) = \frac{-\Psi}{\lambda} \left( \frac{\Theta_s^{1+1/\alpha} - \Theta_i^{1+1/\alpha}}{\Theta_s^{1+1/\alpha} - \Theta_i^{1+1/\alpha}} \right) \]  

(14)

Our approach in evaluating \( G(\theta_s, \theta) \) in the GA framework is to scale the GA wetting-front capillary pressure parameter \( H_s \) by the ratio of (14) to (13)

\[ G(\theta_s, \theta) = \frac{H_s}{\Psi} \left( \frac{\Theta_s^{1+1/\alpha} - \Theta_i^{1+1/\alpha}}{1 - \Theta_i^{1+1/\alpha}} \right) \]  

(15)

Note that in dividing (14) by (13), the bubbling pressure \( \Psi \) is eliminated from (15), and the knowledge of \( H_s \), \( \lambda \), \( \theta_s \), and \( \theta_i \) allows calculation of the capillary drive \( G \) between the initial water content \( \theta_i \) and an arbitrarily higher water content \( \theta_s \). Eq. (15) allows the explicit calculation of the capillary drive term needed in (5) given \( H_s \), or alternatively, \( \Psi \) may be used in combination with an appropriate expression for \( H_s \).

For a given soil, \( H_s \) may be determined experimentally. The literature contains several approximate methods to calculate this parameter (Morel-Seytoux and Khanji 1974). In this study we assume that the GA effective wetting-front capillary pressure parameter is given by integrating the Brooks and Corey (1964) relative conductivity equation over the entire range of capillary pressure

\[ H_s = \int_0^\infty \left( \frac{\Psi}{\lambda} \right)^{2+3\alpha} d\Psi + \Psi_0 \]  

(16)

which, upon integration, becomes

\[ H_s = -\Psi \left( \frac{2 + 3\alpha}{1 + 3\alpha} \right) \]  

(17)

Brakensiek (1977) performed a similar analysis that employed the air exit pressure rather than the bubbling pressure, and followed the work of Bouwer (1969) in assuming that the air exit pressure is equal to \( \Psi_s/2 \).

**TWO WETTING FRONT GA METHODOLOGY**

The multiple-ponding case considered in this development is shown schematically in Fig. 2, which depicts the Richards equation and GAR method soil water profiles at four times during a simulation on a sandy loam soil. Potential infiltration is calculated using (1) for the cases shown in the top two frames of Fig. 2. The top left frame shows the soil water profile before the first ponding, while the top right frame shows the soil water profile after the first ponding but before the first hiatus. The method expressed in (5) is used to redistribute the infiltrated profile during rainfall hiatus after the first ponding, as shown in the lower left frame of Fig. 2. Once the rainfall intensity increases to a value greater than the saturated hydraulic conductivity in the post-haustus stage, a second water profile is formed alongside the earlier profile as shown in the lower right frame of Fig. 2. The original profile is allowed to continue to redistribute using (5). The infiltration rate from surface water or rainfall is assumed to feed only the second profile until the two wetting fronts are at the same depth \( Z_1 = Z_2 \), at which time the profiles merge into one profile. Prior to merging the infiltration rate for the second profile is computed using (1) by replacing \( \Delta \theta \) with \( \Delta \theta' \) and \( F \) with \( F' \) using the following equations:

\[ \Delta \theta' = \theta_s - \theta_i \]  

(18)

where \( \theta_s \) = soil water content of the first profile as shown on the lower-right frame of Fig. 2; and

\[ F' = F - F_1 \]  

(19)

FIG. 2. Wetting-Front Configurations Considered in Two-Wetting Front Scheme (Results Correspond to Simulations on Slit Loam Soil at Times = 0.7 h, 0.9 h, 2.8 h, and 3.1 h, Respectively)

\[ Z_n = \left( \frac{\theta_s - \theta_i}{{\theta_s} - {\theta_i}} \right) Z_1 + \left( \frac{\theta_i - \theta_s}{{\theta_s} - {\theta_i}} \right) Z_2 \]  

(20)

This procedure limits the maximum number of water profiles to two and greatly simplifies the algorithm. From (20) it is apparent that \( Z_n < Z_2 < Z_1 \), so that this method of profile merging represents an averaging of the two soil water profiles. Strictly, this is physically impossible. However, this approximation introduces little error into the redistribution calculations provided that \( \theta_s - \theta_i > \theta_i - \theta_s \), which effectively limits the use of (20) to relatively short redistribution periods.

There remains the issue regarding determination of the soil surface water content prior to the first ponding since there is no explicit provision in (1) for this. The preponding surface moisture content is required if the rainfall rate \( r \) falls below the saturated hydraulic conductivity \( K_r \) prior to the first ponding, creating a hiatus period. Eq. (5) is relied upon to estimate the rising surface saturation from the initial condition \( \theta_0 = \theta_i \) at the onset of rainfall. A fourth-order Runge-Kutta solution methodology is used, with the assumption that \( \theta_0 - \theta_i/Z = 1 \) the very first time step with precipitation. This assumption is required because \( Z = 0 \) by definition at the beginning of rainfall, and there is no method to calculate the portion of the first infinitesimal rainfall that contributes to raising the surface saturation versus increasing the depth of the wetting front. The solution is quite insensitive to this assumption. The GAR method depends on this estimate of \( \theta_0 \) only when \( r \) falls below \( K_r \) prior to the first ponding. In this case, errors in \( \theta_0 \) will not have a significant effect on the redistribution of this profile.
because each term in (5) will be small—except in the case when the surface flux falls below \( K_s \) immediately prior to ponding.

**EVALUATION OF METHOD**

The performance of the GAR technique based on (5), using (15) to calculate the capillary drive during redistribution, is compared with the numerical solution of Richards equation. The 11 soil textural classifications and parameters listed in Table 1 from Rawls et al. (1982) and Rawls et al. (1983) are evaluated with \( H_c \) calculated using (17).

The GNFLUX Richards equation solver was used in this comparison. GNFLUX is a versatile, robust, mass-conservative algorithm for the solution of Richards equation. It is a refinement of the solution algorithm first developed by Smith and Woolhiser (1971) and has been used in a number of studies of infiltration (Smith 1990; Smith et al. 1993; Corradini et al. 1994). GNFLUX uses a weighted implicit, Newton-Raphson finite-difference scheme with a variable time step. The GNFLUX variable time step is selected as to limit the change in \( \theta \) and the change along the \( \theta(V) \) characteristic during each time step to acceptable values to minimize solution error. GNFLUX has the capability to simulate hysteresis effects. This capability was not used in this study to allow comparison of the GAR approach with Richards equation under similar conditions. GNFLUX has several options for the parameterization of the soil water retention and unsaturated hydraulic conductivity relations, including Brooks and Corey (1964), van Genuchten (1980), and an alternative form (Smith et al. 1993). The Brooks and Corey relations were selected for use in this comparison to maintain consistency with the GAR method with parameters \( a = 2 \) and \( b = 3 \) as in (9). The validation of the GNFLUX algorithm was presented in Smith (1990).

Each of the 11 different textural classifications listed in Table 1 was used to test the GAR method in simulations with two pulses of rainfall. Each rainfall pulse had an intensity considerably greater than the saturated hydraulic conductivity of each soil to assure ponding during both pulses. Rainfall pulse duration and intensity are listed in Table 2 for each soil tested. The rainfall rates listed in Table 2 are not intended to be representative of actual rain rates. Each soil is assumed to have an initial volumetric water content \( \theta_i \) equal to the wilting point water content as given in Table 1. The first and second rainfall pulses begin at \( t = 0 \) h and 3 h, respectively. The rainfall rate during hiatus was 0, and excess rainfall is assumed to immediately become runoff.

Fig. 3 shows plots of rainfall rate and infiltration rates predicted by both Richards equation and the GAR method versus time for six of the 11 soil types tested: sand, sandy loam, silt loam, clay loam, sandy clay, and clay. During the first rainfall pulse, the GAR method is identical to the GA method. Note in each plot that the GAR approach consistently overestimates the ponding time and cumulative infiltration during each pulse. Also note on the plots for sandy loam and silt loam that there is a discontinuity in the first derivative of the infiltration rate predicted by the GAR method during the second rainfall pulse. This occurs when \( z_r = z_w \) (see lower right frame of Fig. 2) and the two profiles merge resulting in a reduction in the rate of decrease in \( f \) because of the sudden increase in \( \Delta \theta \) used in (1).

Fig. 4 shows six plots of the time evolution of relative saturation at the soil surface \( \theta_s \) for the same cases shown in Fig. 3. First, consider the preponding rise in surface relative saturation \( \theta_s \) for the first rainfall pulse. The fourth-order Runge-Kutta solution of (5) with the rectangular soil water profile predicts rapid ponding in the sand and sandy loam cases, as does Richards equation. However, the solution of (5) predicts a slower rise in \( \theta_s \) preponding for all other soils compared...
with the Richards equation solution. This solution technique also underpredicts relative surface saturation by approximately 1% to 2% at ponding time on all soils except sand, loamy sand (figure not shown), and sandy loam. This can be seen by observing the sudden increase in $\Theta_s$ at ponding time that is forced by the requirement that $\Theta_s = \Theta_i$ at the GAR ponding time.

During the postponding rainfall hiatus, the solution of (5) using (15) to evaluate the unsaturated capillary drive agrees quite well with the Richards equation solution on the finer soil textures, as the plots on Fig. 4 indicate. There is considerable error in the prediction of $\Theta_i$ in the case of sand, loamy sand (figure not shown), and sandy loam. Note that the piecewise linear features in the Richards equation curves are an artifact of the variable time step employed by the GNFLUX program.

Results from the comparison between the GAR technique and the numerical solution of Richards equation are tabulated in Table 3 for each rainfall pulse on each soil. The numerical solution of Richards equation is assumed to represent the true results for comparison purposes. The soil numbers in column 1 correspond to the soil textural classifications and parameters as listed in Table 1. Column 2 denotes the rainfall pulse to which the values on each row of the table correspond, while column 3 lists the total rainfall during each pulse. Columns 4–6 list the values of ponding time $t_p$, cumulative infiltrated depth $F_p$ at the end of the rain pulse, and the surface relative saturation $\Theta_{s,b}$ at the end of hiatus after each pulse, respectively, for the Richards equation solution. Columns 7–9 list the same three variables denoted as $t_{p,GAR}$, $F_{p,GAR}$, and $\Theta_{s,b,GAR}$ for the GAR solution.

Column 10 represents the difference in hours between the ponding times calculated by the numerical solution of Richards equation and the GAR method. The ponding time difference $\Delta t_p$ is calculated using

$$\Delta t_p = t_{p,R} - t_{p,GAR}$$ (21)

Note that the GAR approach consistently overestimates $t_p$ an average amount of 0.024 h (1.4 min) during the first pulse, and by 0.031 h (1.9 min) during the second pulse on all soil types.

Column 11 lists the error in cumulative infiltrated depth at the end of each pulse $\varepsilon_f$, calculated using

$$\varepsilon_f = \frac{F_{p,R} - F_{p,GAR}}{F_{p,R}} \times 100\%$$ (22)

The GAR method overestimates the cumulative infiltrated depth on all soil types during both rainfall pulses compared with the numerical solution of Richards equation by an amount ranging from 1.05% to 8.18%, with an average overestimation of 2.0% at the end of the first pulse and 4.5% at the end of the second. Note that for the two coarsest soils, $\varepsilon_f$ is smaller for the second rainfall pulse than the first. The opposite is true for the remainder of the soil types tested.

Column 12 in Table 3 gives the difference in soil surface relative saturation $\Delta \Theta_s$ between the numerical solution of Richards equation and the GAR method at the end of rainfall hiatus. Similarly to the ponding time difference, the difference in soil surface relative saturation is calculated using

$$\Delta \Theta_s = \Theta_{s,b,R} - \Theta_{s,b,GAR}$$ (23)
### TABLE 3. Results of Simulations with Richards Equation Solution and GAR Method

<table>
<thead>
<tr>
<th>Soil</th>
<th>Rain pulse</th>
<th>Total rain (cm)</th>
<th>Richards Equation</th>
<th>Green and Ampt Redistribution</th>
<th>$\Delta t_r$</th>
<th>$\epsilon_r$</th>
<th>$\Delta \theta_r$</th>
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<td>(1)</td>
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<td>$t_r$ (h)</td>
<td>$F_r$ (cm)</td>
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</tr>
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<td>3.133</td>
<td>0.55</td>
<td>0.936</td>
<td>3.175</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Average errors

- First pulse: $-0.024$%
- Second pulse: $-0.031$

The quantity $\Delta \theta_r$ represents a bias in the soil surface water content estimate for a given soil. This bias is clearly largest (more negative, indicating an underestimation of the decrease in $\theta_r$ by the GAR method) for both pulses of rainfall on coarser soil textures. This is particularly true for the sand soil texture, where the GAR method underestimates the decrease in soil surface water content by 12% and 19% during the first and second hiatus periods, respectively. Note that the GAR method performs more accurately during the second redistribution period than during the first on the finer soil textures.

### ANALYSIS

Errors in cumulative infiltrated depth listed in column 11 of Table 3 average 2.0% and 4.5% at the end of the first and second rainfall pulses, respectively. The consistent overestimation of ponding time is largely the cause of the consistent overestimation of $t_r$ for the second rainfall pulse. Fig. 5 is a plot of $\epsilon_r$ versus $\Delta t_r$ for both pulses of rainfall on each soil texture tested. Error in cumulative infiltrated depth $\epsilon_r$ is poorly correlated with $\Delta t_r$ for the first rainfall pulse. However, the absolute $\epsilon_r$ for the second rainfall pulse increases with the absolute error in ponding time. The error in cumulative infiltrated depth for the second rainfall pulse is compounded by cumulative errors in $F$ from the first rainfall pulse. The error in ponding time is uncorrelated with all soil hydraulic parameters and thus represents a bias associated empirical nature of the GA equation.

The assumed rectangular soil water profile is largely the cause of the error associated with the application of (5) for predicting the rise of $\theta_r$ before the first ponding. Review of soil moisture profiles provided by the Richards equation solution before the first ponding reveals that without exception the soil moisture profile is quite nonrectangular because of the dominance of sorptivity effects early during rainfall. The rectangular profile assumption forces $\theta_{\text{GAR}}$ to be less than $\theta_{\text{r,ini}}$ prepending, except where the rainfall rate is much larger than $K_s$. This effect can be seen in the upper left frame of Fig. 2 for the silt loam case.

![FIG. 5. Error in Cumulative Infiltrated Depth Predicted by GAR Method versus Error in Ponding Time](image)

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The solution of (5) to predict the rise in $\theta_s$ before the first ponding can lead to errors up to 10% at certain times. In lieu of the solution of (5) an alternate approach derived from the Brooks and Corey (1964) $K(\theta)$ equation may be used that provides a superior "fit" to the Richards equation solution. This equation is

$$\theta_s = \theta_i + \left( \theta_s - \theta_i \right) \left( \frac{r}{f_r} \right)^{1/(3+2/\lambda)}$$  \hspace{1cm} (24)

This equation was derived by solving (9) for $\theta_s$, and substituting the potential infiltration rate $f_r$ for $K_r$. If $K_r$ were used in (24) in place of $f_r$, the equation would be valid only when $r < K_r$. The potential infiltration rate $f_r$ is used because, by definition, $f_r$ is always greater than the rainfall rate before ponding and $f_r$ approaches $K_r$ with increasing $F$. Eq. (24) was found to more closely approximate the increase in $\theta_s$ given by the Richards equation solution than the fourth-order Runge-Kutta solution of (5) assuming a rectangular wetting front. The magnitude of the exponent $1/(3 + 2/\lambda)$ varies between 0.06 and 0.17 for the soils given in Table 1.

The saturated hydraulic conductivity $K_s$ is a key parameter in determining both the saturated and unsaturated flow of water in soil. Errors in the GAR prediction of the soil surface relative saturation $\theta_s$ at the end of rainfall hiatus should be dependent upon the magnitude $K_s$, as the data in Fig. 4 and Table 3 indicate. Fig. 6 is a plot of $\Delta \theta_s$ versus log($K_s$). This figure illustrates that the accuracy of the GAR scheme presented herein for predicting the evolution of soil surface water content correlates well with the magnitude of $K_s$. For $K_s$ less than 1 cm/h, the absolute error in surface relative saturation at the end of both hiatus periods is quite small (<2%). For soils with $K_s$ greater than 1 cm/h the absolute error in surface relative saturation during redistribution increases approximately with log($K_s$). These observations are valid for both the first and second hiatus periods. For soils with larger $K_s$, the absolute error is larger at the end of the second hiatus period than at the end of the first. The converse is true for soils with $K_s$ less than 0.5 cm/h.

Significant errors in surface water content estimates during redistribution in coarse soil textures are due to a rapid drying at the soil surface that is predicted by Richards equation for redistribution with $r < K_r$. Rapid drying at the soil surface is illustrated in Fig. 7 that shows results from the Richards equation and GAR method simulations of infiltration on the sand soil texture (soil 1). The left and right frames of Fig. 7 depict the relative saturation profiles at $t = 0.63$ h during the first hiatus period and $t = 3.36$ h during the second hiatus period, respectively. Note that the GAR scheme cannot accurately track the rapid drying at the soil surface that occurs with coarse textured soils and simultaneously conserve mass. The assumed rectangular profile must be maintained at all times during redistribution, resulting in large $\Delta \theta_s$ on coarse soils with large values of $K_s$.

**CONCLUSIONS**

A Green and Ampt redistribution (GAR) scheme was developed ([53]) that uses the traditional rectangular soil water profile and "piston" wetting-front displacement. A total of three soil infiltration parameters ($K_s$, $H_s$, $\lambda$) and three water contents ($\theta_i$, $\theta_s$, $\theta_f$) are required to use the method. Eq. (15) was derived to scale the GAR wetting front soil suction parameter $H_s$ between arbitrary water contents.

A two wetting-front infiltration scheme similar to the approach developed by Smith et al. (1993) was formulated with the assumption of rectangular soil water profiles as shown in Figs. 1 and 2. This formulation is required for predicting infiltration for multiple periods of ponding.

The GAR approach performs quite well compared with the numerical solution of Richards equation for predicting cumulative infiltration $F$ on all soil textures tested during two intense pulses of rainfall separated by a rainfall hiatus period that varied from 2.0 to 2.75 h in length. The error in the prediction of $F$ is an average of 2.0% at the end of the first rainfall pulse and 4.5% at the end of the second. During the first pulse of rainfall, the GAR method is equivalent to the GA method. The error in cumulative infiltrated depth at the end of the second rainfall pulse generally increases on finer soil textures, and is associated with errors in the estimate of ponding time during the second pulse as shown on Fig. 5.

The GAR approach can accurately predict the evolution of soil surface water content $\theta_s$ during rainfall hiatus for soils with saturated hydraulic conductivity $K_s$ less than 1 cm/h. For soils with larger $K_s$, the error in the prediction of $\theta_s$ increases with log($K_s$) as shown on Fig. 6. Errors in $\theta_s$ on coarser soils are the result of rapid drying near the soil surface as illustrated in Fig. 7.

The ability of the GAR algorithm described in this paper to mimic the numerical solution of Richards equations is quite good. Generally, infiltration excess (Hortonian) runoff production results when rain falls on medium to fine textured soils with an intensity that exceeds the infiltration capacity of the soil. Therefore, the method described in this paper is generally applicable to situations where Hortonian runoff production mechanisms predominate.

Given that the GA approach is empirical, one would not
expect it to perform perfectly when compared with the numerical solution of Richards equation. The GA parameters \( K_r \) and \( H_r \) provide flexibility to perform calibration and reduce the error in ponding time. The GAR method to redistribute soil water during rainfall hiatus periods is explicit—and therefore easy to implement and computationally efficient. This attribute renders the method well suited for distributed hydrologic or irrigation advance modeling with a physically based, distributed framework. The primary benefit from the use of the GAR method is that it allows use of the GA approach when there are two or more ponding periods separated in time by drying phases. This development overcomes the main limitation of the traditional GA infiltration equation. However, the GAR method is limited in that it does not consider soil water retention hysteresis effects, or vertical variability of soil infiltration properties.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- \( F \) = cumulative infiltrated depth (L);
- \( F' \) = cumulative infiltrated depth in second moisture profile (L);
- \( F_i \) = cumulative infiltrated depth contained in first soil water profile (L);
- \( F_{GAR} \) = cumulative infiltrated depth predicted by GAR method (L);
- \( F_h \) = cumulative infiltrated depth at the beginning of rainfall hiatus (L);
- \( f_r \) = potential infiltration rate (L/T);
- \( G \) = unsaturated capillary drive (L);
- \( H_r \) = Green and Ampt wetting front soil suction parameter (L);
- \( K_r \) = initial saturated hydraulic conductivity = \( K(\theta) \) (L/T);
- \( K_a \) = saturated hydraulic conductivity (L/T);
- \( K(\theta) \) = unsaturated hydraulic conductivity (L/T);
- \( r \) = rainfall rate (L/T);
- \( r_h \) = rainfall rate during hiatus (L/T);
- \( S \) = sorptivity (L\(^2\)/T);
- \( t \) = time since beginning of first rainfall pulse (T);
- \( t_i \) = time at beginning of latest rainfall hiatus (T);
- \( t_p \) = ponding time (T);
- \( t_{p,GAR} \) = ponding time predicted by GAR method (T);
- \( t_{p,R} \) = ponding time predicted by Richards equation (T);
- \( Z \) = depth to soil moisture profile under consideration (L);
- \( Z_i \) = depth of first moisture profile (L);
- \( Z_2 \) = depth of second moisture profile (L);
- \( Z_m \) = depth of merged soil water profile (L);
- \( \Delta \) = error in ponding time estimate (T);
- \( \Delta \Theta_0 \) = error in surface relative volumetric water content (dimensionless);
- \( \Delta \Theta \) = soil water deficit \( \theta_i - \theta_s \) (dimensionless);
- \( \Delta \Theta' \) = soil water deficit in second moisture profile \( \theta_i - \theta_2 \) (dimensionless);
- \( \varepsilon_r \) = error in cumulative infiltrated depth (L) (dimensionless);
- \( \Theta \) = relative volumetric water content = \( \theta / \theta_s \) (dimensionless);
- \( \Theta_0 \) = relative volumetric water content of the first soil water profile (dimensionless);
- \( \Theta_{0,R} \) = relative volumetric water content at natural saturation (dimensionless);
- \( \Theta_{0,B} \) = relative initial volumetric water content (dimensionless);
- \( \Theta_{0,S} \) = relative volumetric water content at soil surface (dimensionless);
- \( \Theta_{GAR} \) = relative volumetric water content at soil surface predicted by GAR method (dimensionless);
- \( \Theta_{0,R} \) = relative volumetric water content at soil surface predicted by Richards equation (dimensionless);
- \( \Theta_1 \) = volumetric water content of first moisture profile (dimensionless);
- \( \Theta_0 \) = volumetric water content at natural saturation (dimensionless);
- \( \Theta_1 \) = initial volumetric water content (dimensionless);
- \( \Theta_s \) = volumetric water content at soil surface (dimensionless);
- \( \Theta_c \) = volumetric water content at residual saturation (dimensionless);
- \( \lambda \) = pore distribution index (dimensionless);
- \( \Psi \) = soil water matric potential (L);
- \( \Psi_s \) = air entry pressure head (L); and
- \( \Psi_{c} \) = soil water matric potential at \( \theta = \theta_s \) (L).