Immigration and Pension Benefits in the Host-country. ECONOMICA.

Francisco Lagos, University of Granada
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Juan A. Lacomba and Francisco Lagos

Department of Economics, University of Granada, 18071, Granada, Spain.

Telephone number: +34958249605.

E-mail: jlacomba@ugr.es; fmlagos@ugr.es

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Abstract

This paper examines the role that low-skilled immigration plays in determining pension benefits of the host population. With an overlapping-generations model which allows identifying which groups of native population are in favour or against immigration, we find that despite immigrants having a low average productivity, an open borders policy would be implemented since most of current domestic cohorts gain from immigration. Only younger workers might be against immigration since they will coincide with immigrants in their retirement periods. Moreover, we show how a larger immigrant quota would increase the probability of a Pareto
INTRODUCTION

Nowadays the flow of immigrants towards developed countries with generous welfare systems constitutes one of the most important economic issues. In most of these countries, pay-as-you-go (PAYG) financed public pensions are suffering from demographic problems because of the ongoing aging of their populations in such a manner that immigration is frequently being thought of as a means to mitigating the financial problems of public pensions. However, at the same time, it is argued that immigrants are the net beneficiaries of the welfare state (see Sinn (2002) and Chand and Paldam (2004) among others).

As a result, the public choice approach towards immigration has gained increasing interest within the academic literature. Razin and Sadka (1999), in an overlapping-generations model with two periods, show that even though immigrants may be low-skilled and net beneficiaries of the pension system, all groups living at the time of the immigrants’ arrival stand to gain. This result depends crucially on the assumptions that immigrants have the same birth rate as the native population and that the ability index of immigrant offspring is similarly distributed to that of native offspring. Krieger (2004a) replicates the Razin and Sadka model by relaxing these assumptions. He shows that the Razin and Sadka result is no longer unambiguous. In addition, Leers et al. (2004) show that Razin and Sadka’s positive effect of immigration result might only apply in the long-run but not in the short-run. They find that "if immigrants form a significant proportion of mobile labour, and if immigrants’ fertility declines
less than that of natives, mobile labour may become redundant, leading to a decrease in its wage and, consequently, to incentives for emigration."

As Razin and Sadka (1999) point out, the equal ability distribution assumption is a subject for debate. They suggest that the children of immigrants may have attributes such as relatively low school completion rates that weaken their earnings potential later in life. In much the same line, Djajic (2003) argues that the assimilation of immigrants is a multidimensional process of enormous complexity. Assimilation into each of these dimensions (earnings, human capital, occupational status, fertility, etc.) may occur at rates that differ from those of their children. He also argues that the behavioral pattern of immigrants (in consumption, language proficiency, housing, customs, etc.) might result in a slower assimilation of immigrant children in terms of human capital accumulation and economic performance. For these reasons, the assumption that the ability index of immigrant offspring is distributed similarly to that of the native population is followed less strictly in this paper. We assume that the ability indexes of immigrant and native offspring differ across time.

The present article specifically examines the role that the low-skilled immigrant labor force plays in determining the welfare of the host population. In a country with open borders and a PAYG redistributive pension system, we abstract from the effect immigration may have on very different social and economic issues and concentrate on the effect on pension benefits. To do so, we consider that the preferences of the domestic population with regard to immigration at any given point in time crucially depends on the effect of immigration.
on their pensions. We interpret this as a voting decision on immigration.

As a time horizon is needed to assess the true impact of immigration, this paper takes into account that at some point in the future immigrants will start collecting retirement benefits. Moreover, in order to avoid individuals ignoring the impact of current decisions on future retirement benefits, we assume fully rational individuals, that is, individuals who deem their future income benefits to be as quantitatively important as their present income benefits.

Unlike Razin and Sadka (1999) and Krieger (2004a), our theoretical framework considers an overlapping-generations model in continuous time which allows us to specifically identify the different effects of immigration on the different groups within the domestic population. Furthermore, we also analyze how low-skilled immigration affects pension benefits in the long run, once the population has again stabilized.

The results of this paper show how the impact of the immigration policy on the welfare levels of the host population depend crucially on whether their pension benefits are shared or not with immigrants. When this is not the case (with retirees and workers who are close to retirement), the arrival of immigrants has an unambiguously positive impact on the welfare of the native population. Retirees and older workers of the current domestic population would vote in favor of immigration: immigration increases the number of contributors and thus leads to higher pensions. However, immigration might be disadvantageous to ‘younger’ workers. Provided that they coincide with immigrants during the retirement period and thus share pension benefits, this negative effect may offset
the positive effect from immigrants’ additional contributions. Concretely, we find that younger workers will either gain or lose from immigration depending on the average labor productivity of the immigrants and also on the size of the immigrant quota. The larger the immigrant quota, the more likely that even younger workers will vote in favor of immigration. Consequently, we find that even if the average labor productivity of immigrants is lower than that of the domestic population, an open borders policy will be implemented since most of the current domestic cohorts benefit from this immigration.

Our results also highlight the crucial role of the pension plan considered. Scholten and Thum (1996) and Krieger (2004b), assuming a defined-benefits plan, find that retirees -and older workers- are less likely to support a policy of open borders. In this paper, the opposite result is obtained. We find that retirees and older workers will always be in favor of immigration. As Razin and Sadka (1999), we consider a defined-contribution plan where pension benefits are affected by the arrival of immigrants. This is a plausible assumption since -due to the aging of societies- contribution rates will certainly not fall in the future. At the same time, global tax competition and mobile workers set an upper limit to payroll taxation which does not allow further increasing contribution rates either.³

This paper is organized as follows. Section 2 develops the model. Section 3 analyzes how the arrival of immigrants affects the welfare of the host population. In Section 4 a numerical example illustrates the results obtained in the previous sections. Section 5 summarizes the main results. The proofs appear in the
I. THE MODEL

Following Lacomba and Lagos (2005), we consider an overlapping-generations model in continuous time. At each point in time $t$ a new cohort of individuals is born. We assume a constant birth rate which is normalized to unity. In this model we have a continuous and uniform distribution of agents with regard to age, with no uncertainty on the length of their lives, going from zero to a fixed age, $T$. Furthermore, the wage rate is normalized to unity and a continuous distribution of agents in labor productivity between a minimum and a maximum level $[l_-, l_+]$ is assumed.

The government levies a contribution rate, $\tau$, for a redistributive social security program. This social security program is a defined-contribution "pay as you go" system (PAYG), in which current workers are net contributors while the retired are net beneficiaries.

Subjects have a temporally independent utility function, which is strictly increasing in terms of consumption. Let $\delta$ and $r$ be the subjective rate of time preference and the market rate of interest, respectively. Let $p$ be the annual pension benefits that workers receive when they retire. Let $R \in (T/2, T)$ be the current legal retirement age at which pension benefits are available. Hence the lifetime utility of an individual $i$ can be written as

$$\int_0^T u(c_t^i) e^{-\delta t} dt = \int_0^R u(c_t^i) e^{-\delta t} dt + \int_R^T u(c_t^i) e^{-\delta t} dt$$

(1)
with

\[ \int_{0}^{T} c_i e^{-rt} dt = \int_{0}^{R} (1 - \tau) l_i e^{-rt} dt + \int_{R}^{T} pe^{-rt} dt \quad (2) \]

where \( c_i \) and \( l_i \) are respectively the consumption at period \( t \) and the labor productivity of individual \( i \). Furthermore, it is also assumed that the consumption utility is twice differentiable with \( u' > 0 \) and \( u'' < 0 \).

The pension system is assumed to be unfunded and fully redistributive, consequently the discounted value of total pension benefits received by any individual is as follows

\[ P = \int_{R}^{T} \frac{\tau RL}{T - R} e^{-rt} dt \quad (3) \]

where \( L \) is the average labor productivity of the working population.

Social security systems may range from a flat-rate pension benefits type (usually referred to as a Beveridgean scheme) to an earnings-related pension benefits type (usually referred to as a Bismarckian scheme).\(^6\) In this article we concentrate on a Beveridgean pension system in order to examine the less advantageous benchmark on pension benefits for native individuals. If a Bismarckian pension system were considered then the effects of immigration would be more beneficial for the native population. The more Bismarckian the system, the less the intra-generational redistribution, and thus the lower the impact on pension benefits of the falling average labor productivity due to immigration.\(^7\)

For the sake of simplicity, we assume that there are no returns on savings
and that individuals do not discount the future, so both discount rates are zero
\((\delta = r = 0)\). This assumption implies that each individual will set a constant
consumption per period, that is, individuals are fully rational and hence their
future income benefits become quantitatively as important as their present in-
come benefits. Moreover, this assumption is adopted in order to avoid myopic
individuals (of different ages) ignoring the true impact of current decisions on
their pension benefits. 8

Thus the indirect lifetime utility function of an individual \(i\) of age \(a\) can be
reduced to

\[
U(c_i) \equiv (T - a) u(c_i)
\]

where the optimal consumption is given by

\[
c_i = \frac{1}{T} (R (1 - \tau) l_i + (T - R) p).
\]

At period \(t\), a quota of \(m \in (0, 1)\) immigrants are allowed in. 9 Following
Razin and Sadka (1999), it is assumed that these immigrants are all young
\((a = 0)\) and unskilled workers \((l = 0)\). We focus on low-skill immigration in
order to examine, again, the less advantageous benchmark for native individuals.
Moreover, immigrants have the same preferences and the same fertility rate as
the native population. Hence, in the host country a cohort of \(1 + m\) individuals
is born at each point in time from period \(t\) on.

At this period \(t\), agents are then characterized both by age and labor pro-
ductivity. They also have a different amount of accumulated wealth, \( \pi(a, l_i) \), which is given by the total income earned minus total consumption up to the instant at which the immigrants arrive

\[
\pi(a, l_i) = \left\{ \begin{array}{ll}
a \left(1 - \frac{R}{\tau}\right) ((1 - \tau)l_i - p) & a \leq R; \\
R \left(1 - \frac{R}{\tau}\right) ((1 - \tau)l_i - p) & a > R.
\end{array} \right.
\]  

(6)

This expression describes the pattern of wealth accumulation. Note that there exists a threshold labor productivity \( \tilde{l} \) such that \( \tilde{l}(1 - \tau) = p(\tilde{l}) \). In particular

\[
\tilde{l} = \frac{R\tau L}{(1 - \tau)(T - R)}.
\]  

(7)

We consider that \( l_\tau > \tilde{l} \). Therefore, the accumulated wealth increases linearly with age up to the retirement age and beyond that age agents start to spend their accumulated savings.

II. WELFARE ANALYSIS THROUGH RETIREMENT BENEFITS

Having determined the setting, we shall now turn our attention to how the arrival of immigrants affects the welfare of the host population. Due to immigration, the pension benefits of each native individual will be affected in a different way according to age. These new pension benefits will lead to changes in the optimal consumption. Thus, at period \( t \), the new optimal consumption will now be given by
\[ c_i = \begin{cases} \frac{1}{1-a} ((R-a)(1-\tau)l_i + (T-R)p(a,m) + \pi(a,l_i)) & a \leq R; \\ \frac{1}{1-a} (T-a)p(a,m) + \pi(a,l_i)) & a > R. \end{cases} \] (8)

The basic issue in deriving these expressions is that the optimal consumption adjusts to an income change only in the remaining periods of life.

Analytically, we derive how the utility function of each individual evaluated in the optimal consumption changes with immigration,

\[ \frac{\partial U (\cdot)}{\partial m} = (T-a) u'(c_i) \frac{\partial c_i}{\partial m}. \] (9)

The sign of (9) depends on the sign of \( \frac{\partial c_i}{\partial m} \), that is, the effect of immigration on welfare comes from its effect on consumption levels through the pension benefits.

On the other hand, notice that the preferences of domestic groups with regard to unskilled immigration will depend on how their welfare levels are affected by the arrival of immigrants. As in Krieger (2003), these preferences can be interpreted as a voting decision on immigration. In this manner, we can also examine whether an open borders policy would be implemented or not.

As mentioned above, with the arrival of immigrants at period \( t \), the pension benefits of each native individual will be affected in a different way according to age. Three benchmark cases can be defined.
The retirees

With regard to retirees, that is, native individuals whose age at period $t$ is above the retirement age, $a \geq R$, it is easy to check that they unambiguously gain from immigration. These native individuals do not coincide with immigrants while retirees since they will pass away before the first immigrants start collecting benefits. Thus, the discounted value of total pension benefits received by a retired individual aged $a$ is as follows

$$P(a) = (T - a) \tau RL \frac{R}{T - R} + \int_0^{T - a} \tau mI \frac{I}{T - R} \, dt.$$  

(10)

where $I$ denotes the average labor productivity of immigrants. Hereafter, it is assumed that $I < L$.\textsuperscript{10} The first term of the RHS in (10) is the part of pension benefits coming from native workers’ contributions, and the second term is the increase in pension benefits derived from immigrant workers’ contributions. Needless to say, any positive immigrant quota, $m$, will increase current retirees’ pension benefits.

The "non-sharing pension" workers

With regard to the working population, we have to distinguish between those native workers who will not coincide with immigrants during their retirement period and those who will. The first group is composed of the native workers who will also pass away before the first immigrants start collecting benefits. These native workers are those whose age at period $t$ is higher than the length of the retirement period, $T - R \leq a \leq R$. Thus, in this group, the discounted
value of total pension benefits received by an individual aged $a$ is as follows

$$P(a) = \tau RL + \int_{R-a}^{T-a} \frac{\tau tm I}{T-R} dt$$  \hspace{1cm} (11)$$

where again the first term of the RHS in (11) is the part of pension benefits coming from native workers’ contributions, and the second term is the increase in pension benefits derived from immigrants’ contributions. As above, the only effect here is the increase in total pension benefits due to the arrival of immigrants.\(^\text{11}\) Thus, like retirees, 'older' workers would gain from immigration and therefore they would also vote in favor of any positive immigration quota.\(^\text{12}\)

In short, for those domestic individuals who will not coincide with immigrants during their retirement period, immigrants are net contributors to the social security system. The following proposition can therefore be stated.

**Proposition 1** For any native individual aged $a \in [T - R, T]$, any immigrant quota $m \in (0, 1)$ increases her pension benefits.

Proof: It follows straightforward from (10) and (11).

From Proposition 1 it can be deduced that any native individual who does not coincide with immigrants during her retirement period will increase her welfare and will thus support an open borders policy for any positive immigrant quota.

The "sharing pension" workers

Now we shall turn to 'younger' workers, that is, the domestic population who will coincide with immigrants, at least some years, during their retirement
period. This group is composed of native workers whose age at period $t$ is lower than the length of the retirement period, $0 \leq a \leq T - R$. In this group, the discounted value of total pension benefits received by an individual aged $a$ is as follows

$$P(a) = a \frac{\tau RL}{T-R} + \int_{R-a}^{R} \frac{\tau mI}{T-R} dt + \int_{R}^{T-a} \frac{\tau RL + \tau RmI}{T-R + (t-R)m} dt$$

(12)

The first two components of the RHS in (12) represent pension benefits received before the first generation of immigrants reaches the retirement period. As in the previous cases, native individuals will benefit during these years from the arrival of immigrants. The third component reflects pension benefits received once immigrants start to enter the retirement period. In this third component, besides the usual positive effect on the native population’s pension benefits derived from immigrants’ additional contributions we also observe a negative effect, since now, these pension benefits will have to be shared among a larger number of retirees (natives and immigrants).

Therefore, the final effect on pension benefits is ambiguous. It will depend on the immigrant quota, $m$, the average labor productivity of immigrants, $I$, and the native workers age, $a$.

Let us define $\bar{I}(a, m)$ as the threshold value of the immigrants’ average productivity. Then, we summarize the possible results in the following proposition:
Proposition 2 For any native individual aged $a \in [0, T - R]$, and for any immigrant quota $m \in (0, 1)$, there exists a threshold average labor productivity of immigrants $\bar{I}(a, m) \in (L, T)$ with $0 < L < \bar{I}(a, m) < I < L$, such that:

i) $\frac{\partial p}{\partial m} < (>)0$ for any $I < (>)\bar{I}(a, m)$.

ii) $\frac{\partial \bar{I}(a, m)}{\partial a} < 0$.

iii) $\frac{\partial \bar{I}(a, m)}{\partial m} < 0$.

Proof: See Appendix.

From Proposition 2 it can be deduced that immigration does not always entail a Pareto improvement for the population of the host country. The first point states that for each cohort of these native workers there exists a threshold value $\bar{I}(a, m)$ such that if the immigrants’ average productivity is lower (greater) than this threshold value their pension benefits will be reduced (improved) and so will their welfare. Needless to say, these workers would vote against (in favor of) immigration.

The second point states that the younger native workers are, the higher this threshold value is. That is, if for instance, $I = \bar{I}(\hat{a}, m)$, then domestic workers of age $a < \hat{a}$ will be against immigration while domestic workers with age $a > \hat{a}$ will be in favor of it. The intuition is the following. The younger the native workers are, the larger the part of the retirement period they will share with immigrants and therefore the less they will benefit from immigrant arrival. Consequently, it is more likely that younger native workers vote against an open borders policy.

However, the third point of Proposition 2 tells us that the larger the im-
migrant quota, \( m \), the lower the threshold value \( \bar{I}(a,m) \) is for any given age, and therefore, the more likely the open borders policy be supported even by 'younger' workers. The underlying intuition is the following. When these native workers are retired, the immigrant working population will have already stabilized. As such, the positive effects of the additional immigrant contributions due to a larger immigrant quota \( m \) will be larger than the negative effects of the reduction in pension benefits caused by this larger quota since immigrants will not yet be totally incorporated into the retirement period.

Finally, notice that if the average productivity of immigrants were lower than the threshold value of native workers of age \( a = T - R \), \( I < \bar{I}(T - R, m) \), then the welfare of the whole group of 'younger' domestic workers would be reduced and therefore they would vote against immigration. But since \( R > T/2 \), this group would be composed of less than 50% of native cohorts, and consequently a majority of the population, formed by older workers and retirees, would still support an open borders policy.

On the other hand, if the average productivity of immigrants were larger than the threshold value of native workers of age \( a = 0 \), \( I > \bar{I}(0, m) \), then immigration would entail a Pareto improvement for the domestic population of the host country.

In short, since most of the native population will not coincide with immigrants in their retirement period, or they will coincide for a few years only, an open borders policy would improve their pension benefits leading them to higher welfare levels. Since we assume \( R > T/2 \), the coalition of retirees and 'older'
workers would always be the majority and thus the vote would be in favor of immigration. Moreover, the larger the immigration quota, the more likely that even 'younger' workers be in favor of immigration.

III. NUMERICAL EXAMPLE

In this numerical example we illustrate the gains and losses derived from the arrival of immigrants. The results are summarized in Tables 1 and 2.

Since the effects of immigrants on welfare levels are due to changes produced in the pension benefits of the current domestic population, we focus on the effect of a positive immigrant quota on these pension benefits.

The first column of the two tables describes the different immigrant quotas allowed. We consider three possibilities: an economy with no immigration, that is, where \( m = 0 \); and two positive immigrant quotas, \( m = 0.1 \) and \( m = 0.2 \). The second, third and fourth columns of the two tables contain the value of the sum of total pension benefits received by the three different current domestic groups: retirees, the non-sharing pension workers and the sharing pension workers, respectively. The last columns of the two tables show the sum of the pension benefits of these three groups, that is, the sum of total pension benefits received by the domestic population.

The calculations are carried out for the following values: life length \( T = 62 \); retirement age \( R = 42 \); and contribution rate \( \tau = 25\% \). We consider two levels of average productivity for immigrants: \( I = 0.1L \) (in Table 1), that is, a productivity ten times lower than the domestic population average productivity;
and $I = 0.5L$ (in Table 2), that is, half of the domestic population average productivity.

<table>
<thead>
<tr>
<th>$m$</th>
<th>Retirees</th>
<th>Non-Sh.Workers</th>
<th>Sh.Workers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1155</td>
<td>2100</td>
<td>2310</td>
<td>5565</td>
</tr>
<tr>
<td>0.1</td>
<td>1157.02</td>
<td>2110.5</td>
<td>2301.67</td>
<td>5561.55</td>
</tr>
<tr>
<td>0.2</td>
<td>1159.03</td>
<td>2121</td>
<td>2280.65</td>
<td>5560.68</td>
</tr>
</tbody>
</table>

From Table 1, it can be observed how the introduction of a positive immigrant quota increases the pension benefits received both by retirees and by non-sharing pension workers. However, the reduction in the sum of pension benefits received by the sharing pension workers means that the sum of total pension benefits of the whole domestic population to be lower, the larger the immigrant quota. This negative effect of the immigrant quota on total pension benefits is fundamentally due to the low average productivity of the immigrant population, as can be observed when comparing these results with those of Table 2.

In spite of the reduction in total pension benefits, since retirees and non-sharing pension workers make up the majority of the population, any positive immigrant quota would be implemented in a majority voting process.
Table 2. Pensions of the domestic population with $I=0.5L$

<table>
<thead>
<tr>
<th>m</th>
<th>Retirees</th>
<th>Non-Sh.Workers</th>
<th>Sh. Workers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1155</td>
<td>2100</td>
<td>2310</td>
<td>5565</td>
</tr>
<tr>
<td>0.1</td>
<td>1165.08</td>
<td>2152.5</td>
<td>2376.9</td>
<td>5694.48</td>
</tr>
<tr>
<td>0.2</td>
<td>1175.17</td>
<td>2205</td>
<td>2443.71</td>
<td>5823.88</td>
</tr>
</tbody>
</table>

Table 2 contains simulation results for a higher immigrant average productivity. Here again all retirees and non-sharing workers gain from immigration. But in this case the sum of total pension benefits of sharing pension workers also increases and therefore so does the sum of pension benefits of the domestic population.

What about changes in the welfare levels of agents with different levels of labor productivity? The comparison depends on which utility function is used, but since the relative weight of pension benefits in the utility function is lower the richer an individual is, we can state that when unskilled immigrants arrive, the most skilled members of the domestic population are the least affected and vice versa, which is in line with Razin and Sadka (2000).

IV. CONCLUSIONS

In most developed countries public pension systems are suffering from demographic problems because of the ongoing aging of their population. In this context, immigration could be considered as an attractive resource in mitigating the financial problems of the public pension systems. However, is immigration welcomed by all native generations in the host country?
With an overlapping-generations model in continuous time, which allows us to identify specifically which groups within the native population are better or worse off with immigration and a fully redistributive pension system, we have investigated the impact that low-skill immigration has on the domestic population’s pension benefits.

The model shows that since most of the current native cohorts gain from immigration, the open borders policy will be implemented. A majority of voters (retirees and older workers) will be in favor of immigration while only the younger workers might vote against it. On the one hand, the increasing number of contributors due to immigration will result in higher pension benefits for both retirees and older workers. On the other hand, younger workers, who will coincide with immigrants in their retirement periods, will either gain or lose from immigration depending on the average labor productivity of the immigrants and also on the size of the immigrant quota. The larger the immigrant quota, the more likely that even younger workers vote in favor of immigration. Consequently, as in Leers et al. (2004), while some young native generations may not welcome immigration and prefer a policy of closed borders, others may get benefits from immigrants and prefer the opposite policy.13

Summarizing, we find that in spite of low-skilled immigrants and a fully redistributive social security system, immigration might entail a Pareto improvement for the whole native population. However, as Razin and Sadka (1999) suggest, the children of immigrants may have attributes such as relatively low school completion rates that weaken their earnings’ potential later in life. Thus,
if immigrants maintain low labor productivity over time, the young native generations may lose out from immigration and opt to halt it. What is more, a conflict of interests could arise among younger generations.

**APPENDIX**

*Proof of proposition 2 (for \( a = 0 \))*

After some simplifications, individual annual pension benefits can be rewritten as

\[
p(0) = \frac{\tau R}{T - R} \frac{\ln (1 + m)}{m} (L + mI).
\] (13)

We obtain that

\[
\frac{\partial U(\cdot)}{\partial m} = Tu'(c) \frac{\partial c}{\partial m}
\] (14)

where

\[
\frac{\partial c}{\partial m} = \frac{\tau (m (L + mI) - \ln (1 + m) L (1 + m))}{Tm^2 (1 + m)}
\] (15)

By definition \( u'(c) \) is strictly positive, therefore the sign of (14) depends on the sign of \( \partial c/\partial m \). Let us define \( I = p \ast L \). Then \( \partial U(\cdot) / \partial m \), after some simplifications, can be expressed as follows

\[
\frac{\partial U(\cdot)}{\partial m} = u'(c) \left[ pm^2 + m - (1 + m) \ln (1 + m) \right] \frac{I \tau R}{p (1 + m) m^2}
\] (16)
In order to clarify the definitive sign of $\partial c/\partial m$ and $\partial U (.) / \partial m$ we proceed:

For $p \in [0,1]$ and $m \in (0,1)$ define a function $S (m,p) := pm^2 + m - (1 + m) \ln (1 + m)$. Notice that the sign of $S (m,p)$ is equivalent to the sign of $\partial c/\partial m$ and $\partial U (.) / \partial m$. The function $S (m,p) > 0$ if and only if $pm^2 + m - (1 + m) \ln (1 + m) > 0$, that is, if and only if $p > \frac{(1 + m) \ln (1 + m) - m}{m^2}$. For $m \in (0,1)$, define a function $H (m) := \frac{(1 + m) \ln (1 + m) - m}{m^2}$. We obtain that $\lim_{m \to 0} H (m) = 0.5$ and $H (1) = 0.38$. Let $\overline{p} = 0.5$ and $\underline{p} = 0.38$. Then for any $p > \overline{p}$ we find that $p > H (m)$, $\forall m \in (0,1)$. Consequently, for any $p > \overline{p}$, we find that $S (m,p) > 0$, i.e., $\partial c/\partial m$ and $\partial U (.) / \partial m$ are positive, $\forall m \in (0,1)$.

Analogously, for any $p < \underline{p}$ we find that $p < H (m)$ and thus $S (m,p), \partial c/\partial m$ and $\partial U (.) / \partial m$ are negative $\forall m \in (0,1)$.

Moreover, since $H (m)$ is a continuous, monotone and decreasing function, for any $\underline{p} < p < \overline{p}$ there is a $m^* \in (0,1)$ such that $p = H (m^*)$. Thus $S (m^*, p), \frac{\partial c}{\partial m} |_{m^*}$ and $\frac{\partial U (.)}{\partial m} |_{m^*}$ are equal to zero. Then, for any $m' < m^*$ implies that $H (m') > H (m^*)$ which means that $H (m') > p$. As we showed earlier, this is equivalent to $S (m,p), \partial c/\partial m$ and $\partial U (.) / \partial m$ are negative $\forall m \in (0,m^*)$. Symmetrically, for any $m' > m^*$ this leads to $H (m') < H (m^*)$ which yields that $H (m') < p$. That is, $S (m,p), \partial c/\partial m$ and $\partial U (.) / \partial m$ are positive $\forall m \in (m^*,1)$.

Let $\mathcal{T}$ and $\mathcal{L}$ be the levels of average labor productivity of immigrants for $\overline{p}$ and $\underline{p}$ respectively. Therefore, we can state the following:

i) For any $I \in (0, \mathcal{L})$ then $\partial c/\partial m$ and $\partial U (.) / \partial m$ are strictly negative.

ii) For any $I \in (\mathcal{T}, \mathcal{L})$ then $\partial c/\partial m$ and $\partial U (.) / \partial m$ are strictly positive.
iii) For any $I \in (L, T)$ the signs of $\partial c/\partial m$ and $\partial U(\cdot)/\partial m$ depend on $m$. In this case, there exists a $m^*$ such that for any $m \in (0, m^*)$ the signs of $\partial c/\partial m$ and $\partial U(\cdot)/\partial m$ are negative and for any $m \in (m^*, 1)$ the signs of $\partial c/\partial m$ and $\partial U(\cdot)/\partial m$ are positive.

Let us define $p_0 = H(m_0)$ and $p_1 = H(m_1)$. Let us suppose that $p_1 < p_0$ and $m_1 < m_0$. Since $H(m)$ is a continuous, monotone and decreasing function, $m_1 < m_0$ implies that $H(m_1) > H(m_0)$ or equivalently $p_1 > p_0$, which is a contradiction. Therefore, if $m_1 < m_0$ then $p_1 > p_0$. That is, the larger the quota $m$ is, a lower immigrant labor productivity $I$ is needed to obtain $\partial c/\partial m$ and $\partial U(\cdot)/\partial m$ positive or negative. Q.E.D.

*Proof of proposition 2 (for $0 < a \leq T - R$)*

After some simplifications, the individual annual pension benefits can be rewritten as

$$p(a) = \frac{\tau}{(T - R)} \left[ \frac{a}{T - R} \left( RL + mI \left( R - \frac{a}{2} \right) \right) + RL\alpha + RmI\alpha \right] \quad (17)$$

where $\alpha = \frac{\ln(T - R + nT - mR - ma) - \ln(T - R)}{m}$.

From first order conditions of the utility function we obtain

$$\frac{\partial U(\cdot)}{\partial m} = (T - a) u'(c) \frac{\partial c}{\partial m} \quad (18)$$

where
\[
\frac{\partial c}{\partial m} = \frac{\tau \left( \frac{a}{T-R} I (R - \frac{1}{2}a) + RL\gamma_1 + RL\gamma_2 \right)}{(T-a)} \tag{19}
\]

where

\[
\gamma_1 = \frac{T - R - a}{T - R + m(T - R - a)}
\]

and

\[
\gamma_2 = \frac{T - R - a}{(T - R + m(T - R - a))m} \ln \left( \frac{T - R + m(T - R - a)}{m} \right) - \ln (T - R).
\]

By definition \( u'(c) \) is strictly positive, thus the sign of (18) depends on the sign of \( \partial c/\partial m \). Let us define \( I = p \ast L \). Then \( \partial U(\cdot)/\partial m \), after some simplifications, can be expressed as follows

\[
\frac{\partial U(\cdot)}{\partial m} = u'(c) \left( \frac{a}{T-R} \left( R - \frac{1}{2}a \right) + R\gamma_1 + \frac{1}{L} R\gamma_2 \right) \tau I \tag{20}
\]

In order to clarify the definitive sign of \( \partial c/\partial m \) and \( \partial U(\cdot)/\partial m \) we proceed:

For \( a \in (0, T - R], R \in [T/2, T], p \in [0, 1] \) and \( m \in (0, 1) \), define a function \( T_{a,R}(m, p) := \frac{a}{T-R} \left( R - \frac{1}{2}a \right) + R\gamma_1 + \frac{1}{p} R\gamma_2 \). Notice that the sign of \( T_{a,R}(m, p) \) is equivalent to the sign of \( \partial c/\partial m \) and \( \partial U(\cdot)/\partial m \). The function \( T_{a,R}(m, p) > 0 \) if and only if \( \frac{a}{T-R} \left( R - \frac{1}{2}a \right) + R\gamma_1 + \frac{1}{p} R\gamma_2 > 0 \), that is, if and only if \( p > -\frac{2R\gamma_2 T - 2R^2\gamma_2}{2aR - a^2 + 2R\gamma_1 T - 2R^2\gamma_1} \). For \( a \in (0, T - R], R \in [T/2, T] \) and \( m \in (0, 1) \), define a function \( Q_{a,R}(m) := -\frac{2R\gamma_2 T - 2R^2\gamma_2}{2aR - a^2 + 2R\gamma_1 T - 2R^2\gamma_1} \). Moreover, \( Q_{a,R}(m) \) is a continuous, monotone and decreasing function. Let \( \overline{p} \) be the maximum value of \( Q_{a,R}(m) \) with \( a \in (0, T - R] \) and \( R \in [T/2, T] \). Then for any
we obtain that \( p > Q_{a,R}(m) \), \( \forall m \in [0,1] \). Consequently, for any \( p > \overline{p} \), we have that \( T_{a,R}(m,p) > 0 \), i.e., \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are positive, \( \forall m \in (0,1) \).

Analogously, let \( \underline{p} \) be the minimum value of \( Q_{a,R}(m) \) with \( a \in (0,T-R] \) and \( R \in [T/2,T] \). Then, for any \( p < \underline{p} \) we find that \( p < Q_{a,R}(m) \) and thus \( T_{a,R}(m,p) \), \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are negative \( \forall m \in (0,1) \).

Moreover, since \( Q_{a,R}(m) \) is a continuous, monotone and decreasing function. For any \( \underline{p} < p < \overline{p} \) there is a \( m^* \in (0,1) \) such that \( p = Q_{a,R}(m^*) \) with \( a \in (0,T-R] \) and \( R \in [T/2,T] \). Thus \( T_{a,R}(m^*,p) \frac{\partial c}{\partial m} |_{m^*} \) and \( \frac{\partial U(.)}{\partial m} |_{m^*} \) are equal to zero. Then, for any \( m' < m^* \) implies that \( Q_{a,R}(m') > Q_{a,R}(m^*) \) which means that \( Q_{a,R}(m') > p \). As we showed earlier, this is equivalent to that \( T_{a,R}(m,p) \), \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are negative \( \forall m \in (0,m^*) \). Symmetrically, for any \( m' > m^* \) this leads to \( Q_{a,R}(m') < Q_{a,R}(m^*) \) which yields that \( Q_{a,R}(m') < p \). That is, \( T_{a,R}(m,p) \), \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are positive \( \forall m \in (m^*,1) \).

Let \( T \) and \( L \) be the levels of average labor productivity of immigrants with \( \overline{p} \) and \( \underline{p} \) respectively. Therefore, we can state the following:

i) For any \( I \in (0,L) \) then \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are strictly negative.

ii) For any \( I \in (T,L) \) then \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are strictly positive.

iii) For any \( I \in (L,T) \) the signs of \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) depend on \( m \).

In this case, there is a \( m^* \) such that for any \( m \in [0,m^*) \) the signs of \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are negative and for any \( m \in (m^*,1] \) the signs of \( \partial c/\partial m \) and \( \partial U(.)/\partial m \) are positive.

Let us define \( p_0 = Q_{a,R}(m_0) \) and \( p_1 = Q_{a,R}(m_1) \). Let us suppose that \( p_1 < p_0 \) and \( m_1 < m_0 \). Since \( Q_{a,R}(m) \) is a continuous, monotone and decreasing
function, $m_1 < m_0$ implies that $Q_{a,R}(m_1) > Q_{a,R}(m_0)$ or equivalently $p_1 > p_0$, which is a contradiction. Therefore, if $m_1 < m_0$ then $p_1 > p_0$. Therefore, the larger $m$, a lower $I$ is needed to obtain $\partial c/\partial m$ and $\partial U(\cdot) / \partial m$ positive or negative.

At last, there is no analytical solution to verify the inverse relationship between $I$ and $a$, but numerically one can easily obtained.\(^{14}\) Q.E.D.

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**Notes**

Although immigrants may return to their country of origin before to collecting pension benefits (and thus this fact would reinforce their role of contributors to the social security systems), in our setting we assume that immigrants never return before qualifying for their full pension benefits. See Krieger (2008) for an analysis in detail with effects of return migration on pension systems.

\(^{2}\)In this context, the assumption of rational individuals has been discussed by Lindbeck and Persson (2003), Diamond (2004) or more recently by Cremer et al. (2007). These authors argue the possibility that individuals may be “myopic” and not adequately save for their retirement.
3Haupt and Peters (1998) and Krieger (2003) also show how preferences regarding immigration policy turn upside down when changing from a defined-benefit system to a defined-contribution system. Lacomba and Lagos (2006) also show the relevance of correctly choosing the parameter affected by the dependency ratio, contribution rate or pension benefits, in the design of the Social Security programme.

4Lacomba and Lagos (2005) analyze how the arrival of immigrants affects the optimal legal retirement age of each native individual and, as consequence, what kind of pressure this event could exert on the retirement age of the public pension system. In this paper we focus instead on retirement benefits of each native individual.

5By assuming a legal retirement age $R$ larger than $T/2$, and a uniform distribution of agents, we consider that the number of workers must be larger than the number of retirees. In fact, even in the most pessimistic estimations about aging, the ratio retirees-workers is always less than one.

6See Casamatta et al. (2000) for a classification of several OECD countries depending on the redistribution nature of the Social Security system.

7Cremer and Pestieau (1998) show how two countries choose their social insurance systems (Bismarckian or Beveridgean) in a context of labor mobility and payroll tax competition.

8If $\delta$ and $r$ were equal but different from zero, the consumption would not be affected. However, the calculation of pension benefits would be greatly complicated without adding further insights.

9Although the basis of the immigrant quota might be a share of applications of all immigrants which are permitted or a specific share of the host country population or even a combination of both, in this article the immigrant quota could be thought as a percentage of the host-country population. Furthermore, Leers et al. (2004) find that the aging of native population may eventually lead to emigration ($m < 0$) instead of immigration ($m > 0$). However, we just focus on immigration towards developed countries with generous welfare systems.

10As mentioned earlier, the equal ability distribution assumption is a subject of open debate. For that reason, we assume that the ability index of immigrants and natives differ across time.
Notice that within this group, the younger the native worker, the larger the additional contributions coming from immigrants.

In the second term on the RHS of (10) and (11), $t$ denotes the number of immigrant generations living in the host-country ever since the first generation arrived.

Although in a different theoretical framework, Leers et al. (2004) find that young workers (mobile against immobile) might support opposite policies with regard to immigration.

The numerical verification holds for a wide range of parameter values: $l > 0; \tau \in (0,1); L > 0; I \leq L; m \in (0,1)$ and $a \leq T - R$.

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