Vertical Integration in Unregulated Industries with Essential Facilities

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Vertical Integration in Unregulated Industries with Essential Facilities

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Abstract

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In this paper, we consider a market that is operated by a non-integrated monopoly upstream that owns an important essential facility and an duopolistic market downstream that is facing entry in the monopolistic upstream market. We show that: (i) for small fixed costs of building a new facility the unique equilibrium entails vertical integration for all firms and duplication of essential facilities; (ii) for an intermediate range, the unique equilibrium entails full vertical integration and a shared-facility; and (iii) for large fixed costs, the unique equilibrium entails vertical integration by the incumbent, no integration by the entrant and a shared-facility. In addition, the equilibrium in (i) is always efficient relative to a shared-facility agreement, the equilibrium in (ii) is inefficient relative to duplication of essential facilities for fixed costs lower than certain cutoff and efficient otherwise, and the equilibrium in (iii) is always efficient relative to duplication of essential facilities.

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1 Introduction

It has become a general practice in several unregulated industries to share essential facilities to “avoid” duplication of unnecessary capacity. For instance, it is common in the airline industry to share terminal ground space or ground crews, in the gas and petroleum industry to share pipelines and in the banking industry to share ATM networks. Shared facilities are also important in regulatory analysis of transportation, electricity, telecommunications and mail delivery systems to name a few.\footnote{There are a number of other markets in which network competition vis-a-vis a shared network design are alternative industry structures, such as hardware and software industries, broadcasting, technology and standards, telecommunications, etc. All these industries are beyond the scope of this paper because their main characteristic is the existence of network externalities (see, Leibowitz and Margolis (1994) and Oz (2000) for further details). On the contrary, the main characteristic of the industries to which our results apply is the presence of an essential facility in the absence of consumption externalities, switching costs and lock-ins, etc.} Furthermore, since the 1990s, several of these industries have experienced process of vertical integration that have resulted in that the owner of the essential facility become directly involved in serving final customers. For instance, ports operators are integrated to shipping companies and long-distance carriers are vertically integrated with local telephone companies.\footnote{In 1994 Telefónica-Chile, basically a monopoly in the local telephone with around 95 % of market, was authorized to operate in the long-distance market through a multicarrier system. Although a regulated one.} These trends have merit the attention of antitrust authorities. In fact, antitrust agencies have noticed that sharing facilities and vertically integration can be anticompetitive, and many have been subject to detail regulatory scrutiny. For example, Serra (2001) describes five cases in which essential facilities and vertical integration are involved where the Chilean antitrust authority adopted the following measures to avoid what they believe are the negative consequences of these practices.\footnote{The cases are: (i) Vertical integration in telecommunications; (ii) vertical integration in the electrical sector; (iii) auction in gas pipelines; (iv) auction of the main Chilean seaports; and (v) vertical integration in garbage collection, transfer and disposal.} They: (i) regulate the fees charged to access the essential facility; (ii) promote the existence of more than one essential facility supplier; (iii) increase the autonomy of the ancillary or force it to operate as an independent firm; and (iv) restrict or prohibit the participation of the owner of the essential facility downstream.\footnote{Another example is the October 1999 FTC and the Antitrust Division of the Department of Justice release of a draft of antitrust guidelines for collaboration among competitors.}

To better understand the consequences over competition and welfare of vertical integration and shared facilities, we consider a market that is operated by a non-integrated monopoly upstream that owns an important essential facility and an duopolistic market downstream that is facing changes like a decrease in barriers to foreign investment or deregulations that make entry feasible in the monopolistic upstream market. In particular, we assume that there is an
important input supplier entering in the upstream market that can enter either building its own essential facility (full entry) or buying capacity from the incumbent monopoly upstream (partial entry) and upon entry upstream firms decide whether to vertically integrate with downstream firms.

We show that: (i) for small fixed costs of building a new facility the unique equilibrium entails vertical integration for all firms and duplication of essential facilities; (ii) for an intermediate range, the unique equilibrium entails full vertical integration and a shared-facility; and (iii) for large fixed costs, the unique equilibrium entails vertical integration by the incumbent, no integration by the entrant and a shared-facility. In addition, the equilibrium in (i) is always efficient relative to the preferred shared-facility agreement, the equilibrium in (ii) is inefficient relative to duplication of essential facilities for fixed costs lower than certain cutoff and efficient otherwise, and the equilibrium in (iii) is always efficient relative to duplication of essential facilities. Furthermore, the equilibrium upon entry is always efficient relative to no entry. Thus, a shared-facility agreement, contrary to what many antitrust authorities believe, may be welfare enhancing relative to duplication of essential facilities or no entry. The reason being that a shared facility avoids duplication of fixed costs and, under certain conditions, changes the incentives to vertically integrate.

The results in this paper are also important because they provide a rationale for full liberalization of industries with characteristics of natural monopoly or state-owned monopolies that have goals other than pure profit maximization. In fact, privatization with no regulation becomes a relevant policy design under essential facility based competition, in particular when the cost of regulation is higher relative to the gains from more competition. It also provides advice to competition policy authorities by appealing that vertical integration and shared facilities do not always curb competition. As a matter of fact, vertical integration in industries having the structure considered in this model may be welfare improving. In practice, however, which industries are those is a question that demands a case by case analysis that is out of the scope of this paper.

Because our paper combines the literature on vertical integration with the one concerning essential facilities, but does not deal with the issue of vertical integration as mechanism for recovering the lost market power from degrading quality of access to the essential facility, there are a few papers that are closely related to ours and none of them deals exactly with the same issues. Chen and Ross (2000) show in a monopolistic market facing an entrant that a shared-facility agreement may deter a more aggressive entry that reduces the incumbent market power, yet a shared facility always results in a larger output than the one chosen by a monopoly

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5See, Mandy (2000) and Beard et al. (2001).
but in less output than the one chosen when the entrant builds its own facility. Our paper is also related to the literature concerning excess capacity as a deterrent instrument. For instance, Spence (1977) and Dixit (1980) show how excess capacity is used as a deterrent instrument. Lastly, Brueckner and Whalen (1999) consider the use of market power in the airline industry alliances and show theoretically and empirically that alliance partners charge interline fares that are lower. This effect is due to the existence of economies of traffic densities and that alliances internalize the negative externality from uncoordinated choice of subfares. The other paper that come closest to our paper is Gaudet and Van Long (1997), who show that vertical integration is a dominant strategy when the number of upstream and downstream is the same and lower than 4 and that multiple equilibria exist for more than 4 firms among full vertical integration is one of them. They, as we do, derive their result assuming Cournot competition upstream and downstream, free trade between all the parties involved; that is, no market foreclosure is imposed, and marginal costs are constant and the demand function is linear. The literature on market foreclosure is also related because it deals with vertical integration under different types of competition. For instance, the well-know paper by Ordover, Saloner and Salop (1990) show that vertical foreclosure is possible in equilibrium and thereby vertical integration may have anticompetitive effects. Result that is derived by Chen (2001) under less restrictive assumptions. The reason why less restrictive assumptions are needed is because he realizes that vertical integration may change the pricing incentive of a downstream producer and the incentive of a competitor in choosing an input supplier.

The rest of the paper is as follows. In the next section, we present to examples of Chilean industries that have experienced vertical integration and shared facilities agreements in unregulated industries. In the next section, we present the model. In section 4, we derive the equilibrium when each firm builds its own essential facility, the equilibrium when a shared-facility agreement is reached, and finally, derive the full equilibrium and its welfare properties. In the next section, section 5, we discuss the robustness of our results to different bargaining games, capacity constraints and oligopolies versus duopolies. In the final section concluding remarks are presented.

2 The Chilean Oil and Natural Gas Industries

In this section we present two examples were vertical integration and shared facilities are present. These are the oil and natural gas industries in Chile. While neither of these fits perfectly the issues involved in the model, in both cases there are elements that are related to the model’s structure and predictions and shows that there are industries with important essential facilities
that are not regulated or if they are, the regulation is to ensure open access to the essential facilities.

The oil industry has at least the following vertically related segments of the market: upstream there is exploration and exploitation of the resource and downstream there are refinery, pipelines, storage services, wholesale distribution, and retail distribution. If we concentrate in the downstream segment of this market, we observe that both pipelines and storage services have important scale economies, whereas refinery, wholesale distribution, and retail distribution operate in a more competitive basis. Then, we can think of pipelines and storage services as the essential facility in this industry.

In the Chilean fuel industry, upstream there is a state owned firm, ENAP, that operates in association with third private parties around the world in the exploration segment of the market and as the main importer of petroleum after running out of this resource in Chile. This firm owns the only two refineries in the country, almost 100% of storage services, and is the major shareholder in the existent pipelines. So, in a highly stylized sense ENAP is the incumbent in the essential facility segment of the industry, but this firm does not operate in the distribution segment. Furthermore, ENAP’s commercialization policy provides important rents to its owner (the government) by selling crude oil derivatives to wholesale distributors at the Gulf’s parity price. Thus, ENAP’s rents come from the fact that it is cheaper to buy large amounts of crude oil abroad, transport it to Chile, and make the refinery process in the country rather than to import smaller amounts of each oil derivative abroad and then transport them separately to Chile.6

Currently, ENAP is being threatened by the entry of a vertically integrated important international operator—the Spanish Repsol through its ancillary Repsol-YPF—that entered in the retail distribution a few years ago and is entering in the wholesale market by bringing crude oil derivatives from Argentina to Chile, where it owns an oil refinery with excess of capacity just beyond the Andes Mountains, 200 hundred miles far from Santiago, Chile. Thus, the Chilean oil industry is facing competition in the essential facility segment of the market at least to refineries concern by a vertically integrated firm. We also have casual information that Repsol-YPF is evaluating to build and operate its own pipelines to bring gasoline from its close by refinery. If this occurs and given that Repsol-YPF is a vertically integrated firm, then we should observe either vertical integration taking place between ENAP and some of the three

6In the last 3 to 5 years ENAP has signaled its commitment for reaching strategic alliances that sustain its position in the industry by reaching agreements in the upstream segment of the industry – that is, oil exploration – with several international firms, such as the same Repsol-YPF (Spain-Argentina), ABB (Switzerland), Perez Companc (Argentina), Petrobras (Brazil), Chevron (USA), Ferrostaal (Germany), CFG (France), and so on.
current wholesale distributors,\textsuperscript{7} or ENAP remains as an independent firm, but it looses market share to Repsol-YPF.

Furthermore, the entry of YPF, currently Repsol-YPF, in the middle of 1990s initiated an important vertical integration process between wholesale and retail distribution in the last decade. In 1991 the main three wholesale distribution companies–Copec (belonging to a Chilean group), Esso (an Exxon ancillary), and Shell–practically did not have direct operations in the retail market, having a majority indirect participation by renting machineries, pools, and company flags to independent operators. By 2002, wholesale distribution companies’ direct operations in the retail market rose to above 52% of the market and independent operators’ market participation fell down to 7% of what they had in 1991. Thus, is a sector that is experienced important changes and our model may help us to predict the welfare consequences of these changes and provide some advice to antitrust authorities to how to deal with these changes.

The natural gas with all its advantages was introduced in Chile only in 1996. The regulatory regime adopted, in contrast to ones adopted in other countries like the USA and UK, in which the authority gives exclusive rights to a firm to operate in a given market for a given time allocated through an auction, regulates the access charge to gas transportation, limits the possibilities of vertical integration between production, transportation and distribution, and imposes a series of norms regulating the access to the network to different agents, favors competition among private firms. The main regulation of the sector is to guarantee the open access under a non-discriminatory basis following a most favored nation clause. This implies that the operators have to offer their capacity of transportation to the different users under equal economic, commercial, and technical conditions limiting the possibility of price discrimination. The access price is determined by an open season and if the total capacity of a firm is contracted in the open season, that firm does not have to serve new customers unless it wants to, while if it is not, the firm has to serve the new demand to the price determined in the open season, unless the contracted capacity plus the new demand exceed the total capacity in which case the firm may call to a new open season.

The Chilean authority did not allocate exclusive rights to the transportation companies and limit only to establish a minimum regulation concerning the access rule, construction, security and operation of the pipelines. The current regulation allows competition in exploitation, wholesale, transportation and distribution of natural gas within a given concession that is

\textsuperscript{7}In fact Copec (belonging to a Chilean group), reveal a few weeks ago in the main Chilean newspaper (El Mercurio, 23 of April, 2003) that it is in its interest to vertically integrate with ENAP in the case that the firm were privatized.
determined by a geographical area, yet a concession does not assign exclusive rights to operate in the given geographical area. That is, the authority cannot reject any concession demanded that satisfies the restriction concerning the construction, security and operation of pipelines.

The industry of natural gas in the northern zone of the country consists in two pipelines—Norandino and Gas Atacama—that import and transport natural gas from Argentina. Each of these firms is vertically integrated with combined-cycle gas power-generating firms. Norandino imports 216.3 mill $m^3$/year that sells to the power-generating firm Edelnor (150 mill $m^3$/year) and to Electroandina (54.1 mill $m^3$/year). Gas Atacama imports 524.9 mill $m^3$/year that sells to power-generating firm Taltal (50.7 mill $m^3$/year) and to Nopel (466.9 mill $m^3$/year).

The industry in the central zone of Chile consists only in one pipeline—GasAndes—which imports and transports 1966.6 mill $m^3$/year gas from Argentina. Of this amount 855.1 mill $m^3$/year are bought by Metrogas—a distribution company that serves the capital city of Santiago, and by three combined-cycle gas power-generating firms—Nehuenco (307 mill $m^3$/year), San Isidro (424.9 mill $m^3$/year), and ESSA (349.8 mill $m^3$/year). Metrogas sells 45.6 mill $m^3$/year to Energas—a distribution company that serves the coast cities of Valparaiso and Viña del Mar, 66.7 mill $m^3$/year to Gas Valpo—a distribution company that serves the same cities, and the rest goes to a refinery owned by ENAP (140.1 mill $m^3$/year). Of the 602.8 mill $m^3$/year that metrogas sells to final consumers directly around 80 % goes also to power-generating companies. Thus, as can be easily concluded of all natural gas imported from Argentina more than 85 % goes to power-generating companies.

The main difference with the northern zone is that the demand of the central zone is more than twice the one in the north and there is less vertical integration than in the north. In fact, the controller of GasAndes (totalfinaElf) does not own any power-generating company in the central zone, yet the second largest shareholder (AES gener) owns ESSA. In addition, the percentage not used of installed capacity for GasAndes was 35.6% in 2001, while this number for the two pipelines in the north was 63.7% in the same year. Thus, what we have in Chile today is two different markets, one with a duplication of essential facilities and vertical integration and one without duplication and without vertical integration.

The prices charged for one $m^3$ per kilometer is equal to US$ 0.09 for GasAndes, US$ 0.013 for GasAtacama and US$ 0.016 for Noranadino in the international portion of the pipeline and US$ 0.011 in the national portion of the pipeline. What is surprising here is that the price charged in the Northern area, where there is a facility based competition is larger than the one charged in central area. A possible reason for this is that the government restricted intentionally the number of concession in the central area. When the concessions for the central area where

\footnote{Nopel sells 5.1 mill $m^3$ to other industries and the rest are losts of the system.}
open, two candidate firms asked for a concession. The two companies needed external financing
to build the pipeline from Argentina to Santiago and the World Bank was willing to provide
financing with the consent of Chilean government. The government decided that there were
room for only one pipeline and recommend to the World Bank to provide financing for only one
company, which was GasAndes since this commits to charge a lower price than its competitor.
This may explain why there are two pipelines in the northern area and one in the central one,
and the fact that firms compete the rents away to win the concession may explain why the price
is lower in the central area. In the northern area the government did not intervene and allocate
a concession to each of the firms requiring one.

With respect to domestic consumers what we have is a situation in which just one firm
supplies natural gas to domestic consumers in the capital city whereas in the cities of Valparaiso
and Viña del Mar there are two growing competitive independent networks. The effects of this
difference are quite important. On the one hand, more competition in the coast has pushed to
higher coverage in the residential, commerce, and industry markets with around 12% at the end
of year 2000, whereas this coverage was around 10% in Santiago. On the other hand, consumer
prices in the Valparaiso-Viña del Mar area have been lower than or equal to those in Santiago,
although the demand for natural gas in the capital city is larger than in the coast. For instance,
between years 1999 and 2001, prices were 22% lower. A more detailed look at the market in
the coast region reveals that prices are the consequence of a fierce competition that started
after Energas threatened the incumbency of GasValpo in 1998. Thus, essential facility based
competition has curbed the market power in this segment of the natural gas market. The lesson
is that if the government had no constraint the number of concessions to one in the central area,
we would had have today two pipelines and lower prices.

3 The Basic Model

We consider a market structure that initially has one non-integrated upstream firm, $U_1$, that
owns an essential facility and two downstream firms, $D_1$ and $D_2$. There is a firm, denoted
by $U_2$ in what follows, that is entering the market upstream, which requires to build its own
essential facility or buy access to the facility own by firm $U_1$. The upstream firms, $U_1$ and $U_2$,
provide an homogeneous input to downstream firms denoted by $z_i$ while the downstream firms
produce a final good denoted by $q_i$. Thus, upstream firms confront a derived demand given
by the amount of input demanded by downstream firms. For the sake of simplicity, and as
is commonly done in the literature, we assume a constant return to scale technology of fixed
proportions downstream; that is, to produce one unit of the final good, each firm needs one
unit of the input $z_i$. Thus, $q_i = z_i$.

The only cost that downstream firms incur to produce a unit of the final good is the price paid for a unit of input; that is, the cost of other inputs in the downstream market is normalized to zero. Thus, firm $D_i$’s marginal cost is constant and equal to $c_i$, where $c_i$ is the price paid by firm $D_i$ for each unit of input and all downstream firms are symmetric.

The production of input $z_i$ requires units of capacity, denoted by $y_i$, coming from an essential facility. We also assume a constant return to scale technology of fixed proportions upstream; that is, each input unit requires one unit of capacity. Thus, $z_i = y_i$.

The essential facility when combined with other inputs can produce units of capacity according to the cost function $K_i + m_i y_i$, where $K_i$ is the fixed—not necessarily sunk—cost and $m_i$ is the marginal cost of each unit of capacity. Because we assume that the transfer price for each unit of capacity is set to the efficient level, the marginal cost of producing one unit of input for the firm that owns the essential facility is $m_i$. We assume in what follows that $m_1 = m_2 = m$ and $K_1 = K_2 = K$; that is, the entrant’s essential facility is equally efficient than the incumbent’s facility and the two facilities have the same fixed cost.9

The timing of decisions is as follows. At stage 1, firm $U_2$ enters. At stage 2, firm $U_1$ makes a take-it-or-leave it offer to supply as many units of capacity as firm $U_2$ wants at a price $r$ per-unit of capacity. If firm $U_2$ accepts it does not need to build its essential facility, otherwise it builds its own facility in no time. At stage 3, upstream firms $U_1$ and $U_2$ have an initial opportunity to acquire one of the downstream firms, $D_1$ and $D_2$. If there is a merger, we suppose it to be between firm $U_i$ and $D_i$ and denote the merged firm by $F_i$. At the next stage, stage 4, upstream firms choose the amount of input to be produced and at the final stage downstream firms chooses the amount of final good to be produced.

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9The results of the paper hold as well when the entrants marginal cost is lower than the incumbent’s marginal cost. Yet, we choose equal marginal cost across firms because no new insight is gained by different marginal costs and the conditions for the equilibrium are much messier.
4 The Analysis

4.1 Preliminaries

The inverse demand function that downstream firms confront is assumed to be of the following
form:\textsuperscript{10}

\[ P(Q) = a - bQ, \quad a \geq 0 \text{ and } b \geq 0, \]

where \( Q = q_i + q_j \) and \( q_i \) represents the amount of the final good that firm \( D_i \) sells to consumers.

Since \( D_1 \) and \( D_2 \) compete à-la-Cournot, firm \( D_i \) produces the amount of the final good that
maximizes its profits, subject to its conjecture about \( D_j \)’s production; that is, \( D_i \) maximizes
\[ \pi_i^D = (a - bQ) q_i - c_i q_i, \]

where \( c_i \) is the price paid for the input by firm \( D_i \). It is straightforward
to verify that in equilibrium each downstream firm produces \( q_i (c_i, c_j) = \frac{1}{2b} (a - 2c_i + c_j) \) and
thereby firm \( D_i \)’s profit is

\[ \pi_{D_i}(n) = \frac{(a - 2c_i + c_j)^2}{9b}, \]

where \( n \) represents the number of integrated firms.

In order to ensure positive quantities for all \( m \) it is assumed that \( a \geq 2m \).

In what follows, we define \( Z = \sum_i z_i \) as the total production of the input, with \( z_i \) representing the production of upstream firm \( U_i \). Since downstream firms transform the upstream
product one for one, in equilibrium, we must have \( Z = Q \).

4.2 Duplication of Essential Facilities: Full Entry.

We have to consider all possible market configurations that may arise when each upstream
firm has its own essential facility. These are: (i) No integration; (ii) full integration; and (iii)
integration by firms \( U_i \) and \( D_i \) only. Notice that in this case each upstream firm has its own
essential facility and therefore each upstream firm’s marginal cost of production is \( m \).

If no integration takes place, the derived inverse demand faced by the upstream firms is
\[ c = a - \frac{3}{2} bZ \]

and thereby upstream firm \( U_i \)’s profit is \( (a - \frac{3}{2} bZ - m) z_i \).

Given quantity competition it is easy to establish that the only equilibrium is for each
upstream firm to produce \( z_i^F (0) = \frac{2}{9b} (a - m) \), where \( 0 \) stands for zero integrated firms and \( F \)

\textsuperscript{10}Most of our findings do not depend on the linear demand function assumption. This simplification allows
us to obtain closed-form solutions that greatly facilitates the equilibrium and welfare comparisons of different
market structures.
for full entry. Hence, firm $U_i$’s profit is

$$\pi_U \left( 0, K \right) = \frac{2(a - m)^2}{27b} - K,$$

and firm $D_i$’s profit is

$$\pi_D \left( 0 \right) = \frac{4(a - m)^2}{81b}.$$

Under full integration, there will then be no demand for inputs from independent upstream firms, thereby we have a standard Cournot duopoly, in which firm $F_i$’s marginal cost is $m$. It is straightforward to check that the equilibrium quantities are given by $z_i^F \left( 2 \right) = \frac{4}{3b} (a - m)$ and firm $F_i$’s profit is

$$\pi_F \left( 2, K \right) = \frac{(a - m)^2}{9b} - K,$$

where 2 stands for the two integrated upstream firms.

Lastly, consider the case in which only firm $F_i$ is vertically integrated. In this case the integrated and non-integrated downstream firms simultaneously determine the quantities of the final output in the final good production stage. This stage is preceded by the upstream production stage, during which the non-integrated upstream firm and the integrated firm again compete in quantities taking into account the derived demand resulting from the final good production decisions of the next stage. The decision variable of the non-integrated firm $U_j$ is the quantity it produces of the upstream good $z_j$. The decision that matters for the integrated firm at this stage is its net sales to the non-integrated sector, denoted by $s_i$. We will let the quantity of the input traded between the non-integrated firm and the integrated firm be determined endogenously with no a priori restrictions on the direction of this trade. Thus, the integrated firm may, if it so chooses, to sell inputs to the non-integrated downstream firm or buy inputs from the non-integrated upstream firm, thereby $s_i$ may either be positive or negative. Thus, total profit of the integrated firm $F_i$ is $(a - bQ - m) q_i + (c - m) s_i - K$, the profit of the non-integrated upstream firm $U_j$ is $(c - m) z_j - K$, and the profit of the non-integrated downstream firm $D_j$ is $(a - bQ - c) q_j$.

It follows from the equilibrium conditions for the downstream market that the optimal quantities are given by $q_j^F \left( 1 \right) = \frac{1}{3b} (a - 2c + m)$ and $q_i^F \left( 1 \right) = \frac{1}{3b} (a - 2m + c)$, where 1 stands for only one integrated upstream firm.

The market demand for the upstream input comes from the non-integrated downstream firm $D_j$. This firm will be supplied by the non-integrated upstream firm $U_j$ that produces $z_j$.
and potentially by the integrated upstream firm $U_i$ that have net sales $s_i$. The competition at the upstream state is therefore subject to the derived inverse demand

$$c = \frac{a + m - 3b (s_i + z_j)}{2} \quad (2)$$

Using the envelope theorem, the equilibrium conditions in this case are then given by

$$a - m - 3bs_i - 6bz_j \leq 0, \quad (3)$$

$$\frac{2}{3}c - \frac{1}{3}a - \frac{1}{3}m + s_i \left( -\frac{3}{2}b \right) \leq 0. \quad (4)$$

It readily follows from these equilibrium conditions that the optimal quantities are:

$$z_i^F (1) = \frac{5(a_m)}{24},$$

$$s_i^F (1) = \frac{-1(a_m)}{12},$$

$$z_i^F (1) = \frac{17(a_m)}{384}.$$

Thus, the input price is $c = \frac{1}{16} (5a + 11m) > m$ and the final good price is $p = \frac{1}{16} (7a + 9m)$. Notice the surprising result that the integrated upstream firm buys inputs from the unintegrated upstream firm at a price $c$ larger than its own cost of providing the input $m$. This strategy pushes up the input price for the non-integrated downstream firm, reducing the intensity of competition at the downstream market. This type of strategy is known as raising rivals costs strategy (Salop and Sheffman, 1987) and it has been studied for the case of oligopolies by Gaudet and Van Long (1997).

Firm $U_j$’s profit is

$$\pi_{U_j}^F (1, K) = \frac{25 (a - m)^2}{384b} - K, \quad (5)$$

the independent distributor $D_j$ obtains

$$\pi_{D_j}^F (1) = \frac{(a - m)^2}{64b}, \quad (6)$$

and the integrated firm, denoted by $F_i$, gets

$$\pi_{F_i}^F (1) = \frac{49 (a - m)^2}{256b} - \frac{5 (a - m)^2}{192b} - K \quad (7)$$

Comparing profits from each different market structure the following proposition is shown in the appendix.
**Proposition 1** If duplication of essential facilities occurs, then in equilibrium there is full vertical integration for all $m \geq 0$.

This proposition establishes that the only equilibrium is for both firms to vertically integrate despite the fact that firms are better-off if no one integrates; that is, $\pi_{U_i}^F (0, K) + \pi_{D_i}^F (0) > \pi_{U_i}^F (2, K) + \pi_{D_i}^F (2)$ for $i = 1, 2$. This implies that firms face a prisoner’s dilemma because vertical integration is the unique equilibrium, but each firm would be better-off if no one would integrate. The intuition is as follows. If no one integrates a firm has an incentive to deviate since by integrating it can avoid double-marginalization, and rise its downstream competitor’s cost of the input, decreasing competition downstream. This effect results in that the other firm in equilibrium also wants to integrate to recover its market power, but given that integration increases competition downstream, the increased competition downstream outweigh the gains from eliminating double marginalization.\(^{11}\)

### 4.3 A Shared-Facility Agreement: Partial Entry.

In this section we study the situation in which the entrant does not build its own essential facility, rather it buys units of capacity from the incumbent upstream firm $U_1$. This strategy allows the upstream firm $U_2$ to save on the cost of building its own facility $K$. In particular, we assume that firm $U_1$ sells as many units of capacity $y$ as firm $U_2$ wants at a given price of $r$ per-unit of capacity. Following Chen and Ross (2001), we assume that firm $U_1$ has all the bargaining power and makes a take-it-or-leave-it offer to the upstream firm $U_2$.\(^{12}\) Furthermore, in order to guarantee non-negative quantities in every possible market configuration, $r$ is restricted to the following interval $[0, \frac{a+m}{2}]$.

After the two firms have agreed to share the facility under the contract terms specifying the access to an unlimited units of capacity at a price $r$ per-unit, each firm has to decide how many units of the input to produce. Given that firms choose quantities, quantities in this case are the same as when no agreement is in place, but now firm $U_2$’s marginal cost of production is $r$—the price paid for each unit of capacity and firm $U_1$’s profit is different because when the facility is shared, firm $U_1$ makes extra profit equal to $(r - m) y_2$, where $y_2$ are the units of capacity that firm $U_2$ buys from firm $U_1$.

\(^{11}\)As shown by Gaudet and Van Long (1997) in the case in which the marginal cost is zero for all $i$, this is no longer true for an oligopoly with more than 4 firms upstream and downstream. The reason being that when there are several firms in downstream market the gain from reducing competition in this market cannot compensate the increased marginal cost of production.

\(^{12}\)The consequences of this assumption are discussed at length in the next section.
Consider first the case in which no firm is vertically integrated; that is, \( n = 0 \). In this case, firm \( U_2 \) faces the same problem as when no shared-facility agreement is adopted, yet now it faces a marginal cost of production of the input equal to \( r \), which is the cost of each unit of capacity. Therefore, firm \( U_2 \)'s profit is

\[
\pi_{U_2}^P (0) = \frac{2}{27b} (a - 2r + m)^2
\]

while firm \( U_1 \)'s profit is

\[
\pi_{U_1}^P (0, K) = \frac{2}{27b} \left[ (a - 2m + r)^2 + 3(r - m) (a - 2r + m) \right] - K.
\]

Firm \( D_i \)'s profit for \( i = 1, 2 \) is

\[
\pi_{D_i}^P (0) = \frac{(2a - m - r)^2}{81b}.
\]

Consider next the case in which full vertical integration takes place; that is, \( n = 2 \). In this case, firm \( F_2 \) faces the same problem as when no shared-facility agreement is adopted, but now its marginal cost is \( r \), which is the cost of each unit of capacity. Thus, firm \( F_2 \)'s profit is

\[
\pi_{F_2}^P (2) = \frac{(a - 2r + m)^2}{9b},
\]

and firm \( F_1 \)'s profit is

\[
\pi_{F_1}^P (2, K) = \frac{1}{9b} \left[ (a - 2m + r)^2 + 3(r - m) (a - 2r + m) \right] - K.
\]  \( \text{(8)} \)

Consider now the case in which firms \( U_1 \) and \( D_1 \) remain as independent firms and firms \( U_2 \) and \( D_2 \) integrate to form firm \( F_2 \). Firm \( F_2 \) faces the same problem as when no shared-facility agreement is in place, but now its marginal cost is given by \( r \), which is the priced paid for each unit of capacity. Firm \( U_2 \) can also sell inputs to firm \( D_1 \) at the market price \( c \) or buy produced units of inputs to firm \( U_1 \) also at price \( c \) and thereby firm \( F_2 \)'s total profit is \( (a - bQ - r) q_2 + (c - r) s_2 \). This implies that firm \( U_1 \)'s profit is \( (c - m_1) z_1 + (r - m_1) y_2 \).

By the same analysis as the one for full entry it can be shown that the optimal quantities are:

\[
\begin{align*}
z_1^P (1) &= \frac{5(a+r-2m)}{24b}, \\
s_2^P (1) &= -\frac{1(a+r-2m)}{12b}, \\
z_2^P (1) &= \frac{17a+14m-31r}{48b}.
\end{align*}
\]

It readily follows from this that firm \( U_1 \)'s profit, denoted by \( \pi_{U_1}^P (1, K) \), is

\[
\pi_{U_1}^P (1, K) = \frac{25 (a + r - 2m)^2}{384b} + (r - m) \frac{(7a - 9r + 2m)}{16b} - K,
\]  \( \text{(9)} \)
the independent distributor $D_1$ obtains

$$\pi_{D_1}^P (1, K) = \frac{(a + r - 2m)^2}{64b},$$

and the conglomerate firm $F_2$ gets

$$\pi_{F_2}^P (1) = \frac{(7a - 9r + 2m)^2}{256b} - \frac{(5a - 11r + 6m)(a + r - 2m)}{192b}.$$  

Lastly, consider the case in which firms $U_2$ and $D_2$ remain as independent firms and firms $U_1$ and $D_1$ integrate to form firm $F_1$. Firm $U_2$ faces the same problem as when no shared-facility agreement is in place, but now its marginal cost is given by $r$. By the same analysis as the one for full entry it can be shown that the optimal quantities are:

$$z_1^P (1) = \frac{5(a+m-2r)}{24b},$$
$$s_2^P (1) = -\frac{1(a+m-2r)}{12b},$$
$$z_2^P (1) = \frac{(17a+14r-31m)}{48b}.$$

Thus, firm $U_2$’s profit is

$$\pi_{U_2}^P (1) = \frac{25(a - 2r + m)^2}{384b},$$

while firm $D_2$’s profit is

$$\pi_{D_2}^P (1) = \frac{(a - 2r + m)^2}{64b}.$$

Firm $F_1$’s profit is

$$\pi_{F_1}^P (1, K) = \frac{(7a - 9m + 2r)^2}{256b} - \frac{(5a - 11m + 6r)(a + m - 2r)}{192b} + \frac{(r - m)5(a - 2r + m)}{24b} - K.$$

Now that we have derived firm’s payoffs under each possible market structure we can obtain the equilibrium to the integration game by mean of comparing the joint profits in each case. In the appendix the following is shown.

**Proposition 2**  Suppose a shared-facility agreement $r$ is in place, then in equilibrium there is full vertical integration for all $r \leq \frac{7a+10m}{17}$ and non-integration by firms $U_1$ and $D_1$ and integration by firms $U_2$ and $D_2$ for $\frac{a+m}{2} \geq r > \frac{7a+10m}{17}$. 

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Firms $U_2$ and $D_2$ always integrate because integration is a dominant strategy for the same reason as when firms do not share the essential facility. Integration, however, is not a dominant strategy for firms $U_1$ and $D_1$ for $r > \frac{7a+10m}{17}$ because firm $U_1$, as opposed to the case in which each firm builds its own essential facility, benefits from the units produced by firm $U_2$ through the units of capacity sold. Because when only firms $U_2$ and $D_2$ integrate the amount of the input produced by firm $U_2$ and thereby its demand for units of capacity is larger than when there is full integration, firm $U_1$ has no incentive to integrate when the price stipulated in the shared-facility agreement contract is sufficiently large. Thus, provided that the price per-unit of capacity is sufficiently large the benefit from selling more units of capacity outweigh the benefits of avoiding double marginalization and the decrease in the demand for the input from the non-integrated downstream sector resulting from the use by firms of $U_2$ and $D_2$ of the rising rivals’ cost strategy.

4.4 Equilibrium and Welfare Analysis

In this section, we derive the optimal shared-facility agreement within the class of per unit price contracts. In order to so, we need to make an assumption with respect to how the rents of integration are split between the upstream and downstream firm. We will assume that the upstream firm makes a take-it-or-leave-it offer to the downstream. Thus, in equilibrium, firm $U_1$ makes an offer $r$ that leaves firm $U_2$ exactly indifferent between the shared-facility agreement and his best alternative when he builds his own facility. This results in that the upstream firms gets the total profit from integration minus the downstream profits when no-integration occurs given its rival’s strategy and that firm $U_1$ chooses $r$ to maximize its profits conditional on that the shared-facility agreement is accepted by firm $U_2$. Thus, given the result in proposition 1 in equilibrium the following must hold

$$
\pi_{U_2}^P (n) \geq \pi_{U_2}^F (2, K),
$$

where $\pi_{U_2}^P (n)$ denotes firm $U_2$’s profit under a shared-facility agreement or partial entry when there are $n \in \{0, 1, 2\}$ vertically integrated firms and $\pi_{U_2}^F (2, K)$ is firm $U_2$’s profit when firms $U_2$ and $D_2$ merge and firm $U_2$ invests in its own essential facility.

Consider first the case in which under a shared-facility agreement full vertical integration is the equilibrium in the integration game; i.e., $r \leq \frac{7a+10m}{17}$. In this case, firm $U_1$ solves the following problem:

$$
\begin{align*}
\max_{r \in \mathbb{R}^+} & \quad \pi_{F_1}^P (2, K) - \pi_{D_1}^P (1) \\
\text{subject to} & \quad \pi_{F_2}^P (2) - \pi_{D_2}^P (1) \geq \pi_{F_2}^F (2, K) - \pi_{D_2}^F (1),
\end{align*}
$$

(12)
where \( \pi_{D_1}^P (1) \) is firm \( D_1 \)'s profit when firms \( U_2 \) and \( D_2 \) integrate and firms \( U_1 \) and \( D_1 \) do not and firms \( U_1 \) and \( U_2 \) share the essential facility, \( \pi_{D_2}^P (1) \) is firm \( D_2 \)'s profit when firms \( U_1 \) and \( D_1 \) integrate and firms \( U_2 \) and \( D_2 \) do not and firms \( U_1 \) and \( U_2 \) share the essential facility, and \( \pi_{D_2}^F (1) \) is firm \( D_2 \)'s profit when firms \( U_1 \) and \( D_1 \) integrate and firms \( U_2 \) and \( D_2 \) do not and each upstream firm builds its own essential facility. Furthermore, we assume that \( K \leq K \), where \( K \) is defined as the maximum \( K \) at which firm \( U_1 \) makes positive profits under full integration; that is, \( \pi_{F_i}^F (2, K) - \pi_{D_i}^F (1) = 0, i = 1, 2 \). This assumption guarantees that firm \( U_2 \) chooses to enter in the absence of a shared-facility agreement for all \( K \).

It is straightforward to show that the solution to this problem when the constraint is ignored is

\[
 r = \frac{302a + 356m}{603a} > \frac{7a + 10m}{17}. 
\]

Thus, the optimal solution for problem 12, denoted by \( r (2) \), cannot be given by the \( r \) that maximizes \( \pi_{F_i}^P (2, K) - \pi_{D_i}^P (1) \). Because the objective function is strictly concave and increasing for all \( r \leq \frac{7a + 10m}{17} \) and the restriction is strictly convex and decreasing in \( r \) for all \( r \leq \frac{7a + 10m}{17} \), the optimal \( r (2) \) is set to the minimum between

\[
 \min \left\{ \frac{a + m}{2} - \frac{1}{2} \left( \frac{a - m}{2} - \frac{576}{55} bK \right)^{\frac{1}{2}}, \frac{7a + 10m}{17} \right\},
\]

where the first entry is the \( r \) that satisfies firm \( U_2 \)'s profit constraint with equality.

For full integration with partial entry to be an equilibrium of the whole game the following two necessary and sufficient conditions must be satisfied: (i) \( r (2) \leq \frac{7a + 10m}{17} \); and (ii) \( \pi_{U_1}^P (2, K) - \pi_{D_1}^P (1) \geq 0 \). The second condition tells us that firm \( U_1 \) is better-off sharing its facility with the entrant than when upon entry firm \( U_2 \) builds its own facility.

Notice first that the difference between the \( r \) that satisfies firm \( U_2 \)'s profit constraint with equality and \( \frac{7a + 10m}{17} \) is equal to

\[
 \frac{3(1-a-m)}{4} - \frac{1}{2} \left( (a - m)^2 - \frac{576}{55} bK \right)^{\frac{1}{2}}. 
\]

This difference is positive for \( K = K \), since the second term is zero and negative for \( K = 0 \). Thus, by continuity there exists a \( K \), denoted by \( K^* \), such that for all \( K \leq K^* \equiv \frac{3850 (a-m)^2}{41616} \) firm \( U_2 \)'s profit constraint is satisfied with equality for an \( r \leq \frac{7a + 10m}{17} \). Thus, \( r (2) \) is equal to

\[
 \frac{a + m}{2} - \frac{1}{2} \left( (a - m)^2 - \frac{576}{55} bK \right)^{\frac{1}{2}},
\]

for \( K \leq K^* \) and equal to \( \frac{7a + 10m}{17} \) for \( K > K^* \).

Notice next that \( \pi_{U_1}^P (2, K) - \pi_{U_1}^F (2, K) \) is equal to

\[
 \frac{1}{576} \frac{302a (r (2) - m) - 27m^2 + 356mr (2) - 329r (2)^2}{b},
\]

which is increasing in \( r (2) \) for all \( r (2) \leq \frac{7a + 10m}{17} \), and that \( r (2) \) is increasing in \( K \). Therefore, \( \pi_{U_1}^P (2, K) - \pi_{F_1}^F (2, K) \) is increasing in \( K \). Because at \( K = 0 \), \( \pi_{U_1}^P (2, K) - \pi_{F_1}^F (2, K) = 0 \) and at \( K = K^* \), \( \pi_{U_1}^P (2, K) - \pi_{F_1}^F (2, K) \) is positive, by continuity of \( K \), there exists a \( K < K^* \), denoted by \( K (2) \), such that firm \( U_1 \) is better-off under a shared-facility agreement for all
$K > K(2)$. Thus, for $r \leq \frac{7a+10m}{17}$ a shared-facility agreement under full integration is optimal for all $K > K(2) = \frac{24915}{869928} \frac{(a-m)^2}{b}$.

Consider next the case in which $\frac{a+m}{2} \geq r > \frac{7a+10m}{17}$; that is, under a shared-facility agreement firms $U_1$ and $D_1$ do not integrate while firms $U_2$ and $D_2$ do so. Firm $U_1$ solves the following problem:

$$\max_{(r, r) \in \mathbb{R}^2} \pi_{U_1}(1, K)$$
subject to $\pi_{F_2}(1) - \pi_{D_2}(0) \geq \pi_{F_2}(2, K) - \pi_{D_2}(1), \tag{13}$$

It is straightforward to show the solution to this problem when the constraint is ignored is $r = \frac{218a + 164m}{382} > \frac{a+m}{2}$. Thus, because $\pi_{U_1}(1, K)$ is strictly concave and increasing in $r$ for all $r \leq \frac{a+m}{2}$, the optimal $r$, denoted by $r(1)$, is the maximum price per-unit of capacity that satisfies firm $U_2$’s profit constraint. That is, the maximum $r$ such that

$$\pi_{F_2}(1) - \pi_{D_2}(0) - (\pi_{F_2}(2, K) - \pi_{D_2}(1)) = 0.$$ 

If the profit constraint at $r = \frac{a+m}{2}$ is satisfied, then $r(1) = \frac{a+m}{2}$. One can check that at $K = 0$ firm $U_2$’s profit constraint cannot be satisfied and since it is increasing in $K$ and equal to $\frac{5}{9216} (a-m)^2 > 0$ at $K = \overline{K}$, by continuity there exists a $K < \overline{K}$, denoted by $K'$, such that the constraint is satisfied for all $K > K' \equiv \frac{875}{9216} \frac{(a-m)^2}{b}$ at $r = \frac{a+m}{2}$. Thus, $r(1) = \frac{a+m}{2}$ for all $K > K'$.

For $K \leq K'$, firm $U_2$’s profit constraint is binding but since this is strictly convex and decreasing in $r$ for all $r \leq \frac{a+m}{2}$, there exists an $r < \frac{a+m}{2}$ such that the profit constraint is satisfied. This $r$ is equal to

$$\frac{4267a + 3226m}{7493} - \frac{6}{7493} \left(417299 (a+m)^2 - 1669196ma - 4315968bK\right)^{\frac{1}{2}}.$$

In order for this $r$ to be a solution, we need to check that it belongs to $\left(\frac{7a+10m}{17}, \frac{a+m}{2}\right]$ for all $K$. It is easy to check that for $K = 0$, the solution is lower than $\frac{7a+10m}{17}$; that is, there is no $r$ in the allowed range that satisfies firm $U_2$’s profit constraint. Since the LHS of the profit constraint increases continuously with $K$, there exists a $K$, denoted by $K''$, such that for all $K > K'' \equiv \frac{14599}{106564} \frac{(a-m)^2}{b}$ there is an $r > \frac{7a+10m}{17}$ satisfying firm $U_2$’s profit constraint.\(^{13}\)

Next, we need to find conditions under which $\pi_{U_1}(2, K) - (\pi_{F_1}(2, K) - \pi_{D_1}(1)) \geq 0$. Because this difference is strictly concave, increasing in $r$ for all $r \leq \frac{a+m}{2}$ and depends on $K$ only through the optimal value of $r$, which is increasing in $K$, this difference is increasing in $K$ for\(^{13}\) Notice that $K'' < \overline{K}$.

\(^{13}\) Notice that $K'' < \overline{K}$.  

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all $r \leq \frac{a+m}{2}$. Thus, there exists a $K$, denoted by $K(1)$, such that a shared-facility agreement in which firms $U_1$ and $D_1$ do not integrate while firms $U_2$ and $D_2$ do so is preferred for all $K > K(1)$.

It is easy to show that $\pi^P_{U_1}(2, K) - (\pi^F_{F_1}(2, K) - \pi^F_{D_1}(1))$ at $r = \frac{a+m}{2}$ is equal to $\frac{505 (a-m)^2}{18496}$ and therefore it is positive for all $K > K'$ while for $K \leq K'$ it is positive for all $K > K(1) \equiv 0.0015269 \frac{(a-m)^2}{b}$. While for $r > \frac{7a+10m}{14}$ a shared-facility agreement under full integration is optimal for all $K > K(1)$.

So far, we have obtained necessary condition for a shared-facility agreement to be preferred to full entry, but we have not yet found conditions under which a shared-facility agreement with $r \leq \frac{7a+10m}{14}$ is preferred to a one with $r > \frac{7a+10m}{14}$. This is key since there is range for $K$ in which the necessary conditions are satisfied for the two cases, then we need to find conditions under which $\pi^P_{U_1}(1, K) - \pi^P_{U_1}(2, K) \geq 0$ for $K \geq K''$; that is, a shared-facility agreement with $r \leq \frac{7a+10m}{14}$ is preferred.

Notice that

$$\pi^P_{U_1}(1, K) - \pi^P_{U_1}(2, K) = -\frac{5}{1152b} \left(7a^2 - 10ar - 4am - 17r^2 + 44rm - 20m^2\right) = 0$$

It is easy to check that at $r = 0$ this difference is negative, at $r = \frac{7a+10m}{14}$ is 0 and for $\frac{a+m}{2} \geq r > \frac{7a+10m}{14}$ is positive. This implies that when a shared-facility agreement is preferred, firm $U_1$ offers a contract with $r > \frac{7a+10m}{14}$ for all $K > K''$ and since $K^* > K''$ this leads to the following result.

**Proposition 3 (1)** In equilibrium full entry is observed for all $K \leq K(2)$, a shared-facility agreement with $r(2)$ and full vertical integration is observed for all $K(2) < K \leq K^*$, and a shared-facility agreement with $r(1)$, non-integration by firms $U_1$ and $D_1$ and integration by firms $U_2$ and $D_2$ is observed for all $K > K^*$; and (2) $Q^F(2) > Q^P(2)$ and $Q^F(2) > Q^P(1)$.

This proposition tells us that by agreeing to share the essential facility with the new entrant, the incumbent firm achieves two goals: (i) it restricts the production of its rival by mean of increasing the rival’s marginal cost; i.e., $r(2) \geq m$ and $r(1) \geq m$ for all $K$ and (ii) it avoids the prisoner’s dilemma problem that arises under full entry for $K > K^*$.

Notice that the less efficient is the essential facility; i.e., the larger is $m$, the smaller are $K^* - K(2)$, $K(2)$ and $K^*$. Consequently, the less efficient is the essential facility, the more

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14 Notice that $K(1) - K' = -9.3417 \times 10^{-2} \frac{(a-m)^2}{b} < 0$ and $K(1) - K'' = -8.6174 \times 10^{-2} \frac{(a-m)^2}{b} < 0$. Furthermore, $K(1) - K(2) = -2.7246 \times 10^{-2} \frac{(a-m)^2}{b} < 0$, $K'' - K(2) = \frac{1006179629}{1383552024} \frac{(a-m)^2}{b} > 0$, $K^* - K'' = \frac{89}{18496} (a - m)^2 > 0$ and $K^* - K' = -\frac{6475}{2603.424} \frac{(a-m)^2}{b} < 0$. 

15.
likely that a shared-facility agreement takes place and the more likely is that the owner of
the essential facility does not integrate with the entrant. Furthermore, as expected \( r(2) \) and
\( r(1) \) increase with \( m \). The intuition being that the less efficient is the essential facility the less
competitive is the entrant and therefore, the less profitable is to build its own essential facility.

Next, we study the efficiency of partial entry versus full entry. The effect over total welfare
depends on two factors. First, a shared-facility agreement avoids duplication of the essential
facilities and hence generate a cost saving of \( K \) and second, a shared-facility agreement decreases
the intensity of competition through the increased marginal cost since in equilibrium \( r > m \).
The final effect will depend on which force dominates.

Notice first that \( Q^F(2) = \frac{2}{35} (a - m) \geq Q^P(2) = \frac{1}{35} (2a - m - r(2)) \) because \( r(2) \geq m \)
and \( Q^F(2) > Q^P(1) = \frac{1}{480} (27a - 22m - 5r(1)) \) since \( r(1) > m \). Thus, consumers’ welfare is
always lower under a shared-facility agreement. The reason being that under partial entry total
output is restricted relative to total output under full entry.

Total welfare when each firm builds its own essential facility is \( W^F(2, K) \equiv \pi^F_1(2, K) + \pi^F_2(2, K) + \frac{b[Q^F(2)]^2}{2} \), while when a shared-facility agreement takes place is \( W^P(n, K) \equiv \pi^P_1(n, K) + \pi^P_2(n) + \frac{b[Q^P(n)]^2}{2} \) with \( n = 2 \) when full integration takes place and \( n = 1 \) when
firms \( U_1 \) and \( D_1 \) do not integrate and firms \( U_2 \) and \( D_2 \) do so.

Given the results in proposition 3, total welfare under full entry minus total welfare under
partial entry is given by

\[
\begin{align*}
\left\{ \begin{array}{ll}
\frac{1}{180} (2ar - 4mr + r^2 - 2am + 3m^2 - 18bK) & \text{for } K(2) < K \leq K^*, \\
\frac{1}{46080} (185a^2 - 484am + 114ar + 532m^2 - 580mr + 233r^2 - 4608Kb) & \text{for } K > K^*.
\end{array} \right.
\]

If we evaluate this equation at the optimal for \( K(2) < K \leq K^* \) we get that is equal to

\[
\frac{1}{660b} \left( 55 (a - m)^2 - (a - m) \left( 3025 (a - m)^2 - 31680Kb \right) \right)^{\frac{1}{2}} - 756Kb.
\]

Because this is strictly concave in \( K \) and has two real roots equal to \( K = 0 \) and \( K = \frac{715}{7938} (a - m)^2/b \),
it follows that total welfare under full entry is larger for all \( K \leq \hat{K} \equiv \frac{715}{7938} \frac{(a-m)^2}{b} < K^* \) and
smaller otherwise.\(^{15}\)

Consider next the case for \( K > K^* \). If we evaluate total welfare at the maximum \( r = \frac{a+m}{2} \);
\( i.e., K > K' \), then total welfare under partial entry is larger than under full entry for all
\( K > \frac{1201}{18432} (a - m)^2. \(^{16}\)

\(^{15}\) \( K^* - \hat{K} = \frac{22385}{7938} \frac{(a - m)^2}{b} > 0 \) and \( K(2) - \hat{K} = \frac{4200625}{7938} \frac{(a - m)^2}{b} < 0 \).

\(^{16}\) \( -\frac{1201}{18432} (a - m)^2 - K' = -\frac{61}{201} \frac{(a - m)^2}{b} < 0 \).
Furthermore, if we evaluate total welfare at \( r(1) \) for all \( K^* < K \leq K' \) we get that it is equal to

\[
\frac{1}{646790964486} \left( 5443579087 (a - m)^2 - 4263936 (a - m) \left( 417299 (a - m)^2 - 4315968Kb \right)^{\frac{1}{2}} - 73729681344Kb \right)
\]

Notice that this equation is strictly concave in \( K \) and reaches its maximum at

\[
\frac{15645884959 \ (a - m)^2}{168093440064} < K'.
\]

Furthermore at its maximum is equal to \(-\frac{296843 \ (a - m)^2}{9839808}\), which is negative. This leads to the following proposition.

**Proposition 4** (i) For \( K \leq K^* (2) \), full entry is observed and it is efficient relative to partial entry; (ii) for \( K^* (2) < K \leq K \), partial entry is observed and it is inefficient relative to full entry while for \( \hat{K} < K \leq K^* \) is efficient relative to full entry; and (iii) for \( K > K^* \), partial entry takes place and it is efficient relative to full entry.

This shows that partial entry is efficient for \( K \) sufficiently large. This being the result of two forces: (i) partial entry avoids duplication of an essential facility; and (ii) it changes the incentives to vertically integrate. Furthermore, the region in which partial entry is inefficient decreases as the essential facility becomes more inefficient.

## 5 Discussion

In this section we discuss what are the consequences over the equilibrium when (i) essential facilities have a limited capacity; (ii) the entrant has all the bargaining power; and (iii) there is an oligopoly instead of a duopoly.

### 5.1 Capacity Constraints

In this section we assume that when each firm invests in its own essential facility there is no capacity constraints; that is, the existent capacity for each essential facility is enough to cover the optimal amount produced in each of the market structures considered in section 4.2, yet under a shared-facility agreement, the limited capacity, denoted by \( \bar{y} \), is such that firm \( U_1 \)'s essential facility by itself is not capable of serving the whole demand for inputs in each of the possible market structures. The first part of this assumption requires that \( \bar{y} \in \left[ \frac{21}{1456} (a - m), \frac{4}{95} (a - m) \right] \) and the second that \( \bar{y} \in \left[ \frac{10}{135} (a - r), \frac{2}{35} (a - r) \right] \). Thus, we need
to derive the optimal quantities only in the case in which firms decide to share firm $U_1$’s essential facility.

Consider first the case in which no firm is vertically integrated. If no integration takes place the derived inverse demand faced by the upstream firms is therefore $c = a - \frac{3}{2}bZ$. Thus, firm $U_1$ is faced with the following problem

$$\max_{z_1 \in \mathbb{R}^+} \left( a - \frac{3}{2}bZ - m \right) z_1 + (r - m) y_2 \text{ subject to } z_1 \leq \bar{y} - y_2,$$

where $y_2 = z_2$ are the units of capacity that firm $U_2$ buys, while firm $U_2$’s problem is

$$\max_{z_2 \in \mathbb{R}^+} \left( a - \frac{3}{2}bZ - r \right) z_2.$$ 

Given quantity competition and the assumption that the capacity constraint is binding, then from the first-order condition for firm $U_2$ it follows that $z_2^P(0, \bar{y}) = \frac{2(a - r) - 3b\bar{y}}{3b}$ and $z_2^P(0, \bar{y}) = \bar{y} - z_2^P(0, \bar{y})$. Thus, firm $U_1$’s profit is

$$\pi_{U_1}^P(0, K, \bar{y}) = \left( a - \frac{3}{2}b\bar{y} - m \right) \left( \frac{6b\bar{y} - 2(a - r)}{3b} \right) + (r - m) \frac{2(a - r) - 3b\bar{y}}{3b} - K,$$

firm $U_2$’s profit is

$$\pi_{U_2}^P(0, K, \bar{y}) = \frac{1}{6b} (2a - 2r - 3b\bar{y})^2,$$

and firm $D_i$’s profit is

$$\pi_{D_i}^P(0, \bar{y}) = \frac{1}{4}b\bar{y}^2.$$

Consider next the case of full integration. In this case there will then be no demand for inputs from independent upstream firms, thereby we have a standard Cournot duopoly, in which firm $F_1$’s marginal cost is $m$ and firm $F_2$’s marginal cost is $r$. Thus, firm $F_1$ is faced with the following problem

$$\max_{q_1 \in \mathbb{R}^+} (a - bQ - m) q_1 + (r - m) y_2 \text{ subject to } q_1 \leq \bar{y} - y_2,$$

where $y_2 = z_2 = q_2$ are the units of capacity that firm $F_2$ buys, while firm $F_2$’s problem is

$$\max_{q_2 \in \mathbb{R}^+} (a - bQ - r) q_2.$$
Again given the assumption that the capacity constraint is binding, it readily follows from the first-order condition for firm $F_2$ that $z_2^P(2, \bar{y}) = \frac{a - r - b\bar{y}}{b}$ and $z_1^P(2, \bar{y}) = \bar{y} - z_2^P(2, \bar{y})$. Thus, firm $F_1$’s profit is

$$\pi^P_{F_1}(2, K, \bar{y}) = (a - b\bar{y} - m) \left( \frac{2b\bar{y} - a + r}{b} \right) + (r - m) \left( \frac{a - r - b\bar{y}}{b} \right) - K, \quad (14)$$

while firm $F_2$’s profit is

$$\pi^P_{F_2}(2, \bar{y}) = \frac{1}{b} (a - b\bar{y} - r)^2. \quad (15)$$

Consider next the case in which firms $U_1$ and $D_1$ remain as independent firms while firms $U_2$ and $D_2$ integrate to form firm $F_2$. Thus, firm $U_1$ is faced with the following problem

$$\max_{z_1 \in \mathbb{R}^+} (c - m)q_1 + (r - m)y_2 \text{ subject to } z_1 \leq \bar{y} - y_2,$$

where $y_2 = z_2$ are the units of capacity that firm $F_2$ buys, while firm $F_2$’s problem is

$$\max_{(q_2, s_2) \in \mathbb{R}^2_+} (a - bQ - r)q_2 + (c - r)s_2,$$

where $s_2$ are the net sales to the non-integrated sector.

The market demand for the upstream input comes from the non-integrated downstream firm; i.e., $D_1$, thereby, $c = \frac{a + r - 3bq_1}{2}$, where $q_1 = \bar{y} - y_2 + s_2$; that is, the input produced by firm $U_1$ plus the net sales of the integrated sector to the non-integrated sector. It follows from the first-order condition for $s_2$ and the fact that $q_1^P(1, \bar{y}) = \bar{y} - y_2^P(1, \bar{y}) + s_2^P(1, \bar{y})$, that $s_2^P(1, \bar{y}) = \frac{2(a - r - 2b\bar{y})}{3b}$ and $z_2^P(1, \bar{y}) = \frac{5(a - r - 7b\bar{y})}{3b}$. Therefore, $q_2^P(1, \bar{y}) = \frac{a - r - b\bar{y}}{b}$ and $q_1^P(1, \bar{y}) = \frac{2b\bar{y} - (a - r)}{b}$. It readily follows from this that firm $U_1$’s profit is

$$\pi^P_{U_1}(1, K, \bar{y}) = \frac{5(2a - r - 3b\bar{y} - m)(2b\bar{y} - a + r)}{3b} + (r - m)\frac{5(a - r) - 7b\bar{y}}{3b} - K, \quad (16)$$

the independent distributor $D_1$ obtains

$$\pi^P_{D_1}(1, \bar{y}) = \frac{(2b\bar{y} - a + r)^2}{b}, \quad (17)$$

and the integrated firm $F_2$ gets

$$\pi^P_{F_2}(1, \bar{y}) = \frac{(a - r - b\bar{y})^2}{b} + \frac{2(2a - 2r - 3b\bar{y})(a - r - 2b\bar{y})}{3b}. \quad (18)$$

Finally, consider the case in which firms $U_2$ and $D_2$ remain as independent firms and firms $U_1$ and $D_1$ integrate to form firm $F_2$. Thus, firm $U_1$ is faced with the following problem

$$\max_{(q_1, s_1) \in \mathbb{R}^2_+} (a - bQ - m)q_1 + (c - r)s_1 + (r - m)y_2 \text{ subject to } z_1 \leq \bar{y} - y_2,$$
where \( y_2 = z_2 \) are the units of capacity that firm \( U_2 \) buys, while firm \( U_2 \)'s problem is

\[
\max_{z_2 \in \Re_+} (c - r) z_2,
\]

where \( s_2 \) are the net sales to the non-integrated sector.

The market demand for the upstream input comes from the non-integrated downstream firm; i.e., \( D_2 \), thereby, \( c = a - b q_1 - 2 b q_2 \). Notice that \( q_1 = \bar{y} - y_2 - s_1 \); that is, the input produced by firm \( U_1 \) plus the net sales from the integrated sector to the non-integrated sector, and \( q_2 = z_2 + s_1 \). It follows from the first-order condition for \( s_1 \) and the fact that \( q_1^P (1, \bar{y}) = \bar{y} - y_2 - s_1 \) that \( s_1^P (1, \bar{y}) = -\frac{w_2}{2} \) and \( z_2^P (1, \bar{y}) = \frac{2(a - r - b \bar{y})}{3b} \). Therefore, \( q_2^P (1, \bar{y}) = \frac{a - r - b \bar{y}}{3b} \) and \( q_1^P (1, \bar{y}) = \frac{4 b \bar{y} + r - a}{3b} \). It readily follows from this that firm \( F_1 \)'s profit, denoted by \( \pi_{F_1}^P (1, K, \bar{y}) \), is

\[
\pi_{F_1}^P (1, K, \bar{y}) = \frac{(a - b \bar{y} - m)(4 b \bar{y} - a + r)}{3b} + (r - m) \frac{2(a - r - b \bar{y})}{3b} - \frac{(2a + r - 2 b \bar{y} - 3m)(a - r - b \bar{y})}{3b} - K, \tag{19}
\]

the independent distributor \( D_2 \) obtains

\[
\pi_{D_2}^P (1, \bar{y}) = \frac{(a - r - b \bar{y})^2}{9b}, \tag{20}
\]

and the conglomerate firm \( U_2 \) gets

\[
\pi_{U_2}^P (1, \bar{y}) = 4 \frac{(a - r - b \bar{y})^2}{9b}. \tag{21}
\]

Comparing profits from the different market configurations the following proposition can be easily shown.

**Proposition 5** Suppose a shared-facility agreement such that the following holds \( \bar{y} \in \left[ \frac{10}{13b} (a - r), \frac{7}{3b} (a - r) \right] \), then in equilibrium integration by firms \( U_2 - D_2 \) and non-integration by firms \( U_1 - D_1 \) is observed.

The reason being that when the capacity is small, firms \( U_1 - D_1 \) cannot benefit from integration because they cannot cover the extra market share they gain by avoiding double marginalization and outweighed the cost of rising rival’s cost strategy.

Given the results in proposition 5, firm \( U_1 \) solves the following problem:

\[
\max_{(r_0, r) \in \Re_+^2} \pi_{F_2}^P (1, \bar{y}) - \pi_{D_2}^P (0, \bar{y}) \tag{22}
\]

subject to \( \pi_{F_1}^P (2, \bar{y}) - \pi_{D_2}^P (0, \bar{y}) \geq \pi_{F_2}^F (2, K) - \pi_{D_2}^F (1) \),
It is straightforward to show the solution to this problem when the constraint is ignored is \( r = a - \frac{8}{7}b\hat{y} \). For this price per-unit of capacity to be optimal firm \( U_2 \)'s profit constraint must be non-binding. If we evaluate the constraint at this \( r \), then it is easy to show that the constraint is non-binding for all \( K \geq \hat{K} \equiv K = \frac{16}{7}b\hat{y}^2 \). Therefore, \( r = a - \frac{8}{7}b\hat{y} \) is the optimal solution, denoted by \( r (1, \hat{y}) \), for all \( K > \hat{K} \).

Because the constraint is strictly convex and decreasing for all \( r \leq a - \frac{10}{7}b\hat{y} \) and the objective function is strictly concave and increasing in \( r \) for all \( r \leq a - \frac{8}{7}b\hat{y} \), then the optimal \( r \) for all \( K \leq \hat{K} \) is the maximum \( r \in [a - 2b\hat{y}, a - \frac{8}{7}b\hat{y}] \) satisfying firm \( U_2 \)'s profit constraint. That is, the maximum \( r \) is equal to

\[
 r (1, \hat{y}) = a - \frac{10}{7}b\hat{y} - \frac{1}{168} \left( 144(b\hat{y})^2 + 1155(a - m)^2 - 12096Kb \right)^{\frac{1}{2}}.
\]

We need to check that \( r (1, \hat{y}) \geq a - 2b\hat{y} \). It is easy to check that this holds for all \( K \geq \hat{K} \equiv \frac{K - 3}{4}b\hat{y}^2 \). Thus, firm \( U_2 \)'s profit constraint can be satisfied by an \( r \) in the relevant range if and only if \( K \geq \hat{K} - \frac{3}{4}b\hat{y}^2 \), otherwise there is no acceptable shared-facility agreement.

Lastly, we need to find conditions under which \( \pi_{U_1}^F (2, K) - (\pi_{F_1}^F (2, K) - \pi_{D_1}^F (1)) \geq 0 \). Because this difference is strictly concave it has a maximum at \( r = a - \frac{8}{7}b\hat{y} \). At its maximum it is positive and thereby for all \( K \geq \hat{K} \) a shared-facility agreement is optimal. While for \( 
\hat{K} \leq K < \hat{K} \), there exists a \( K \), denoted by \( \bar{K} \), such that a shared-facility agreement in which firms \( U_1 \) and \( D_1 \) do not integrate while firms \( U_2 \) and \( D_2 \) do so is preferred for all \( K > \bar{K} \).

Let define \( K (1, \hat{y}) \equiv \max \{ \hat{K}, \bar{K} \} \), then the next result follows from the discussion so far.\(^{17}\)

**Proposition 6** (i) If \( K > K (1, \hat{y}) \), in equilibrium a shared-facility agreement with \( r (1, \hat{y}) \) is observed, otherwise the entrant builds its own facility; and (ii) \( Q^F (2) > Q^P (1) = \hat{y} \).

The differences with the model in which there is no capacity constraints is that the incumbent monopoly will never choose to integrate with a downstream firm upon entry of firm \( U_2 \) and \( K (1, \hat{y}) \) does not depend on \( m \). The reason being that with capacity constraints the opportunity cost from transferring one unit to an ancillary at \( m \) is the price that the integrated sector is willing to pay for that unit, which is \( c > m \). The other results are robust to capacity constraints. That is, for a sufficiently large \( K \) a shared-facility agreement is reached and the price per unit of capacity is increasing with \( m \). Notice that \( K (1, \hat{y}) \) decreases with \( \hat{y} \); that is, the less restrictive the capacity constraint is, the more likely that a shared-facility agreement is reached. The intuition being that as \( \hat{y} \) increases the cost in terms non-produced units from not building a new facility decreases.

\(^{17}\)If \( \hat{y} \leq \left( \frac{1}{3} + \frac{1}{48} \sqrt{34} \right) \left( \frac{a - m}{b} \right) \), then \( K (1, \hat{y}) = \hat{K} \) while if \( \hat{y} > \left( \frac{1}{3} + \frac{1}{48} \sqrt{34} \right) \left( \frac{a - m}{b} \right) \), \( K (1, \hat{y}) > \hat{K} \)
5.2 Bargaining Power

We have assumed that the incumbent firm has all the bargaining power; that is, firm $U_1$ makes a take-it-or-leave-it offer to firm $U_2$ to share the essential facility. In this case we consider the other polar case in which the entrant, firm $U_2$, has all the bargaining power and therefore it makes a take-it-or-leave-it offer to firm $U_1$ to share the essential facility.

Consider first the case in which the unique equilibrium in the integration game is full integration; that is, $r \leq \frac{7a+10m}{17}$. Thus, firm $U_2$ chooses $r$ to maximize its profits conditional on that the shared-facility agreement is accepted by firm $U_1$. That is, firm $U_2$ solves the following problem:

$$
\max_{r \in \mathbb{R}_+} \pi_{F_2}^U(2) - \pi_{D_2}^U(1)
$$

subject to $\pi_{F_1}^U(2, K) - \pi_{D_1}^U(1) \geq \pi_{F_1}^U(2, K) - \pi_{D_1}^U(1)$,

(23)

Given that the objective function is strictly convex and decreasing in $r$ for all $r \leq \frac{a+m}{2}$, firm $U_2$ chooses the lowest $r$ that satisfies firm $U_1$’s profit constraint. Consequently, the optimal $r$, denoted by $r_2(2)$, is equal to $m$.

Notice next that $\pi_{U_2}^P(2, K) - \pi_{U_2}^F(2, K)$ is equal to

$$
\pi_{U_2}^P(2) - \pi_{U_2}^F(2, K) = \frac{55(a-m)^2}{576b} - \left(\frac{55(a-m)^2}{576b} - K\right) = K \geq 0
$$

Thus, for all $r \leq \frac{7a+10m}{17}$ a shared-facility agreement under full integration is the unique equilibrium for all $K \geq 0$.

Consider next the case in which $\frac{a+m}{2} \geq r > \frac{7a+10m}{17}$, that is, firms $U_1$ and $D_1$ do not integrate while firms $U_2$ and $D_2$ do so. Firm $U_2$ solves the following problem:

$$
\max_{(r_0, r) \in \mathbb{R}_+^2} \pi_{F_2}^U(1) - \pi_{D_2}^U(0)
$$

subject to $\pi_{U_1}^F(1, K) \geq \pi_{F_1}^U(2, K) - \pi_{D_1}^U(1)$,

(24)

Because the objective function is strictly convex and decreasing in $r$ for all $r \leq \frac{a+m}{2}$ and the constraint is strictly concave and increasing in $r$ for all $r \leq \frac{a+m}{2}$, the solution to this problem is the minimum $r$ that satisfies firm $U_1$’s profit constraint. It is easy to check that at $r = \frac{7a+10m}{17}$ the constraint is equal to $\frac{19817(a-m)^2}{166364} < 0$. Thus the optimal $r$, denoted by $r_2(1)$, is equal to $\frac{7a+10m}{17}$.

Lastly, we need to find conditions under which $\pi_{F_2}^U(1) - \pi_{D_2}^U(0) \geq \left(\pi_{F_1}^U(2, K) - \pi_{D_1}^U(1)\right)$. If we evaluate this equation at the optimal $r$, we obtain that it is equal to $\frac{7583(a-m)^2}{83232b} + 83232Kb$. Thus, it is positive for all $K > K_2(1)$, where $K_2(1) \equiv \frac{7583(a-m)^2}{83232b} < K$.  

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Because, for $r \leq \frac{7a+10m}{17}$ a shared-facility agreement is always preferred and for $r \geq \frac{7a+10m}{17}$ a shared-facility agreement is optimal for all $K > K_2(1)$, we need to find conditions under which firm $U_1$ offers a contract with $r \leq \frac{7a+10m}{17}$; that is, conditions under which $\pi_{F_2}^P(2) - \pi_{D_2}^P(1) - \left( \pi_{F_2}^P(1) - \pi_{D_2}^P(0) \right) \geq 0$. It is straightforward to check that this difference is equal to $\frac{3061}{166464} \left( a-m \right)^2 > 0$.

This leads to the following result.

**Proposition 7** *In equilibrium: (i) a shared-facility agreement and full integration with $r_2(2) = m$ is observed for all $K$; and (ii) $Q_F^P(2) = Q_D^P(2)$.*

This proposition tells us that a shared-facility agreement with full integration is the unique equilibrium. The reason being that the entrant prefers the lowest price per-unit of capacity and since firm $U_1$ can reject the agreement, firm $U_2$ cannot offer to pay less than $m$ because at $r = m$, firm $U_1$ makes no profit from sharing the facility. Given this, by sharing the facility firm $U_2$ can save the fixed cost and get the input at the same price as if it builds its own facility and thereby sharing facilities is always better. This also implies that firm $U_1$‘s optimal strategy is to vertically integrate when firm $U_2$ does so since otherwise it will suffer from double marginalization and the use of the rising rival’s cost strategy by its rival.

In this case total welfare is always larger under a shared-facility agreement since total output is not restrict and the duplication of the essential facility is avoided.

The results here suggests that if neither the incumbent monopoly nor the entrant has all the bargaining power the results derived under the assumption that the incumbent has all the bargaining power are the same, but the optimal price per-unit of capacity will be smaller, a shared-facility agreement would take place more often and welfare would be larger. In additional, if the entrant’s bargaining power is close enough to have all the bargaining power, then it is likely that the equilibrium in which firms $U_1$ and $D_1$ integrate disappears.

### 5.3 Oligopoly versus Duopoly

Ngo and VanLong (1997) show that the unique equilibrium when firms are symmetric and have a marginal cost equal to zero is full vertical integration when there are less than five firms and there are multiple equilibria when there are five or more firms. When there are multiple equilibria, full vertical integration is one of the equilibria and the other one is one in which no one integrates. Thus, our results are robust to more than two firms since full integration remains one of the equilibria by all possible number of firms. Yet, for a large number of firms there is another kind of equilibria which entails no-integration for all firms. The reason for no integration to be an equilibrium is that as the number of firm increases, the mark-up in the
downstream market decreases and so does the gain from unilaterally integrating a pair of firms decreases.

There is not much that one can say when the equilibria is no-integration. However, we conjecture that because under a shared-facility agreement full integration is less likely to be the equilibrium, it is even less likely when there is a large number of firms since the benefits from using a rising rival’s cost strategy decrease. This implies that there is a range of number of firms under which is likely that the unique equilibrium under full entry is full integration, while the equilibrium under a shared-facility agreement is no integration. If that is the case, then a shared-facility agreement is more likely to be the equilibrium for a larger set of values of \( K \) relative to the case analyzed in the paper because total profits are larger under no-integration.

6 Conclusions

In this paper, we have shown that the existence of essential facilities may change the incentives to vertically integrate and that in equilibrium shared-facility agreements will be observed when the cost of building a facility are sufficiently large. Furthermore, when the building costs are large a shared-facility agreement is efficient despite the fact that it constraints the total amount of the final good produced because it avoids duplication of essential facilities.

This results suggests certain actions to antitrust authorities that sometimes opposed to the actions usually taken. Recall that Chilean antitrust authority has adopted the following measures to avoid either vertical integration or shared-facilities or both of these practices. They: (i) regulate the fees charged to access the essential facility; (ii) promote the existence of more than one essential facility supplier; (iii) increase the autonomy of the ancillary or forced it to operate as an independent firm; and (iv) restrict or prohibit the participation of the owner of the essential facility downstream.

The model suggests that actions (ii) and (iv) are not necessarily actions that will improve total welfare since vertical integration by the entrant and shared facilities may be efficient relative to no integration and full entry. The rule of thumb for this decision is the magnitude of the fixed costs and the efficiency of the essential facility. If fixed costs are large, sharing the facility may be optimal. This may be attenuated if the facility may work with a demand that results in a capacity constraint. Action (ii) may be welfare enhancing since the model show that there is a range of values for the fixed costs in which share the essential facility is inefficient relative to full entry. Action (iii) coupled with the obligation of sharing the essential facility can also be welfare enhancing when applied only to the owner of the essential facility since the model predicts that when the cost of building another facility are large shared-facility coupled
with integration by the entrant and non-integration by the incumbent yield a larger welfare than duplication of facilities and vertical integration by both, the incumbent and the entrant.

There are two predictions of the model that always have positive effects on total welfare. These are: (i) never stop an entrant that wants to vertically integrate with a downstream firm; and (ii) increase the bargaining power of the entrant when the facility is no longer dangerous of facing capacity constraints. A way of doing so is capping the fee that the owner can charge for access to the essential facility. For instance, in the case of natural gas in Chile the authority could put an upper limit to the access fee agreed on the open season.

References


7 Appendix

Proof. of proposition 1: Consider first firms $U_2$ and $D_2$’s best response to non-integration by firms $U_1$ and $D_1$. Notice that

$$\pi^F_{U_2} (1, K) - [\pi^F_{U_2} (0, K) + \pi^F_{D_2} (0)] = \frac{1}{20736b} 869 (a - m)^2.$$

Thus, vertical integration is the best response to non-integration by firms $U_1$ and $D_1$ for all $m$.

Consider next firms $U_2$ and $D_2$’s best response to integration by firms $U_1$ and $D_1$. Notice that

$$\pi^F_{U_2} (2, K) - [\pi^F_{U_2} (1, K) + \pi^F_{D_2} (1)] = \frac{35}{1152b} (a - m)^2.$$

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms $U_1$ and $D_1$ for all $m$.

Consider now firms $U_1$ and $D_1$’s best response to non-integration by firms $U_2$ and $D_2$. Notice that

$$\pi^F_{U_1} (1, K) - [\pi^F_{U_1} (0, K) + \pi^F_{D_1} (0)] = \frac{869}{20736} (a - m_1)^2.$$

Thus, vertical integration is the best response to non-integration by firms $U_1$ and $D_1$ for all $m_2 \leq m_1$.

Finally, consider firms $U_1$ and $D_1$’s best response to integration by firms $U_2$ and $D_2$. Notice that

$$\pi^F_{U_1} (2, K) - [\pi^F_{U_1} (1, K) + \pi^F_{D_1} (1)] = \frac{35}{1152b} (a - m)^2.$$

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms $U_2$ and $D_2$ for all $m$.

Proof. Consider first firms $U_2$ and $D_2$’s best response to non-integration by firms $U_1$ and $D_1$. Notice that

$$\pi^P_{U_2} (1) - [\pi^P_{U_2} (0, K) + \pi^P_{D_2} (0)] = \frac{1}{20736b} (869a^2 - 2390ar + 652am + 1349r^2 - 308rm + 172m^2).$$

It is easy to check that the difference in joint profits is continuous and strictly convex in $r$ and equal to $\frac{1}{20736b} \frac{869a^2 + 652am - 172m^2}{b} > 0$ at $r = 0$. Furthermore, notice that this expression has two real roots given by $r = \frac{154}{1349} m_1 + \frac{1195}{1349} a \pm \frac{48}{1349} \sqrt{111} (a - m)$. The two roots are positive.
and larger than $\frac{a+m}{\gamma}$, which is the maximum value allowed for $r$. Thus, vertical integration is the best response to non-integration by firms $U_1$ and $D_1$ for all $r \leq \frac{a+m}{\gamma}$.

Consider next firms $U_2$ and $D_2$’s best response to integration by firms $U_1$ and $D_1$. Notice that

$$\pi^{P}_{F_2} (2) - [\pi^{P}_{U_2} (1) + \pi^{P}_{D_2} (1)] = \frac{35}{1152b} (a + m - 2r)^2.$$ 

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms $U_1$ and $D_1$ for all $r \leq m$.

Therefore, vertical integration is a dominant strategy for all $r \leq \frac{a+m}{\gamma}$.

Consider now firms $U_1$ and $D_1$’s best response to non-integration by firms $U_2$ and $D_2$. Notice that

$$\frac{1}{20736b} \left( 869a^2 + 364ar - 2102am + 404r^2 - 1172rm + 1637m^2 \right).$$

It is easy to check that the difference in joint profits is continuous, strictly convex in $r$ and equal to $\frac{869a^2-2390am+3149m^2}{20736}$ when $r = 0$, which is always positive since $\frac{a}{\gamma} \geq m$. Furthermore, this expression has no real roots for $r$. Thus, vertical integration is the best response to non-integration by firms $U_1$ and $D_1$ for all $r \geq 0$.

Finally, consider firms $U_1$ and $D_1$’s best response to integration by firms $U_2$ and $D_2$. Notice that

$$\pi^{F}_{F_1} (2, K) - [\pi^{F}_{U_1} (1, K) + \pi^{F}_{D_1} (1)] = \frac{5}{1152b} \left( 7a^2 - 10ar - 4am - 17r^2 + 44rm - 20m^2 \right).$$

Notice that this difference is a strictly concave function of $r$ and has two real roots given by $\{r = -a + 2m, r = \frac{7}{17}a + \frac{10}{17}m \}$. The first one is negative since $m_1 \leq \frac{a}{\gamma}$ and the second one is positive and lower than $\frac{a+m}{\gamma}$. Thus, given the strict concavity of the function, then integration is the best response to integration by firms $U_2$ and $D_2$ for all $r \leq \frac{7}{17}a + \frac{10}{17}m$ and non-integration is the best-response otherwise.

**Proof.** of proposition 2: Consider first firms $U_2$ and $D_2$’s best response to non-integration by firms $U_1$ and $D_1$. Notice that

$$\pi^{P}_{F_2} (1, \bar{y}) - \left( \pi^{P}_{U_2} (0, \bar{y}) + \pi^{P}_{D_1} (0, \bar{y}) \right) = \frac{1}{12} \frac{20a^2 - 40ar - 56ab\bar{y} + 20r^2 + 56b\bar{y}r + 39 (b\bar{y})^2}{b} = 0$$

It is easy to check that the difference in joint profits is continuous and strictly convex in $r$ and that this equation has two real roots given by $a - \frac{13}{10}b\bar{y}$ and $a - \frac{3}{2}b\bar{y}$. Thus, vertical integration
is the best response to non-integration by firms \( U_1 \) and \( D_1 \) for all \( r \in \left[ a - \frac{13}{10}b\bar{y}, a - \frac{3}{2}b\bar{y} \right] \), otherwise non-integration is the best-response.

Given that \( r \in \left[ a - \frac{13}{10}b\bar{y}, a - \frac{3}{2}b\bar{y} \right] \) then vertical integration is the best response to non-integration by firms \( U_1 \) and \( D_1 \) for all feasible \( r \).

Consider next firms \( U_2 \) and \( D_2 \)'s best response to integration by firms \( U_1 \) and \( D_1 \). Notice that

\[
\pi_{F_2}^P (2, \bar{y}) - \left( \pi_{U_2}^P (1, \bar{y}) + \pi_{D_2}^P (1, \bar{y}) \right) = \frac{4}{9} \left( -a + r + b\bar{y} \right)^2 > 0
\]

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms \( U_1 \) and \( D_1 \) for all \( r \).

Therefore, vertical integration is a dominant strategy for firms \( U_2 \) and \( D_2 \) for all feasible \( r \).

Given that vertical integration is a dominant strategy for firms \( U_2 \) and \( D_2 \), consider firms \( U_1 \) and \( D_1 \)'s best response to integration by firms \( U_2 \) and \( D_2 \). Notice that

\[
\pi_{F_1}^P (2, K, \bar{y}) - \left( \pi_{U_1}^P (1, K, \bar{y}) + \pi_{D_1}^P (1, \bar{y}) \right) = \frac{2}{3} \left( -7ab\bar{y} + 2a^2 - 4ar + 6b^2\bar{y}^2 + 7b\bar{y}r + 2r^2 \right)
\]

It is easy to check that the difference in joint profits is continuous and strictly convex in \( r \) and has two real roots in equal to \( a - \frac{13}{10}b\bar{y} \) and \( a - \frac{3}{2}b\bar{y} \). Thus, vertical integration is the best response to integration by firms \( U_2 \) and \( D_2 \) for all \( r \in \left[ a - \frac{13}{10}b\bar{y}, a - \frac{3}{2}b\bar{y} \right] \), otherwise non-integration is the best-response to integration by firms \( U_2 \) and \( D_2 \). Given that \( r \in \left[ a - \frac{13}{10}b\bar{y}, a - \frac{3}{2}b\bar{y} \right] \) then non-integration is the best response to integration by firms \( U_1 \) and \( D_1 \) for all feasible \( r \).