Winter October, 2009

Uncertainty, Pay for Performance, and Asymmetric Information

Felipe Balmaceda, Assoc Prof., *Diego Portales University*

Uncertainty, Pay for Performance, and Asymmetric Information

Felipe Balmaceda*
Universidad de Chile

This article develops a new rationale for the emergence of pay-for-performance contracts where the labor market is competitive, workers are risk averse, and firms are risk neutral and unaware of workers' productivities. The article shows that the prevalence of pay for performance rises and the pay-for-performance sensitivity falls as environmental uncertainty increases. This empirical regularity is unaccounted for alternative models such as the standard agency model. (*JEL* D86, L2, M5, J3)

1. Introduction

The standard rationale for linking pay to performance is that it helps to align workers' incentives with those of the firm—the well-known incentive effect. Pay for performance imposes risk on a risk-averse worker that results in higher wage costs. The risk increases with the uncertainty of the environment, thereby giving rise to a negative trade-off between risk and incentives. Although appealing, this prediction is not borne out by the data. For many occupations, the evidence suggests that pay for performance is more prevalent the more uncertain the environment.[1]

The evidence for a positive relationship between uncertainty and incentives, reviewed in Prendergast (2002b), is based on four different occupations: executives, agricultural sharecropping, franchise holders, and salespersons. The most cited examples are for executives, where the evidence about the negative trade-off is mixed. Some authors like Aggarwal and Samwick (1999) confirm the relationship, whereas others such as Garen (1994) find none. Agricultural sharecropping clearly points to a positive relationship (Allen and Lueck 1995, 2000) as fixed rent contracts are more likely to be observed in crops with

*Centro de Economía Aplicada, Departamento de Ingeniería Industrial, Universidad de Chile, República 701, Santiago 6521122, Chile. E-mail: fbalmace@dii.uchile.cl.

I would like to thank the detailed comments of two anonymous referees and Nicolas Figeroa and participants in the regular seminar at the University of Chile for comments to an earlier version. As usual, all remaining errors are my own responsibility. This article received financial support from Fondecyt (grant no. 1060207).

1. As mentioned by Lazear (2000) and Prendergast (2002), casual evidence also seems to suggest that incentive pay is used more frequently in more uncertain industries, such as the use of bonuses in the financial sector.

The Journal of Law, Economics, & Organization, Vol. 25, No. 2,
doi:10.1093/jleo/ewn022
Advance Access publication November 25, 2008

greater yield variance. Among franchise holders strong output-based contracts are the norm, whereas in company owned stores variable pay is usually absent or minimal. Given the positive relationship between uncertainty and pay for performance, franchise holders are much more common in industries characterized by uncertainty (see, Lafontaine and Slade 2001). Finally, the literature on salesforce compensation finds little evidence of either a negative or positive relationship.

The purpose of this article is to build a theory that is consistent with the pattern found in the data and cannot be explained by the standard rationale in which pay for performance is used to induce agents to exert effort. Thus, a model is developed in which incentives for effort plays no role, and identical risk-neutral firms compete for risk-averse workers whose productivity can be either high or low. Each worker knows his productivity level, but firms only know the proportion of high productivity workers. Workers' preferences exhibit nonincreasing absolute risk aversion, and the distribution of output for a worker depends on his productivity and a parameter that captures environmental uncertainty. An increase in this parameter is associated to a rise in environmental uncertainty, understood as mean-preserving spread (MPS) of the initial distribution of output. In addition, firms are allowed to offer a menu of contracts.

The proposed model is a competitive screening model, similar to the competitive insurance model developed by Rothschild and Stiglitz (1976), but it does not suffer from the absence of equilibrium when the proportion of high-ability workers is large because a different timing, suggested by Hellwig (1987), is adopted. That is, first, firms offer a menu of contracts; second, each worker bids for a contract; and then firms, observing other firms' offers, either accept or reject applicants.

When the proportion of high-productivity workers is smaller than a given threshold, firms offer a menu with a straight-salary and a pay-for-performance contract. High-productivity workers self-select into pay-for-performance jobs and low-productivity workers into straight-salary jobs. As in Rothschild-Stiglitz' (1976) competitive insurance model, high-productivity workers (low-risk consumers) self-select into pay-for-performance jobs (partial insurance contracts) because they generate larger expected output and it is less costly for them to bear the risks associated with pay for performance. In contrast, when the proportion of high-productivity workers is higher than the given threshold, all firms offer the same pay-for-performance contract. Both high- and low-productivity workers accept that contract and receive a compensation equal to the average workers' expected output. The rationale is as follows. If straight-salary contracts were offered to all workers, firms could counter offer with profitable pay-for-performance contracts imposing higher compensation risk that only high-productivity workers would be willing to accept. Thus, pooling contracts with straight salaries do not arise in equilibrium because cream-skimming high-productivity workers is always profitable. It then follows that any pooling equilibrium must involve some, however small, compensation risk. In short, when the proportion of productive workers is small, pay-for-performance and straight-salary contracts coexist and workers self-select into different jobs, whereas when that is large, only pay-for-performance contracts are observed since self-selection results in too much compensation risk.[2]

This result is used to study the relationship between risk (environmental uncertainty) and the prevalence as well as the power of pay for performance. With linear contracts and plausible restrictions on workers' preferences, the threshold for the proportion of high-productivity workers above which the equilibrium is pooling falls as environmental uncertainty rises. In other words, in stable environments, pay-for-performance and straight-salary contracts are more likely to coexist and workers more likely to self-select into different jobs, whereas in unstable environments, only pay-for-performance contracts are more likely to be observed since self-selection results in too much compensation risk.

The model also yields the following empirical predictions: (1) workers paid by output earn and produce, on average, more than salaried or hourly workers in the same job; (2) pay-for-performance sensitivity and environmental uncertainty are negatively related; and (3) pay-for performance sensitivity in jobs where all workers are paid by output could be quite small.

The outline for the rest of the article is as follows. Section 2 discusses to some extent the pay-for-performance literature. Section 3 describes the model and the equilibrium concept and provides the full information benchmark. Section 4 derives the equilibrium. Section 5 studies the relationship between incentives and environmental uncertainty when contracts are assumed to be linear on output. Section 6 discusses the basic model's empirical predictions and compares them with those from the linear agency model and the evidence supporting them. Concluding remarks are presented in the last section.

2. Literature Review

The vast pay to performance literature can be split into two broad categories: incentive pay for performance and nonincentive-based theories.[3] Incentive-based theories are best represented by the linear agency model of Holmstrom and Milgrom(1987). It predicts a negative trade-off between risk and incentives as well as a decreasing pay-for-performance sensitivity and increasing compensation in uncertain environments. In a multitasking setting, Holmstrom and Milgrom (1991) demonstrate that general contracts induce a misallocation of effort across tasks, and when this is sufficiently severe, the theory predicts that incentive pay should not be used. Prendergast (2002b), in a model with risk-neutral workers and multitasks, explains why variable pay should be more prevalent in uncertain environments. He argues that in stable environments,

2. Because in the model here firms, after observing other firms' offers, may reject some of the applicants an equilibrium always exists.

3. For excellent surveys concerning incentive-based theories, see Prendergast (1999) and Gibbons (1998).

firms are content to assign workers to specific tasks and monitor their effort. In uncertain environments, firms delegate the choice of tasks to workers and use variable pay to align their incentives and constrain their discretion. Thus, output-based contracts are more likely to be observed in uncertain environments. The driving force behind this model is the assumption that there is no correlation between measured output and environmental uncertainty—contrary to the main assumption in the linear agency model. Prendergast shows that when such correlation exists, pay by output is not necessarily more likely to take place in uncertain environments. The present article can be seen as complimentary to Prendergast; whereas his explanation is based on monitoring output versus input, this is based on sorting and a different set of assumptions.

Nonincentive-based theories are best represented by Lazear (1986). He shows that the least productive workers self-select into firms offering straight salaries and the most productive choose firms offering piece rates.[4] His model for self-selection is quite different from the model in this article. In his article, the more productive workers self-select into pay-for-performance jobs because for them it is worth indirectly paying the (higher) monitoring costs associated with piece rates, whereas here self-selection occurs only when the environment is sufficiently certain that it is worth facing compensation risk instead of being pooled with low-productivity workers. Lazear's article cannot explain why pay for performance is more prevalent in uncertain environments and the predictions of his model are not easily compared with those from the linear agency model.

Lazear (2005) focuses on pay for performance as a way to extract worker's private information with regard to firm's prospects and retaining them. Arya and Mittendorf (2005) show that stock options are granted to distinguish workers. Together, these two articles show that a manager confident of his contribution to a firm's value is willing to accept a contract loaded with stock options. Cadenillas et al. (2005) consider the problem of a risk-neutral firm that tries to hire a risk-averse executive of unknown ability. Executives affect stock price dynamics through the choice of volatility and by applying costly effort. In this setting, they show that the use of options discourage low-ability executives from applying to the firm since the implicit risk of an option will make its value lower than low-ability executive's reservation wage. Thus, Cadenillas' et al. (2005) argument is similar to the one here with the difference that in their model the variance of the stock price (output) is controlled by the executive and they do no provide an explanation for the use of heterogeneous compensation methods within a job.

Oyer (2004) proposes a rationale for pay for performance that has neither incentive nor selection effects. He shows that the use of such contracts helps to satisfy workers' participation constraints. He argues that when a worker's outside option and firm's performance are positively correlated, by indexing wages to performance, the firm avoids worker loss. This is relevant when a worker's resignation is expensive for the firm and the subsequent ex-post

4. See Matutes et al. (1994) for a similar result.

adjustment of wages is also costly. Neither article, however, makes predictions about the relationship between environmental uncertainty, the adoption of pay for performance and its sensitivity.

Lastly, there are few articles that deal with both incentive and sorting effects. Moen and Rosen (2005) analyze a multitasking model with risk-neutral agents, a distorted performance measure, and private information about the agent's cost of effort. They show that in equilibrium pay-for-performance contracts are too high powered as they can result in a misallocation of effort across tasks. This article makes no prediction about the relationship between uncertainty and pay for performance, although like this article it also shows that sorting requires pay for performance. They show that pay for performance induces too much effort while it imposes compensation risk on risk-averse workers. Prendergast (2002a), in a model where compensation is based on supervisors performance appraisal, demonstrates that firms when choosing incentives weigh the benefits of higher effort against the cost of loosing information about workers' skills as consequence of biased appraisal. In uncertain environments, supervisors' reports are not particularly valuable for sorting purposes, so the marginal cost of increased incentives is lower.[5] In contrast to the predictions of the linear agency model and the sorting model, it is implied that pay-for-performance sensitivity increases with the uncertainty.

In short, this article complements the existing literature on sorting effects of pay-for-performance contracts by deriving a link between environmental uncertainty and the prevalence and strength of pay-for-performance contracts that is consistent with the empirical evidence.

3. The Basic Model and The Equilibrium Concept

3.1 The Basic Model

Identical risk-neutral firms compete for a fixed number of workers of unknown productivity. Workers come in two types, high-productivity workers (H) and low-productivity workers (L), with each firm having the same production technology. The output, denoted by y, is assumed to be contractible and its price is normalized to 1. The technology is such that the i-worker's output is distributed with a continuous and differentiable density function $f^i(y \mid r)$, with support $Y \equiv [\underline{y}, \bar{y}]$, $\underline{y} \geq 0$, and expected output given by $\theta_i = \int y f^i(y|r)\mathrm{d}y$ for $i \in \{L, H\}$. In this setting, the parameter $r \in [0, \infty]$ is a risk measure, which need not correspond to variance, and that does not affect the mean of the distribution. In particular, an increase in r results in an MPS of the original distribution and is associated with a more uncertain environment. This implies, as in the standard principal agent model, that there is a positive correlation between the quality of the performance measure and the uncertainty of the environment. For simplicity, $f^i(\cdot \mid r)$ is assumed twice continuously differentiable for all r.

5. Their definition of sorting differs from this article as neither workers nor firms know, at the time of contracting, the skills of a worker. The only information they receive comes from the performance appraisals.

Let define the likelihood ratio $\ell(y|r) \equiv \frac{f^L(y|r)}{f^H(y|r)}$. Then the following is assumed.

(A1) (MLRP) $\ell(y \mid r)$ is decreasing in y for all $y \in Y$ and $r \in [0, \infty]$.

Observe that this assumption implies that $\theta_H > \theta_L$—that is, a high-productivity worker's average output is greater than a low-productivity worker's average output.[6] In addition, this assumption implies that high-productivity worker's distribution of output first-order stochastic dominates (FSD) a low-productivity worker's distribution of output.

Employers know only that a worker's productivity can be either high or low and that the proportion of workers who have a high productivity is μ. In contrast, workers know their own productivity. All the rest is common knowledge.

Workers are assumed to be risk averse and have a utility function $U(w(y))$ that is twice continuously differentiable with first derivative $U'(\cdot) > 0$ and $U''(\cdot) < 0$, where $w(y)$ is total compensation when output y is realized. In addition, a worker's outside opportunity provides him with an expected utility equal to $U(0)$ which is assumed to be lower than $U(\theta_i)$ for $i \in \{L, H\}$.

In addition, the following technical restriction on $U(\cdot)$ is imposed.

(A2) $\lim_{w \to \infty} U(w) \to \infty$ and $\lim_{w \to -\infty} U(w) \to -k \ll 0$.

Each firm is allowed to offer a menu of wage contracts in which a contract yields a total compensation equal to $w(y)$ when output y is realized. When $w(y) = w$ for all $y \in Y$, C is a fixed wage or straight-salary contract and when $w(y) = y$ for all y, C is a pure piece-rate contract. Any other contract in which $w(y)$ is nondecreasing in y for all $y \in Y$ and strictly increasing for a subset of Y of positive measure is referred to as a pay-for-performance contract. In what follows, the contract (0, 0) is denoted by C_0.

An i-worker's expected utility when he accepts contract $C = \{w(y)\}$ is given by:

$$V_i(C) \equiv \int U(w(y)) f^i(y|r)\mathrm{d}y, \tag{1}$$

and a firm's expected profit from employing an i-worker under the same contract C is given by:

$$\pi_i(C) \equiv \int (y - w(y)) f^i(y|r)\mathrm{d}y. \tag{2}$$

The timing of decisions used here was suggested by Hellwig (1987) and is as follows. At Stage 1, firms are symmetrically informed and simultaneously offer a menu of wage contracts that includes either a pay-for-performance or straight-salary contract or both for the upcoming period. At Stage 2, after offers have been made, each worker applies to a particular firm for the

6. The expectation of $\ell(y \mid r)$ with respect to $F^H(\cdot \mid)$ is 1 and, because $\ell(\cdot \mid r)$ is decreasing, its covariance with y must be negative; hence

$$0 > \int y(\ell(y|r) - 1) f^H(y|r)\mathrm{d}y = \int y\left(f^L(y|r) - f^H(y|r)\right)\mathrm{d}y = \theta_L - \theta_H.$$

upcoming period. In the case that more than one firm offers the same contract, workers choose randomly between firms. At Stage 3, after each worker has chosen a contract and *firms have observed other firms' offers*, firms then have the opportunity to either accept or reject a worker's application. Once a worker has agreed to work for a particular firm and has been accepted, the terms of the agreement become binding for that period. At the final stage, output is produced and compensation takes place as specified in the contract.

3.2 The Equilibrium Concept

This section briefly explains the equilibrium concept used and the importance of the timing adopted. Under the standard equilibrium concept (Perfect Bayesian Equilibrium) and the standard timing for screening games (Stages 1 and 2), this type of model suffers from *non-existence of equilibrium* for some parameter values. A classic example of this is competitive insurance model by Rothschild and Stiglitz (1976), where an equilibrium does not exist when the proportion of low-risk individuals is sufficiently large.[7]

Hellwig (1987) added the third stage to the two-stage screening game to solve the problem of the *nonexistence* of equilibrium.[8] However, Hellwig's last two stages mimic a signaling game, so the timing effectively trades the problem of nonexistence for the problem of multiple equilibria. As Cho and Kreps (1987) show, signaling games have a plethora of Perfect Bayesian Equilibrium (hereafter, PBE) that are supported by unreasonable off-the-equilibrium path beliefs. In this article, I adopt a signaling equilibrium refinement proposed by Mailath et al. (1993) that eliminate equilibria that are based on unreasonable beliefs. In particular, it will be required that any PBE of the signaling subgame (Stages 2 and 3) must be *undefeated* among all possible PBEs that arise from any first-stage contract offers. This equilibrium refinement picks only those PBE that give the highest payoff to high-productivity workers or only those equilibria that are constrained Pareto efficient.

I adopt the Undefeated Equilibrium refinement rather than the Intuitive Criterion or Divinity. For the later, the equilibrium selected remains unchanged for any positive proportion of high-productivity workers, so it is not sensitive to the proportion of high-productivity workers unless this is exactly equal to 1. Yet, it seems unreasonable that the outcome of a game with a one worker with a one-million-chance of being a low-productivity worker differs significantly from a game in which there is no chance of being such a worker. In addition, as the distribution of workers types will not be certain, the model and its equilibrium will only be useful if the predicted outcome is not overly sensitive to the description of the environment, in particular to the proportion of high-productivity workers.

7. There are several other equilibrium concepts that differ from that used in this article and that deal with the nonexistence of equilibrium problem. The most common are Riley's Reactive Equilibrium and Wilson's Anticipatory Equilibrium. In addition, Dasgupta and Maskin (1984) derived conditions that guarantee the existence of a mixed strategy equilibrium.

8. Grossman (1979) was the first to discuss this specification but in a nonsequential setting.

3.3 The Full-Information Benchmark

Equations (1) and (2) together with risk aversion tell us that under complete information, all contracts should be efficient straight-salary contracts. Furthermore, competition among employers forces them to pay each type his expected productivity. That is, the first-best contract for a high-productivity worker is $C_H^* = \{w(y) = \theta_H \text{ for all } y \in Y\}$ and that for a low-productivity worker is $C_L^* = \{w(y) = \theta_H \text{ for all } y \in Y\}$. Because of this, it is efficient for both types of worker (low and high productivity) to participate; that is,$V_i(C_i^*) > V_i(C_0) for\, i \in \{H, L\}$.

4. Asymmetric Information

4.1 Preliminaries

Now consider the case in which employers do not know workers' productivity. First, note that the first-best contracts (C_H^*, C_L^*) are not incentive compatible since C_H^* involves no risk and pays a higher straight-salary than C_L^*. That is, $V_L(C_H^*) = V_H(C_H^*) > V_L(C_L^*)$. Second, for any given contract $w(y)$ that is nondecreasing in y, a high-productivity worker's expected utility is greater than or equal to that for the low-productivity worker; that is,

$$\int U(w(y))f^H(y|r)\mathrm{d}y \geq \int U(w(y))f^L(y|r)\mathrm{d}y.$$

This is a direct consequence of the monotone likelihood ratio property since this implies that $F^H(y \mid r)$ FSD $F^L(y \mid r)$, the fact that $U'(\cdot)$ is increasing and $w(y)$ rises with y.

The next result shows that whenever a low-productivity worker is indifferent between two contracts with different risk, a high-productivity worker's expected utility is greater under the riskier contract. All proofs are placed in the Appendix.

Lemma 1. Consider two arbitrary contracts C_1 and C_2 such that $V_L(C_1) = V_L(C_2)$, and assume that C_1 single-crosses C_2 from below. Then, $V_H(C_1) > V_H(C_2)$.

This lemma shows that the trade-off of the return on risk for a given pay-for-performance contract is more favorable for the H type than for the L type: the same risk exposure cost caused by a steeper contract is lower for the H type than for the L type. This implies that firms could use pay for performance as a screening device. This intuition is explored in the next section.

4.2 Separating Equilibrium

Suppose that separation of the two types occurs at equilibrium, competition will require firms to offer a menu of contracts that contains a contract that maximizes high-productivity workers' expected utility (cream-skimming) and another that maximizes low-productivity workers' expected utility subject to (i) no one type of worker has incentive to choose the contract designed for the other type and (ii) no contract guarantees expected positive profits for the firm when chosen by the proper type.

Condition (i) implies that the following incentive compatibility constraints must be satisfied,

$$\int U(w^H(y))f^H(y|r)\mathrm{d}y \geq \int U(w^L(y))f^H(y|r)\mathrm{d}y, \tag{1}$$

and

$$\int U(w^L(y))f^L(y|r)\mathrm{d}y \geq \int U(w^H(y))f^L(y|r)\mathrm{d}y. \tag{2}$$

Adding these two incentive constraints and rearranging terms, a necessary condition for separation to be an equilibrium is:

$$\int [U(w^H(y)) - U(w^L(y))][1 - \ell(y|r)]f^H(y|r)\mathrm{d}y \geq 0, \tag{3}$$

The following lemma which is direct consequence the diffidence theorem of Gollier and Kimball (1996) provides the necessary and sufficient conditions for equation (5) to hold when the wage schedule is nondecreasing in y.

Lemma 2. Suppose that $w^i(y)$ is nondecreasing in y for $i \in \{L, H\}$, $w^H(y)$ and $w^L(y)$ cross only once at y_0, and low-ability workers' incentive constraint is binding, then the necessary and sufficient condition for equation (5) to hold is

$$\ell(y|r) \geq \ell(y0|r) \quad \text{if} w^H(y) < w^L(y),$$
$$\ell(y|r) \leq \ell(y0|r) \quad \text{if} w^H(y) > w^L(y).$$

This lemma shows that if low-ability workers' incentive constraint is binding, MLRP is necessary and sufficient to guarantee that there is a pair of incentive compatible contracts, each of them involving different degrees of pay for performance.

When the two incentive constraints are satisfied, workers self-select and so competition forces employers to pay each productivity type the corresponding expected output. Any contract different from C_L^* that either breaks even or makes positive profits when chosen by low-productivity workers only yields them a lower expected utility than C_L^*.[9] This implies that the contract tailored to low-productivity workers is the full information contract C_L^*—that is, $w_L^*(y) = \theta_L$ for all $y \in Y$.

9. Note that a firm offering C_L^* will make non-negative profits in this contract regardless of who applies to this contract.

Because the optimal contract for a low-productivity worker is C_L^*, the optimal contract for a high-productivity worker, denoted by C_H^s, will be solution to

$$\max_{w(y)\in\Re} \int U(w(y))f^H(y|r)\mathrm{d}y$$

subject to

$$\begin{aligned} &\int U(w(y))f^L(y|r)\mathrm{d}y \leq U(\theta_L) \text{ and} \\ &\int w(y)f^H(y|r)\mathrm{d}y \leq \theta_H. \end{aligned} \tag{4}$$

Letting λ and ρ, respectively, be the Lagrange multipliers on the constraints, the first-order condition is

$$U'(w(y))[f^H(y|r) - \lambda f^L(y|r)] - \rho f^H(y|r) = 0. \tag{5}$$

Observe that $\lambda > 0$ otherwise the contract would be a fixed wage, which together with the second constraint, will not be consistent with sorting.[10] Solving the first-order condition yields

$$U'(w(y)) = \frac{\rho}{1 - \lambda\ell(y|r)}. \tag{6}$$

As the full-information contract C_H^* cannot sort workers out when offered together with C_L^*, it follows from this and MLRP that the contract tailored to high-productivity workers must be a pay-for-performance contract. Because the worker is risk averse and thereby exhibits decreasing marginal utility, it follows from equation (8) that MLRP implies that the unique solution to the problem in (6) must be increasing in y. This is formally shown in the next lemma.

Let us denote the contract that solves equation (8) by C_H^s and the optimal contract for low-productivity workers by C_L^s, where s stands for separation. Then the following result obtains.

Lemma 3. (i) C_H^s is a pay-for-performance contract and $C_L^s = C_L^*$; (ii) the contract C_H^sexists and is unique; and (iii) λ and ρ are implicitly defined as continuously differentiable functions of r and hence $V_H(C_H^s)$ is continuously differentiable in r.

This result establishes that firms use pay-for-performance contracts to skim the cream (the risk imposed by a pay-for-performance contract is less attractive for low-productivity workers because they have a lower expected output).

Notice that this equilibrium is equivalent to the separating equilibrium in the competitive insurance model by Rothschild and Stiglitz (1976). High-risk consumers (low-productivity workers) choose a full insurance contract (straight-salary contract), whereas low-risk consumers (high-productivity workers) choose a partial insurance contract (pay-for-performance contract).

10. The first-order condition is necessary and sufficient since the Lagrangian is strictly concave.

4.3 Pooling Equilibrium

Next suppose that the equilibrium is pooling—that is, high- and low-productivity workers choose the same contract. In this case firms must believe that a worker is a high-productivity worker with probability μ since contract choice does not reveal any information. We know from Rothschild and Stiglitz that, when the two-stage timing is used, this cannot be an equilibrium. A firm offering a slightly steeper contract than the pooling contract would attract high-productivity workers only and thus it makes a profit, whereas the nondeviating firms lose money. This intuition will not hold given the timing adopted in this article. The reason is that no worker would ever apply for the pooling contract offered by the nondeviating firms, as they know that they are going to be rejected in the third stage. Nondeviating firms, after seeing a cream-skimming offer, will know that their offers could only attract low-productivity workers and so they would loose money. Workers, anticipating this rejection, would apply to the deviating firm offering the cream-skimming contract and which in turn would make this deviation unprofitable. Thus, no firm has an incentive to deviate from the pooling contract.

Furthermore, among all those pooling contracts that break-even at the population's average productivity, $\hat{\theta} \equiv \mu\theta_H + (1-\mu)\theta_L$, the equilibrium refinement selects the contract with the largest expected utility for high-productivity workers. Let us denote this contract by C^p, where p stands for pooling. There are other pooling contracts that can only be sustained as PBEs by assuming unreasonable off-the-equilibrium path beliefs. A pooling contract different from C^p is only possible as a PBE if a deviating firm believes that this new pooling contract attracts an above average group of workers. This is unreasonable since by definition C^p offers the largest expected utility to high-productivity workers.

The equilibrium contract C^p in the pooling equilibrium will be the solution to

$$\max_{w(y)\in\Re} \int U(w(y)) f^H(y|r)\mathrm{d}y$$

subject to

$$\int w(y)(\mu f^H(y|r) + (1-\mu) f^L(y|r))\mathrm{d}y \leq \mu\theta_H + (1-\mu)\theta_L. \tag{7}$$

Letting γ be the Lagrange multiplier on the constraint, the first-order condition is

$$U'(w(y)) - \gamma[\mu f^H(y|r) + (1-\mu) f^L(y|r)] = 0.$$

Hence,

$$U'(w^p(y)) = \gamma[\mu + (1-\mu)(y|r)]; \tag{8}$$

and thereby $w(y)$ rises with y since it is being assumed that $\ell(y\,|\,r)$ falls with y.

This shows that the best pooling contract from the high-productivity agent's point of view is a pay-for-performance contract.

The next lemma formalizes the discussion above.

Lemma 4. (i) C^p is a pay-for-performance contract, (ii) the contract C^p exists and it is unique, and (iii) γ is implicitly defined as continuously differentiable function of r and μ and hence $V_i(C^p)$ is continuously differentiable in r and μ.

It is interesting to remark once again that C^p is a pay-for-performance rather than a straight-salary contract as the optimal risk allocation might lead us to conclude. For, if a straight-salary contract is offered, it can be broken by any firm offering a contract with higher expected compensation and reduced risk which only high-productivity workers would be willing to bear. So, the pooling straight-salary contract becomes unprofitable and cannot be sustained as an equilibrium.

4.4 The Equilibrium

In a pooling equilibrium, high-productivity workers' expected compensation is lower than their expected output, whereas in a separating equilibrium, their expected compensation is equal to their expected output. This suggests that a high-productivity worker would be better-off when cream skimming is pursued, unless the risk offered by the pooling contract is sufficiently lower.

In addition to this, as the proportion of high-productivity workers rises, a high-productivity worker compensation in the pooling equilibrium increases. This suggests that as μ rises, it is more likely that the best equilibrium for both high- and low-productivity workers is the pooling equilibrium. In fact, the next proposition shows that the equilibrium is pooling when μ exceeds a given critical value and separating otherwise. This implies among other things that contract choice in each case is such that the equilibrium is constrained Pareto efficient.

Let us define $\mu(r)$ as the minimum proportion of high-productivity workers on the population that leaves high-productivity workers indifferent between contract C^p and contract C_H^s and leaves low-productivity workers better-off than when they choose C_L^s. That is $V_H(C_H^s) = V_H(C^p)$ and $V_L(C_L^s) < V_L(C^p)$. Then the following is proposed and formally demonstrated in the Appendix.

Proposition 1. Suppose that $w_H^s(y)$ single-crosses $w^P(y)$ from below.[11] Then, there exists a critical value for the proportion of high-ability workers denoted by $\mu(r)$ such that: (i) if $\mu \leqq \mu(r)$, then in equilibrium firms offer a menu with two contracts: the straight-salary contract C_L^s and the pay-for-performance contract C_H^s. Low-productivity workers self-select into straight-salary jobs, whereas high-productivity workers self-select into pay-for-performance jobs; whereas (ii) if $\mu > \mu(r)$, then in equilibrium all firms offer the pay-for-performance contract C^p and both types of workers participate.

Two remarks are in order. First, the equilibrium refinement implies that the equilibrium is unique and constrained Pareto efficient. Second, if we restrict ourselves to either pure separating or pure pooling equilibrium, with more than

11. When contracts are linear as in the next section this is trivially satisfied.

two worker types, the analysis shows that for a separating equilibrium only the lowest productivity type receives a straight salary, and all other productivity types are paid by pay-for-performance contracts. Because of MLRP, the steepness of the optimal contract increases with the productivity parameter, that is, in equilibrium, for any two workers with productivity parameters θ_i and $\theta_{i'}$ with $\theta_i > \theta_{i'}$, $w_i^s(y)$ is steeper almost everywhere than $w_{i'}^s(y)$. In the pooling equilibrium, all firms offer the same pay-for-performance contract, and workers are indifferent between firms. The contract offered is the one that yields the highest expected utility to the highest productivity workers among the contracts that break even at the average productivity and is accepted by all workers. As in the two type cases, a unique pooling equilibrium exists. This occurs when each worker's productivity type prefers the pooling equilibrium rather than the separating one, with strict preferences given to at least one worker's type. Therefore, with more than two productivity types, the results are similar to the ones in proposition 1.

5. The Risk and Incentives Relationship with Liner Contracts

In this section, I focus on the relationship between risk and the prevalence and power of pay for performance. More precisely, how the threshold for the proportion of high-productivity workers and the steepness of the optimal contract vary as environmental uncertainty rises.

As known since the seminal article by Grossman and Hart (1983), the existence of optimal incentive contracts with economically meaningful properties requires assumptions such as MLRP and the concavity of the distribution function property. In addition, there is no unambiguous prediction with regard to the relationship between risk—environmental uncertainty—and the power of incentives. This has lead researchers to focus on linear contracts. However, focusing on linear contracts is not free of problems. Mirrlees (1974) showed that the best linear contract is worse than various nonlinear contracts. In particular, a step-function contract, where the worker earns w_H if $y \geq \tilde{y}$, but w_L otherwise, can perform very well in the sense that it approaches the twin goals of full incentives and full insurance in the limit (as $\tilde{y}$ and w_L fall in appropriate fashion, so that the worker almost surely gets w_H and yet has incentives from the fear of receiving the very low payment w_L). Although linear contracts may not be optimal, tractability reasons together with the fact that complex contracts may impose significant writing costs, and they can be subject to gaming in ways no modeled here, but which are important in the real world, force me to limit the attention to linear contracts of the following form: $w(y) = \alpha + \beta y$, with $\beta \in (0, 1)$.

In addition to this, linear contracts permit simpler comparative static analysis, since the pay-for-performance sensitivity induced by the optimal contract is specified by a single parameter; allow equilibrium contracts to be characterized in terms of the main parameter of interest, which is the one that captures environmental uncertainty; and allow comparisons of the shape of the optimal contracts to those from the standard linear agency model.

It is also assumed that

$$F^i(\cdot|r) = r[G_1(\cdot) - G_0(\cdot)] + H^i(\cdot) \text{ for } i \in \{L, H\},$$

where $r \geq 0$, $G_1(\cdot)$ is a mean-preserving spread of $G_0(\cdot)$, and both distributions have a mean equal to $\tilde{y}$. Then, the family generated by $F^i(\cdot|r)$ as r increases is a linear path or sequence of riskier distribution from $H^i(\cdot)$ to $F^i(\cdot|r)$. Thus, r is a measure of risk in the sense that a greater r means a more uncertain environment. Furthermore, $F^H(\cdot|r)$ FSD $F^L(\cdot|r)$for all r since $H^H(\cdot)$ FSD $H^L(\cdot)$.

Let also define the coefficient of absolute risk aversion as $A(w) = -\frac{U''(w)}{U'(w)}$, the coefficient of relative risk aversion as $A_R(w) = wA(w)$, the coefficient absolute prudence as $P(w) = -\frac{U'''(w)}{U''(w)}$, and that of relative prudence as $P_R(w) \equiv wP(w)$.

(A3) Preferences exhibit nonincreasing absolute risk aversion, that is, $A'(w) \leq 0$ for all w.

This assumption ensures that the wealthier an agent, the smaller the amount that the agent is willing to pay to escape a given additive risk. Because $A'(w) = A(w)(A(w) - P(w))$, this assumption entails $P(w) \geq A(w)$. In particular, the agent is prudent (i.e., $U'''(\cdot) > 0$).

Prudence is the sensitivity of the optimal choice of a decision variable to risk. This term is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to risk aversion, which is how much one dislikes uncertainty and would turn away from uncertainty if possible.

First, suppose the equilibrium is separating. Then the firm is faced with the following problem:

$$\max_{\beta_H^s \in [0,1]} \int U(\theta_H + \beta_H^s(y - \theta_H)) f^H(y|r) \mathrm{d}y$$

subject to

$$\int U(\theta_H + \beta_H^s(y - \theta_H)) f^L(y|r) \mathrm{d}y = U(\theta_L). \tag{1}$$

The first-order condition for this problem is

$$\int U'(\theta_H + \beta_H^s(r)(y - \theta_H))(y - \theta_H)(1 - \lambda \ell(y|r)) f^H(y|r) \mathrm{d}y = 0, \tag{2}$$

and the optimal slope $\beta_H^s(r)$ is fully determined by the incentive compatibility constraint.

When the equilibrium is pooling, the firm is faced with the following problem:

$$\max_{\beta^p \in [0,1]} \int U(\hat{\theta} + \beta^p(y - \hat{\theta})) f^H(y|r) \mathrm{d}y. \tag{3}$$

The first-order condition for this problem is

$$\int U'(\hat{\theta} + \beta^p(r)(y - \hat{\theta}))(y - \hat{\theta})f^H(y|r)\mathrm{d}y = 0, \tag{4}$$

where $\beta^p(r)$ is the optimal pay-for-performance sensitivity.

This problem is similar to the standard portfolio problem in which an investor decides how to divide his wealth between a risky and arisk-free asset. The comparative statics literature on the properties of the solution to this problem is vast and summarized in great detail in Gollier (2001). Basically, the main conclusion is that one must restrict either the preference set or the set of mean-preserving increases in risk to sign the comparative statics results in accordance with economic intuition, that is, the share invested in the risky asset falls with r. When mean-preserving spreads are considered, several restrictions on the preference set are needed, while when no restriction other than concavity is imposed, Gollier (1995) showed that the necessary and sufficient condition to get the expected signs on the comparative statics exercises requires to focus on a particular kind of increases in risk known as Central Dominance.[12] Moreover, as pointed out by Gollier (1995), second-order stochastic dominance (SSD) is neither necessary nor sufficient for central dominance (CD). In fact, SSD and CD cannot be compared to each other. Thus, in what follows I will focus on the more standard approach of restricting the preference set to obtain economically meaningful comparative statics.

The next proposition provides conditions for the optimal pay-for-performance sensitivity to fall as environmental uncertainty rises.

Proposition 2. (i) $\beta^s_H(r)$ falls as r rises; (ii) $\beta^p(r)$ falls as r rises if either (a) $U'''(\cdot) \geq 0$ and $P_R(w) \leq 2 + \hat{\theta}P(w)$; or (b) $A_R(w) \leq 1 + \hat{\theta}A(w)$, $A_R'(w) \geq 0$ and $A'(w) \leq 0$.

The intuition when the equilibrium is separating is as follows. Because an increase in environmental uncertainty has not effect on low-productivity workers' optimal contract (since they are paid a fixed wage), greater risk makes the pay-for-performance contract more amenable to high-productivity workers and less attractive to low-productivity workers and; thus, a lower pay-for-performance sensitivity is needed to induce sorting.

In the pooling equilibrium, however, this holds only when the agent is prudent and $U'(w)w$ is concave. These conditions guarantee that the slope of the indifference curve in the (α, β)—space rises with r and thereby for a given drop in β, the worker is now willing to give up a lower amount of the fixed

12. Gollier (1995) derives a necessary and sufficient condition, called CD, for this to hold for any concave utility function. This requires the existence of an scalar m such that

$$\int^k y\mathrm{d}F(y|r_1)\mathrm{d}y \geq m\int^k y\mathrm{d}F(y|r_0)\mathrm{d}y \text{ for all } k \in Y.$$

Observe that when $m > 1$, the mean under $F(y|r_1)$ must be greater than the mean under $F(y|r_0)$.

wage relative to the initial situation. In this case, a high-productivity worker needs less compensation risk as the reduction in the cross-subsidy obtained by using pay for performance cannot compensate for the extra compensation risk.

It is worthwhile to remark that the concavity of $U'(w)w$ is guaranteed by the following restriction on the coefficient of relative prudence: $P_R(w) \leq 2$ for all $w \geq 0$. This implies that for large values of w the worker must behave as if he is nearly risk neutral.

It is also clarifying to know that in the case of mean-variance preferences, the condition under which $\beta^p(r)$ falls with r is the same as the one that guarantees that the slope of the indifference curve rises (i.e., the indifference curve becomes flatter) as the variance of the performance measure rises.

In the next proposition I state conditions under which $\beta^p(r)$ falls with the population share of high-productivity workers, μ.

Proposition 3. $\beta^p(r)$ falls with μ if $A_R(w) \leq 1 + yA(w)$ for all $y \in Y$.

An increase in μ rises total expected output and the slope of the average iso-profit curve. This creates a sort of income and substitution effect. Because of nonincreasing absolute risk aversion, the former results in a greater fixed wage, which decreases the worker's distaste for risk. The substitution effect results in a lower pay-for-performance sensitivity. The change in $\beta^p(r)$ depends on which effect dominates. The substitution effects dominates when the indifference curve becomes sufficiently flat. This occurs when relative risk aversion is small.

Again by mean of an analogy with the mean-variance framework, $A_R(w) \leq 1$ ensures that the slope of the indifference curve rises as average output rises.

Now I turn to the main question of this section which is how $\mu(r)$ changes with r. In particular, the next proposition states conditions under which an increase in environmental uncertainty results in a lower $\mu(r)$. In terms of the equilibrium of the model this means that in stable environments, pay-for-performance and straight-salary contracts are more likely to coexist and workers more likely to self-select into different jobs, whereas in unstable environments, only pay-for-performance contracts are more likely to be observed since self-selection results in too much compensation risk. In short, it provides a condition for an increase in risk (understood as a MPS) to result in pay-for-performance contracts being more prevalent.

A useful preliminary result, which shows that MLRP implies the single-crossing property, is stated in the next lemma.

Lemma 5. A high-ability worker's indifference curve in the (α, β) single-crosses a low-ability worker's indifference curve only once and it does it from above.

This proves that the MLRP property implies that the single-crossing property holds in the (α, β) space for all strictly concave utility functions.

Before stating the main result of this section, the following assumption is needed.

(A4) (i) $A_R(w_H^s(y)) < C(y)$ for all $y > \theta_H$ and $A_R(w_H^s(y)) \geq C(y)$ otherwise; and (ii) $A_R'(\theta_H) \leq \frac{C'(\theta_H)}{\beta_H^s(r)}$, where

$$C(y) \equiv A(\theta_H)(\theta_H - y_H^*)\beta_H^s(r) + A(w_H^s(y))((1 - \beta_H^s(r))\theta_H + \beta_H^s(r)y_H^*) - \frac{(1-\lambda)[\ell(y|r) - \ell(\theta_H|r)]}{(1-\lambda\ell(\theta_H|r))(1-\lambda\ell(y|r))}K(r)\beta_H^s(r),$$

$K(r)$ is a positive constant, and y_i^* satisfy $y_i^* \int U'(w)\mathrm{d}F^i = \int U'(w)y\mathrm{d}F^i$.[13]

First, observe that $C(\theta_H) = A_R(w_H^s(\theta_H))$, since $w_H^s(\theta_H) = \theta_H$. Thus, $A_R(w_H^s(y))$ and $C(y)$ cross each other at $y = \theta_H$. Second, because of MLRP, the third term in $C(y)$ is negative for all $y > \theta_H$ and positive otherwise, and rises with y. Together with the fact that preferences exhibit nonincreasing absolute risk aversion, this implies that $C(y)$ may either rise or fall with y.

Because assumption A4 requires that relative risk aversion single-crosses the function $C(y)$ once from above, if $C(y)$ does not fall with y, nonincreasing relative risk aversion is a sufficient condition for assumption A4 to hold. In this case also admits increasing relative risk aversion as long as this increases with y less than $C(y)$ does it. On the contrary, if $C'(y) < 0$, then relative risk aversion must be decreasing and it must fall at a higher rate than $C(y)$ does it. Thus, the more the likelihood ratio falls with y, the less stringent is assumption A4.

Now, I am ready to state the result.

Proposition 4. Suppose that $U'(w_H^s(y))(y - y_H^*)$ is increasing and concave in y, and assumption A4 holds. Then, the threshold for the share of high-ability workers $\mu(r)$ above which the equilibrium is pooling falls as the environmental uncertainty rises, that is, $\frac{\partial\mu(r)}{\partial r} < 0$.

The intuition for this result can be better understood with the help of Figure 1. The figure depicts the separating equilibrium for a given level of environmental uncertainty (represented by the continuous lines) and the separating equilibrium after environmental uncertainty rises (represented by dotted lines). That is, as r goes from r_0 to r_1, with $r_1 > r_0$. Dot A in the figure corresponds to the pay-for-performance contract with slope $\beta_H^s(r_0)$ and fixed wage equal to $\alpha_H^s(r_0)$. This contract provides a low-productivity worker with the same utility as the contract that pays a fixed wage equal to θ_L when environmental uncertainty is given by r_0, whereas dot C represents the equivalent contract when environmental uncertainty is r_1. Dot B corresponds to the tangency between the slope of the average iso-profit curve and the indifference curve for a high-productivity worker in the initial separating equilibrium, $IC_H(r_0)$, whereas dot D is the analog when environmental uncertainty is r_1. Because the slope of the

13. Observe that $\int U'(w)(y - y_i^*)\mathrm{d}F^i = 0$ and the integrand changes sign once from negative to positive.

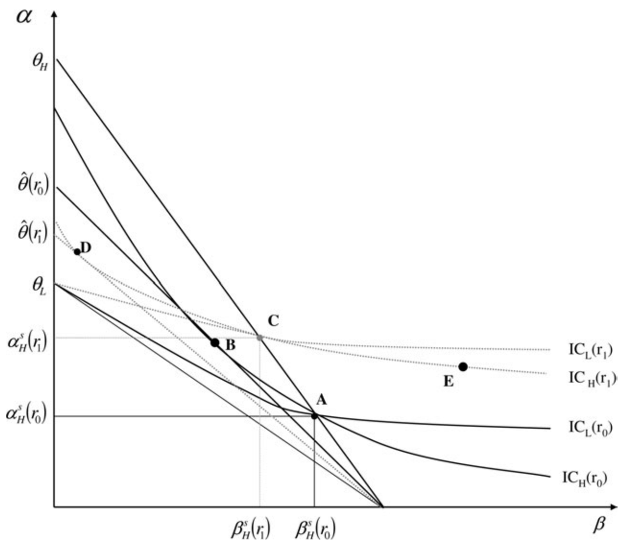

Figure 1. Separating equilibrium before and after environmental uncertainty rises.

average iso-profit curve falls with μ, the slope at the tangency provides the smaller share of high-productivity workers above which a high-productivity worker's utility under the optimal pooling contract is at least as large as the one he obtains in the separating equilibrium. Because the slope in dot *D* is greater than that in dot *B*, the figure depicts a case in which an increase in environmental uncertainty results in a decrease in $\mu(r)$.

A change in environmental risk changes the slope of the indifference curves in the (α, β) space and the shape of the optimal contracts. These two effects goes in the opposite direction. On the one hand, if $U'(w(y))(y - y_i^*)$ is concave in y as assumed, an increase in r makes both low- and high-productivity workers' indifference curve flatter in the (α, β) space. This implies that low-productivity workers dislike risks more after an increase in r. As a result of this, sorting can be achieved by paying high-productivity workers a higher fixed wage and by lowering the pay-for-performance sensitivity of the optimal contract—that is, $\beta_H^s(r_0) > \beta_H^s(r_1)$ and $\alpha_H^s(r_0) < \alpha_H^s(r_1)$—up to the point where the expected wage equals the expected output. This results in a lower compensation risk, which in turn makes, from the high-productivity worker's viewpoint, sorting more attractive than pooling.

The increase in environmental risk also decreases the pay-for-performance sensitivity of the optimal pooling contract. This makes the pooling contract more attractive for both low- and high-productivity workers. Whether or not pooling is more likely to arise as environmental uncertainty rises depends on which effects dominate. In order for the threshold for the share of high-ability workers $\mu(r)$ above which the equilibrium is pooling to fall, the increase in environmental risk must make pooling more attractive than sorting for high-productivity workers. This is more likely to be accomplished when the drop in the absolute value of the slope of the indifference curve in the (α, β) space for a high-productivity worker is relatively greater than that for a low-productivity worker.

This depends on two different effects. First, the switch in the indifference curve from $IC_H(r_0)$ to $IC_H(r_1)$, which entails moving across a ray from origin that goes through A and E. The slope of the indifference curve as this switches from A to E remains constant when relative risk aversion is constant and rises (falls) when that is decreasing (increasing). Second, the movement along the indifference curve $IC_H(r_1)$ from point E to point C. The slope of indifference curve as one moves up through the indifference curve remains constant when absolute risk aversion is constant and rises (falls) when that is increasing (decreasing). For a given r, the slope of the indifference curve as the optimal contract moves from A to C depends on the sign of the following term:

$$\int U'(w)(y - y_i^*)(A_R(w) - \theta_H A(w))\mathrm{d}F^i.$$

Because $\int U'(w)(y - y_i^*) = 0$, this term is zero if $A_R(w) - \theta_H A(w)$ is constant and positive (negative) if $A_R(w) - \theta_H A(w)$ is increasing (decreasing) in y. Observe that this is increasing if and only if

$$A_R'(w) - \theta_H A'(w) > 0.$$

Because $A'(w)$ is nonpositive, if relative risk aversion is increasing, the whole term is positive, whereas if relative risk aversion is negative, this could be negative when y is sufficiently large, but this is never the case for $y \leq \theta_H$, since $w_H^s \leq \theta_H$ for all $y \leq \theta_H$. In fact, if $A'(w)$ is not too negative, $A_R(w) - \theta_H A(w)$ rises with y.

If $A_R(w) - \theta_H A(w)$ is increasing, the slope rises when an increase in environmental uncertainty takes place. This implies that the fall in the optimal pay-for-performance after an increase in r is large, which makes sorting more attractive from a high-productivity worker's viewpoint. On the other hand, this makes high-productivity workers' indifference curve $IC_H(r_1)$ flatter, which makes the indifference curve $IC_H(r_1)$ more likely to cross the initial average iso-profit curve from below.

By how much relative risk aversion is allowed to rise depends on how much $\beta_H^s(r)$ changes with r. When the change in $\beta_H^s(r)$ is small, increasing relative risk aversion may be allowed, while when the change is high, relative risk aversion must be decreasing. In fact, if either $\ell(y|r)$ falls sufficiently fast with y or $A'(w)$ does not fall drastically with y, then $C'(y) \geq 0$ for all $y \in Y$, which in turn implies that increasing relative risk aversion is allowed. On the contrary, if $C'(y) \leq 0$, relative risk aversion must be decreasing.

Before ending this section, it is worthwhile to notice that the result in proposition (4) also holds when output is distributed normal with mean θ_i and variance r^2, preferences exhibit constant absolute risk aversion equal to A, and the following parametric restriction is satisfied: $Ar^2 > \Delta\theta \equiv \theta_H - \theta_L$.

This case is interesting in its own right since contrary to the assumption made on the article, the support of the distribution is unbounded. In addition to this closed form solutions can be obtained. In fact, one can easily show that

$$\beta_H^s(r) = \frac{-\Delta\theta + (\Delta\theta^2 + 2Ar^2\Delta\theta)^{\frac{1}{2}}}{Ar^2},$$

and

$$\beta^p(r) = \frac{(1-\mu)\Delta\theta}{Ar^2}.$$

Observe that both $\beta_H^s(r)$ and $\beta^p(r)$ fall with r (i.e., the variance of output) and A. In addition, $\beta^p(r)$ falls with μ and $\lim_{\mu\to 1}\beta^p(r)\to 0$. Furthermore, one can show that

$$\mu(r) = 1 - \frac{Ar^2}{\Delta\theta} - \frac{1}{\Delta\theta}\left(A^2r^4 - \left[(\Delta\theta^2 + 2\Delta\theta Ar^2)^{\frac{1}{2}} - \Delta\theta\right]^2\right)^{\frac{1}{2}}.$$

Differentiation of $\mu(r)$ with respect to r leads to

$$\frac{\partial\mu(r)}{\partial r} = -\frac{2Ar}{\Delta\theta}\left[1 + \frac{\left(1 - \Delta\theta(\Delta\theta^2 + 2\Delta\theta Ar^2)^{-\frac{1}{2}}\beta_H^s(r)\right)}{(1-\beta_H^s(r)^2)^{\frac{1}{2}}}\right] < 0,$$

because the parameters are such that $\beta_H^s(r)\in(0,1)$.

6. Empirical Predictions and Evidence

This section compares the model's empirical predictions with the linear agency model's predictions and examine them in light of the empirical evidence.

6.1 Empirical Predictions

I begin by focusing on the well-known predictions from the static linear agency model. This model, like that presented in Section (5), has the advantage that the pay-for-performance sensitivity depends on a unique measure of risk, which is the variance of output. The optimal contract within the class of linear contracts has a pay-for-performance sensitivity equal to $\beta_i^M = \frac{1}{1+Ac\sigma_\varepsilon^2}$, when output is assumed to be $y = e + \varepsilon$, where ε is distributed normal with zero mean and variance σ_ε^2, the cost of effort is $\frac{ce^2}{2}$, the market is competitive, and preferences exhibit constant absolute risk aversion.[14]

14. In this case, the optimal effort is $e = \frac{\beta}{c}$ and the principal's problem is $\max_{(\alpha,\beta)\in\Re^2} \alpha + \beta e - \frac{ce^2}{2} - \frac{A}{2}\beta^2\sigma_\varepsilon^2$ subject to $e = \frac{\beta}{c}$ and $e(1-\beta) - \alpha \geq 0$. Because competition implies that firms obtain zero expected profits, $\alpha = \frac{\beta}{c}(1-\beta)$. Then, the principal problem becomes: $\max_{\beta\in\Re^2} \frac{1}{c}\beta\frac{1}{2c}\beta^2 - \frac{A}{2}\beta^2\sigma_\varepsilon^2$. The solution to this problem is the one stated in the text.

The relationship between risk and pay-for-performance sensitivity in both the pure incentive and pure sorting models is stated in the next result.

Proposition 5. (Pure Incentive Model) The optimal pay-for-performance β_i^M falls with environmental uncertainty for $i \in \{H, L\}$;

(Pure Sorting Model) (i) Separating equilibrium. The optimal pay-for-performance for a high-productivity worker (i.e., $\beta_H^s(r)$) falls with environmental uncertainty and is independent of μ, whereas the optimal contract for a low-productivity worker entails no pay for performance (i.e., $\beta_L^s(r) = 0$); (ii) Pooling equilibrium. The optimal pay-for-performance $\beta^P(r)$ falls with environmental uncertainty if and only if $U'''(\cdot) \geq 0$ and $P_R(w) \leq 2 + \hat{\theta}P(w)$ and falls with μ if $A_R(w) \leq 1 + yA(w)$ for all $y \in Y$; and (iii) $\beta^p(r) < \beta_H^s(r)$.

The main conclusion of this proposition is that environmental uncertainty affects the pay-for-performance sensitivity in the same way in both the pure sorting and pure incentive models. That is, there is a negative trade-off between risk (understood as the uncertainty of the environment) and incentives as (measured by the pay-for-performance sensitivity).[15] Both models predict that on average pay-for-performance workers earn and produce more than straight-salary workers. However, the pure sorting model predicts that in uncertain environments, pay-for-performance contracts are more prevalent. The pure incentive model does not make such prediction.

The model also predicts that ceteris-paribus in occupations where the proportion of high-productivity workers is greater, the likelihood of observing only pay-for-performance workers is higher.

Only Prendergast (2002a,b) is concerned with the link between pay for performance and uncertainty and he (2002a) predicts that pay-for-performance sensitivity is increasing with the variance of output, but he makes no prediction concerning the prevalence of pay for performance. Prendergast (2002b) shows that input monitoring coupled with fixed wages is more prevalent than output monitoring coupled with pay for performance in stable environments. Given his assumptions, pay-for-performance sensitivity should be unresponsive to the output variance. In contrast, the pure sorting story model proposed here makes predictions concerning both the prevalence of pay-for-performance contracts and the pay-for-performance sensitivity. Although the prediction concerning the prevalence is consistent with Prendergast (2002b) that concerning the pay-for-performance sensitivity is different from that in Prendergast (2002a) and consistent with that from the linear agency model.

6.2 Empirical Evidence

In discussing the empirical evidence it is useful to bear in mind that the models' predictions are less applicable to ongoing renegotiable relationships such as sharecropping and situations in which uncertainty affects a company-wide

15. The same holds when risk is understood as a worker's degree of risk aversion.

performance measure such as stock options for employees whose performance does not move the stock price—which is, for instance, so prevalent in high-tech firms and startup companies.[16] This, however, is less the case for top executives since their performance has a direct impact on firm value (stock price) and long-term relationships such as some franchise deals and other forms of royalty systems.[17]

The evidence from executive data shows that the negative relationship between pay-for-performance sensitivity and the variance of the performance measure is at best mixed (see, Prendergast 2002, table 1). Prendergast reports that three articles find a statistically significant negative effect, two a significant positive relationship and five find no relationship between risk and incentives. This evidence presents something of a puzzle for the pure sorting and the pure incentive theories, as it does for Prendergasts, since it is unlikely that monitoring costs and supervisors'-biased performance appraisals are important components when determining top executives' compensation.

The prediction that pay for performance is more likely to be found in uncertain environments is sustained in research about franchising. Lafontaine and Slade (2001), who summarize and carefully discuss the evidence, find that there is positive relation between the decision to franchise and risk; they conclude that "these results suggest a robust pattern that is unsupportive of the standard agency model" (p. 10). Lafontaine (1992) studies how uncertainty affects both (i) the decision to franchise (high pay for performance) versus store retention by companies (with little or no pay for performance) and (ii) the royalty rate offered to franchisees. She examines 548 franchisors in 14 different sectors of which 117 franchise all their retail outlets. She reports that the decision to franchise is positively related and significant to uncertainty, measured as the likelihood of bankruptcy (table 5), whereas the royalty rate is negatively related and statistically insignificant. This suggests that risk plays a different role between the decision to use pay for performance and its intensity. In fact, the role is consistent with the pure sorting model proposed here, but not with the pure incentive model, just as explained by Prendergast's (2002b) monitoring input versus output story. The evidence available, however, does not allow definitive judgment between either.

Another prediction of the model is that occupations in which only pay-for-performance workers are observed, pay-for-performance sensitivity tend to be small. This result can also be explained by the linear agency model when the variance of output is sufficiently large, but it says little about when that may occur. In contrast, the pure sorting model not only predicts that pay-for-performance sensitivity decreases with the uncertainty but also that only pay-for-performance workers will be observed in high uncertain environments.

16. In ongoing renegotiable relationships past output provides information on workers' productivity and thus firms improve their information about a worker's productivity over time. In particular, this implies that once a worker's type is revealed, the model predicts that there is no need to use pay-for-performance contracts unless there is also a moral hazard problem.

17. I thank one of the referees for pointing this out to me.

Some support for these prediction is to be found in Murphy's (1999) review of the CEO literature, concluding the estimated pay-for-performance sensitivity for CEO's is rather small, between 0.001 and 0.007.[18] Evidence for occupations like salespersons and auto-workers, where both method of pay coexist, show a much greater pay-for-performance sensitivity. In addition, Lafontaine and Slade (2001) report evidence from several studies in favor of this prediction and the executive compensation studies, when controlling for firm size, also find a negative relationship.

The prediction that workers under a straight salary have both a lower average productivity and compensation compared to pay-for-performance workers is fully borne-out in the data. For example, on average, pay-for-performance workers earn more and have a higher average productivity than straight-salary workers. The compensation and productivity differences ranged roughly between 5% and 37%.[19]

Finally, there is evidence that directly highlights the importance of sorting. In particular, Foster and Rosenzweig (1996), using agricultural labor market data find that when workers can work in either a piece-rate or a straight-salary work, the more productive workers work in the piece-rate sector while the less productive in the straight-salary sector.

7. Conclusions

Pay-for-performance contracts make up a significant portion of all compensation contracts. The standard rationale is incentives. Although incentives for effort are important, incentive theory does not mesh well with a number of empirical facts. In this article, an alternative rationale for linking pay to performance has been advanced that is broadly consistent with the empirical evidence. In particular, this theory help explains why variable pay is more prevalent in more uncertain environments. This has strong implications for empirical work. If one could credible isolate workers by productivity class and adequately measure the power of incentives built in their contracts, then one would expect to find the traditional trade-off between uncertainty and incentives. If, on the other hand, the strength of incentives is measured as the prevalence of incentive contracts, then one could expect the relationship to change sign.

In short, the pure sorting model appears more consistent with the data than the pure incentive model and therefore more attention should be paid to the effects of asymmetric information on pay-for-performance contracts. Yet, it would be a mistake to discard the incentive model. Lazear (2000), using detailed data from an autoworkers company, found that the switch from a fixed-wage system to a piece-rate one resulted in an increase in productivity of about 44%, half being attributed to incentives and the other half to sorting.[20] This

18. Similar evidence is found in Kaplan (1994), Gibbons and Murphy (1990), and Murphy (1985, 1986).

19. See, for example, Brown 1992, Lazear 2000, Petersen 1991 and 1992, Seiler 1984.

20. See, also Shearer (2004) and Paarsh and Shearer (2000) for similar evidence.

suggests that the evidence could be more satisfactorily explained by developing a model that deals with key features of sorting, incentives, and monitoring.

Appendix

The Equilibrium Concept: Notation and Definitions

Before defining an undefeated equilibria, several definitions are in order.

1. $\hat{\mu}(i|C_k)$ denotes a firms' belief about the probability that a worker applying to contract C_k is of type $i \in I \equiv \{H, L\}$.
2. $V_i(\Upsilon)$ denotes, with some abuse of notation, the expected payoff that an i-worker gets in the pure strategies PBE Υ.[21]
3. $\sigma_i(C_k) = 1$ when the i-worker applies to the C_k contract and 0 otherwise, and $\rho_j(C_k) = 1$ when firm j accepts applications to the C_k contract and 0 otherwise.
4. G denotes the signaling subgame starting in the second stage and PSE(G) the set of pure strategies PBEs for the signaling subgame G.

Definition 1. The pure strategies PBE for the signaling subgame $\wedge$ defeats the pure strategies PBE for the signaling subgame $\wedge'$ if there is a contract C_k such that:

C1: $\forall i \in I$: $\sigma_i'(C_k) = 0$ and $K \equiv \{i \in I{:}\sigma_i(C_k) = 1\} \neq \phi$,
C2: $\forall \in i \in K$: $V_i(\Lambda) \geq V_i(\Lambda')$ and $\exists i \in K{:}V_i(\Lambda) > V_i(\Lambda')$, and
C3: $\exists i \in K : \hat{\mu}'(i|C_k) \neq \frac{\mu(i)\beta(i)}{\sum_{h\in I}\mu(h)\beta(h)} \equiv \mu(i,\ \beta(i))$ for any $\beta : I \to [0, 1]$ satisfying,

(i) $h \in K$ and $V_h(\Lambda) > V_h(\Lambda')$, $\beta(h) = 1$ and
(ii) $h \notin K$, $\beta(h) = 0$.

Definition 2. A PBE Λ is undefeated if there is no other PBE Λ' that defeats Λ.

Definition 3. The three-stage screening game has an equilibrium if the set of contracts offer give rises to an undefeated PBE of the signaling subgame; that is, stages 2 and 3, with respect to all possible PBEs that may arise from any feasible set of contracts that firms may offer in Stage 1.

Proof of lemma 1.

Proof. Let $\tilde{y}$ the output level at which $w_1(y)$ single-crosses $w_2(y)$ from below.

21. The same result holds when mix strategies are allowed. For the sake of simplicity, we focus on pure strategies. For a more formal justification of why focus only in pure strategies, see Mailath (1992).

The hypothesis $V_L(C_1) = V_L(C_2)$ implies

$$\begin{aligned}
0 &= \int [U(w_1(y)) - U(w_2(y))] f^L(y|r)\mathrm{d}y \\
&= \int [U(w_1(y)) - U(w_2(y))]\, \ell(y|r) f^H(y|r)\mathrm{d}y \\
&= \int_{\underline{y}}^{\tilde{y}} [U(w_1(y)) - U(w_2(y))]\, \ell(y|r) f^H(y|r)\mathrm{d}y \\
&\quad + \int_{\tilde{y}}^{\bar{y}} [U(w_1(y)) - U(w_2(y))]\, \ell(y|r) f^H(y|r)\mathrm{d}y \\
&< \ell(\tilde{y}|r) \left\{ \int_{\bar{y}}^{\tilde{y}} [U(w_1(y)) - U(w_2(y))] f^H(y|r)\mathrm{d}y \right. \\
&\quad \left. + \int_{\tilde{y}}^{\bar{y}} [U(w_1(y)) - U(w_2(y))] f^H(y|r)\mathrm{d}y \right\} \\
&= \ell(\tilde{y}|r) \int [U(w_1(y)) - U(w_2(y))] f^H(y|r)\mathrm{d}y,
\end{aligned}$$

where the inequality follows from the fact that $\ell(y \mid r)$ decreases monotonically with y and $w_1(y)$ single-crosses $w_2(y)$ from below.

It readily follows from this that

$$\int_Y U(w_1(y)) f^H(y|r)\mathrm{d}y > \int_Y U(w_2(y)) f^H(y|r)\mathrm{d}y.$$

Proof of lemma 2.

Proof. First note that a sufficient condition for equation (5) to hold when the low-ability worker's incentive constraint is binding is

$$\int \left[U(w^L(y)) - U(w^H(y))\right] f^H(y|r)\mathrm{d}y \leq 0.$$

To show that this holds, I will make use of The Diffidence theorem due to Gollier and Kimball (1996). Since, I will make use of this theorem several times in the rest of the article, I will state it formally here.

The diffidence theorem states that the necessary and sufficient conditions for

$$Eg_1(y) \leq g_1(\tilde{y}) \Rightarrow Eg_2(y) \leq g_2(\tilde{y}), \tag{1}$$

where $\tilde{y}$ is a $y \in Y$ such that $Eg_1(\tilde{y}) = g_1(\tilde{y})$ and $Eg_2(\tilde{y}) = g_2(\tilde{y})$, to hold are the following:

$$NSC : g_2(y) \leq \frac{g_2'(\tilde{y})}{g_1'(\tilde{y})} g_1(y) \text{ for all } y \in Y.$$

$$NC1 : \frac{g_2'(\tilde{y})}{g_1'(\tilde{y})} > 0.$$

$$NC2 : g_2'(\tilde{y}) \leq \frac{g_2''(\tilde{y})}{g_1''(\tilde{y})} g_1'(\tilde{y}).$$

The $NC1$ condition is required only when the condition in (A1) is with an inequality.

Let define $g_1(y) = [U(w^L(y)) - U(w^H(y))]\ell(y|r)$ and $g_2(y) = U(w^L(y)) - U(w^H(y))$. Let y_0 a point at which $w^L(y0) = w^H(y0)$.[22]

Substituting in the corresponding values in the NSC condition, the following is obtained

$$U(w^L(y)) - U(w^H(y)) \leq \frac{\ell(y|r)}{\ell(y_0|r)} \left[U(w^L(y)) - U(w^H(y))\right] \text{for all } y \in Y.$$

Because $(y \mid r) > 0$ for all $y \in Y$, the $NC1$ condition is satisfied. The NSC condition can be written as follows:

$$\ell(y|r) \geq \ell(y0|r) \quad \text{if } w^H(y) \leq w^L(y),$$

$$\ell(y|r) \leq \ell(y0|r) \quad \text{if } w^H(y) > w^L(y).$$

Observe that if $w^H(y)$ crosses $w^L(y)$ only once from below at $y0$, then this condition holds if the likelihood ratio is monotonically decreasing in y for all $y \in Y$.

The $NC2$ is given by

$$\frac{\ell'(y0|r)}{\ell(y0|r)} \left(\frac{\partial U(w^L(y0))}{\partial y} - \frac{\partial U(w^H(y0))}{\partial y} \right) \geq 0.$$

Observe that if $w^H(y)$ crosses $w^L(y)$ from below at $y0$, then this condition holds if the likelihood ratio is monotonically decreasing in y at $y0$, which is the case because of MLRP.

Proof of lemma 3.

Proof. Let us first to prove uniqueness. Assume that there exists λ and ρ satisfying the first-order condition. Assume that there is another contract $C' = \{w'(y)\}$ that satisfies both constraints, differs from C_H^s on a set of positive measure, and yields a greater expected utility to the high-productive worker. That is, $V_H(C_H^s) > V_H(C')$.

22. It is trivial to show that such a point exists.

Let $w^{\varepsilon}(y)$ be implicitly defined by

$$U(w^{\varepsilon}(y)) = (1-\varepsilon)U(w_H^s(y)) + \varepsilon U(w'(y)).$$

So, $w^{\varepsilon}(y)$ is the certainty equivalent of a $(1-\varepsilon,\varepsilon)$ lottery between $w_H^s(y)$ and $w'(y)$. Note that

$$\frac{\partial w^{\varepsilon}(y)}{\partial\varepsilon} = \frac{1}{U'(w^{\varepsilon}(y))}\left[U(w'(y)) - U(w_H^s(y))\right].$$

Since the agent is risk averse, it follows that $w^{\varepsilon}(y)$ is convex in ε and strictly so if C' differs from C_H^s on a set of positive measure. This implies that $C_H(w^{\varepsilon}(y)) \equiv \int_Y w^{\varepsilon}(y) f^H(y|r)\mathrm{d}y$ is strictly convex in ε. Thus, since $C_H(w'(y)) = C_H(w_H^s(y)) = \theta_H$,

$$\frac{\partial C_H(w^{\varepsilon}(y))}{\partial\varepsilon}\Big|_{\varepsilon=0} = 0,$$

Observe that

$$\begin{aligned}
\frac{\partial C_H(w^{\varepsilon}(y))}{\partial\varepsilon}\Big|_{\varepsilon=0} &= \int \frac{\partial w^{\varepsilon}(y)}{\partial\varepsilon}\Big|_{\varepsilon=0} f^H(y|r)\mathrm{d}y \\
&= \int \frac{1}{U'(w_H^s(y))}\left[U(w'(y)) - U(w_H^s(y))\right] f^H(y|r)\mathrm{d}y \\
&= \int \frac{1-\lambda\ell(y|r)}{\rho}\left[U(w'(y)) - U(w_H^s(y))\right] f^H(y|r)\mathrm{d}y \\
&= \frac{1}{\rho}\int\left[U(w'(y)) - U(w_H^s(y))\right] f^H(y|r)\mathrm{d}y \\
&\quad - \frac{\lambda}{\rho}\int\left[U(w'(y)) - U(w_H^s(y))\right] f^L(y|r)\mathrm{d}y > 0. \qquad (2)
\end{aligned}$$

Note that the second term in the last line of (2) is zero since both contracts C' and C_H^s satisfy low-productivity worker's incentive compatibility constraint, whereas the first term in the last line is positive by hypothesis. This implies that any other contract that provide more expected utility than C_H^s cost more than θ_H which contradicts $C_H(w'(y)) = C_H(w_H^s(y)) = \theta_H$.

The existence of λ and ρ is guaranteed by continuity assumptions, the intermediate value theorem and the fact that $\lim_{w\to\infty} U(w) \to \infty$ and $\lim_{w\to-\infty} U(w) \to K \ll 0$. In fact, one can easily show that

$$\lambda = \frac{Cov_H(U',U)}{Cov_L(U',U) - (E_H U - U(\theta_L))E_L U'} < 1 \text{ and } \rho = E_H U' - \lambda E_L U' > 0$$

Furthermore, by the implicit function theorem, λ and ρ are continuously differentiable in r, and hence $V_H(C_H^s)$ is as well.

From (7) we have that

$$g_1(\rho, \lambda, r) = \int U\left[(U')^{-1}\left(\frac{\rho}{1-\lambda\ell(y|r)}\right)\right] f^L(y|r)\mathrm{d}y - U(\theta_L) = 0 \text{ and}$$
$$g_2(\rho, \lambda, r) = \int (U')^{-1}\left(\frac{\rho}{1-\lambda\ell(y|r)}\right) f^H(y|r)\mathrm{d}y - \theta_H = 0.$$

Differentiation yields

$$\frac{\partial g_1}{\partial \lambda} = -\int \frac{(U'(w_H^s(y)))^3}{U''(w_H^s(y))} \frac{\ell^2(y|r)}{\rho} f^H(y|r)\mathrm{d}y$$
$$\frac{\partial g_2}{\partial \lambda} = -\int \frac{(U'(w_H^s(y)))^2}{U''(w_H^s(y))} \frac{\ell(y|r)}{\rho} f^H(y|r)\mathrm{d}y$$
$$\frac{\partial g_1}{\partial \rho} = \int \frac{(U'(w_H^s(y)))^2}{U''(w_H^s(y))} \frac{\ell(y|r)}{\rho} f^H(y|r)\mathrm{d}y$$
$$\frac{\partial g_2}{\partial \rho} = \int \frac{U'(w_H^s(y))}{U''(w_H^s(y))} \frac{1}{\rho} f^H(y|r)\mathrm{d}y.$$

Consider the determinant

$$\Delta \equiv \begin{vmatrix} \frac{\partial g_1}{\partial \lambda} & \frac{\partial g_1}{\partial \rho} \\ \frac{\partial g_2}{\partial \lambda} & \frac{\partial g_2}{\partial \rho} \end{vmatrix}$$

Let $K_1 = \sqrt{-\frac{\left(U'(w_H^s(y))\right)^3}{U''(w_H^s(y))} \frac{1}{\rho} \ell(y|r)}$ and $K_2 = \sqrt{-\frac{U'\left(w_H^s(y)\right)}{U''(w_H^s(y))} \frac{1}{\rho}}$.

Then

$$\Delta = EK_1^2 EK_2^2 - (EK_1K_2)^2 > 0$$

by the Cauchy-Schwarz inequality. By MLRP K_1 and K_2 are not linearly related and thereby the implicit function theorem can be used. Since this holds for all r, the conclusion above holds.

Differentiating the first-order condition one gets the following

$$\frac{\partial w_H^s}{\partial y} = \frac{\lambda U' \ell'}{U''(1-\lambda\ell)} > 0,$$

since $U'' < 0$ and $\ell' < 0$.

In addition, differentiating again with respect to y one gets

$$\frac{\partial^2 w_H^s}{\partial y^2} = \frac{\partial w_H^s}{\partial y}\left[\frac{\partial w_H^s}{\partial y}\left(2\frac{U''}{U'} - \frac{U'''}{U''}\right) + \frac{\ell''}{\ell'}\right].$$

Finally, it is easy to check that the high-productivity worker's incentive compatibility constraint is satisfied. This follows from noticing that

$$V_H(C_L^*) = V_L(C_L^*) = \int U(w_H^s(y)) f^L(y|r)\mathrm{d}y < \int U(w_H^s(y)) f^H(y|r)\mathrm{d}y,$$

where the inequality is due to FSD and the fact that $U(w_H^s(y))$ increases with y.

Proof of lemma 4.

Proof. The existence of γ is guaranteed by continuity assumptions, the intermediate value theorem and the assumptions on U, and the uniqueness can be proven in the same way as before.[23] In fact, one can easily show that

$$\gamma = E_H U',$$

since $\int \ell(y|r) f^H(y|r)\mathrm{d}y = 1$.

Furthermore, by the implicit function theorem, γ is continuously differentiable in r and μ, and hence $V_i(C^P)$ as well.

Let us define the function $\phi(\cdot) = (U')^{-1}(\gamma[\mu + (1-\mu)\ell(y|r)])$ and note that $\phi'(\cdot) = \frac{1}{U''(\cdot)}$.

From Equation (8) we have that

$$g(\gamma, \mu, r) \equiv \int \phi(y)(\mu f^H(y|r) + (1-\mu) f^L(y|r))\mathrm{d}y - \hat{\theta} = 0.$$

Differentiation yields

$$\frac{\partial g}{\partial \gamma} = \int_Y \phi'(y)(\mu f^H(y|r) + (1-\mu) f^L(y|r))\mathrm{d}y < 0.$$

Thus, $\gamma(\mu)$ is implicitly defined by $g(\gamma(\mu), \mu) = 0$.

Applying the implicit function theorem one gets that

$$\frac{\partial \gamma(\mu)}{\partial \mu} = -\frac{\partial g \partial g}{\partial \mu \partial \gamma} < 0$$

since

$$\frac{\partial g}{\partial \mu} = \int_Y (\phi'(\cdot)U'(\phi'(\cdot)) + \phi'(\cdot) - y)(f^H(y|r) - f^L(y|r))\mathrm{d}y < 0.$$

The term in parenthesis is decreasing in y because $U'''(w^p(y)) \geq 0$, and therefore the inequality follows from the fact that $F^H(y|r)$ FSD $F^L(y|r)$.

Differentiating the first-order condition one gets the following

$$\frac{\partial w^p}{\partial y} = \frac{\gamma(1-\mu)\ell'}{U''} > 0,$$

since $U'' < 0$ and $\ell' < 0$.

Proof of proposition 1.

Proof.

Lemma 6. Contracts C_L^* and C_H^s are a PBE of the signaling subgame.

23. For the sake of brevity, the details are omitted but they can be filled in by following the proof lemma (3).

Proof. Consider the following strategies and beliefs.

Stage 1: Each firm offers the following menu (C_L^*, C_H^s, C').

Stage 2: $\sigma_H(C_H^s) = 1$ and $\sigma_L(C_L^*) = 1$, that is, low-productivity workers apply to C_L^*, whereas high-productivity workers apply to C_H^s.

Stage 3: All the applicants to contract C_H^s and C_L^* are accepted.

On-the-equilibrium-path beliefs: $\hat{\mu}(H|C_H^s) = 1$ and $\hat{\mu}(L|C_L^s) = 1$.
Off-the-equilibrium-path beliefs: $\hat{\mu}(H|C') = 0$, $\forall C' \neq C_H^s$.
It is easy to check that these strategies satisfy the PBE requirements.

Claim 1. In any PBE, denoted by Υ, low-productivity workers' equilibrium payoff is at least as large as the payoff that they would obtain under perfect information, that is, $V_L(\Upsilon) \geq V_L(C_L^*)$.

Proof. We will prove this by contradiction. Suppose not, then there exists a PBE, Υ', such that $V_L\Upsilon') < V_L(C_L^*)$. Then by continuity of preferences and risk aversion, there is a straight-salary contract, C', that provides low-productivity workers with at least the same expected payoff than Υ' does, $V_L(C') \geq V_L\Upsilon')$, and yields positive expected profits when is chosen by either low- or high-productivity workers or both. This implies that C' yields positive expected profits for any beliefs that firms could hold, and therefore, all firms offering C' accept all the applicants to C'. Then a firm offering a menu that contains C', make positive profits for any $\hat{\mu} \in [0, 1]$, and attract all low-productivity workers and may be some high-productivity workers. Therefore, no matter which beliefs the deviating firm holds it has a profitable deviation contradicting that Υ' is PBE. $\square$

Claim 2. In any fully separating PBE, denoted by Υ, low-productivity workers' equilibrium payoff, is $V_L(C_L^*)$, the payoff that they would obtain in the perfect information case.

Proof. Let the contract chosen only by low-productivity workers be C_L^s. By claim 7, the equilibrium payoff of this contract must be so that $V_L(C_L^s) \geq V_L(C_L^*)$. Observe that any contract $C_L \neq C_L^s$ that satisfies $V_L(C_L) \geq V_L(C_L^s)$ yields negative expected profits when chosen by low-productivity workers only since either pays a straight salary greater than θ_L or offers risk and thus in order to yield at least $V_L(C_L^s)$ must promise an expected salary greater than θ_L. Thus, in any separating equilibrium, applicants to contract C_L are rejected and thus it must be the case that $V_L(C_L^s) \leq V_L(C_L^*)$, contradicting claim 7. Therefore, the only possible contract that is chosen only by low-productivity workers and applications are accepted in equilibrium is C_L^*. This proves that $V(C_L^s) = V_L(C_L^*)$.

Claim 3. In any fully separating UPBE, denoted by Λ, high-productivity workers' equilibrium payoff is at least as large as $V_H(C_H^s)$.

Proof. Let $\{C_1', C_L^*, C_1\}$ be the set of contract offered in the fully separating equilibrium Λ' and Λ. Furthermore, suppose that $V_H(C_1) \geq V_H(C_H^s) > V_H(C_1')$, where C_1' is the contract chosen by high-productivity workers in Λ' and C_1 is the contract chosen by high-productivity workers in Λ.

Because Λ' is a separating equilibrium, it must be the case that $V_H(C_1') > \max\{V_H(C_L^*), V_H(C_1)\}$, $V_L(C_L^*) \geq \max\{V_L(C_1'), V_L(C_1)\}$. Thus, $\sigma_H'(C_1') = 1$, $\sigma_L'(C_1') = 1$, and firms' beliefs in this equilibrium must be: $\hat{\mu}'(L|C_L^*) = 1$ and $\hat{\mu}'(H|C_1') = 1$. Because, $V_H(C_1) > V_H(C_H^s)$, Λ' is an equilibrium only when applications to C_1 are not accepted and $V_L(C_L^*) \geq V_L(C_1)$. The former requires that $\pi(\hat{\mu}'(H|C_1), C_1) < 0$, which implies that firms' beliefs must satisfy the following: $\hat{\mu}'(H|C_1) \in (0, \hat{\mu}'^*(H|C_1))$, where $\hat{\mu}'^*(H|C_1)$ solves $\pi(\hat{\mu}'(H|C_1), C_1) = 0$. Notice that $\hat{\mu}'^*(H|C_1) < \mu$, otherwise the firms offering C_1 will accept all the applicants to C_1 and at least break even since all high-productivity workers and may be some low-productivity workers will apply to C_1. Note also that $V_H(C_1) \geq V_H(C_H^s) > V_H(C_1')$ implies that $V_L(C_1) \geq V_L(C_1')$.

Because the equilibrium Λ is a separating equilibrium, it must be the case that $V_H(C_1) > V_H(C_L^*)$ and $V_L(C_L^*) \geq V_L(C_1)$. Hence, $\sigma_H(C_1) = 1$ and $\sigma_L(C_L^*) = 1$, and firms' beliefs in this equilibrium must be: $\hat{\mu}(L|C_L^*) = 1$ and $\hat{\mu}(H|C_1) = 1$.

Since in Λ', $\sigma'_i(C_1) = 0$, $\forall i \in I$, and in Λ, $\sigma_H(C_1) = 1$, $K = \{H\}$, which satisfies condition C_1 of the refinement. Condition C_2 is satisfied because $V_L(\Lambda) = V_L(\Lambda')$ and $V_H(\Lambda) > V_H(\Lambda')$. Because, $K = \{H\}$ condition C_3 imposes that $\beta(H) = 1$, $\beta(L) = 0$, therefore $\mu(H, \beta(H)) = 1$ which is different from $\hat{\mu}'(H|C_1)$. Therefore, Λ defeats any separating PBE, Λ', in which $V_H(\Lambda') < V_H(C_H{}^s)$ and thus in any fully separating UPBE high-productivity workers get at least a payoff equal to $V(C_H^s)$.

Claim 4. In any fully separating UPBE, denoted by Λ, high-productivity workers' equilibrium payoff is equal to the expected payoff from contract C_H^s, $V_H(C_H^s)$.

Proof. Suppose there exist a fully separating PBE, denoted by Λ', so that $V_H(\Lambda') > V_H(C_H^s)$. By claims 3 and 2, in any fully separating equilibrium, $\forall C_k$ such that $\sigma_L(C_k) > 0$, $V_L(C_k) = V_L(C_L^*)$ and $\forall C_k$ such that $\sigma_H(C_k) > 0$, $V_H(C_k) \geq V_H(C_H^s)$ and $\pi_H(C_k) \geq 0$. If a contract C_k such that $\sigma_H(C_k) > 0$, $V_H(C_k) > V_H(C_H^s)$ and $\pi_H(C_k) \geq 0$ exists, then C_H^s cannot be the contract that maximizes high-productivity workers' expected utility when only high-productivity workers apply to this contract. This plus the fact that in any UPBE $V_H(\Lambda) \geq V_H(C_H^s)$ implies that there is no fully separating UPBE where $V_H(\Lambda') \neq V_H(C_H^s)$.

These four claims prove the lemma.

Claim 5. The contract C^p is a PBE of the signaling subgame.

Proof. Let us define C^p as the solution to problem 7. To prove that there is a PBE of the SSG that sustain C^p as a PBE, notice first that by definition, C^p maximizes high-productivity workers' expected utility and breaks even only at the population average probability of success, therefore $\pi_L(C^p) < 0$. To prove that C^p can be supported as PBE consider the following strategies:

Stage 1: Each firm offers the menu $\{C^p, C'\}$.

Stage 2: $\sigma_H(C^p) = 1$ and $\sigma_L(C^p) = 1$, that is, high- and low-productivity workers apply to contract C^p.

Stage 3: Firms accept all applicants to C^p and firms accept all applicants to C', $\forall C'$ such that $\pi_L(C') \geq 0$.
On-the-equilibrium-path beliefs: $\hat{\mu}(H|C^p) = \mu$.

Off-the-equilibrium-path beliefs: $\hat{\mu}(H|C') = 0$, $\forall C' \neq C^p$ and $\pi_L(C') \geq 0$.
It is easy to check that these strategies satisfy the PBE requirements.

Claim 6. In an undefeated pooling equilibrium, high-productivity workers payoff is $V_H(C^P)$ and low-productivity workers payoff is $V_L(C^p)$.

Proof. Suppose there exists a pooling PBE denoted by $\Lambda^{p\prime}$ that defeats Λ^p. Recall that by definition, C^p is, among all possible pooling contracts, the one that yields the highest payoff to high-productivity workers.

Let $\{C^p, C^{p\prime}\}$ be the contracts offered in the PBE $\Lambda^{p\prime}$ and $\sigma_i'(C^{p\prime}) = 1$ $\forall i \in I$, and let $\{C^p, C^{p\prime}\}$ be the contracts offered in the PBE Λ^p, where $\sigma_i(C^p) = 1$ $\forall i \in I$. In this case, $K = I$, and $C2$ fails since any pooling PBE $\Lambda^{p\prime}$ different from Λ^p, it is true that $V_H(\Lambda^{p\prime}) < V_H(\Lambda^p)$.

So far it has been shown that there is no pooling equilibrium that defeats the one in which all types choose C^p and that there is no fully separating equilibrium that defeats the one in which low-productivity workers choose C_L^s and high-productivity workers choose C_H^s. It rests to show that when $\mu \leq \mu(r)$, the fully separating equilibrium in which low-productivity workers choose C_L^s and high-productivity workers choose C_H^s is undefeated by the pooling equilibrium in which all types choose C^p and the opposite occurs when $\mu > \mu(r)$.

Let Λ^s to denote the fully separating PBE of G that sustains the contracts C_H^s and C_L^s as an UPBE and Λ^p to denote the pooling PBE of G that sustain the contract C^p as an UPBE.

Suppose that in both equilibria, Λ^s and Λ^p, the following contracts are offered $\{C_H^s, C_L^*, C^p\}$.

By definition of $\mu(r)$, when $\mu \leq \mu(r)$, the highest payoff that high-productivity workers get in Λ^p is such that $V_H(C^p) < V_H(C_H^s)$.

Suppose that Λ^p defeats Λ^s and $\mu \le \mu(r)$. Notice that $V_L(C^p) > V_L(C_L^*)$, and thus $\sigma_L^s(C^p) = \sigma_H^s(C^p) = 0$ and $\sigma_i^p(C^p) = 1$, $\forall i \in I$. Thus, $K = I$. Because $V_L(C^p) > V_L(C_L^*)$ and $V_H(C^p) < V_H(C_H^s)$, condition C_2 of undefeated equilibrium is immediately violated. Therefore, when $\mu \le \mu(r)$, there is no PBE that defeats Λ^s.

Suppose next that $\mu > \mu(r)$.Because $K = I$ and now $V_L(C^p) > V_L(C_L^*)$ and $V_H(C^p) > V_H(C_H^s)$, condition C_2 is satisfied. Now let us check if condition C_3 holds or not. Because $K = I$ and $V_L(C^p) > V_L(C_L^*)$ and $V_H(C^p) > V_H(C_H^s)$, $\beta(H) = \beta(L) = 1$. Recall that $\hat{\mu}^s(H|C^p) = 0$ and $\hat{\mu}^s(L|C^p) = 1$ and thus $\hat{\mu}^s(H|C^p) \ne \frac{\mu(H)\beta(H)}{\mu(H)\beta(H)+\mu(L)\beta(L)} = \mu$. This implies that condition C_3 is satisfied and thus Λ^p defeats Λ^s.

Lastly, I need to show that $\mu(r)$ exists.

Observe first that $V_H(C_H^s)$ is independent of μ and that

$$\begin{aligned}\frac{\partial V_H(C^p(\mu))}{\partial \mu} &= \gamma\left(\int_Y (w^P(y) - y)[f^H(y|r) - f^L(y|r)]\mathrm{d}y\right)\\ &= \gamma\left(\int_Y w^P(y) f^L(y|r)\mathrm{d}y - \theta_L\right) > 0\end{aligned}$$

where the first equality follows from the envelope theorem and the second from the fact that the optimal contract $C^p(\mu)$ satisfies the wage constraint with equality. The inequality follows from the fact that in a pooling equilibrium a low-ability worker must be paid more than his expected output, otherwise he prefers contract C_L^* since this pays θ_L for sure. Thus, the high-ability worker's expected utility rises as μ increases.

Let us define $\mu(r)$ as the minimum μ such that $V_H(C^p(\mu)) = V_H(C_H^s)$. The existence of this threshold is guaranteed by the fact that $V_H(C^p(\mu))$ is continuously increasing in μ and that at $\mu = 1$, $V_H(C^p(\mu)) = V_H(C_H^*) > V_H(C_H^s)$.

However, we need to check that for any μ greater than $\mu(r)$ low-ability workers prefer contract $C^p(\mu)$ than C_L^*.

The hypothesis $V_H(C_H^s) \ge V_H(C^p(\mu))$ implies

$$\begin{aligned}0 &\le \int \left[U(w^P) - U(w_H^s)\right] f^H(y|r)\mathrm{d}y\\ &= \int \left[U(w^P) - U(w_H^s)\right][1/\ell(y|r)] f^L(y|r)\mathrm{d}y\\ &= \int_{\underline{y}}^{\tilde{y}} \left[U(w^P) - U(w_H^s)\right]\frac{1}{\ell(\tilde{y}|r)} f^L(y|r)\mathrm{d}y + \int_{\tilde{y}}^{\bar{y}} \left[U(w^P) - U(w_H^s)\right]\frac{1}{\ell(\tilde{y}|r)} f^L(y|r)\mathrm{d}y\\ &< \frac{1}{\ell(\tilde{y}|r)}\left\{\int_{\tilde{y}}^{\tilde{y}} \left[U(w^P) - U(w_H^s)\right] f^L(y|r)\mathrm{d}y + \int_{\tilde{y}}^{\bar{y}} \left[U(w^P) - U(w_H^s)\right] f^L(y|r)\mathrm{d}y\right\}\\ &= \frac{1}{\ell(\tilde{y}|r)}\int \left[U(w^P) - U(w_H^s)\right] f^L(y|r)\mathrm{d}y,\end{aligned}$$

where the inequality follows from the fact that $1/\ell(y \mid r)$ increases monotonically with y and $w_H^s(y)$ single-crosses $w^P(y)$ from below.

It readily follows from this that $\int U(w^P(y))f^L(y|r)\mathrm{d}y > \int U(w_H^s(y))f^L(y|r)$ $\mathrm{d}y = V_L(C_L^*)$ and thereby whenever contract $C^p(\mu)$ is preferred to contract C_H^s by the high-ability type, $C^p(\mu)$ is also preferred to contract C_L^* by the low-ability type.

Finally, continuity and the implicit function theorem implies that $\mu(r)$ is continuously differentiable in r.

Proof of proposition 2.

Proof. First observe that the incentive compatibility constraint is not satisfied at $\beta^s = 0$ since the left-hand side becomes equal to $U(\theta_H) - U(\theta_L) > 0$, whereas at $\beta^s = 1$, that becomes

$$\int U(y)f^L(y|r)\mathrm{d}y < U(\theta_L),$$

where the inequality follows from the strict concavity of $U(y)$. Thus, $\beta_H^s(r) \in (0, 1)$.

Using the implicit function theorem one can show that

$$\frac{\partial \beta_H^s(r)}{\partial r} = -\frac{\int U(\theta_H + \beta_H^s(r)(y - \theta_H))\triangle g \mathrm{d}y}{\int U'(\theta_H + \beta_H^s(r)(y - \theta_H))(y - \theta_H)f^L(y|r)\mathrm{d}y}.$$

Because $U(\cdot)$ is strictly concave and $G_1(\cdot)$ is a mean-preserving spread of $G_0(\cdot)$, the numerator is negative.

Next, it is necessary to show that the denominator is also negative.

$$\begin{aligned}
&\int U'(\cdot)(y - \theta_H)f^L(y|r)\mathrm{d}y \\
&= \int^{\theta_H} U'(\cdot)(y - \theta_H)f^L(y|r)\mathrm{d}y + \int_{\theta_H} U'(\cdot)(y - \theta_H)f^L(y|r)\mathrm{d}y \\
&\leq \int^{\theta_H} U'(\theta_H)(y - \theta_H)f^L(y|r)\mathrm{d}y + \int_{\theta_H} U'(\theta_H)(y - \theta_H)f^L(y|r)\mathrm{d}y \\
&= U'(\theta_H)\int(y - \theta_H)f^L(y|r)\mathrm{d}y < 0.
\end{aligned}$$

The first inequality follows from the concavity $U(\cdot)$ and the last from the fact that $\theta_H > \theta_L$.

Showing that $\beta^p(r)$ falls with r is more cumbersome.

Observe first that the left-hand side of first-order condition (2) when evaluated at $\beta = 0$ is given by

$$U'(\hat{\theta})\int(y - \hat{\theta})f^H(y|r)\mathrm{d}y > 0,$$

while when evaluated at $\beta = 1$ is given by

$$\int U'(y)(y-\hat{\theta})f^H(y|r)\mathrm{d}y$$

$$< U'(\hat{\theta})\int_Y (y-\hat{\theta})f^H(y|r)\mathrm{d}y$$

$$= \theta_H - \hat{\theta}.$$

Thus, we need to impose that the utility function is such that

$$\int U'(y)(y-\hat{\theta})f^H(y|r)\mathrm{d}y \leq 0.$$

Using the implicit function theorem one can show that $\beta^p(r)$ falls with r if and only if $\frac{\partial V^p(r)}{\partial\beta^p(r)\partial r} < 0$.

It follows from the first-order condition that

$$\frac{\partial V^p(r)}{\partial\beta^p(r)\partial r} = \int U'(\hat{\theta}+\beta^p(r)(y-\hat{\theta}))(y-\hat{\theta})\triangle g\mathrm{d}y.$$

Integrating by parts twice, one gets that

$$\frac{\partial V^p(r)}{\partial\beta^p(r)\partial r} = \beta^p(r)\int\left[U'''(\cdot)\beta^p(r)(y-\hat{\theta})+2U''(\cdot)\right]\Gamma(y)\mathrm{d}y$$

Noticing that the term in square brackets can be written as:

$$-U''(w)\left[P(w)\beta^p(r)(y-\hat{\theta})-2\right]$$

Thus, the concavity of $U'(w)(y-\hat{\theta})$ guarantees the negativity of $\frac{\partial V^p(r)}{\partial\beta^p(r)\partial r} < 0$. The proof that this is also a necessary condition is omitted since it is presented in Hadar and Seo (1990).

To end observe that the concavity of $U'(w)(y-\hat{\theta})$ is guaranteed if $U'(w)w$ is concave and $U'(\cdot)\geq 0$. The concavity of $U'(x)x$ implies that $U'''(x)x + 2U''(x) < 0$, whereas the concavity of $U'(w)(y-\hat{\theta})$ implies that $U'''(\cdot)\beta^p(r)(y-\hat{\theta})+2U''(\cdot)<0$. Thus,

$$\begin{aligned}\left[U'''(\cdot)\beta^p(r)(y-\hat{\theta})+2U''(\cdot)\right] &= \beta^p(r)[U'''(w)w+2U''(w)] \\ &\quad - U'''(\cdot)\beta^p(r)\hat{\theta}\end{aligned}.$$

Thus, if $P_R(w)\leq 2$, then $U'(w)w$ is strictly concave. Furthermore, a sufficient condition for $\frac{\partial V^p(r)}{\partial\beta^p(r)\partial r} < 0$ is that $P_R(w)\leq 2+\hat{\theta}P(w)$

This completes the proof.

Proof of proposition 3.

Proof. Again, using the implicit function theorem showing that $\beta^p(r)$ falls with μ entails to show that $\frac{\partial V^p(r)}{\partial\beta^p(r)\partial\mu} < 0$.

It follows from the first-order condition that

$$\frac{\partial V^p(r)}{\partial \beta^p(r)\partial \mu} = (\theta_H - \theta_L)\int_Y \Big[U''(\cdot)(1-\beta^p(r))(y-\hat{\theta}) - U'(\cdot)\Big] f^H(y|r)\mathrm{d}y.$$

Notice that the term in square brackets can be written as:

$U'(w)[A_R(w) - 1 - yA(w)]$,

Thus, a sufficient condition for $\frac{\partial V^p(r)}{\partial \beta^p(r)\partial \mu} < 0$ to hold is that $A_R(w) \leq 1 + yA(w)$ for all $y \in Y$.

Proof of lemma 5.

Proof. Totally differentiation of the i-ability worker's expected utility leads to

$$\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right)_i = -\frac{\int_Y U'(w^H)y\mathrm{d}F^i(y|r)}{\int_Y U'(w^H)\mathrm{d}F^i(y|r)}. \tag{A3}$$

It remains then to show that $\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right)_H < \left(\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right)_L$ for all r.

First observe that F^H FSD F^L plus the concavity of $U(\cdot)$ imply that

$$\int_Y U'(w^H)\mathrm{d}F^H < \int_Y U'(w^H)\mathrm{d}F^L.$$

It remains to show then that

$$\int_Y U'(w^H)y\mathrm{d}F^L < \int_Y U'(w^H)y\mathrm{d}F^H.$$

Using the diffidence theorem, this leads to find conditions under which the following holds:

$$\int_Y U'(w^H)(1-\ell(y|r))\mathrm{d}F^H \leq 0 \Rightarrow \int_Y U'(w^H)y(1-\ell(y|r))\mathrm{d}F^H \geq 0.$$

Let y_0 be the point at which $(y_0\,|\,r) = 1$. Then the *NSC* condition is given by

$(y - y_0)(1-\ell(y|r)) \geq 0$,

which holds true always since for all $y \geq y_0$, $\ell(y\,|\,r) \leq 1$ and for all $y < y_0$, $(y\,|\,r) > 1$.

The *NC*1 condition is satisfied and the *NC*2 condition entails the following,

$-2\ell'(y_0|r)U'(w^H(y_0)) \geq 0$,

which holds true since $\ell'(y_0 \mid r) < 0$.

Proof of proposition 4.

Proof. The first step is to derive the threshold $\mu(r)$. The following conditions must hold at $\mu = \mu(r)$,

$$
\begin{aligned}
&(\mathrm{i})\alpha_L = \theta_L \text{ and } \beta_L = 0,\\
&(\mathrm{ii}) \int U(\alpha_H + \beta_H y) f^L(y|r)\mathrm{d}y = U(\theta_L),\\
&(\mathrm{iii}) \int U(\alpha_H + \beta_H y) f^H(y|r)\mathrm{d}y = V^p(r),
\end{aligned}
\tag{4}
$$

where condition (i) is the full insurance condition for low-ability types, the second is the incentive compatibility constraint for the low-ability type, and the third ensures that the high-ability type is indifferent between pooling and sorting.

The average expected profit is given by

$$\mu(\theta_H - \int(\alpha_H + \beta_H y)f^H(y|r)) + (1-\mu)(\theta_L - \int(\alpha_L + \beta_L y)f^L(y|r)).$$

I need to find the effect on the profit function of changing the contract offers subject to constrains (i), (ii), and (iii), since this provides the lowest average iso-profit that leaves a high-ability worker indifferent between separation and pooling. The procedure is the same as the one adopted by Stiglitz (1977).

Let us define $w^H \equiv \alpha_H + \beta_H y$.

Observe first that, by totally differentiation of (iii)

$$\left(\frac{\mathrm{d}\alpha_H}{\mathrm{d}\beta_H}\right)_H = -\frac{\int U'(w^H)y\mathrm{d}F^H(y|r)}{\int U'(w^H)\mathrm{d}F^H(y|r)}. \tag{5}$$

Note also that

$$\frac{\mathrm{d}\beta_L}{\mathrm{d}\alpha_L} = 0. \tag{6}$$

Next, by totally differentiation of (ii), using (A5) to substitute for $\left(\frac{\mathrm{d}\alpha_H}{\mathrm{d}\beta_H}\right)_i$ and (A6) to substitute for $\frac{\mathrm{d}\beta_L}{\mathrm{d}\alpha_L}$ one gets that

$$\frac{\mathrm{d}\beta_H}{\mathrm{d}\alpha_L} = \frac{U'(\theta_L)\int U'(w^H)\mathrm{d}F^H}{\int U'(w^H)\mathrm{d}F^H \int U'(w^H)y\mathrm{d}F^L - \int U'(w^H)y\mathrm{d}F^H \int U'(w^H)\mathrm{d}F^L}. \tag{A7}$$

Totally differentiation of the average expected profit function gives, using (6), (5), and then (7)

$$-\mu\left(\left(\frac{d\alpha_H}{d\beta_H}\right)_H + \theta_H\right)\frac{d\beta_H}{d\alpha_L} - (1-\mu) = 0.$$

Solving for μ one obtains

$$\mu(r) = \frac{1}{1+\Phi(r)}$$

where

$$\Phi(r) \equiv \frac{U'(\theta_L)\int U'(w^H)(y-\theta_H)dF^H}{\int U'(w^H)dF^H \int U'(w^H)y dF^L - \int U'(w^H)y dF^H \int U'(w^H)dF^L}.$$

Notice that $\mu(r) < 1$ requires $\Phi(r) > 0$. Because $\int U'(w^H)(y-\theta_H)dF^H \leq 0$, this implies that the term in the denominator must be negative, which is a direct consequence of lemma (5).

Because $\mu(r)$ falls as $\Phi(r)$ rises, it is necessary to find conditions under which $\Phi(r)$ increases as r rises when evaluated at the optimal contracts that achieve separation. That is,

$$\frac{d\Phi(r)}{dr} = \frac{\partial\Phi(r)}{\partial r} + \frac{\partial\Phi(r)\partial\beta_H}{\partial\beta \quad \partial r} \geq 0.$$

Evaluation of $\Phi(r)$ in the optimal contracts and then differentiation of $\Phi(r)$ with respect to r leads to:

$$\begin{aligned}\frac{d\Phi(r)}{dr} = {} & \left[\int U'(w)(y-\theta_H)d\triangle G + \frac{\partial\beta_H^s(r)}{\partial r}\int U''(w)(y-\theta_H)^2 dF^H\right] * \\ & \times \left(\int U'(w)dF^H \int U'(w)y dF^L - \int U'(w)y dF^H \int U'(w)dF^L\right) \\ & - \int U'(w)(y-\theta_H)dF^H \Bigg\{ \left(\int U'(w)y d\triangle G \int U'(w)dF^H\right. \\ & \left. + \int U'(w)d\triangle G \int U'(w)y dF^L\right) - \left(\int U'(w)y d\triangle G \int U'(w)dF^L\right. \\ & \left. + \int U'(w)d\triangle G \int U'(w)y dF^H\right) \\ & + \frac{\partial\beta_H^s(r)}{\partial r}\left[\left(\int U''(w)y(y-\theta_H)dF^L \int U'(w)dF^H + \int U'(w)y dF^L\right.\right. \\ & \left. \times \int U''(w)(y-\theta_H)dF^H\right) - \left(\int U''(w)y(y-\theta_H)dF^H \int U'(w)dF^L\right. \\ & \left.\left. + \int U'(w)y dF^H \int U''(w)(y-\theta_H)dF^L\right)\right]\Bigg\}.\end{aligned}$$

Let us define ΔF as $F^H - F^L$. Then, after a few steps of simple algebra $\frac{d\Phi(r)}{dr}$ can be written as follows:

$$
\begin{aligned}
&-\int U'(w)(y-\theta_H)\mathrm{d}\triangle F\bigg(\int U'(w)\mathrm{d}F^H\int U'(w)y\mathrm{d}\triangle G\\
&-\int U'(w)y\mathrm{d}F^H\int U'(w)\mathrm{d}\triangle G\bigg)\\
&+\frac{\partial\beta^s_H(r)}{\partial r}\bigg\{\int U'(w)(y-\theta_H)\mathrm{d}F^L\bigg(\int U''(w)y(y-\theta_H)\mathrm{d}F^H\int U'(w)\mathrm{d}F^H\\
&-\int U''(w)(y-\theta_H)\mathrm{d}F^H\int U'(w)y\mathrm{d}F^H\bigg)\\
&-\int U'(w)(y-\theta_H)\mathrm{d}F^H\bigg(\int U''(w)y(y-\theta_H)\mathrm{d}F^L\int U'(w)\mathrm{d}F^H\\
&-\int U''(w)(y-\theta_H)\mathrm{d}F^L\int U'(w)y\mathrm{d}F^H\bigg)\bigg\}.
\end{aligned}
$$

One can show then that $\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r}\geq 0$ if and only if the following holds

$$
\begin{aligned}
&-\int U'(w)(y-\theta_H)\mathrm{d}\triangle F\int U'(w)(y-y^*_H)\mathrm{d}\triangle G\\
&-\frac{\partial\beta^s_H(r)}{\partial r}\bigg\{\int U'(w)(y-\theta_H)\mathrm{d}F^L\int A(w)U'(w)(y-y^*_H)(y-\theta_H)\mathrm{d}F^H\\
&-\int U'(w)(y-\theta_H)\mathrm{d}F^H\int A(w)U'(w)(y-y^*_H)(y-\theta_H)\mathrm{d}F^L\bigg\}\geq 0.
\end{aligned}
$$

Using the first-order condition for β^s, the value of $\frac{\partial\beta^s_H(r)}{\partial r}$, and combining terms, one can show that $\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r}\geq 0$ if the following holds

$$
\begin{aligned}
&\int U'(w)(y-y^*_H)\mathrm{d}\triangle G\int U'(w)(y-\theta_H)(1-\lambda)(y|r)\mathrm{d}F^H\\
&\quad\geq -\int U(w)\mathrm{d}\triangle G\int A(w)U'(w)(y-y^*_H)(y-\theta_H)(1-\lambda(y|r))\mathrm{d}F^H.
\end{aligned}
$$

Let us define $K(r)\equiv\frac{\int U'(w)(y-y^*_H)\mathrm{d}\triangle G}{\int U(w)\mathrm{d}\triangle G}$. Observe that this term is positive since the concavity of $U'(w)\big(y-y^*_H\big)$ implies that $\int U'(w)\big(y-y^*_H\big)\mathrm{d}\triangle G<0$, and the concavity of $U(\cdot)$ results in that $\int U(w)\mathrm{d}\triangle G<0$. Then, this can be written as follows

$$
\int U'(w)(y-\theta_H)m(y)(1-\lambda(y|r))\mathrm{d}F^H\geq 0. \tag{8}
$$

where

$$
m(y)\equiv\int U(w)\mathrm{d}\triangle G\left[\frac{(1-\lambda)(y|r)}{(1-\lambda(y|r))}K(r)+A(w)(y-y^*_H)\right].
$$

First observe that the optimal λ is given by

$$\lambda = \frac{\int U'(w_H^s(y))(y-\theta_H)f^H(y|r)\mathrm{d}y}{\int U'(w_H^s(y))(y-\theta_H)f^L(y|r)\mathrm{d}y} > 0.$$

Notice that $\lambda < 1$ if $U'(w_H^s(y))(y-\theta_H)$ is increasing. This follows from the fact that $F^H(y|r)$ FSD $F^L(y|r)$. That is, $U''(w_H^s(y))\beta_H^s(y-\theta_H)+ U'(w_H^s(y)) > 0$, which results in that $R(w_H^s) = 1 \leq 1 + \theta_H A(w_H^s)$.

Second, observe that $y_H^* < \theta_H$ for all $\beta_H^s(r) > 0$.

Third, making use of the diffidence theorem to show that condition (8) holds entails showing the following:

$$\int U'(w)(y-\theta_H)(1-\lambda\ell(y|r))\mathrm{d}F^H = 0 \Rightarrow$$
$$\int U'(w)(y-\theta_H)m(y)(1-\lambda\ell(y|r))\mathrm{d}F^H \geq 0.$$

The *NSC* condition is given by

$$U'(w)(y-\theta_H)m(y)(1-\lambda\ell(y|r)) \geq$$
$$\frac{U'(\theta_H)(1-\lambda(\theta_H|r))m(\theta_H)}{U'(\theta_H)(1-\lambda(\theta_H|r))}U'(w)(y-\theta_H)(1-\lambda\ell(y|r)).$$

Because $U'(w) > 0$ and $1-\lambda(y|r) > 0$ for all $y \in Y$, it is easy to show that the *NSC* condition holds if and only if the following is satisfied:

$$(y-\theta_H)(m(y)-m(\theta_H)) \geq 0 \text{ for all } y \in Y.$$

Then, after a few steps of simple algebra this condition can be written as follows

$$(y-\theta_H)\left\{\begin{array}{c}\dfrac{(1-\lambda)[\ell(y|r)-\ell(\theta_H|r)]}{(1-\lambda\ell(\theta_H|r))(1-\lambda\ell(y|r))}K(r)+ \\ \left[A(w)y - A(\theta_H)\theta_H + y_H^*(A(\theta_H)-A(w))\right]\end{array}\right\} \leq 0 \text{ for all } y \in Y. \tag{9}$$

Observe that $\ell(y|r) \gtreqless \ell(\theta_H|r)$ for $y \lesseqgtr \theta_H$ since $\ell(y|r)$ decreases with y, and $A(w(y)) \gtreqless A(\theta_H)$ for $y \gtreqless \theta_H$ since preferences satisfy DARA. This implies that the first term in curly brackets is negative for all $y > \theta_H$ and positive otherwise, while the second term could be either negative or positive otherwise.

It can then be shown after a few steps of simple algebra that condition (A9) is equivalent to the following

$$A_R(w) < C(y) \text{ for all } y > \theta_H,$$
$$A_R(w) \geq C(y) \text{ for all } y \leq \theta_H,$$

where

$$C(y) \equiv A(\theta_H)(\theta_H - y_H^*)\beta_H^s(r) + A(w)((1-\beta_H^s(r))\theta_H + \beta_H^s(r)y_H^*) \\ - \frac{(1-\lambda)[\ell(y|r) - \ell(\theta_H|r)]}{(1-\lambda\ell(\theta_H|r))(1-\lambda\ell(y|r))}K(r)\beta_H^s(r).$$

Observe that $C(\theta_H) = R(\theta_H)$ and

$$C'(y) = A'(w)\beta_H^s(r)((1-\beta_H^s(r))\theta_H + \beta_H^s(r)y_H^*) + \\ - \frac{(1-\lambda)\ell'(y|r)}{(1-\lambda\ell(y|r))^2}K(r)\beta_H^s(r).$$

The $NC1$ condition is irrelevant since the restriction is with equality and the $NC2$ condition imposes that $m'(\theta_H) \geq 0$. That is,

$$\frac{(1-\lambda)\ell'(\theta_H|r)}{(1-\lambda\ell(\theta_H|r))^2}K(r) + A'(\theta_H)\beta_H^s(r)(\theta_H - y_H^*) + A(\theta_H) \leq 0.$$

Using the fact that $A_R'(\theta_H) = A'(\theta_H)\theta_H + A(\theta_H)$, one can easily see that this condition is satisfied if

$$A_R'(\theta_H) \leq \frac{C'(\theta_H)}{\beta_H^s(r)}.$$

.

References

Arya, A., and B. Mittendorf. 2005. "Offering Stock Options to Gauge Managerial Talent," 40 *Journal of Accounting and Economics* 189–210.

Brown, C. 1990. "Firms' Choice of Method of Pay," 43 *Industrial and Labor Relations Review* 165S–182S.

———. 1992. "Wage Levels and the Method of Pay," 23 *Rand Journal of Economics* 367–75.

Cadenillas, A., Cvitanic, J., and Zapatero, F. 2005. "Stock Options with Unknown Executive Type," Working Paper.

Cho, I., and Kreps, D. 1987. "Signaling Games and Stable Equilibria," 102 *Quarterly Journal of Economics* 179–222.

Foster, A. D., and Rosenweig, M. R. 1996. "Comparative Advantage, Information and the Allocation of Workers to Tasks: Evidence from an Agricultural Labor Market," 63 *Review of Economic Studies* 347–74.

Gollier, C. 1995. "The Comparative Statics of Change in Risk Revisited," 66 *Journal of Economic Theory* 522–36.

———. 2001. *The Economics of Risk and Time.* Cambridge, MA: MIT Press.

Gollier, C., and Kimball, M. S. 1996. "Towards a Systematic Approach to the Economics of Uncertainty: Characterizing Utility Functions." Discussion Paper, University of Michigan.

Grossman, H. 1979. "Adverse Selection, Dissembling, and Competitive Equilibrium," 10 *Bell Journal of Economics* 336–43.

Grossman S., and Hart, O. 1983. "An Analysis of the Principal-Agent Problem," 51 *Econometrica* 7–45.

Hellwig, M. 1987. "Some Recent Developments in the Theory of Competition in Markets with Adverse Selection," 31 *European Economic Review* S319–25.

Holmstrom, B., and Milgrom, P. 1987. "Aggregation and Linearity in the Provision of Intertemporal Incentives," 55 *Econometrica* 303–28.

Jensen, M. C., and Murphy, K. J. 1990. “Performance Pay and Top Management Incentives,” 98 *Journal of Political Economy* 225–65.

Kaplan, S. 1994. “Top Executive Rewards and Firm Performance: A Comparison of Japan and United States,” 102 *Journal of Political Economy* 510–46.

Kimball, M. S. 1990. “Precautory Savings in the Small and the Large,” 58 *Econometrica* 911–20.

Lafontaine, F. 1992. “Agency Theory and Franchising: Some Empirical Results,” 23 *Rand Journal of Economics* 263–83.

Lafontaine, F., and Slade, M. 2001. “Incentive Contracting and the Franchise Decision,” in K. Chatterjee and W. Samuelson, eds., *Advances in Business Applications of Game Theory.* Boston, MA: Kluwer Academic Press.

Lazear, E. P. 1986. “Salaries and Piece-rates,” 59 *Journal of Business* 405–31.

———. 2000a. “Performance Pay and Productivity,” 90 *American Economic Review* 1346–61.

———. 2000b. “The Power of Incentives,” 90 *American Economic Review, Papers and Proceedings* 410–14.

———. 2005. “Output-Based Pay: Incentives or Sorting?” 23 *Research in Labor Economics* 1–25.

Mailath, J. M, Okuno-Fujiwarwa, M., and Postlewaite, A. 1993. “Belief-Based Refinements in Signaling Games,” 60 *Journal of Economic Theory* 241–76.

Matutes, C., Regibeau, P., and Rockett, K. 1994. “Compensation Schemes and Labor Market Competition: Piece Rates versus Wage Rate,” 3 *Journal of Economics and Management Strategy* 325–53.

Mirrlees, J. 1999. “The Theory of Moral Hazard and Unobservable Behavior: Part I,” 66 *Review of Economic Studies* 23–38.

Moen, E., and Rosén, A. 2005. “Performance Pay and Adverse Selection,” 107 *Scandinavian Journal of Economics* 279–98.

Murphy, K. 1999. “Executive Compensation,” in Orley Ashenfelter and David Card, eds., *Handbook of Labor Economics*, Chap. 38, 2485–2563. Vol. 3b. North Holland, Amsterdam: Elsevier Science.

Oyer, P. 2004. “Why Do Firms Use Incentives That Have No Incentive Effects,” 59 *Journal of Finance* 1619–50.

Paarsh, H., and Shearer, B. S. 2000. “Piece Rates, Fixed Wages, and Incentive Effects: Statistical Evidence from Payroll Records,” 40 *International Economic Review* 59–92.

Petersen, T. 1991. “Reward Systems and the Distribution of Wages,” 7 *Journal of Law, Economics and Organization* 131–57.

———. 1992. “Payment Systems and the Structure of Inequality: Conceptual Issues and a Analysis of Salespersons in Department Stores,” 98 *American Journal of Sociology* 76–104.

Prendergast, C. 1999. “The Provision of Incentives in Firms,” 37 *Journal of Economic Literature* 7–63.

———. 2002a. “Uncertainty and Incentives,” 20 *Journal of Labor Economics* 115–37.

———. 2002b. “The Tenuous Trade-off between Risk and Incentives,” 110 *Journal of Political Economy* 1071–102.

Rothschild, M., and Stiglitz, J. 1976. “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, 629–49.

Shearer, B. 2004. “Piece Rates, Fixed Wages and Incentives, Evidence from a Field Experiment,” 71 *Review of Economic Studies* 513–34.