Modified robust external force control with disturbance rejection with application to piezoelectric actuators
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Abstract
In micromanipulation applications, controlling the force exerted on the object is of great importance. In such cases, any uncontrolled forces may damage the object or cause system failure. However, the presence of disturbances such as impedance uncertainties and hysteresis can strongly degrade force control performance and even lead to instability. Therefore, accurate force control when internal and external disturbances occur is a significant challenge. Conventional control methods usually have a number of restrictive conditions especially on the disturbance bounds. To rectify these issues, a modified robust disturbance rejection-based force control approach is proposed in this paper. For this purpose, an appropriate disturbance observer is utilized to estimate the disturbance effect regardless of amplitude. Then a robust control method is employed to achieve the disturbance-free desired dynamic. A modification is also performed to rectify the need for acceleration measurement in the control design. Finally, the force control for an unknown environment in the presence of disturbances is accomplished. The efficiency of the proposed approach is evaluated through simulation studies and compared with the well-known PI method. The experimental results validate the force control performance for the micropositioning piezoelectric actuator.

Keywords
Disturbance rejection, force control, hysteresis, observer, piezoelectric actuators.

Introduction
Micromanipulation by piezoelectric actuators has been an interesting research field in the last decade. These actuators possess special properties, such as high natural frequency, fine resolution and response time (Huang and Chiu, 2009). However, their performance suffers from the effect of hysteresis non-linearity. This non-linear characteristic makes the control process a challenging problem (Ghafarirad et al., 2014). Several open- and closed-loop control schemes have been proposed for piezoelectric micropositioning (Ang et al., 2007; Xu and Li, 2010). Impedance control, sliding mode control and robust control coupled with observers and adaptive structures have also been presented (Bashash and Jalili, 2009; Sheikh Sofa et al., 2011; Xu, 2013; Zareinejad et al., 2009).

However, in many micromanipulation applications, the piezoelectric actuator is in contact with the environment. In such cases, the intermediate force between the actuator and the environment should be precisely controlled. In sensitive processes such as cell injection and micro assembly, exerting a specified force on the object is a crucial objective. Any uncontrolled force can damage the object or cause failure in the system. As a result, an efficient control strategy should be designed to control the external force (Siciliano et al., 2009; Spong et al., 2006).

One of the main obstacles in the force control process is the existence of disturbances such as impedance uncertainties and hysteresis, which can severely degrade performance and even lead to instability. They can also cause exertion of uncontrolled forces on the object. Therefore, proposing a robust, stable force control approach with the capability of rejecting disturbance is the primary goal in this paper.

To this end, several robust force control approaches have been proposed. A number of core conservative assumptions are usually considered as follows. First, some approaches are specified for particular cases of disturbance such as parameter uncertainties, and are not applicable for other types like external disturbances (Li and Zhang, 2010). Second, the disturbances should be bounded, and bounds are required for robust control design (Khadraoui et al., 2010). Third, the

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robust approach should compensate for all disturbances and it can degrade controller efficiency (Chen, 1991). Fourth, the environment dynamic and its impedances are required for the force control process (Davis et al., 2011; Komada et al., 1993).

To rectify these issues, disturbance estimation and compensation is a potential alternative method. For this purpose, a sliding-based disturbance observer has been presented (Daly and Wang, 2009). The sliding nature of the proposed observer may, however, degrade its performance. Also, an unknown input observer has been proposed for a piezoelectric actuator (Rakotondrabefy and Lutz, 2009), although the necessary conditions for the proposed observer are restrictive. One of the most well known non-linear disturbance observers has been introduced for a two-link revolute manipulator (Chen et al., 2000; Gupta and O’Malley, 2011). This observer can also be utilized for external force estimation purposes (Amin et al., 2013). It has further been improved for disturbance estimation in general revolute manipulators (Nikooeein and Haghighi, 2009). Mohammadi et al. (2013) have recently revisited the observer for the general structure of manipulators.

By considering such an observer, a robust observer-based force control approach is proposed in this paper to rectify the aforementioned conservative assumptions. Therefore, the disturbance observer is utilized for estimation and feedforward compensation of existing disturbance regardless of source (first assumption). The proposed observer has no restriction on the estimated disturbance bounds either. It means the observer would estimate and compensate for the disturbance regardless of magnitude. The only condition is the boundedness of time variation of disturbances (second assumption). Then a robust technique is utilized to eliminate only the effect of disturbance estimation error and not all disturbances (third assumption). As a result, a new disturbance-free desired dynamic model can be achieved. Regarding the new achieved dynamic, any environment-independent force control approach can be utilized (fourth assumption). Therefore, the force control of a micropositioning piezoelectric actuator in the presence of disturbances can be appropriately implemented. Overall system stability in the presence of estimated disturbances has analytically been demonstrated. Simulation and experimental results validate the efficiency of the proposed controller for disturbance compensation and accurate force control. The performance of this approach is confirmed with a case study and compared against a conventional PI control approach.

**General non-linear dynamic modelling**

A general $m$ degree-of-freedom non-linear dynamic system in contact with the environment is represented as:

$$\ddot{x} + C(x, \dot{x})\dot{x} + G(x) = T - F_{ext}$$  \hspace{1cm} (1)

where $x = [x_1, x_2, \ldots, x_n]^T$ is the vector of $n$ generalized coordinates; $M(x)$ represents the symmetric and positive-definite inertia matrix; $C(x, \dot{x})$ is Coriolis and centripetal matrix and $G(x)$ denotes the gravity matrix; $T$ and $F_{ext}$ are the input and external force vectors, respectively.

Due to parametric uncertainties, unmodelled dynamics and identification errors, the dynamic parameter matrices are described as:

$$\dot{M} = M + \Delta M$$
$$\dot{C} = C + \Delta C$$
$$\dot{G} = G + \Delta G$$  \hspace{1cm} (2)

where $M$, $C$, $G$ are the known parts and $\Delta M$, $\Delta C$, $\Delta G$ are the unknown parts of $M$, $C$ and $G$, respectively. Therefore, the dynamic model can be modified as follows:

$$M(x)\ddot{x} + C(x, \dot{x})\dot{x} + G(x) = T - F_{ext} - (\Delta M\ddot{x} + \Delta C\dot{x} + \Delta G)$$  \hspace{1cm} (3)

With regard to the dynamic model (3), the total uncertainties may be represented as a disturbance term such as $d(t) = -(\Delta M\ddot{x} + \Delta C\dot{x} + \Delta G)$. As a result, the system dynamic model would be simplified as:

$$M(x)\ddot{x} + C(x, \dot{x})\dot{x} + G(x) = T - F_{ext} + d(t)$$  \hspace{1cm} (4)

**Control design**

The control design objective is accurately to control the external force with regard to the existence of all disturbances. Disturbances may degrade the accuracy of the external force control process. For this reason, the effect of disturbances should be compensated. Figure 1 shows the overall block diagram of the proposed observer-based external force control for micropositioning actuators.

The mentioned approach includes two stages:

- **Stage 1**: Observer-based robust control for disturbance rejection. An appropriate observer is proposed to estimate disturbance regardless of its magnitude. Then the estimated disturbance is utilized for feedforward disturbance rejection. With regard to observer estimation error, a robust control approach is designed to eliminate the estimation error effect. Consequently, a disturbance-free desired dynamic can be achieved.
- **Stage 2**: External force control for an unknown environment. An environment-independent force control approach is proposed to control the external force for the achieved desired dynamic model.

These stages are further elaborated subsequently.

**First control stage: observer-based robust control for disturbance rejection**

A modified disturbance observer is utilized to estimate all disturbances regardless of magnitude (Chen et al., 2000).

$$\dot{d} = -Ld + L\left[(M(x)\ddot{x} + C(x, \dot{x})\dot{x} + G(x)) - T + F_{ext}\right]$$  \hspace{1cm} (5)

where $d$ and $L$ express the estimated disturbance and positive definite observer gain matrix, respectively.
By defining the estimation error as $\tilde{d} = \hat{d} - d$, the observer closed-loop dynamic (6) may be achieved.

\[ \dot{\tilde{d}} - \hat{d} = -L\dot{d} 
+ L[(M(X)\dot{X} + C(X, \dot{X})\dot{X} + G(X)) - T' + F_{ext}] - \hat{d} \]
\[ = -L\hat{d} + Ld - \hat{d} \]
\[ \tilde{d} = \tilde{d} + L\hat{d} = -\hat{d} \]

(6)

where $\dot{d}$ denotes the disturbance time variation. With regard to the positive definiteness of $L$, the closed-loop dynamic (6) can be considered a stable system with input $\dot{d}$. It may be assumed that the time variation of the disturbance is bounded as $\|\tilde{d}\| \leq \tilde{d}$, where $\tilde{d}$ is a positive constant. This assumption is a conventional point mentioned in different references (Mohammadi et al., 2013; Peng and Chen, 2009).

To investigate closed-loop stability, a positive definite Lyapunov function $V(t) = \frac{1}{2} \tilde{d} \dot{\tilde{d}}$ is considered. The time derivative would be:

\[ \dot{V}(t) = \frac{1}{2} \tilde{d} \dot{\tilde{d}} = -\tilde{d}^{T}L\tilde{d} - \tilde{d} \dot{\tilde{d}} \]

(7)

Based on the quadratic property of $\lambda_{\min}(L)\|\tilde{d}\|^2 \leq \tilde{d}^{T}L\tilde{d} \leq \lambda_{\max}(L)\|\tilde{d}\|^2$, the ultimate boundedness of the closed-loop dynamic can be deduced.

\[ \dot{V}(t) = -\tilde{d}^{T}L\tilde{d} + \tilde{d} \hat{d} \leq -\lambda_{\min}(L)\|\tilde{d}\|^2 
+ \|\tilde{d}\| \|\hat{d}\| \leq 0 \quad \text{if} \quad \|\tilde{d}\| \geq \|\tilde{d}\|\lambda_{\max}(L) \]

(8)

As a result, the disturbance estimation error would be bounded as $\|\tilde{d}\| \leq \|\hat{d}\|\lambda_{\min}(L) \leq \tilde{d}$. Similarly, it can be deduced that $\|\tilde{d}\| \leq \|\hat{d}\|\lambda_{\max}(L)$. This means that for any disturbance with little variation, the observer can effectively estimate disturbance with small estimation error.

Remark 1. In the case of slow, time-varying disturbances, i.e. $\dot{d}(t) = 0 \Rightarrow \tilde{d}(t) = 0$, the proposed disturbance observer approach would generate an asymptotic closed-loop dynamic $d(t) + L\dot{d}(t) = 0$. Thus, the asymptotic convergence of estimated disturbance to the real value may be concluded as $\tilde{d}(t) \rightarrow 0$.

Based on the introduced disturbance observer, a robust control is designed to attain the disturbance-free desired dynamic to facilitate an accurate external force control process.

For this purpose, a suitable sliding surface $S$ containing the desired dynamic should be defined:

\[ S = \int_{0}^{t} [(M(X)\dot{X} + C(X, \dot{X})\dot{X} + G(X) - T' + F_{ext}) \, dt \]

(9)

where $T'$ represents the control input for the desired dynamic.

Theorem. With regard to the general dynamic model (4), the robust control input (10) including the estimated disturbance (5) and sliding surface (9) is proposed:

\[ T = T' - \hat{d} - \eta_{1}S - \eta_{2}Sgn(S) \]

(10)

where $\eta_{1}$ and $\eta_{2}$ are positive definite diagonal control gain matrices. Accordingly, the disturbance-free desired dynamic can be achieved as:

\[ M(X)\ddot{X} + C(X, \dot{X})\ddot{X} + G(X) = T' - F_{ext} \]

(11)

Proof. By differentiating the sliding surface (9) and substituting the dynamic model (4), Equation (12) is derived.

\[ \dot{S} = M(X)\dot{\dot{X}} + C(X, \dot{X})\dot{\dot{X}} + G(X) - T' + F_{ext} \]

\[ = T + \dot{d}(t) - T' \]

(12)

By merging the control input (10) with the sliding surface derivative (12) generates the following closed-loop dynamic:

\[ \dot{S} = T + \dot{d}(t) - T' \]
\[ = -\dot{d} - \eta_{1}S - \eta_{2}Sgn(S) + \dot{d}(t) \]

(13)

By defining the estimation error as $\tilde{d}(t) = \dot{d}(t) - \dot{d}(t)$, the closed-loop dynamic is obtained.
\[ \dot{S} + \eta_1 S + \eta_2 \text{Sgn}(S) = -\dot{d}(t) \]  
(14)

The positive definite Lyapunov function candidate \( V = \frac{1}{2} S^T S \) is considered. The closed-loop dynamic is substituted in the Lyapunov function time derivative as follows:

\[
\dot{V} = S^T \dot{S} = -S^T \eta_1 S - S^T \eta_2 \text{Sgn}(S) - S^T \dot{d}(t)
= \sum_{i=1}^{n} (-\eta_1 |S_i|^2 - \eta_2 |S_i| |\dot{d}_i| + S_i \dot{d}_i)
\leq \sum_{i=1}^{n} (-\eta_1 |S_i|^2 - \eta_2 |S_i| |\dot{d}_i|) 
\]
(15)

With respect to the boundedness of disturbance estimation error \( (|\dot{d}_i| \leq \dot{d}/\lambda_{\text{max}}(L)) \), the negative definiteness of \( \dot{V} \) can be achieved by choosing \( \eta_{2i} = \eta_{2i} + \dot{d}/\lambda_{\text{max}}(L) \).

\[ \dot{V} \leq \sum_{i=1}^{n} (-\eta_1 |S_i|^2 - \eta_2 |S_i| |\dot{d}_i|) \leq 0 \]  
(16)

Thus asymptotic stability of the closed-loop dynamic system is guaranteed. As a result, \( \dot{V} \) tends toward zero as \( t \to \infty \) and the appropriate disturbance-free dynamic is achievable (Cho and Park, 2005).

**Remark 2.** To alleviate the chattering phenomena caused by the discontinuous nature of the signum function, the saturation function \( \text{Sat}(S/s) \) can be utilized as the continuous form of \( \text{Sgn}(S) \).

### Second control stage: external force control for a disturbance-free dynamic system

Considering the achieved disturbance-free desired dynamic model (11), a force control approach can be designed. In some approaches, it is assumed that the environment has known impedances, but in many applications, such as micromanipulation systems, the manipulator comes into contact with a wide range of environments with unknown impedances. In fact, environmental dynamic behaviour can only be assumed and its impedances are unknown. Consequently, the conservative assumption of a known environment should be rectified in the control design.

To this end, the desired dynamic control input \( T \) is proposed in a way so as to linearize the system.

\[ T' = C(X, \dot{X}) \dot{X} + G(X) + F_e + M(X) U \]  
(17)

Therefore, the system dynamic would be transformed to the decoupled double integrator configuration:

\[ \ddot{X} = U \]  
(18)

The dynamic of any individual degree of freedom is represented by \( \ddot{x}_i = u_i \). To simplify the control design, index \( i \) would be eliminated in the rest. The control input \( u \) should be designed to yield proper external force control.

Considering the double integrator dynamic (18), the control input (19) would inject the desired impedance parameters \((M_e, C_e)\) to the system (Spong et al., 2006).

\[ u = \frac{1}{M_e} [-C_e \ddot{x} + F_d - F_e] \]  
(19)

As a result, the closed-loop system guarantees external force control caused by contact with an unknown environment.

By substituting the control input (19) in the linearized dynamic system (18), the closed-loop dynamic would be:

\[ M_e \ddot{x} + C_e \dot{x} = Z_e(s) \dot{x} = F_d - F_e \]
\[ Z_e(s) = M_e s + C_e \]  
(20)

where \( Z_e(s) \) and \( s \) represent the desired impedance and Laplace transform variable, respectively. As a basic condition, the desired impedance should not contain any gravity like impedance. The equivalent electrical description of the closed-loop system is shown in Figure 2, where \( Z_e(s) \) represents the impedance of the environment. A general mass-damper-spring dynamic (21) is assumed for the environment dynamic model. Impedances are entirely unknown.

\[ F_e = Z_e(s) \dot{x} \]
\[ Z_e(s) = M_e s + C_e \]  
(21)

Considering Figure 2, force relation (22) is derived.

\[ \frac{F_e(s)}{F_d(s)} = \frac{Z_e(s)}{Z_e(s) + Z_s(s)} \]  
(22)

In view of the definition of desired dynamic and environment impedances, the steady-state error converges to zero (23) and external force control may be demonstrated.

\[ \epsilon = \lim_{s \to 0} s E(s) = \frac{Z_e(0)}{Z_e(0) + Z_s(0)} \Rightarrow F_e - F_d \]  
(23)

The proposed control structure guarantees an appropriate force control process.

### Control design modification by acceleration measurement rectification

The proposed disturbance observer structure (5) and the designed robust control (10) require the acceleration signal. The existence of an acceleration signal is a drawback in control systems. In fact, acceleration sensors are not widely applicable due to their performance. Also, numerical
differentiation significantly deteriorates the signal. To solve this problem, filtering (integrating) is an alternative solution for rectifying the necessity for a measurement. With this method, the order of the system would decrease and the acceleration signal can be changed to the velocity order signal. For this purpose, an auxiliary parameter as the filtered (integrated) equivalent of an acceleration term should be defined. Thus, two modifications are proposed as follows.

**Modified observer**

In the proposed observer structure, a new auxiliary parameter \( z_i \) is defined as \( z_i(t) = \dot{x}_i(t) - p_i(t) \) (Chen et al., 2000). By differentiating both sides and substituting the observer structure (5), a new dynamic can be achieved for the observer:

\[
\dot{z}_i(t) = \dot{x}_i(t) - p_i(t) \\
\ddot{z}_i(t) = -L \ddot{x}_i + L [M(x) \ddot{x}_i + C(x, \dot{x}_i) \dot{x}_i + G(x)] - T + F_{ext} \\
- p_i(t) \\
\dddot{z}_i(t) = -L z_i(t) + p_i(t) \\
\ddot{z}_i(t) = \dot{L} z_i(t) + L [C(x, \dot{x}_i) \dot{x}_i + G(x)] - T + F_{ext} - p_i(t) \\
\frac{dp_i}{dt} = LM(x) \ddot{x}_i \\
\frac{dp_1}{dt} = LM(x) \ddot{x}_i \\
\frac{dp_2}{dt} = -M(x) \ddot{x}_i
\]

If parameter \( p_i(t) \) is designed such that \( \frac{dp_i}{dt} = LM(x) \ddot{x}_i \), the new observer dynamic is derived as:

\[
\dot{z}_i(t) = -L z_i(t) + L [C(x, \dot{x}_i) \dot{x}_i + G(x)] - T + F_{ext} - p_i(t)
\]

As a result, the observer structure is modified as:

\[
\dot{a}(t) = z_i(t) + p_i(t) \\
\ddot{z}_i(t) = -L z_i(t) + L [C(x, \dot{x}_i) \dot{x}_i + G(x)] - T + F_{ext} - p_i(t) \\
\frac{dp_2}{dt} = -M(x) \ddot{x}_i
\]

Therefore, by properly defining parameter \( p_i(t) \) the acceleration measurement will be relaxed. In linear systems, it can be simply chosen as \( p_i(t) = L ax \).

**Modified controller**

Achieving the sliding surface \( S(t) \) also requires the acceleration signal. To mitigate this problem, another auxiliary state is introduced: \( z_2(t) = \ddot{x}_2(t) + p_2(t) \). By differentiating both sides and substituting the sliding surface a new dynamic can be attained for the observer as follows:

\[
\dddot{z}_2(t) = \dddot{x}(t) + \frac{dp_2}{dt} \\
= \dddot{x}_i(t) + C(x, \dot{x}_i) \dot{x}_i + G(x) - T' + F_{ext} + \frac{dp_2}{dt}
\]

If parameter \( p_2(t) \) is designed in a case where \( \frac{dp_2}{dt} = -M(x) \ddot{x}_i \), then signal \( S(t) \) can be derived as follows:

\[
S(t) = z(t) - p_2(t) \\
\dddot{z}_2(t) = C(x, \dot{x}_i) \dot{x}_i + G(x) - T' + F_{ext} \\
\frac{dp_2}{dt} = -M(x) \ddot{x}_i
\]

Finally, the complete control design for a general non-linear dynamic model is:

\[
T = [C(x, \dot{x}_i) \dot{x}_i + G(x) + F_s + M(x) U] - \dot{a} - \eta_1 S - \eta_2 \text{Sign}(S)
\]

\( U, \dot{a} \) and \( S \) are derived from equations (19), (26) and (28).

**Control design for micropositioning piezoelectric actuators**

The proposed controller should be implemented on a piezoelectric actuator. The first step entails suitable dynamic modelling of the actuator including non-linear hysteresis behaviour. A second-order dynamic is utilized for piezoelectric actuators. The model is divided into two parts. The first part is a second-order linear dynamic that refers to the mass-spring-damper system and the second part describes the non-linear portion of the dynamic, i.e. the hysteresis non-linearity effect. Figure 3 shows the linear second-order dynamic of a piezoelectric actuator that would be added by the hysteresis non-linearity effect in the input.

The governing equation in free motion is represented by:

\[
m \ddot{x} + c \dot{x} + k x = k H_0(v) - F_s + \dot{d}(t)
\]
where \( x(t) \) and \( v(t) \) represent the actuator displacement and input voltage, respectively; \( m, c \) and \( k \) denote mass, damping and stiffness gain, respectively; \( H_o(v(t)) \) expresses the hysteresis relation between the electrical input voltage and actuator position; and \( F_e \) is the external force exerted on the actuator. To consider uncertainties, hysteresis estimation error and likely unmodelled dynamics, a perturbation term \( d(t) \) is added to the dynamic model.

A generalized Prandtl–Ishlinskii (PI) model is used for both hysteresis identification and compensation (Ghafarirad et al., 2012). The most important advantages of this model is its simplicity and that its inverse can be calculated analytically. The key idea of inverse feedforward compensation hysteresis is to cascade the inverse hysteresis operator \( H_o^{-1} \) with the actual hysteresis as shown in Figure 4.

As a result, the non-linear hysteretic behaviour would be compensated and the actuator dynamic model would change to the standard form of general dynamic systems. Therefore, the proposed control algorithms can be implemented on the actuator. Otherwise, the hysteresis effect has to be considered a time-varying disturbance and it can degrade controller performance.

By utilizing the inverse PI hysteresis model and choosing the input voltage as:

\[
v = H_o^{-1} \left( \frac{1}{k} T(t) \right)
\]

apparently, the actuator dynamic would be linearized as follows:

\[
m \ddot{x} + c \dot{x} + kx = T - F_e + d
\]

where \( T \) is the control input.

Equation (32) implies the linear structure of the actuator due to using the inverse PI model. The objective is accurate external force control. The disturbance observer is proposed as defined by:

\[
\begin{align*}
\dot{z}_1 &= z_1 + p_1 \\
\dot{z}_2 &= -L \dot{z}_2 + L \left[ c \ddot{x} + k \dot{x} - T + F_e - p_1 \right] \\
p_1 &= -m \ddot{x}
\end{align*}
\]

The robust observer-based control is implemented as derived in the theorem given previously.

\[
T = T' - \dot{d} - \eta_1 S - \eta_2 \text{sgn}(S)
\]

\[
T' = c \dot{x} + kx + F_e + mu
\]

\[
u = \frac{1}{m} [-C_2 \dot{x} + F_d - F_e]
\]

where the sliding surface is defined as:

\[
S = z_2 + p_1
\]

\[
z_2 = c \dot{x} + kx - T' + F_e
\]

\[
p_2 = -m \ddot{x}
\]

where \( \eta_1 \) and \( \eta_2 \) are positive coefficients and \( \text{sgn}(.) \) is the signum function. As mentioned previously, auxiliary parameters can basically be achieved as \( p_1 = -L \dot{m} \dot{x} \) and \( p_2 = -m \ddot{x} \) independent of acceleration. Finally, the general control input would be:

\[
v = H_o^{-1} \left( \frac{1}{k} \left\{ \left[ c \dot{x} + kx + F_e + m \left[ \frac{1}{m} [-C_2 \dot{x} + F_d - F_e] \right] \right] \right\} \\
-\dot{d} - \eta_1 S - \eta_2 \text{sgn}(S)) \right)
\]

**Performance evaluation for the proposed control method**

A simulation case study was carried out to investigate the performance of the proposed controller. In this case, a linearized model of a piezoelectric actuator used in the experimental investigation is studied. Initially, it is assumed that there is no external disturbance and the system is fully identified. A 50-mN desired force is considered. A simple external force control is implemented on the manipulator and the result is shown in Figure 5.

This result serves to compare the effect of external disturbance and the proposed control performance.

Control performance is evaluated for two different cases. In the first case, disturbance is generated by parameter uncertainties caused by identification error. This is a very common problem in mechanical systems such as piezoelectric actuators. In the second case, disturbance is generated from the actuator source as external disturbance.
**Disturbance caused by uncertainties**

In this circumstance, it is assumed that the identified parameters deviate by roughly 20% compared with real impedances. This deviation may degrade force control efficiency (Figure 6).

Activating the disturbance estimation and robust disturbance rejection may effectively compensate for the deviation in force control results. The proposed observer and control result is depicted in Figure 7.

**Disturbances caused by actuators**

Actuator disturbance is a common defect in some mechanical systems. In this study, a harmonic external disturbance with a frequency of 1 Hz and amplitude 5% of the control input is added to the actuator input, as shown in Figure 8.

Figure 9 indicates the effect of disturbance on force control.

It is obvious that external disturbance can absolutely disrupt performance. To improve the control result, the proposed robust disturbance rejection approach is implemented to the system. Figures 10(a) and (b) depict external disturbance estimation and proper force control, respectively.

**Comparative study**

To evaluate the performance of the proposed control approach, a comparative study was performed between the well-known PI controller and robust disturbance rejection-based force control. An external harmonic disturbance with a frequency of 1 Hz and amplitude 5% of the control input is considered. Figure 11 shows the result of each controller in force tracking and its error.

It is clear that the proposed approach can efficiently diminish the disturbance effect. To magnify this ability, the disturbance frequency was increased to 20 Hz.

Figure 12 depicts the result.

As seen in Figure 12, the PI control performance would significantly degrade at high frequencies.

**Experimental case study**

The proposed controller was implemented experimentally, with the experimental set-up shown in Figure 13. The set-up contains a P-615 NanoCube piezoelectric actuator with 420 nm maximum displacement in the X and Y directions. A DS1104 dSPACE data acquisition card and controller board were used to capture data. Matlab/Simulink software was