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The Poker-Litigation Game

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Abstract: Is civil and criminal litigation a search for truth, like science or philosophy, or a game of skill and luck, like the game of poker? Although the process of litigation has been modeled as a Prisoner's Dilemma, as a War of Attrition, as a Game of Chicken, and even as a simple coin toss, no one has formally modeled litigation as a game of poker. This paper is the first to do so. Specifically, we present a simple “poker-litigation game” and find the optimal strategy for playing this game.

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The poker-litigation game

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There is an undeniable, though imperfect, symmetry between law practice and poker (Lubet, 2005, p. 59).

1. Introduction

Is civil and criminal litigation a search for the truth, like science or philosophy, or a game of skill and luck, like the game of poker? If the former, how do we explain the existence of frivolous claims (or “negative-expected value” lawsuits) and the occurrence of prosecutorial misconduct? If the latter, how do we explain the demand for costly methods of dispute resolution and the critique of random methods of justice?

Although the process of litigation has been modeled as a Prisoner’s Dilemma (Gilson & Mnookin, 1994, pp. 514-522), as a War of Attrition (Klemperer, 2000, pp. 4-9), as a Game of Chicken (Guerra-Pujol, 2010a, pp. 595-597), and even as a simple coin toss (Guerra-Pujol, 2011, pp. 46-48), and although, in the words of one scholar, “there is an undeniable … symmetry between law practice and poker” (Lubet, 2005, p. 59), no one has formally modeled litigation as a game of poker. This paper is the first to do so. Specifically, we present a simple “poker-litigation game” and find the optimal strategy for playing this game.

The remainder of this paper is organized as follows: Section 2 briefly summarizes the similarities between litigation and the game of poker. Section 3 then presents our model of the poker-litigation game. Next, Section 4 presents our solution of the game based on the work of McAdams, 2012. Section 5 concludes.

2. Similarities between poker and litigation

In summary, the process of litigation has many important features in common with the game of poker, for example:

(i) both poker and litigation are “strategic” games in which the players/litigants must make their choices independently of each other (see, e.g., Nash, 1951, p. 286);

(ii) both poker and litigation are “zero-sum, non-cooperative” games in which the economic interests of the players/litigants are opposed (see, e.g., Baird, et al., 1994, pp. 220-224);

(iii) both are games of “incomplete information”: just as a player in a game of poker does not know with certainty when another player is “bluffing,” a litigant in a civil or criminal case may not know with certainty the strength of

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2 For an overview of poker, see Scarne, 1965, pp. 224-240.
his adversary’s case during pre-trial negotiations (see, e.g., Lubet, 2006, pp. 92-93);

(iv) both games involve elements of chance or luck: e.g. random assignment of the cards in poker; random selection of judges and jurors in civil and criminal cases (see generally Duxbury, 1999).

In addition, poker has a rich history of study in other academic fields, including mathematics (Nash, 1951, pp. 293-294; Kuhn, 1950), game theory (von Neumann & Morgenstern, 1953, ch. 19), computer science (Billings, et al., 2000), and law (Lubet, 2006). This paper, however, is the first to formally model litigation as a game of poker. Specifically, we present a simple model of the “poker-litigation game” in section 3 below and then find the optimal strategy for playing this game in section 4.

3. The model

Building on the work of McAdams (2012), von Neumann & Morgenstern (1953, ch. 19), and Nash (1951, pp. 293-294), we model litigation as a simple game of poker, which we call the poker-litigation game. In summary, our poker-litigation game proceeds in four successive stages as follows:

(i) time T₁ . . . a opening round in which each player is dealt one card
(ii) time T₂ . . . a quiet round in which the players examine their hole cards
(iii) time T₃ . . . a betting round in which the players place their bets
(iv) time T₄ . . . a final round in which the bets are paid to the winner

Further, the rules and payoff structure of the poker-litigation game are as follows:

1. There are three players, two main parties, plus a neutral judge: (i) player P, the plaintiff, (ii) player D, the defendant, and (iii) player J, the dealer/judge. The objective of players P and D is to win the poker-litigation game (by maximizing their payoffs from the game), while the objective of the dealer/judge, by contrast, is to shuffle and deal the cards, collect and pay out the bets, and enforce and administer the rules of the game.

3 For a historical overview and general description of poker, see, e.g., Scarne, 1965, ch. 22.
4 We assume the reader is familiar with basic poker terminology. For a glossary of poker terms and phrases, see Billings, et al., 2000, pp. 237-239. See also Scarne, 1965, pp. 228-232.
5 Previous mathematical models of the game of poker take the role of the judge or dealer for granted (see, e.g., McAdams, 2012; von Neumann & Morgenstern, 1953, pp. 186-219; Nash, 1951, pp. 293-294; Nash & Shapley, 1950, pp. 105-116; and Kuhn, 1950, pp. 93-103). In our model, by contrast, the judge is an essential player, since his role is to detect cheating and monitor compliance with the rules of the game.
2. Each player, P and D, is dealt a card “face down” by the dealer/judge at the start of play, i.e. time $T_1$. In summary, these “hole” cards (or private cards) represent the strength or weakness of each player’s case. For simplicity and mathematical tractability, we assume that the values of the players’ private cards are independent and identically distributed (i.i.d.) random variables on the interval $[0, 1]$. We further assume that cards with higher values (i.e. values nearer to 1) are deemed stronger (i.e. are worth more) than cards with lower values (values approaching 0).

3. The players may (but are not required to) examine their face-down private cards at time $T_2$, but the values of their private cards are not revealed until after the betting round. In other words, each player/litigant knows the strength or weakness of his case but does not know the strength or weakness of the other litigant’s case.

4. After examining their private hole cards, the players must simultaneously make their bets or “bids” at time $T_3$. Specifically, all bets must be placed in separate sealed envelopes and handed over to the judge, who will then award the combined bets to the winning player at the end of play at time $T_4$ (see Rule #6 below).

5. The players are allowed to make only two possible bets, a “high” bet, $a$, or a “low” bet, $b$, where $a > b > 0$, and all bets made at time $T_3$ are final.

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6 That is, in place of a standard and finite deck of cards with values 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, and Ace, we assume an infinite deck of cards with values randomly ranging from 0 to 1. We borrow these simplifying assumptions from McAdams (2012). See also von Neumann & Morgenstern’s model of poker, 1953, pp. 187-188: “each player draws a number $s + 1, \ldots, S$ instead. The idea is that $s = S$ corresponds to the strongest possible hand, $s = S - 1$ to the second strongest hand, etc., and finally $s = 1$ to the weakest ... Thus the game begins with two chance moves: The drawing of number $s$ for player 1 and the for player 2, which we denote $s_1$ and $s_2$” (footnote omitted).

7 See Rule #4 below. Thus, not even the judge is allowed to see the private hole cards of the players until after the betting round.

8 Compare von Neumann & Morgenstern, 1953, p. 188: “The next phase of the general game of Poker consists of the making of ‘Bids’ by the players.”

9 As an aside, the bets of the players in this simple game can be compared to the “investment levels” of the litigants. That is, during the process of the litigation, each litigant must decide (independently of the decision of the other litigant) how much effort to invest in his or her case. Unlike litigation, however, where the level of investment arguably has some effect on the outcome of the case, in poker the size of one’s bet has no effect on the strength of one’s hole cards.

10 Again, compare von Neumann & Morgenstern, 1953, pp. 189-190: “We shall express ... restrictions on the size of bids ... in the simplest possible form: We shall assume that the two numbers $a, b$

$$a > b > 0$$
Thus, there is only one round of betting, and the players are not allowed to “call” or “reraise.”\textsuperscript{12} Also, for further simplicity and tractability, we assume $a = 2b$.\textsuperscript{13}

6. Lastly, the players reveal their cards at time $T_4$, and the dealer/judge declares a winner based on the following sub-rules:

a. if both players P and D have submitted high bets, the player with the highest card wins both bets for a net gain of $+a$ (in the event of a tie, the players get back their original bets and play again);

b. if both players have submitted low bets, the player with the highest card wins both bets for a net gain of $+b$ (in the event of a tie, the players get back their original bets and play again);

c. if one player had submitted a high bet and the other a low bet, the player submitting the high bet automatically (by default) wins both bets for a net gain of $+b$, regardless of the values of the players’ cards.

To recap and summarize, our model of the poker-litigation game isolates two important variables of the game: (i) the size of each player’s bet or level of investment in his case (captured by the variables $a$ and $b$), and (ii) the relative strength of his case (captured by the variable $t$). Moreover, our model captures two essential features that poker and litigation have in common: both are strategic games of incomplete information. Specifically, the poker-litigation game is a strategic game because the players must choose their strategies (i.e. place their bets) independently, without communicating with each other, and their choices, once made, are final.\textsuperscript{14} In addition, the poker-litigation game is a game of incomplete information, and this condition makes the solution of the game non-trivial. If the values of the players’ private cards were common knowledge (that is, if the players could see each other’s hole cards before placing their bets), the solution would be trivial. The players would always make “optimal” bets: bet low

\textsuperscript{12} We make these assumptions for mathematical tractability and ease of exposition. For an overview of such simplifying assumptions in poker, see von Neumann & Morgenstern, 1953, pp. 186-188.

\textsuperscript{13} That is, a high bet is like a “double-or-nothing” bet, since a high bet is twice as large as a low bet.

\textsuperscript{14} Or in the words of John Nash, 1951, p. 286: “each participant acts independently, without collaboration or communication with any of the others.” See also Osborne & Rubinstein, 1994, p. 14: “For a situation to be modeled as a strategic game ... the players [must] make decisions independently, no player being informed of the choice of any other player prior to making his own decision.”
when the value of one’s card is lower than the other player’s card to minimize one’s losses.\textsuperscript{15}

Given this simple set of rules (see Rules #1-6 above) and given the temporal and strategic structure of the game, what is the optimal or best strategy in the poker-litigation game?\textsuperscript{16} Put another way, given that the poker-litigation game is a game of incomplete information, when should a player bet high or bet low in order to minimize his losses and maximize his gains? Stated formally, does any strategy in this game guarantee a player a non-negative expected payoff (cf. McAdams, p. 1)? Below, we proceed to find the solution or symmetric equilibrium (in pure or mixed strategies) of the poker-litigation game.

4. Solution

The most well-known solution concept in game theory is the “Nash equilibrium” (Nash, 1951).\textsuperscript{17} Stated formally, a game has an equilibrium point when “no player can profitably deviate [i.e. play a different strategy], given the actions of the other players” (Osborne & Rubenstein, 1994, p. 15). Stated simply, a strategy is a Nash equilibrium when it is a “best response” given the possible choices of the other players.

Following McAdams (2012), who applies Nash’s contradiction method of analysis (Nash, 1951, p. 293), we conjecture that an equilibrium exists with a threshold $t$ and probability $p$ such that a player always bets high given any card whose value is greater than $t$ (i.e. when such player has a “strong” case) and bets low with probability $p$ given any card whose value is lesser than $t$ (when such player has a “weak” case).

To test this conjecture, we consider three different treatments or litigation scenarios: (i) case #1, when the value of a player’s private card is equal to $t$ (in other words, when the player has a “marginal” case in which he is just as likely to win as to lose); (ii) case #2, when the value of his hole card is below $t$ by some unknown quantity $x$, or $t - x$ (i.e. when the player has a “weak” case); and (iii) case #3 when the value of his hole card exceeds the threshold $t$ by $x$, or $t + x$ (the player has a “strong” case).

4.1. case #1: $t$

Assume that a given player (say, player P, the plaintiff) likes to hedge his bets: he bets high, $a$, when the value of his hole card is greater than or equal to a certain

\textsuperscript{15} For an overview of the role of information in game theory, see Roth & Malouf, 1979. See also Baird, et al., 1994, ch. 3.

\textsuperscript{16} Stated formally, what is the equilibrium strategy (i.e. Nash equilibrium) of players P and D?

\textsuperscript{17} For an overview and formal presentation of this solution concept, see, e.g., Osborne & Rubinstein, 1994, pp. 14-15. See also Baird, et al., 1994, pp. 19-23.
threshold \( t \) and bets low, \( b \), when his card is below this threshold.\(^{18}\) In plain English, the intuition behind this strategy is that one should place larger bets the stronger one’s card is in order to maximize one’s gains, since one has a higher chance of winning the game when one has a strong card.\(^{19}\)

Now, assume that the value of Player P’s card is equal to \( t \) and, in addition, assume that his opponent, player D, also likes to hedge his bets.\(^{20}\)

Player P must consider two possible scenarios: either player D’s card is greater than \( t \) (in which case player D bets high, \( a \)) or it is lesser than \( t \) (in which case D bets low, \( b \)). If player D bets low \( b \) because his hole card is lesser than \( t \), then player P will win \(+b\) regardless whether he, player P, makes a high or low bet. By contrast, if the value of player D’s card is greater than \( t \), then player P will lose his bet (regardless whether he, player P, has made a high or low bet), but because player P loses more in this scenario when he bets high than when he bets low, player P should prefer to bet low in this case in order to minimize his losses, since \( b < a \). But notice that this preference is inconsistent with player P’s strategy of making high bets when the value of his hole card is greater than or equal to the threshold \( t \); therefore, the \( m \)-type strategy cannot be an equilibrium strategy or best response for player P.

This analysis leads us to a larger point about our game: a player can still lose even with a high-value hole card, depending on the hole card of the other player, since the ultimate outcome of the game depends on the respective values of the hole cards of both players (and conversely, a player can still win even when he holds a low-value hole card). In other words, like litigation, playing this game and making bets are risky activities because there are costs and benefits of making high bets (and low bets). In equilibrium, these costs should be equal to the benefits—thus we proceed to define these costs and benefits to find the equilibrium strategy of this game (again, our analysis is based on the solution in McAdams, 2012).

First, suppose that the value of player P’s card is equal to \( t \) and that the other player’s card (player D’s card) is greater than \( t \), an event which occurs with probability \( 1 – t \).\(^{21}\) Player P will thus lose the game regardless whether he bets high or bets low, but he loses \(-a\) when he bets high and \(-b\) when he bets low.

\(^{18}\) For reference, we shall designate this strategy as an “\( m \)-type” or mixed strategy.

\(^{19}\) And conversely, one should make smaller bets the weaker one’s card is in order to minimize one’s losses.

\(^{20}\) Stated formally, assume player D is also an \( m \)-type player. That is, assume player D is playing the same \( m \)-type strategy against player P that P is playing against D.

\(^{21}\) This event occurs with probability \( 1 – t \) because of Rule #2 of the poker-litigation game, or stated formally, because we are assuming that the possible values of each player’s card in this game are independent and identically distributed (i.i.d.) random variables on the interval \([0, 1]\). Thus, if \( t \) is probability that player P or D’s card is equal to \( t \), then \( 1 – t \) is the probability that player D’s card is greater than \( t \).
(That is, in this case player P loses an additional amount, the difference between \(a\) and \(b\), and this differential loss is equal to \(b\) since we have previously assumed that \(a = 2b\).) Thus, from player P’s perspective, the true cost a making a high bet in this scenario is \(b(1 - t)\).

Next, suppose player D’s private card is below the threshold \(t\), an event which occurs with probability \(t\).\(^{22}\) Also, suppose player D bets high in this scenario with probability \(p\). When player D bets low in this case, player P wins \(+b\) (the value of player D’s low bet) regardless whether player P himself bets high or low. But when player D bets high (an event which occurs with probability \(p\)), then player P wins \(+a\) (or \(2b\) since \(a = 2b\)).\(^{23}\)

Thus, the benefit to player P from making a high bet in this scenario (i.e. when player D’s card is below \(t\)) can be stated formally as follows:

\[
(b + a)(t)(p), \text{ or } 3b \times t \times p \text{ (since } a = 2b), \text{ or } 3btp.
\]

And in equilibrium, the costs and benefits of making high bets in both scenarios must be the same:

\[
b(1 - t) = 3btp, \text{ or equivalently (after simplification): } 1/t = 1 + 3p
\]

Furthermore, in equilibrium this equality must hold not only at the threshold \(t\) but also everywhere above and below \(t\). Thus, we consider two additional scenarios or cases: case #2 in subsection 4.2 below, when the value of a player’s hole card is below \(t\) (i.e. when such player has a “weak” case), and case #3 in subsection 4.3, when his card exceeds \(t\) (when he has a “strong” case).

### 4.2. case #2: \(t - x\)

Next, we proceed to determine a player’s best strategy or best response when the value of his hole card is less than \(t\).

For simplicity, consider this second type of case from player P’s perspective.\(^{24}\) Assume the value of player P’s private card is below the threshold \(t\) by some unknown quantity \(x\), or \(t - x\). In summary, player P has two options in his strategy set: he can either make a high bet \(a\) or make a low bet \(b\). But what are

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\(^{22}\) See the preceding footnote. This event occurs with probability \(t\) because, by definition, if player D’s card is above \(t\) with probability \(1 - t\), then his card will be below \(t\) with probability \(t\).

\(^{23}\) As an aside, it is worth asking, why would player D ever bet high with a low-value card? Because had player P placed a low bet instead of a high one, then player D would have won \(+b\) instead of losing \(-a\).

\(^{24}\) Recall, however, that our analysis also applies to player D since this game is symmetrical.
player P’s payoffs for each such strategy in this case compared to his payoffs in the first type of case (i.e. when the value of player P’s private card is equal to t)?

In summary, if player P bets high in this second type of case (i.e. when the value of his hole card is $t - x$), then he, player P, loses $-a$ rather than winning $+a$ only when two conditions are met: when (i) the value of player D’s private hole card lies on the interval $[t - x, t]$ and (ii) player D bets high. Since player D will bet high in this particular scenario (i.e. when his hole card is on the interval $[t - x, t]$) with probability $p$ times $x$ (recall that by assumption a player bets high with probability $p$ when the value of his hole card is below $t$), player P’s payoff in this scenario is $-2apx$ lower (or stated equivalently: $-4bpx$ lower, since $a = 2b$) than when the same scenario occurs in the first type of case (when the value of player P’s hole card is equal to $t$).

If, however, player P bets low in this second type of case, then he loses $-b$ rather than winning $+b$ only when two conditions are met: (i) when the value of player D’s card lies on the interval $[t - x, t]$, and (ii) when player D bets low. Since player D will bet low with probability $(1 - p)x$, player P’s payoff is now $-2b(1 - p)x$ lower in this scenario than when the same scenario occurs in the first type of case (threshold = $t$).

Next, to find player P’s best response, we set player P’s revised payoffs for both strategies equal to each other and simplify as follows:

$$2p = (1 - p), \text{ or } p = \frac{1}{3}$$

In plain words, randomizing between high and low bets is a best response given any card less than $t$. Or, stated formally, a player (player P and, by symmetry, player D) is indifferent between making high or low bets for all values of $x$ only when the other player is placing high bets with probability $1/3$. Otherwise, if one of the players were placing high bets with a probability greater than or lesser than $1/3$, the other player could adjust his betting strategy accordingly to increase his gains (or reduce his loses). Furthermore, given that $p = 1/3$, when we substitute this value for $p$ in our original equilibrium equation, $1/t = 1 + 3p$, and solve for $t$, we see that $t = 1/2$ or $0.5$. Thus, the optimal threshold is $0.5$ and a player should bet high with probability $1/3$ (or bet low with probability $2/3$) when the value of his hole card is below this critical threshold.

4.3. case #3: $t + x$

Lastly, we wish to confirm a player’s best strategy (best response) when the value of his hole card is greater than $t$.

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25 We set these “$t - x$” payoffs equal to each other because in equilibrium, a player (player P and, by symmetry, player D) should be indifferent between making high and low bets.
Again, for simplicity, consider this third type of case from a particular player’s perspective, player P, although the same analysis also applies to player D since this game is symmetrical, but now, assume the value of player P’s private card exceeds the threshold \( t \) by some unknown quantity \( x \), or \( t + x \). As before, player P has two possible moves in this game (i.e. he can make a high bet \( a \) or a low bet \( b \)), so given this case (when the value of player D’s card lies on the interval \([t, t + x]\)), we proceed to find player P’s payoffs for each such strategy compared to his payoffs in the first type of case (when the threshold is set to \( t \)).

If player P makes a high bet in this case (i.e., when his card exceeds \( t \) by an amount \( x \)), then he wins \(+a\) rather than losing \( -a \) when the value of player D’s card lies on the interval \([t, t + x]\), since by assumption a player always makes a high bet a when his card is greater than \( t \). Thus, player P’s payoff in this case is \(+2ax\) higher (or, \(+4bx\) higher, since \( a = 2b \)) than his payoff when the same scenario occurs in the first type of case (when threshold = \( t \)).

If, however, player P bets low in this scenario, he wins \(+b\), which is the same amount he would have won in this scenario in the first type of case (when threshold = \( t \)), when the same two conditions are met: when (i) the value of player D’s hole card is less than \( t \) and (ii) player D makes a low bet. Since these are the same conditions under which player P wins in the first type of case, player P’s payoff in this third scenario (case #3) is identical to his payoff when this same scenario occurs in case #1.

Since the payoff from making a high bet is increasing over the interval \([t, 1]\), and since the payoff from making a low bet is constant, since by definition \( b = b \), player P (and, by symmetry, player D) strictly prefers to bet high when the value of his hole card is above \( t \). Further, since this is a symmetrical game, this same conclusion applies to player D.

To recap, given the rules and payoff structure of our game, a player should bet high with probability \( 1/3 \) (or bet low with probability \( 2/3 \)) when the value of his private card is below \( t \) and bet high in all other cases.

Before concluding, we wish to say a few words about the artificiality of our poker-litigation game.\(^{26}\) Admittedly, our poker-litigation game is a simple representation or model of a more complex activity (civil and criminal litigation), but parsimony and simplicity can also be a virtue for several reasons (see, e.g., von Neumann & Morgenstern, 1953, pp. 186-188; see also Guerra-Pujol, 2010b, pp. 630-631). One is tractability. A simple model is easier to analyze and work with than the “real world” is. That is, a simpler representation of poker (or

\(^{26}\) Or in the words of Nate Silver, 2012, p. 225, we wish to explain why our model is “sophisticatedly simple” (emphasis in original, footnote omitted). In this respect, consider the work of John von Neumann and Oskar Morgenstern (1953), John Nash (1951), Lloyd Shapley (1950), and Harold Kuhn (1950), all of whom invented simplified poker games to illuminate fundamental principles of game theory.
litigation) is more tractable than the actual real-world game being modeled, or in the words of John von Neumann and Oskar Morgenstern (1953, p. 186), the founders of game theory, “actual Poker [like actual litigation, we would add] is really much too complicated a subject for an exhaustive discussion.” Another virtue of simplicity is clarity. The process of creating a simple model forces us to identify the players and their strategy sets, define our terms, and state our assumptions, and unlike a purely verbal description of reality, a simple formal model allows us to make falsifiable predictions about the real world. But most importantly, a simple well-designed model can capture the essence of a strategic interaction that is present in a more complex real-world situation. Specifically, our simple model of litigation can help us isolate and demonstrate some fundamental features of the legal process.

5. Conclusion

In this paper, we have presented a simple model of the process of litigation, the “poker-litigation game,” based on the premise that litigation has much in common with the game of poker. Our model is useful because it isolates two important variables of the game: (i) the size of each player’s bet or level of investment in his case (captured by the variables $a$ and $b$), and (ii) the relative strength of his case (captured by the variable $t$). In addition, our model provides an alternative explanation of the existence of frivolous claims (or “negative-expected value” lawsuits) as well as prosecutorial misconduct, for one of the main lessons of our model is that (from the perspective of the players) there is an optimal level of bluffing in the poker-litigation game, with bluffing defined as placing a high bet probabilistically or randomly even when the strength of one’s case is weak.

But what is this optimal level? In the case of the poker-litigation game, the optimal amount of bluffing (given the rules and payoff structure of our game) is to bet high with probability $1/3$ when the value of one’s private card is below the critical threshold $0.5$. Likewise, in real-world litigation games, we would expect the optimal level of bluffing (i.e. frivolous claims) to be a function of two variables: (i) the amount at stake, or in the language of our model, the sum of the bets placed, and (ii) the relative strength and weakness of player P and D’s cases.

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Appendix

As an aside, I set forth in this appendix my initial (and failed) attempt to solve the poker-litigation game (i.e. to find the optimal or best strategy in this game). I include my false start to contrast it with the correct solution to the game, which I obtained from McAdams (2012).

In summary, in my initial failed attempt to solve the game, I considered three types of players: “a-type” players who always make high bids, “b-type” players who always make low bids, and “m-type” players whose bets depend on the value of their hole cards.

a-type players

First, I considered an “a-type” player who always submits a high bid, a. An a-type player will lose his bet only when two conditions are met: (i) when the other player has made a high bid, and (ii) when his card has a lower value than the card of the other player. Otherwise, an a-type player always win (as per sub-rule #6c above), and thus, if such a player plays the litigation game an infinite number of times, his expected payoff is equal to the sum of \( pa - (1 - p)(a) \) when the other player makes a high bid—where \( p \) is the probability that his card is higher than the other player’s card—, and \(+a\) when the other player makes a low bid.

b-type players

Next, I considered a “b-type” player whose strategy is to always submit a low bid, b. In summary, a b-type player wins the game only when two conditions are met: (i) when his card has a higher value than the card of the other player, and (ii) when the other player has submitted a low bid. Otherwise, when neither of these conditions are met, a b-type player will always lose. Thus, if a b-type player plays the litigation game an infinite number of times, his expected payoff is equal to the sum of \(-b\) when the other player submits a high bid and \( pb - (1 - p)(-b) \) when the other player submits a low bid, where \( p \) is the probability that his card is higher than the other player’s card.

m-type players

Lastly, I considered an m-type player who plays a mixed or probabilistic strategy, that is, a player who makes a low bid, b, when his card is below a certain threshold, namely 0.5, and who makes a high bid, a, when his card is above this critical threshold.\(^{27}\) An m-type player has many ways of winning or losing the litigation game, depending on what strategy the other player is choosing and on

\(^{27}\) Such a mixed strategy is probabilistic in nature because the value of one’s hole card is a uniform independent random number on the interval \([0,1]\) as per Rule #2 of the poker-litigation game.
the value of other player’s card. Thus, we find an \( m \)-type player’s expected payoff against \( a \)-type players, \( b \)-type players, and other \( m \)-type players as follows:

First, an \( m \)-type player’s expected payoff against an \( a \)-type player, i.e. a player who always makes high bids, is equal to the following value:

\[
E(m|a) = q(-b) + (1 - q)[pa - (1 - p)(-a)]
\]

where \( E(m|a) \) is the expected payoff of an \( m \)-type player against an \( a \)-type player, \( q \) is the probability that the \( m \)-type player’s card is below the critical threshold value 0.5 and where \( p \) is the probability that his, the \( m \)-type player’s, card is higher than the other player’s card. In other words, when playing against a high-bid, \( a \)-type player, an \( m \)-type player loses his bet \( b \) when the value of his hole card is below the critical threshold, i.e., when \( q < 0.5 \), and wins \(+a\) with probability \( p \) but loses \(-a\) with probability \( 1 - p \) when the value of his hole card exceeds the critical threshold, i.e. when \( q > 0.5 \).

Next, what happens when an \( m \)-type player plays against a \( b \)-type player, that is, a player who always makes low bids? In this case, the \( m \)-type player’s expected payoff is equal to:

\[
E(m|b) = q(pb - (1 - p)(b)) + (1 - q)(b)
\]

where \( E(m|b) \) is the expected payoff of an \( m \)-type player against an \( b \)-type player and \( q \) is the probability that the \( m \)-type player’s card is below the threshold value 0.5. In plain English, when playing against a low-bid, \( b \)-type player, an \( m \)-type player wins \(+b\) with probability \( p \) but loses his bet \( b \) with probability \( 1 - p \) when the value of his hole card is below the critical threshold, i.e. when \( q < 0.5 \), and he wins the bid \(+b\) when the value of his hole card exceeds the critical threshold, i.e. when \( q > 0.5 \).

Third and last, what happens when an \( m \)-type player plays against another \( m \)-type player? That is, what happens when his opponent also plays the same mixed or probabilistic strategy? Now, the \( m \)-player’s expected payoff not only depends on \( q \), the probability that the \( m \)-type player’s card is below the critical threshold value 0.5; his expected payoff is also a function of \( r \), the probability that the other player’s card is below this threshold. Stated formally, when playing against another \( m \)-type player, an \( m \)-type player’s expected payoff is equal to the following value:

\[
E(m|m) = qr[pb - (1 - p)(b)] + q(1 - r)[-b] + (1 - q)(r)[b] + (1 - q)(1 - r)[pa - (1 - p)(-a)]
\]

where \( E(m|m) \) is the expected payoff of an \( m \)-type player against an another \( m \)-type player, \( q \) is the probability that the \( m \)-type player’s hole card is below the critical threshold value 0.5, and \( r \) is the probability that the other player’s hole card is below this threshold. Since this is such a lengthy equation, we shall break
it down into its four constituent parts and explain the substance of each part in plain words as follows:

\[ qr[pb – (1 – p)(b)] + q(1 – r)[−b] + (1 – q)(r)[b] + (1 – q)(1 – r)[pa – (1 – p)(−a)] \]

Notice that each part of this lengthy expected payoff equation corresponds to one of following four possible scenarios:

1. Scenario #1 . . . both players’ hole cards are below the critical threshold.
2. Scenario #2 . . . the first m-type player’s hole card is below the threshold; the other player’s hole card is above the threshold.
3. Scenario #3 . . . the first m-type player’s hole card is above the threshold, while the other player’s hole card falls below the threshold.
4. Scenario #4 . . . both players’ hole cards exceed the critical threshold.

Thus, depending on which scenario occurs, that is, depending on the values of the hold cards of the players, an m-type player will earn the following payoffs:

1. In scenario #1, an m-type player wins +b with probability p but loses his bet b with probability 1 − p.
2. In scenario #2, an m-type player loses −b.
3. In scenario #3, an m-type player wins +b.
4. And in scenario #4, an m-type player wins +a with probability p but loses −a with probability 1 − p.

To recap, in my initial failed attempt to solve the poker-litigation game, I considered three types of players (or three types of strategies)—“a-type” players who always bid high, “b-type” players who always bid low, and “m-type” players who bid high or low depending on the value of their hole card—and I also figured out the expected payoffs of the strategies of each type of player. But it was at this stage that I was “stumped,” unable to determine which strategy is the optimal strategy (and this unable to find what type of player earns the highest expected payoffs in this game).

In sum, because of my inability to solve the game using traditional methods, I instead turned to McAdams’s (2012) elegant solution to steer me in the right direction.

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