Elasticity of substitution and the stagnation of Italian productivity

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The aim of this paper is to investigate the roots of the stagnation in the Italian total factor productivity (TFP). The analysis focuses on the specific pattern of technical progress in determining the dynamics of the TFP. This analysis cannot be done with Cobb-Douglas technology, but requires the employment of a constant elasticity of substitution (CES) production function that allows distinguishing between the direction and the bias of technical progress. We employ a CES specification embodying both labor- and capital-augmenting technical change, with a $\sigma$ less than 1. We obtain three main results. (1) There seems to have been a structural break around the mid-1990s in the direction and bias of technological change; (2) The first half of the sample features a labor-augmenting technical change and a capital bias; and (3) In the second part of the sample, both these characteristics seem to disappear, and the evolution of factor endowments assumes a key role. This fact may be seen as one of the potential causes of the stagnation in Italian productivity.

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Introduction

In this paper, we build on the nonlinear dynamic model of the Italian economy of Saltari et al. (2012) and Saltari, Wymer, and Federici (2013). The main result of those papers is that the weakness of the Italian economy in the last two decades has been the total factor productivity (TFP) slowdown. The aim of this paper is to investigate the roots of this phenomenon. The analysis focuses on the specific pattern of technical progress in determining the slowdown of TFP growth. Of course, this analysis cannot be done with the Cobb-Douglas technology, where technical progress is only Hicks neutral, but requires a constant elasticity of substitution (CES) production function that allows distinguishing between the direction and the bias of technical progress.1

In addition, the Cobb-Douglas production function assumes that factor shares are constant. The stability of the income labor share is a key foundation in most macroeconomic models taken for granted until very recently. Recent empirical evidence shows that since the 1980s, the labor share has dramatically changed its behavior. Differently from the stylized fact of aggregate factor shares constancy, the last three decades have viewed a continuous decline of the labor share owing to a range of factors (e.g. internationalization of production process, regulatory and institutional changes in the labor market, privatization program; see, for instance, Abraham, Konings, and Vanormelingen 2008), thus casting doubt on the shares invariance.

The decline of labor share is not limited to Italy, but occurred in the large majority of industrialized countries (see Elsby, Hobijn, and Şahin 2013; Karabarbounis and Neiman 2013). Figure 1 shows the dynamics of aggregate labor share in Italy, together with those of France, Germany and the USA.
starting from 1970. Actually, until the 1970s, the labor share was approximately constant in almost all the countries, thus confirming one of the stylized fact highlighted by Kaldor (1961).

Starting from the 1980s, the decline of labor share becomes evident: for the period 1980–2011, the reduction is 11% for Italy and France, 8% in Germany, and 6% in the USA. One of the advantages of using CES production function is that it dispenses with the factor shares constancy.

Unlike most of the literature, this investigation employs a CES specification with both labor- and capital-augmenting technical change. While for labor input we keep the traditional constant growth rate representation, for capital stock, we impose a particular structure with Information and Communication Technologies (ICT) capital playing a key role. To the best of our knowledge, the contribution of the ICT sector to the productivity dynamics has not been explicitly modelled. The bulk of the literature assumes that technical progress grows at a constant rate without giving a specific structure (a partial exception is Klump, McAdam, and Willman 2007, 2008).

In our model, we take a stance about how ICT impacts on technical progress: particularly, we assume that the productivity of the traditional capital stock is augmented by the ICT capital stock. This makes a difference with respect to the traditional approach, in that the effect of ICT is not constant, but reflects the pace of investment in innovative technologies. As we will see below, the evolution of the TFP follows closely the one of ICT capital stock. Moreover, very recent econometric estimates of IMF (2015) suggest that lower product market regulation and more intense use of high-skilled labor and ICT capital inputs, as well as higher spending on Research and Development activities, contribute positively and with statistical significance to TFP (chapter 3, p. 104–105).

In this exercise, we do not calibrate the parameters of the CES production function, but use the previous estimation results of Saltari et al. (2012), whose main estimates are reported in the next section.

It should be stressed that they imply an elasticity of substitution significantly less than 1. Such a value is by now well grounded in the empirical literature (see, for instance, Klump, McAdam, and Willman 2008; León-Ledesma, McAdam, and Willman 2010; Mallick 2012; for a critical discussion of the traditional methodology of estimating the elasticity of substitution, see Federici and Saltari, forthcoming). On theoretical grounds, an elasticity of substitution larger than 1 implies that any amount of
output can be produced with either zero amount of capital or zero amount of labor, which is clearly absurd (note that the Cobb-Douglas almost shares this last property).

We obtain three main results: (1) There seems to have been a structural break in the direction and bias of technological change around the mid-1990s, that is, at the midpoint of the sample; (2) The first half features a labor-augmenting technical change and a capital bias; and (3) In the last part of the sample, both these characteristics seem to disappear, and the evolution of factor endowments assumes a key role. The disappearance of the contribution from technical progress may be viewed as one of the potential causes of the stagnation in Italian productivity.

The paper is organized as follows. The next section briefly recalls our production function and normalizes it. Section 3 compares the Cobb-Douglas and CES computation of TFP; it also discusses the determinants of technological progress. Section 4 describes the evolution of the direction and factor bias. Section 5 concludes.

The technology

Our theoretical framework is one of dynamic disequilibrium with traditional and ICT (which includes communication equipment, hardware and software) investment functions, skilled and unskilled labor sectors, and price determination under imperfect competition (see Appendix B for an overview of the model presented in Saltari et al.2012).2

The production technology is given by the following CES aggregate production function:

$$Y_t = \beta_3 [(C_t^\gamma K_t)^{-\beta_1} + (\beta_2 e^{\mu t} L_t)^{-\beta_1}]^{-\frac{1}{\beta_1}}. \quad (1)$$

In Equation (1), $Y$ is the output, $\beta_3$ is a measure of the TFP and $\beta_1$ defines the elasticity of substitution through the relation $\sigma = 1/1 + \beta_1$. Moreover, we have two factor-augmenting technical progress. The efficiency of traditional capital is augmented by ICT capital, $C$, with a weighting factor equal to $\gamma$, a proxy of the relative share of the ICT in total capital. As for labor-augmenting technical progress, we follow the bulk of the literature in assuming that it grows at a constant rate $\mu = \lambda_K + \gamma \lambda_C$, where $\lambda_K$ and $\lambda_C$ are the rates of technical progress in the use of capital $K$ and innovative (i.e. ICT) capital, $C$, with $\beta_2$ as a scaling factor. That way, labor efficiency partly depends on the growth of ICT capital through $\gamma \lambda_C$. Thus, unlike most of the literature, labor efficiency is closely linked to capital efficiency. Finally, $L$ denotes total employment, defined as a Cobb-Douglas function of the skilled and unskilled labor components.

The model allows us to estimate, among other things, the parameters of the production function for the sample period 1981:Q4–2005:Q2.3 For the reader’s convenience, the estimates of the parameters of the production function are given in Table 1. Estimates of the parameters were found by a Gaussian estimator of the nonlinear model subject to all constraints inherent in the model.4

Normalization

We normalize the production function so that the variables are independent of the unit of measure, that is, they are in index number form. Moreover, normalization defines specific ‘families’ of CES functions whose members all share the same base period but are distinguished by the elasticity of substitution (and only the elasticity of substitution). This is a particularly appealing feature for us

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\beta_1$</th>
<th>$\sigma = \frac{1}{1+\beta_1}$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\gamma$</th>
<th>$\lambda_K$</th>
<th>$\lambda_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.54</td>
<td>0.65</td>
<td>67.04</td>
<td>0.44</td>
<td>0.04</td>
<td>0.003</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.015)</td>
<td>(0.734)</td>
<td>(0.048)</td>
<td>(0.007)</td>
<td>(0.001)</td>
<td>(0.006)</td>
</tr>
</tbody>
</table>

Notes: Notice that $\beta_3, \lambda_K, \lambda_C$ and thus $\mu$ are all expressed on a quarterly basis. Thus, for instance the yearly growth rate of labor efficiency is $\mu = 0.3\% \cdot 4 = 1.2\%$.

Standard errors in parenthesis.
as we compare the CES specification of the production function with the Cobb-Douglas. Finally, normalization is necessary for a number of reasons, such as securing the basic property of CES production (the strictly positive relation between the elasticity of substitution and the level of output; see Grandville 2009), and is useful to determine the direction and bias of technical progress. The first paper that used a normalized CES production function to estimate the elasticity of substitution and a (flexible) specification for technological progress is Klump, McAdam, and Willman (2007).

We set the base period for the normalization at the middle of the sample (which coincides with that of the sample averages), that is, \( t = 48 \) corresponding to 1991:Q4. We denote it by the index 0. Normalization implies that all the variables are expressed in terms of their baseline values, that is, \( K_0, L_0 \) and \( Y_0 \).

To normalize the production function, we start with our production function written as:

\[
Y_t = \beta_3 \left( (\text{KIT}_t)^{1-\beta_1} + (\beta_2 e^{\mu (t-t_0)} L_t)^{-\beta_1} \right)^{-\frac{1}{\beta_1}},
\]

where \( t_0 \) is the base period used for normalization, and to simplify notation we set \( \text{KIT}_t = C_{\gamma} K_t \).

Under imperfect competition, factor compensation is subject to a constant mark-up, denoted by \( \beta_{13} \), so that in any period \( t \) the following relation holds:

\[
i_t \text{KIT}_t + w_t L_t = Y_t,
\]

where \( i_t \) is the real interest rate and \( w_t \) is the wage rate.

In the reference period, capital compensation is:

\[
i_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial \text{KIT}_0} = \frac{(\beta_3)^{-\beta_1}}{\beta_{13}} \left( \frac{Y_0}{\text{KIT}_0} \right)^{1+\beta_1} \beta_i,
\]

so that total capital compensation over total factor income, or the capital share (\( \pi_0 \)), in the base period is:

\[
\pi_0 = \frac{i_0 \text{KIT}_0}{Y_0} \beta_{13} = \frac{(\beta_3)^{-\beta_1}}{\beta_{13}} \left( \frac{Y_0}{\text{KIT}_0} \right)^{\beta_i}.
\]

Proceeding in the same way for the labor share and substituting in Equation (2), we get the normalized production function:

\[
Y_t = \left[ \pi_0 (\text{KIT}_t)^{1-\beta_1} + (1-\pi_0) L I T_t^{-\beta_1} \right]^{-\frac{1}{\beta_1}},
\]

where output, labor and capital are already expressed in index form, and \( L I T_t = e^{\mu (t-t_0) L_t} \). In the normalized production function, the only crucial parameter is \( \beta_1 \).

Of course, in the Cobb-Douglas case where \( \beta_1 = 0 \), the production function becomes:

\[
Y_{t}^{\text{CD}} = (\text{KIT}_t)^{\gamma} (L I T_t)^{1-m_0}.
\]

**Technical progress**

The rate of growth of output is determined by the time log derivative of Equation (4):

\[
\frac{\dot{Y}_t}{Y_t} = \epsilon_{r, \text{KIT}} \left( \frac{\dot{K}_t}{K_t} + \gamma \frac{\dot{C}_t}{C_t} \right) + \epsilon_{r, \text{LIT}} \left( \frac{\dot{L}_t}{L_t} + \mu \right)
\]

\[
= \pi_0 \left( \frac{Y_t}{\text{KIT}_t} \right)^{\beta_1} \left( \frac{\dot{K}_t}{K_t} + \gamma \frac{\dot{C}_t}{C_t} \right) + (1-\pi_0) \left( \frac{Y_t}{L I T_t} \right)^{\beta_1} \left( \frac{\dot{L}_t}{L_t} + \mu \right),
\]

where \( \epsilon_{r, \text{KIT}} = \frac{\partial Y}{\partial \text{KIT}} / \frac{Y}{\text{KIT}} \) and \( \epsilon_{r, \text{LIT}} = \frac{\partial Y}{\partial L I T} / \frac{Y}{L I T} \) are the elasticities of output with respect to inputs in efficiency units. In this framework, the capital-augmenting technical change is \( \pi_0 \left( \frac{Y_t}{\text{KIT}_t} \right)^{\beta_1} \gamma \frac{\dot{C}_t}{C_t} \), while the
labor-augmenting factor is \((1 - \pi_0)(\frac{Y_t}{L_t})^\beta_1 \mu\). The contribution of each input-augmenting factor to the rate of growth of output can be split into two components: one is the pure technical progress \((\gamma_C, \mu)\); the other is the sensitivity of output to technical change \((\pi_0(\frac{Y_t}{L_t})^\beta_1, (1 - \pi_0)(\frac{Y_t}{L_t})^\beta_1)\). In the Cobb-Douglas case, \(\beta_i = 0\), and the elasticities are simply the income shares.

It is worth noticing that, unlike the traditional specification, capital-augmenting technical progress depends on the dynamics of the stock of ICT capital. This choice of capital-augmenting technical progress is motivated by the key role played by ICT in the dynamics of productivity in industrialized countries at least since the 1990s. The relevance of ICT is particularly important for Italy (although in a negative sense as shown below). However, by the impossibility theorem of Diamond, McFadden, and Rodriguez (1978), we cannot separately identify this role from that of the elasticity of substitution unless one imposes a specific structure on technical change. In defining this structure, we abandon the traditional specification of a constant rate of growth of technical progress.

In particular, our model assumes that the efficiency of the traditional capital stock is augmented by ICT capital according to a weighting factor equal to \(\gamma\). Since labor-augmenting is defined as \(\mu = \lambda K + \gamma \lambda C\), the same factor also increases labor efficiency. This way, we are assuming that ICT investment also improves labor productivity. As far as we know, this specification of technical progress was first introduced in the Kaldor (1957) growth model.\(^5\)

### The advantage of using a CES production function

The contribution of technical progress to the growth of output (which is the result of growth accounting exercises) is computed through the Solow residual. To see the relevance of the elasticity of substitution, let us compare the computation of TFP using the Cobb-Douglas production function with that obtained with the CES. To this end, we calibrate Equation (5) with the three key parameter estimates given in Table 1 \((\sigma, \gamma, \mu)\).\(^6\)

\[
\text{TFP}_{\text{CES}} = \frac{\dot{Y}_t}{Y_t} - \left( e_{Y, K} K_t + e_{Y, L} L_t \right).
\](6)

In the Cobb-Douglas case, the TFP\(_{\text{CD}}\) becomes

\[
\text{TFP}_{\text{CD}} = \frac{\dot{Y}_t}{Y_t} - \left( \pi_0 K_t + (1 - \pi_0) L_t \right).
\](7)

The result of these two growth accounting exercises is illustrated in Figure 2.

A notable feature of the graph is that in the first part of the sample period, the TFP from the Cobb-Douglas lies above that of the CES, while in the second part, they essentially overlap. A plausible interpretation is that our estimated \(\sigma\) is about two-thirds, while the Cobb-Douglas technology has a \(\sigma\) equal to 1. It follows that, from the property of general means (see Grandville 2009), the Cobb-Douglas output and correspondingly its growth rate is higher than in our CES case. Thus, there is a different weighting of the input growth rates in the two functions: the Cobb-Douglas uses fixed weights (equal to the income shares), while the CES uses the time-variable output-factor elasticities. As we will see, the gap between the two TFPs, and its narrowing until it vanishes at the middle of the sample period, can be explained by splitting the TFP into its components.

The TFP growth rate when calculated with a CES production function is declining over the full sample with an average (quarterly) growth rate of 0.25%. A similar result is reached by Klump, McAdam, and Willman (2008, p. 662) where the average estimated TFP growth rate is 0.28% for the Eurozone using aggregate euro-area data from 1971:Q1–2005:Q3. Its dynamics has a strong resemblance with that of ICT capital stock, reported in Figure 3. The high correlation between the two variables seems to confirm the key role of ICT, and thus the specification of the CES production function given above.
The decomposition of TFP

Whereas the Cobb-Douglas allows the computation of TFP only residually, a further advantage of the CES function is the possibility of decomposing the TFP. This decomposition can best be done if we come back to our original framework. The tools are the output elasticities with respect to the inputs, which represent a key feature of the CES production function. Indeed, they allow distinguishing between the contribution to the output growth rate of the different factors in factor-augmenting

![Figure 2. The dynamics of TFP (Quarterly rates).](image1)

![Figure 3. The dynamics of ICT (Yearly rates).](image2)
technical change. To appreciate the relevance of this property, we analyze the pattern of technical change of the Italian economy.

Let us start with the labor contribution to technical change, $e_{Y,LT} \cdot \mu$. Its dynamics is represented in Figure 4.

It is straightforward to see that the labor contribution features two quite distinct patterns. In the first half of the sample period (1981:Q4–1994:Q2), labor augmentation is steadily increasing. It is more troubling to detect a clear behavior in the second half. Indeed, it remains approximately constant. Hence, in the mid-1990s, there seems to be a structural break. The occurrence of such a break is confirmed by a simple Chow's breakpoint test. How sensitive is this result to changes in the value of \( \sigma \)? As a robustness check of the break timing, we tried values of \( \sigma \) closer or equal to 1 without finding any relevant differences.

A regime shift seems to be confirmed by the development of capital-augmentation, $e_{Y,KT} \cdot \gamma L$. Its time evolution is quite volatile with a number of peaks; indeed, a test based on global information criteria indicates the existence of multiple breaks. However, a simple visual inspection of Figure 5 shows that the relevant break occurs around the middle of the 1990s.

**Factor bias**

The CES production function sheds light on another aspect, the factor bias, which is defined by the ratio of the marginal productivities of the inputs (not in efficiency units). From Equation (4), we have:

$$\frac{\partial Y}{\partial K} = \frac{\pi_0}{1 - \pi_0} \left( \frac{C_t}{e^{u(t-t_0)}} \right)^{-\beta_1} \left( \frac{K_t}{L_t} \right)^{-1}$$

Technical progress is biased toward a factor if it increases its marginal product more than the other factor’s. Following Acemoglu (2002), the bias can be divided into two parts. One is the traditional
substitution effect \((\left(\frac{L_t}{K_t}\right)^{1-\sigma})\), determined by the relative endowments of the two inputs, that favors the scarcer factor. The other component, that can be referred to as the technical change effect, depends on the relative weight of the factor-augmenting technical change \((\left(\frac{C_t}{K_t}\right)^{\sigma-1}\sigma^\sigma)\) and whether the elasticity of substitution is less than, equal to or greater than 1. This second effect is obviously absent in the Cobb-Douglas case.

**Figure 5.** Capital-augmenting technical change (Quarterly rates).

**Figure 6.** The technological bias (Quarterly rates).
The bias is clearly linked to the size of the elasticity of substitution. In our case, where \( \sigma = 0.65 \) is less than 1, the factor inputs are gross complements. It follows that the dominance of labor-augmenting technical change in the first half of the sample implies that technical change is capital biased. Intuitively, the presence of capital bias means that technical change favors capital input.\(^8\)

In Figure 6, the contribution of technical change to capital bias is given by the positive vertical distance separating the CES and the Cobb-Douglas (which includes only the substitution effect). Looking at the graph, it is worth noting that although present, the capital bias progressively reduces until it vanishes in the middle of the 1990s.\(^9\) To clarify this point, the vertical distance, a measure of the contribution of technical progress, is graphed in Figure 7.

In fact, the graph clearly shows not only the disappearance of technical change but also verifies the occurrence of a structural break around the middle of the 1990s seen above. As in our technology representation (4), technical change is predominantly driven by ICT investment (see the definition of \( \mu \) and of the capital-augmenting factor), the disappearance of the technical change contribution can be viewed as a failure to effectively employ innovative technologies in the Italian economy. The Italian productivity was initially favored by the diffusion of ICT, mostly through the adoption of new hardware and software. The marginal product of traditional and innovative capital stocks shows that by the end of the 1980s and the beginning of the 1990s, this improvement in efficiency came to a stop. Many factors (such firm’ size, ownership structure largely family controlled, an underdeveloped financial system predominantly bank-based) contributed to this standstill, but in our view, it was caused mainly by the failure to adopt new forms of organization needed to fully exploit the productivity-enhancing potential (Evangelista and Vezzani 2010, Bank of Italy 2009).

Conclusions

Most analyses of the current economic Italian stagnation focus on a TFP slowdown without delving into its causes. In this paper, we take a step further, looking at the determinants of TFP. To this end, we use our previous CES specification and estimated parameters. We find evidence of a structural break in the mid-1990s in the impact and nature of technical change. Labor augmentation and capital bias

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**Figure 7.** The contribution of technological progress (Quarterly rates).
are found to have been dominant in the first half of the sample period, while no evidence of technological progress of any type seems to be present in the second half. We believe that these results can be relevant not only for theoretical purposes but also for policy choices. In addition, we plan to extend our analysis to other European economies.

Disclosure statement
No potential conflict of interest was reported by the authors.

Notes
1. In this paper, we do not address the microeconomic aspects of the TFP slowdown. For a recent survey of the microeconomic aspects of technological bias and the correlated induced innovations, see Acemoglu (2015).
2. The model assumes that the Italian economy can be described by behavioral functions derived by the intertemporal optimization of objective (profit) function subject to constraints. Institutional and market structures are also incorporated in the model as constraints. These constraints represent the adjustment costs, which hamper the instantaneous equality between factor marginal products and their prices. For instance, differently from the traditional approach, the capital stocks adjust more slowly to their marginal products. These rates of adjustment reflect the costs and risks of firms changing their capital stock. Analogously, it is not assumed that labor market instantaneously clears but rather that there are imperfections and frictions.
3. The dataset is available from the authors upon request.
4. The parameter estimates employed in the paper are taken from Saltari et al. (2012); these were obtained by a full-information maximum-likelihood procedure with all of the constraints implicit in the model being imposed by the estimator. All parameters are identified uniquely and, in fact, the model is heavily over-identified with the same parameters appearing in several equations but not necessarily in the same way.
5. Kaldor is explicit in affirming that one specific characteristic of his growth model is that “[…] it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labour and those induced by technical invention or innovation-i.e., the introduction of new knowledge. The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques” (p. 595).
6. Employing the observed data for capital, labor and output and our parameter estimates, the capital share for the Italian economy in the reference period, using Equation (3), is

$$\pi_0 = (\beta_3)^{-\beta_1} \left( \frac{Y_0}{K_0T_0} \right)^{\beta_1} = 0.25$$

so that the labor income share is

$$1 - \pi_0 = 0.75.$$  

Since these estimates are quite close to those present in several different databases (such as OECD, EU KLEMS, AMECO), we decided to adopt these values of the income shares for the reference period.
7. One of the main causes of the structural break might be the labor market reforms inaugurated in the early 1990’s in Italy.
8. In the same vein, Klump, McAdam, and Willman (2008, p. 663) finds for the Eurozone that “until the end of 1997 technical progress […] is asymptotically labor augmenting with the contribution of capital augmenting progress gradually fading out. Thereafter there is a break in the nature of factor-augmenting technical progress.”
9. Antonelli and Barbiellini Amidei (2011) reach a similar conclusion. Their analysis is based on the income shares evolution (over the period 1950–1992), interpreted as an indicator of the directionality or bias of the technological change. They show that technological change was indeed non-neutral in the sample period. In particular, they argue that in the mid-1990s, a new labor-intensive phase started (p. 159, fn. 3).
10. Estimates of $\beta_{11}$ were not significantly different from 1 showing there is no money illusion in the determination of real wages. In the final model, $\beta_{11}$ was set to 1.

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Acemoglu, D. 2015.

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Appendix

A1. Data

The data used are of the Italian economy, quarterly from 1981:Q4 to 2005:Q2. GDP and NDP, fixed Capital, and total remuneration are defined as € bn., employment in millions of employees, any parameters of variables such as interest rates, rate of time preference, rates of growth, and so on as rates per quarter in natural numbers. All real variables are defined with base year 2000. The stock of fixed capital is calculated from net capital formation divided by the GDP deflator and accumulated from a base stock of 3572.4 (€ bn.) in 2000:Q2. The ICT capital stock is calculated from annual data for gross real investment less depreciation for each of three sub-sectors (office machinery, communication devices and software), each separately interpolated to provide quarterly observations, and total net investment cumulated on a base figure of 80,717 (€ bn.) in 2000:Q4. The sources of the data are ISTAT, Bank of Italy, EU KLEMS, OECD, AMECO, European Commission.
A2. The model

The core of the model is composed by the following equations, where, for brevity, we omitted the
time index (for more details, see Saltari et al. 2012):

1. Production functions:
   (a) General production function
   \[ Y = f(C, K, L_s, L_u) = \beta_3 [ (C^\gamma K)^{-\beta_1} + (\beta_2 e^{(\lambda_s + \gamma \lambda_u) L_s L_u} \gamma)^{-\beta_1} \frac{1}{\beta_4}]. \]  
   (A1)

   (b) ICT production function
   \[ l = f_l(C, L_{Is}) = \beta_2 [ C^{-\beta_6} + (\beta_8 \exp(\lambda_c) L_{Is})^{-\beta_6} \frac{1}{\beta_7}]. \]  
   (A2)

   where \( L_{Is} \) is the skilled labor employed in the ICT sector.

2. Investment functions:
   (a) Traditional capital
   \[ \dot{k} = \alpha_1 [ \alpha_2 \frac{\partial f}{\partial K} - (r - \beta_7 D \ln p + \beta_8) ] - (k - \mu_K). \]  
   (A3)

   where \( k = D \ln (K) \), \( \dot{k} = D^2 \ln (K) \).

   (b) ICT capital
   \[ \dot{k} = \alpha_1 \left[ \alpha_2 \left( \frac{\partial f}{\partial K} - (r - \beta_7 D \ln p + \beta_8) \right) - (k - \mu_K) \right]. \]  
   (A4)

   where in Equation (A3) \( \mu_K = \lambda_K + (\gamma_s - \gamma) \lambda_s + \gamma_u \lambda_u \) and in Equation (A4) \( \mu_c = \lambda_s + \lambda_c \) are the steady-state values of the two capital stocks, and \( c = D \ln (C) \), \( \dot{c} = D^2 \ln (C) \).

3. Skilled labor:
   (a) Demand for skilled labor
   \[ \dot{\ell}_s = \alpha_5 \left[ \alpha_6 \left( \frac{\partial f}{\partial L_s} \left( \frac{W_s}{P} \right) \right) + \alpha_6' \left( \frac{\partial f}{\partial L_{Is}} \left( \frac{W_s}{P} \right) \right) - (\ell_s - \lambda_s) \right]. \]  
   (A5)

   where \( W_s \) is the wage of the skilled labor, and \( \dot{\ell}_s = D \ln (L_s) \), \( \dot{\ell}_s = D^2 \ln (L_s) \)

   (b) Skilled wages (\( W_s \))
   \[ D^2 \ln W_s = \alpha_7 \left[ \alpha_8 \left( \frac{\partial f}{\partial L_s} \left( \frac{W_s}{P} \right) \right) + \alpha_8' \left( \frac{\partial f}{\partial L_{Is}} \left( \frac{W_s}{P} \right) \right) - \right] \]
   \[ (\alpha_7 + \alpha_8 + \alpha_8') (D \ln W_s - \beta_{11} D \ln p - \lambda_K - \gamma \lambda_c) \]  
   (A6)

   where \( \beta_{11} \) measures money illusion.\(^{10}\)

4. Unskilled labor:
   (a) Employment
   \[ \dot{\ell}_u = \alpha_9 \alpha_{10} \ln \left( \ell_u^{d/L_u} \right) - (\alpha_9 + \alpha_{10}) (\ell_u - \lambda_u), \]  
   (A7)

   where \( \ell_u = D \ln (L_u) \), \( \dot{\ell}_u = D^2 \ln (L_u) \), and \( L_u^d \) is the demand for unskilled labor.

   (b) Unskilled wages (\( W_u \))
   \[ D^2 \ln W_u = \alpha_{11} \left[ \alpha_{12} \left( \frac{L_u^d}{L_u} \right) - (D \ln W_u - \lambda_K - \gamma \lambda_c) \right]. \]  
   (A8)

   where the labor supply is \( L_u^d = L_u \left( \frac{W_u}{P} \right)^{\beta_{13}} e^{\lambda_u t} \). In the model, changes in the unskilled labor supply depends on the real wage, with elasticity \( \beta_{12} \).

5. Price determination
The marginal cost of labor is obtained in the usual way as a ratio between the mean wage and the marginal product of labor, where labor is defined as a Cobb-Douglas function of the two labor components, \( L = L_s^\gamma L_u^\gamma \). The short-term marginal cost is a weighted average of skilled and unskilled wage rates

\[
mc \left( \frac{\partial L}{\partial Y} \right) = \left( \frac{w_s L_s}{\gamma_s} + \frac{w_u L_u}{\gamma_u} \right) L_s^{-\gamma_s} L_u^{-\gamma_u} (\beta_2 \beta_3)^{-1} e^{-(\lambda_k + \gamma \lambda_c) t} \left[ 1 + (\beta_2 e^{(\lambda_k + \gamma \lambda_c) t} \psi)^{\beta_1} \right]^{1 + \beta_1 / \beta_3},
\]

where \( \psi = \frac{L}{K} \).

The dynamics of price determination are described by a second-order process:

\[
D^2 \ln(p) = \alpha_{15} \ln \left( \frac{\beta_{31}^\prime \text{mc} \left( \frac{\partial \log Y}{\partial Y} \right)}{p} \right) + \alpha_{13} \left( D \ln \left( \frac{w_s}{p} \right) - \lambda_c \right) + \\
+ \alpha_{14} \left( D \ln \left( \frac{w_u}{p} \right) - (\lambda_k + \gamma \lambda_c) \right) + \alpha_{16} \ln \left( \frac{v M}{P Y} \right)(1 + \lambda_m - \lambda_m e^{-\lambda_m t}),
\]

where \( \beta_{13} \) is the mark-up and \( \tau \) is the indirect tax rate. \( M \) is the volume of money \((M_2)\), \( v \) is the velocity of money increasing by a factor \((1 + \lambda_m - \lambda_m e^{-\lambda_m t})\). This factor is one when \( t = 0 \) and increases at a reducing rate to an asymptote \((1 + \lambda_m)\). Hence, it is assumed that \( v \) increases over time from a base level at \( t = 0 \) owing to more efficient banking services.