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\[ \frac{G_{E_p}}{G_{M_p}} \]

Ratio by Polarization Transfer in \( \text{ep} \rightarrow \text{ep} \)

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$G_{E_p}/G_{M_p}$ Ratio by Polarization Transfer in $\bar{e}p \rightarrow e\bar{p}$


The ratio of the proton’s elastic electromagnetic form factors, $G_{E_p}/G_{M_p}$, was obtained by measuring $P_t$ and $P_e$, the transverse and the longitudinal recoil proton polarization, respectively. For elastic $\bar{e}p \rightarrow e\bar{p}$, $G_{E_p}/G_{M_p}$ is proportional to $P_t/P_e$. Simultaneous measurement of $P_t$ and $P_e$ in a polarimeter provides good control of the systematic uncertainty. The results for the ratio $G_{E_p}/G_{M_p}$ show a systematic decrease.
as $Q^2$ increases from 0.5 to 3.5 GeV$^2$, indicating for the first time a definite difference in the spatial distribution of charge and magnetization currents in the proton.

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Understanding the structure of the nucleon is of fundamental importance in nuclear and particle physics; ultimately such an understanding is necessary to describe the strong force. Certainly, for any QCD-based theory, its ability to predict the pion and nucleon form factors correctly is one of the most stringent tests of its validity, and hence precise data are required. The electromagnetic interaction provides a unique tool to investigate the structure of the nucleon. The elastic electromagnetic form factors of the nucleon characterize its internal structure; they are connected to its spatial charge and current distributions.

The earliest investigations of the proton form factor by Chambers and Hofstadter [1] established the dominance of the one-photon exchange process in the elastic $ep$ reaction. It indicated that the Dirac, $F_{1p}$, and Pauli, $F_{2p}$, form factors depend only on four-momentum transfer squared which for elastic scattering is in the spacelike region. $F_{1p}$ and $F_{2p}$ were found to have approximately the same $Q^2$ dependence up to $\approx 0.5$ GeV$^2$, where $Q^2 = 4E_eE'_e\sin^2\theta_e/2$, $E_e$ and $\theta_e$ are the scattered electron’s energy and angle, respectively, and $E_p$ is the incident beam energy. The data were fitted with a dipole shape, $G_D = (1 + Q^2/0.7F)^{-2}$, characteristic of an exponential radial distribution.

The elastic $ep$ cross section can be written in terms of the electric, $G_{Ep}(Q^2)$, and magnetic, $G_{Mp}(Q^2)$, Sachs form factors, which are defined as

\[
G_{Ep} = F_{1p} - \tau \kappa_p F_{2p} \quad \text{and} \quad G_{Mp} = F_{1p} + \kappa_p F_{2p},
\]

(1)

where $\tau = Q^2/4M^2$, $\kappa_p$ is the anomalous nucleon magnetic moment, and $M$ is the mass of the proton. In the limit $Q^2 \to 0$, $G_{Ep} = 1$ and $G_{Mp} = \mu_p$, the proton magnetic moment. The unpolarized $ep$ cross section is

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e^2 \cos^2\theta_e}{4E_e^3 \sin^4\theta_e/2} \left[ \frac{\tau}{G_{Ep}} + \frac{1}{1 + \tau} \right] \left( \frac{1}{1 + \tau} \right),
\]

(2)

where $\epsilon$ is the virtual photon longitudinal polarization, $\epsilon = [1 + 2(1 + \tau)\tan^2(\theta_e/2)]^{-1}$.

In the Rosenbluth method [2], the separation of $G_{Ep}^2$ and $G_{Mp}^2$ is achieved by measuring the cross section at a given $Q^2$ over a range of $\epsilon$ values that are obtained by changing the beam energy and scattered electron angle. In Eq. (2) the $G_{Mp}$ part of the cross section is multiplied by $\tau$; therefore, as $Q^2$ increases, the cross section becomes dominated by $G_{Ep}$, making the extraction of $G_{Ep}$ more difficult. Figure 1 shows measurements of proton form factors obtained by using this method. For $Q^2 < 1$ GeV$^2$, the uncertainties in both $G_{Ep}$ and $G_{Mp}$ are only a few percent, and one finds that $G_{Mp}/\mu_p G_D \approx G_{Ep}/G_D = 1$. For $G_{Ep}$ above $Q^2 = 1$ GeV$^2$, the large uncertainties and the divergence in results between different experiments, as seen in Fig. 1a, illustrate the difficulties in obtaining $G_{Ep}$ by the Rosenbluth method. In contrast, the uncertainties on $G_{Mp}$ remain small up to $Q^2 = 31.2$ GeV$^2$ [10].

The combination of high energy, current, and polarization, unique to the Continuous Electron Beam Accelerator Facility of the Jefferson Laboratory (JLab), makes it possible to investigate the internal structure of the nucleon with higher precision and different experimental techniques. This experiment used the powerful technique of polarization transfer. For one-photon exchange, the scattering of longitudinally polarized electrons results in a transfer of polarization to the recoil proton with only two nonzero components, $P_t$ perpendicular to, and $P_c$ parallel to the proton momentum in the scattering plane, given by [11]

\[
I_0P_t = -2\sqrt{\tau(1 + \tau)} G_{Ep} G_{Mp} \tan\frac{\theta_e}{2},
\]

(3)

\[
I_0P_c = \frac{1}{M} (E_e + E_c) \sqrt{\tau(1 + \tau)} G_{Ep}^2 \tan^2\frac{\theta_e}{2},
\]

(4)

where $I_0 = G_{Ep}^2 + \frac{\tau}{\epsilon} G_{Mp}^2$. Equations (3) and (4) together give

\[
\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_c} \frac{(E_e + E_c)}{2M} \tan\left(\frac{\theta_e}{2}\right).
\]

(5)

The ratio $G_{Ep}/G_{Mp}$ is obtained from a simultaneous measurement of the two recoil polarization components in the


1399
polarimeter. Neither the beam polarization nor the polarimeter analyzing power needs to be known, which results in small systematic uncertainties. This method was first used recently by Milbrath et al. [9] at MIT-Bates to measure the ratio $G_{E_p}/G_{M_p}$ at low $Q^2$. Our experiment was done in Hall A at JLab. Longitudinally polarized electron beams with energies between 0.934 and 4.090 GeV were scattered in a 15-cm-long circulating liquid hydrogen (LH$_2$) target, refrigerated to 19 K. The kinematic settings are given in Table I. For the four highest $Q^2$ points, a bulk GaAs photocathode, excited by circularly polarized laser light, produced beams with $\sim0.39$ polarization and currents up to $\sim115$ $\mu$A; the helicity was flipped at 30 Hz. For the lower $Q^2$ points, a strained GaAs crystal was used, and typical polarizations of $\sim0.60$ were achieved with currents up to $\sim15$ $\mu$A; the helicity was flipped at 1 Hz. The beam polarization was measured with a Mott polarimeter in the injector line and with a Møller polarimeter in Hall A [12]. Elastic $ep$ events were selected by detecting the scattered electrons and protons in coincidence in the two identical high resolution spectrometers (HRS) of Hall A [12]. The HRS deflect particles vertically by $45^o$ and accept a maximum central trajectory momentum of 4 GeV/c with a 6.5 msr angular acceptance, $\pm5\%$ momentum acceptance, and $\sim10^{-4}$ momentum resolution. The two vertical drift chambers installed close to the focal plane of each HRS give precise reconstruction of the positions and angles at the target. The trigger was defined by a coincidence between the signals from two scintillator planes in each of the two HRS. A focal plane polarimeter (FPP) was installed in the hadron HRS. In the FPP, two front straw chambers define the incident proton trajectory and two rear straw chambers define the proton trajectory after scattering in the graphite analyzer [13]. The graphite analyzer consists of five sets of graphite plates which can be moved out to collect straight-through trajectories for alignment of the FPP chambers. Graphite thicknesses between 11 and 50 cm were used in order to optimize the FPP figure of merit.

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>$\Delta Q^2$ (GeV$^2$)</th>
<th>$\langle\chi\rangle$ (deg)</th>
<th>$\mu_p G_{E_p}/G_{M_p}$</th>
<th>$\Delta_{stat}$</th>
<th>$\Delta_{syst}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.49 \pm 0.04$</td>
<td>0.934</td>
<td>105</td>
<td>0.966 $\pm$ 0.022</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>$0.79 \pm 0.02$</td>
<td>0.934</td>
<td>118</td>
<td>0.950 $\pm$ 0.015</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>$1.18 \pm 0.07$</td>
<td>0.821</td>
<td>136</td>
<td>0.869 $\pm$ 0.014</td>
<td>0.027</td>
<td></td>
</tr>
<tr>
<td>$1.48 \pm 0.11$</td>
<td>3.395</td>
<td>150</td>
<td>0.798 $\pm$ 0.033</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>$1.77 \pm 0.12$</td>
<td>3.395</td>
<td>164</td>
<td>0.728 $\pm$ 0.026</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td>$1.88 \pm 0.13$</td>
<td>4.087</td>
<td>168</td>
<td>0.720 $\pm$ 0.031</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>$2.47 \pm 0.17$</td>
<td>4.090</td>
<td>196</td>
<td>0.726 $\pm$ 0.027</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>$2.97 \pm 0.20$</td>
<td>4.087</td>
<td>218</td>
<td>0.612 $\pm$ 0.032</td>
<td>0.056</td>
<td></td>
</tr>
<tr>
<td>$3.47 \pm 0.20$</td>
<td>4.090</td>
<td>239</td>
<td>0.609 $\pm$ 0.047</td>
<td>0.045</td>
<td></td>
</tr>
</tbody>
</table>

The azimuthal angular distribution after a second scattering in the analyzer of the FPP is given by

$$N_p(\theta, \varphi) = N_p(\theta)\{1 + [h A_1(\theta) P_{1FPP} + \alpha] \sin \varphi - [h A_2(\theta) P_{2FPP} + \beta] \cos \varphi\},$$

where $h$ is the electron beam polarization, $N_p(\theta)$ is the number of protons scattered in the polarimeter, $\theta$ and $\varphi$ are the polar and azimuthal angles after scattering, and $A_1(\theta)$ is the analyzing power; $P_{1FPP}$ and $P_{2FPP}$ are the in-plane polarization components, transverse and normal, respectively, at the FPP analyzer. Instrumental asymmetries ($\alpha$ and $\beta$) are canceled by taking the difference of the azimuthal distributions for positive and negative electron beam helicity. Fourier analysis of this difference distribution gives $h A_1(\theta) P_{1FPP}$ and $h A_2(\theta) P_{2FPP}$.

The proton spin precesses in the fields of the magnetic elements of the HRS, and therefore the polarizations at the target and the FPP are different; they are related through a spin transport matrix $P_{FPP} = (S) \times P$, where $P_{FPP}$ and $P$ are polarization column vectors ($n$, $t$, $\ell$) at the FPP and target, respectively, and (S) is the spin transport matrix. A novel method was developed to extract the values of the polarization components $P_n$ and $P_\ell$ at the target from the FPP azimuthal distribution; the integrals in the Fourier analysis were replaced with sums weighted by the values of the matrix elements, $S_{ij}$, of each event [14]. The matrix elements $S_{ij}$ depend upon the angular ($\theta$ and $\phi$) and spatial ($\gamma$) coordinates at the target and proton momentum ($p$). The $S_{ij}$s were calculated for each event from the reconstructed $\theta$, $\phi$, $\gamma$, $p$ using the spin matrix determined by a magnetic transport code. Both the ray-tracing code SNAKE [15] and the differential-algebra-based code COSY [16] were used, and the spin precession corrections from both methods agree within experimental uncertainties. The stability of the method was studied in detail for all $Q^2$. The data were analyzed in bins of each one of the four target variables, one at a time. The results showed that the extracted $G_{E_p}/G_{M_p}$ ratio is independent of each of these variables.

The results for the ratio $\mu_p G_{E_p}/G_{M_p}$ are shown as filled circles in Fig. 2a, and as the ratio $Q^2 F_2/F_1$, obtained from Eq. (1), in Fig. 2b; in both figures only the statistical uncertainties are plotted as error bars. The data are tabulated in Table I, where both statistical and systematic uncertainties are given for each data point. Three sources contribute to the systematic uncertainty: measurement of the target variables, positioning and field strength of the HRS magnetic elements, and uncertainties in the dipole fringe-field characterization. The systematic uncertainties would shift all data points in the same direction, either up or down. No radiative correction has been applied to the results. External radiative effects are canceled by switching the beam helicity. The internal correction is due to hard photon emission, two-photon exchange, and higher-order contributions. A dedicated calculation [17] predicts the first to be of the order of a few percent. Preliminary indications
The most important feature of the data is the sharp decline of the ratio \( \mu_p G_{E_p}/G_{M_p} \) as \( Q^2 \) increases, which indicates that \( G_{E_p} \) falls faster than \( G_{M_p} \). Furthermore, \( G_{M_p}/\mu_p G_D \) is approximately constant; it follows that \( G_{E_p} \) falls more rapidly with \( Q^2 \) than the dipole form factor \( G_D \).

Results from this experiment are consistent with the earlier results of Refs. [4–6] which have much larger uncertainties. Our results are compatible with the SLAC data of Ref. [8] up to about \( Q^2 \) of 2.5 GeV\(^2\), considering the larger uncertainties, but our results are in definite disagreement with the older results of Ref. [7] from SLAC, as seen in Fig. 2b.

The \( Q^2 F_2/F_1 \) ratio shown in Fig. 2b indicates a continuing increase with \( Q^2 \), contradicting earlier observations based on the data of Refs. [7, 8] that it might have reached a constant value as predicted in perturbative QCD (pQCD): \( F_1 \sim 1/Q^2 \) and \( F_2 \sim 1/Q^2 \) [18]. It would be of great interest to explore the larger \( Q^2 \) region where pQCD will dominate. Extension of this experiment to larger \( Q^2 \) has become a necessity.

So far, all theoretical models of the nucleon form factors are based on effective theories; they all rely on a comparison with existing data and their parameters are adjustable. Much work has been done with the goal of bridging the low and high \( Q^2 \) regimes. There are two quite different approaches to calculate nucleon form factors. In the first approach, the mesonic degrees of freedom are explicit, as in calculations based on vector meson dominance (VMD) [19–22], models comprising a three-quark core dressed with pseudoscalar mesons [23], and a calculation based on the solitonic nature of the nucleon [24]. The second approach consists of QCD-based quark models; these include models such as relativistic constituent quark (RCQM) [25–27], diquark [28], cloudy bag [29], and QCD sum rule [30]. Calculations of the nucleon form factors from lattice QCD are in progress [31].

In the earliest study of the RCQM, Chung and Coester [25] investigated the effect of the constituent quark masses, the anomalous magnetic moment of the quarks, \( F_{2q} \), and the confinement scale parameter. Recently Coester introduced a form factor for \( F_{2q} \) to reproduce the present data; the result is the solid curve in Fig. 2 [26]. This illustrates how the new \( G_{E_p}/G_{M_p} \) data can help constrain the basic inputs to a particular model. The dashed-dotted curve in Fig. 2 shows the recently reevaluated diquark model prediction of Kroll et al. [28]. In the limit \( Q^2 \to \infty \) this model is equivalent to the hard-scattering formulation of pQCD. Calculations based on the cloudy bag model predict the right slope for \( G_{E_p}/G_{M_p} \) shown as a dotted curve in Fig. 2; this model includes an elementary pion field coupled to the quarks inside the bag such that chiral symmetry is restored [29].

Recent theoretical developments indicate that measurements of the elastic form factors of the proton to large \( Q^2 \) may shed light on the problem of nucleon spin. This connection between elastic form factors and spin has been demonstrated within the skewed parton distribution (SPD) formalism by Ji [32]. The first moment of the SPD taken in the forward limit yields, according to the angular momentum sum rule [32], a contribution to the nucleon spin from the quarks and gluons, including the orbital angular momentum. By subsequently applying the sum rule to the SPD, it should become possible to estimate the total contribution of the valence quarks to the proton spin [33, 34].

In conclusion, we have presented a new measurement of \( G_{E_p}/G_{M_p} \) obtained in a polarization transfer experiment with unprecedented accuracy. The results demonstrate for the first time that the \( Q^2 \) dependence of \( G_{E_p} \) and \( G_{M_p} \) is significantly different. The quality of the JLab data will place a tight constraint on the theoretical models. Results
from this experiment combined with future measurements of the neutron form factors will bring us closer to a single description of the structure of the nucleon.

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[26] F. Coester (private communication).