PROSPECTIVE TEACHERS’ USE OF CHIP MODEL

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Prospective Teachers’ Use of Chip Model

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Ten elementary and middle school prospective teachers (PTs) participated in clinical interviews where they modeled integer addition and subtraction number sentences with two-colored chips. The PTs constructed various models using the two-colored chips that both matched and did not match the number sentences presented to them. Although the prospective teachers sometimes created models that did not match the integer addition and subtraction number sentences, some recognized these inconsistencies. The results highlight spaces of PTs’ accomplishments and struggles with using two-colored chips for certain integer number sentences. Implications of this study support facilitating PTs’ construction of models and leveraging their thinking and uses of models in instruction.

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Understanding the ways that prospective teachers (PTs) reason is important so that we can leverage discourse and instructional experiences in teacher education. PTs use various manipulatives, such as the two-colored chips, as they engage in activities around the teaching and learning of integers. Yet, we know little about the ways PTs construct models for integer operations with two-colored chips. This research report highlights the results of an investigation that sought to make sense of the following research question:

In what ways to elementary and middle school PTs use two-color chips to model integer addition and subtraction?

Conceptual Framework

The minus symbol has multiple meanings (i.e. unary, binary, opposite) that are often confounded (Bofferding, 2014; Vlassis, 2004, 2008). In fact, students often operate with the negative integers by simply omitting or “ignoring” the minus symbol and adding it later (e.g., Ayres, 2000; Bell, O’Brien, Shiu, 1980). Confounding the meaning of the symbol or omitting it altogether will interfere with the ways that integer addition and subtraction is modeled with the chips.

Subtraction can be interpreted in two different ways, with take-away or distance (Selter, Prediger, Nührenbörger & Hußmann, 2012). The take-away interpretation of subtraction aligns best with chip models, as discrete objects are used and may be removed; and distance interpretations align with other models, such as number lines. The use of a subtraction model does not automatically insure success; knowing how the model is connected to the mathematics is what is needed to develop conceptual understanding (e.g., Kamii, Lewis, & Kirkland, 2001).

In this study, we focus on a subset of models (Van Den Heuvel-Panhuizen, 2003), manipulatives, and specifically the use of a two-colored chips. The use of two-colored chips is often referenced as the chip model, but these manipulatives can be used in differing ways (e.g., Murray, in press). Thus, there are many chip models, and we explore the various different models that PTs may construct in this study.

Using two-colored chips to model integer addition and subtraction has both affordances and constraints, or places where the model breaks down (Murray, in press). No model is vigorous
enough to support all problem types (Vig, Murray, & Star, 2014). Integer number sentences like \(3 - 5 = □\), for example, presents a constraint in using two-colored chips. If one starts with 3 chips of one color, representing the positive 3, and tries to remove five chips of the same color, this does not work. Consequently, the model needs to be adapted—such as adding in “zero pairs” to the model (i.e., representing \(1 + -1\) with two different colored chips).

Consistency refers to how things like contextual situations match number sentences (Wessman-Enzinger, in press). Similarly, consistency can also refer to how a model matches a number sentence. The use of discrete objects for \(2 + 3\) may not necessarily match \(2 - -3\). Attention to PTs’ consistency as they use two-colored chips to model integer addition and subtraction will highlight potential affordances and hindrances of models (Vig et al., 2014).

**Methods**

Ten elementary and middle school PTs volunteered to participate in structured, task-based interviews (Goldin, 2000). The PTs participated in the study as freshmen, during their first mathematics content course for teaching elementary and middle school mathematics, prior to using two-colored chips in their university coursework.

The PTs worked in pairs within the interviews to better elicit natural discourse about tasks. Although in pairs, each PT received the number sentence on a single sheet of paper, with colored markers, and two-colored chips (i.e., two-sided red and yellow chips). The following number sentences were given to the PTs: \(-5 + 9 = □\), \(1 - 5 = □\), \(7 + -2 = □\), \(-5 - 4 = □\), \(2 - -3 = □\), \(-8 - -3 = □\), \(-5 - -7 = □\). The PTs solved each integer addition or subtraction number sentence any way they wanted, modeled the number sentence with a number line, and then modeled the number sentence with two-colored chips. This research brief focuses on the ways PTs used the two-colored chip models for integer addition and subtraction.

**Results**

The results are described in two parts: (a) ways that PTs used two-colored chip models that are *consistent* with the number sentences and (b) ways that PTs used two-colored chip models that are *inconsistent* with the number sentences.

**Consistent Uses of Two-Colored Chips**

**Zero pairs.** Half of the PTs consistently used zero pairs with the two-colored chips for the number sentences \(-5 + 9 = □\) and \(7 + -2 = □\) in this study. Brooke and Sayoni, a pair of PTs, matched up two sets of chips. Brooke used 9 red chips and 5 yellow chips for \(-5 + 9 = □\); Sayoni used 9 yellow chips and 5 red chips for \(-5 + 9 = □\). Although they represented the \(-5\) flexibly with either red or yellow chips, they paired the chips up similarly. Brooke stated, “I know these cancel out,” pointing to the pairs of chips. And, Sayoni highlighted the solution, “I have these 4 left” after the canceled pairs.

**Comparing two sets.** PTs compared two sets of chips with \(-5 - -7 = □\) only twice. Kaye compared two sets of chips for \(-5 - -7 = □\). She constructed two rows, placing five red chips to represent \(-5\) in one row and seven red chips to represents \(-7\) in another row. Kaye reflected:

I do not know how to show this in a way that universally makes sense … Since you are subtracting, this equal zero somehow (pulls away five pairs of red/red chips)...They are not negative, they are positive (points at the remaining two red chips), that’s the part that I am stuck on.
As Kaye compared the two sets, she confounded zero pairs (1 + -1 = 0) with her reflection on how -1 - -1 = 0, which presented a challenge in determining how the solution she knew (-5 - -7 = 2) matched the remaining two red chips in her model. Although she confounded zero pairs, her comparison aligned to the number sentence.

**Taking away from one set.** The only consistent use of chip model with a take-away interpretation of subtraction included taking chips away from one set of chips for the number sentence -8 - -3 = □. Rochelle started with eight red chips and took away three red chips:

You have the 8 negative chips. You are subtracting the negative three, so you are taking away three of the negatives (pulls away 3 red chips). And that leaves you with negative five.

**Inconsistent Uses of Two-Colored Chips**

**Zero pairs.** Each of the PTs used zero pairs, combining two different colored chips to represent additive inverses (i.e., 1 + -1 = 0). However, in doing so the PTs often changed the number sentences (e.g., changing 1 - 5 = □ to 1 + -5 = □). Crystal, for example, modeled 1 - 5 = □ with one yellow chip representing +1 and five red chips representing -5. She combined the chips, paired a red and yellow chip together, pulled the pair off to the side, and pointed to the four remaining red chips as the solution. Crystal, although she constructed this model for 1 - 5 = □, recognized that this model did not fit the original addition number sentence well: “one minus five is hard to do with the chip model because you are taking away from a smaller number.”

**Joining.** Joining two sets of chips was a common strategy when PTs solved the number sentences -5 - -4 = □ and 2 - -3 = □. However, the PTs often changed these number sentences before modelling the chips with joining. In particular, the PTs changed -5 - -4 = □ to -5 + -4 = □ and 2 - -3 = □ to 2 + 3 = □. Kacee, for instance, modeled 2 - -3 = □ by using 2 red chips and 3 red chips. She joined these two chips, illustrating a solution of 5. Kacee, without prompting, shared: “If I did not already know it was five, I think it would be a lot harder to figure out what I am supposed to do with them [reference to the chips].” Although she recognized that that she did not think this was the best model to use with the chips, she did not provide an alternative model.

**Taking away from one set.** Starting with a set of chips of a singular color, the PTs modeled integer number sentences by taking away chips. For 1 - 5 = □, Jakob started with five red chips and took away one red chip—illustrating 5 - 1 = □ rather than 1 - 5 = □. Jakob referenced this take-away strategy while recognizing that his model did not incorporate subtraction. Even so, he did not offer an alternative model.

**Flipping chips.** PTs occasionally flipped chips as they used them. At times, this method occurred as the PTs employed another strategy, and other times it stood alone. Kaye, flipped chips as a stand alone method for 2 - -3 = □: “If you were to add these, it would be negative. But since you are subtracting it, they’re all like (and flips chips over to yellow).” Kaye started with 2 yellow chips and 3 red chips. She flipped the red chips to yellow. In some ways this matches 2 - -3 = □ in the sense that, her model matched the relationship between subtraction and adding the opposite (e.g., 2 - -3 = 2 + 3).

**Concluding Remarks**

Although the PTs’ focus on procedures interfered with their ability to use models consistently (e.g., changing 2 - -3 = □ to 2 + 3 = □), the PTs often expressed unprompted recognition that their model did not match the number sentences. These types of observations

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have potential to be spaces where mathematics teacher educators can build on PTs’ thinking, facilitating discussion about models and issues of consistency.

It is important for PTs to be consistent with number sentences and models; they need to connect the mathematics with the model (Kamii et al., 2001). Even so, when PTs provide an inconsistent model, mathematics teacher educators can leverage this as a way to encourage discussion. Given the challenges for children transitioning to models from whole number to integers, it seems likely these may be spaces that PTs will need to have discussions in their own classrooms in the future.

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References


