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Risk Considerations in Product Bundling

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Abstract

While bundling literature focuses on risk neutral decision makers (retailers), in this study, we portray a new perspective by addressing risk considerations in a bundling problem. We consider a retailer who has the option of selling a bundle of two products. We use a Mean-Variance approach to include retailer's risk through her profit variability when maximizing the expected value of profit. In this research, we also address the product selection problem, in which the retailer chooses the characteristics of the products to be bundled. We study the impact of the correlation between the reservation prices of the two products. We also consider the impact of the heterogeneity in the range of reservation prices of the two products. Among other findings, we show that optimal price made by a risk-averse decision maker cannot be larger than the one made by a risk neutral decision maker.

Keywords: Pricing Management; Pure Bundling; Risk Analysis; Mean-Variance Analysis

1. Introduction

There is a distinction between product bundling versus price bundling (Stremersch and Tellis, 2002). The sale of two or more separate products in a package at a discount is defined as product bundling while price bundling is used as a temporary price adjustment to the main product to prevent any negative impact on the product's perceived quality, without bundling the products in a package. Whether to bundle or not depends on many different parameters. For instance, it has been shown that a better price discrimination can be achieved especially when customers' evaluations of products are negatively correlated. Furthermore, bundling can help save costs especially when customer valuations are positively correlated. Bundling has been shown to play as a competitive mechanism by preserving the power for deterring a potential entrant, as well. Of course, there are certain situations in which no bundling is preferred, either to enhance the profit or to keep distance from legal concerns. Overall, bundling is extensively used in different industries. Bundling of vacation packages, software applications, insurance packages, restaurant menus, consumer products, electronic journals, telecommunication packages, etc. are some of the common applications in daily life related to both manufacturing and service segments. The trend of using bundles is even increasing over time due to emergence of offering bundles of services with products, in particular for business segments (Dukart, 2000; Swartz, 2000).

Despite significant research on bundling and applications in many industries, to the best of our knowledge, risk aspects of bundling have not been addressed properly in the literature. Our objective in this study is to present risk considerations in bundling decisions.

We consider a monopolist retailer selling two products to a market whose customers have different valuations for the products. We present a customer's valuation for a product through a reservation price, which indicates the maximum price a customer is willing to pay for a product.

Hence, the customers' valuation for a product, from the retailer's point of view, is a random variable. In accordance with the majority of bundling studies, we assume uniformly distributed reservation prices. That is, the reservation price of each customer for a product is a draw from a uniform distribution. However, as opposed to most studies, who consider reservation prices normalized between 0 and 1, we consider a general case of any arbitrary range for reservation prices. Although this more general model makes the derivation of results more difficult, as we will show, there are some characteristics (such as product heterogeneity) which can be captured only when we consider this general form. Using our general model, we can also provide insights on the special case when marginal cost is zero; in order to investigate risk of bundling of information goods.

The retailer applies pure bundling policy, in which the products are offered only in the form of a bundle, and not separately. The bundling literature is rich with the papers which compare pure bundling and no-bundling policies. In this research, however, we focus only on the pure bundling policy and study the product selection problem, through which we investigate the impact of the characteristics of the two products to be bundled. We derive expected and variance of profit as well as optimal prices for each case. To include the impact of risk on bundling decisions, we use a *Mean-Variance* (MV) approach. Compared to other risk related parameters, the expected and variance of profit are most readily available to decision makers. Hence, the MV method can be considered as the most practical approach. We also look at Coefficient of Variation (COV) as a measure of risk since it measures dispersion of profit distribution and represents significance of variation relative to mean of variations. Among other results, we show if the price that maximizes the expected profit results in a profit variance higher than what the

retailer is ready to accept, then she must use a lower price to achieve the maximum expected profit constrained by the acceptable variance level.

While customers' reservation prices are independent from each other, the reservation prices of an individual customer for the two different products can be correlated. To capture the impact of this correlation, we present our results for a continuous spectrum of correlation coefficients for reservation prices ranging from -1 (perfectly negatively correlated) to +1 (perfectly positively correlated), including the case of 0 (independent reservation prices). Note that in the literature often three extreme scenarios of independent, perfectly positively correlated, and perfectly negatively correlated reservation prices have been studied. Yet, they have been studied only as an exogenous parameter and not as a decision variable. In this research we look at these scenarios, from a decision maker's point of view (product selection). The above mentioned continuous spectrum of correlation coefficients can present a wide range of product selection. The positively (negatively) correlated reservation prices present bundle of complementary (substitute) products and the independent scenario presents a bundle of two products with independent demands. We compare the performance of product bundling for bundles of products with different reservation price correlations and offer related managerial insights.

The rest of this paper is organized as follows. In section 2, we briefly review the related literature. Then, in section 3, we describe the model and the structure of our analysis. Specifically, in this section, we characterize the purchasing probability and its sensitivity to product correlation. In sections 4 and 5, we respectively analyze scenarios of risk neutral and risk averse decision makers. In either scenario, we derive optimal prices with the corresponding expected value and variance of profit. Then, we provide numerical examples illustrating our

main findings. Finally, we conclude the paper by summarizing key managerial insights and areas for future research. Proofs of all propositions are in Appendix.

2. Literature review

The literature on bundling can be categorized from different viewpoints such as product types, number of products, bundling types, market structures, and contexts. From a product perspective, there are two mainstreams of papers in the bundling literature: bundling of goods for which the marginal costs are explicitly modeled and bundling of information goods for which the marginal cost can be easily neglected. With respect to the number of products, most research studies limit their scope to dealing with two products to gain managerial insights. There are, however, other studies that consider general cases of more than two products. Pure bundling in which no component of a bundle is offered separately, no bundling, and mixed bundling in which components of bundle are also offered parallel to bundled goods are three main categories of bundling schemes. The market structure in which the bundle is sold is another way to categorize bundling works. While most literature considers a monopoly market environment, there are some studies on duopoly and oligopoly structures, as well.

With respect to the context, the research studies with an economical perspective and a quantitative flavor form the majority of the literature on bundle pricing (Stremersch and Telis, 2002); conducted by either economists or marketers. Literature on the economics of bundling can be segregated into three broad groups: benefits of bundling as a tool for price discrimination (McAfee et al., 1989), as a cost saving mechanism (Evans and Salinger,2005), and finally as a means of entry deterrence (Carlton and Waldman, 2002; Nalebuff, 2004).

Traditionally, economists have explained bundling as an effective tool for price discrimination since it helps a monopolist to reduce heterogeneity in customer valuations (Bakos and Brynjolfsson, 1999). This means the advantage of bundling is especially apparent when the values of products are negatively correlated. In this case, bundling leads to more homogeneous valuations among customers and thus greater portion of customer surplus can be captured by the monopolist. McAfee et al. (1989) showed that even bundling of independent products can still be better than not bundling.

Another theme of studies on bundling has been about transactional cost reduction; mostly in the form of bundle discounts from which customers can benefit (Dewan and Freimer, 2003; Janiszewski and Cunha, 2004; Sheng et al., 2007). In a more recent study, Evans and Salinger (2008) provided a model for the size of discount and highlighted critical role of cost in explaining bundling and tying behavior in comparison with the role of demand in the previous studies. The third advantage of bundling is entry deterrence, which is beyond the scope of this study. The number of such studies is escalating over time (See Whinston, 1990; Carlton and Waldman, 2002; Nalebuff, 2004; Choi and Stefanadis, 2006; Hubbard et al., 2007; Peitz, 2008).

Bundling of information goods has been a common practice for a while due to cost savings in production and distribution of physical media such as CDs and DVDs and it is attracting more attention over time. However, due to technological progresses and significant cost reduction in reproduction and distribution of information goods, traditional benefits or formats of bundling may not be quite applicable. For instance, Bakos and Brynjolfsson (1999) have shown advantage of pure bundling of a large number of information goods. Or, Hitt and Chen (2005) have proposed the concept of customized bundling by which customers may select a fixed number of goods out of the total goods available for a fixed price. Their work was later extended by Wu et

al. (2008) using a nonlinear mixed-integer programming approach. For a comprehensive review of bundling literature see Stremersch and Tellis (2002).

McCardle et al. (2007) is the closest research to ours. Similar to their work, we consider the impact of bundling products on retail merchandising. Our work, however, is different from that study in several aspects. First, we consider only basic products since our objective is to address risk considerations of bundling, not comparing bundles of fashion and basic products. Second, we only consider pure bundling policy as our intention is not to compare pure bundling with no bundling policy. However, as opposed to McCardle et al. (2007)and most other studies considering normalized reservation prices between 0 and 1, we generalize reservation prices by considering arbitrary upper and lower limits. Specifically, the range of reservation prices of one product considered by McCardle et al. (2007) is a subset of the other one. There is no such restriction in our model. As we will see later, some results such as investigating the role of product heterogeneity cannot be observed when reservation prices are between 0 and 1. We also generalize results of McCardle et al. (2007) by considering a continuous spectrum of correlation coefficients ranging from -1 to +1. We look at such a wide spectrum of correlations from a decision making point of view to provide managerial insights on what type of products should be bundled together.

Finally, with respect to the subject of risk, we use an MV approach. The MV formulation has become a fundamental theory for risk management in finance, introduced first by Markowitz (1959). The MV approach and the Von Neumann–Morgenstern Utility (VNMU) approach are two practical methodologies for studying optimization problems with risk considerations (Choi et al., 2008). Even though VNMU approach is a more precise approach, its application is limited since finding an accurate form of utility functions for individual decision makers is quite

difficult. In contrast, the MV approach is more practical since it needs only mean and variance. In this study, we consider the optimal pricing which maximizes the expected profit subject to a variance constraint. We also look at COV of profit as another measure of risk, which represents profit standard deviation normalized by expected value of profit (Miller and Bromiley, 1990).

3. Model Formulation

In a homogeneous market whose size is *M* and customers' purchasing behaviors are independent of each other, we consider a monopolist retailer selling two products *A* and *B* under a *Pure Bundling* policy. In a pure bundling policy only a bundle of two products *A* and *B* is offered to the market. This policy is called *pure* bundling since the products are not offered separately along with the bundle.

A customer's valuation of product *i* is represented by his reservation price for that product, r_i, which indicates the maximum price he is willing to pay to buy it. The customers' reservation prices for a given product are assumed to be independent of each other. That is, the valuation of a customer for product *i* is independent of the valuation of another customer for the same product. However, for a given customer, the reservation prices of the two products *A* and *B* are not necessarily independent of each other. Specifically, from the retailer's perspective, the customer's reservation price for a product is a random variable. Following the common practice in the bundling literature where reservation prices are assumed to be uniformly distributed, we assume reservation prices for product *A* are uniformly distributed: $r_A \sim U[l_A, u_A]$, where $0 \le l_A < u_A$. Our model, however, considers the most generic form of uniform distribution; as opposed to most of the existing studies assuming reservation prices normalized between 0 and 1.

We intend to define the reservation price of product B, r_B, as a random variable on $[l_B, u_B]$, $0 \le l_B < u_B$, with any desired correlation coefficient with r_A. We therefore define r_B as:

$$r_B = \begin{cases} [K(r_A - l_A) + l_B]\lambda + (1-\lambda)\delta & \text{if } 0 \le \lambda \le +1 \\ [K(r_A - l_A) - u_B]\lambda + (1+\lambda)\delta & \text{if } -1 \le \lambda < 0 \end{cases} \tag{1}$$

, where $K = b/a$, $a = u_A - l_A$, and $b = u_B - l_B$. In this definition, $\delta \sim U[l_B, u_B]$ is independent of r_A. Without loss of generality, we assume $K \le 1$ (for $K > 1$, definition of products A and B can be swapped). It is easy to verify that:

a) r_B is a continuous and differentiable random variable for any $\lambda; -1 \le \lambda \le +1$.

b) Domain of r_B is the same as δ for $-1 \le \lambda \le +1$.

c) The correlation coefficient is: $\rho(r_A, r_B) = K\lambda \dfrac{\sigma(r_A)}{\sigma(r_B)} = \dfrac{\lambda}{\sqrt{1 + 2(\lambda^2 - |\lambda|}}$.

d) $\dfrac{\partial \rho}{\partial \lambda} > 0$, for $-1 < \lambda < +1$.

According to (d), special values of -1, 0, and +1 for λ are respectively corresponding to correlation coefficient of -1 (perfectly negatively correlated or where we have highly substitutable products), 0 (independent), and +1 (perfectly positively correlated or where we have highly complementary products). Given property (d), there is a one-to-one relation between λ and ρ, so through the rest of this paper we use λ as a representative of ρ. Figure 1 illustrates ρ versus λ.

Consider c as the unit cost of bundle and p as the selling price of bundle. Assuming positive net profit for each bundle sold and defining $U = u_A + u_b$ and $L = l_A + l_b$, we have the following relations:

$$0 \le Max(c, L) < p < U \tag{2}$$

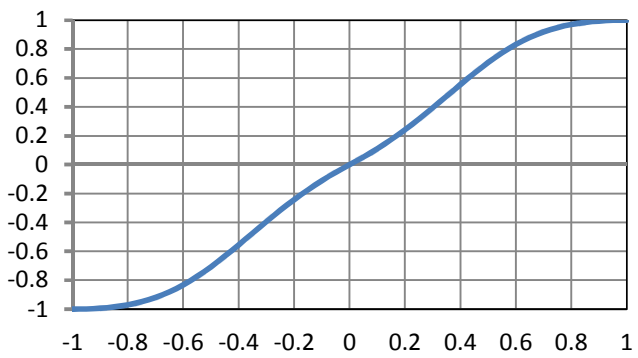

Figure 1 – Correlation coefficient ρ versus λ

We use π as the total profit earned from each individual customer, and Π as the retailer's total profit. Due to the homogeneity of customers and the fact that each customer's purchasing behavior is independent of other customers' purchasing behavior, the expected value and variance of the total profit are, respectively: $E[\Pi] = M.E[\pi]$, and $V[\Pi] = M.V[\pi]$. So, through the rest of the paper, we focus only on the expected and variance of retailer's profit from each individual customer (expected and variance of *total* profit can simply be derived by multiplying by *M*).

A customer buys the bundle if and only if the bundle price is not more than the sum of his reservation prices for each product individually. Hence the probability that a customer buys the bundle is: $\Pr(AB) = \Pr(p \le r_A + r_B)$. The profit function can then be written as:

$$\pi = \begin{cases} e & \text{with probability:} \Pr(AB) = \Pr(p \le r_A + r_B) \\ 0 & \text{with probability:} 1\text{-}\Pr(AB) \end{cases}, \tag{3}$$

, where $e = p - c$.

Using (3), the expected value and variance of the retailer's profit can be derived as follows:

$$E[\pi] = e.\Pr(AB) \tag{4}$$

$$V[\pi] = e^2.\Pr(AB)(1 - \Pr(AB)) \tag{5}$$

Through the rest of this section, we characterize the purchasing probability of a bundle and its sensitivity to bundle price and products coefficient of correlation, λ.

<u>Proposition 1</u>: *Probability of purchasing* $\Pr(AB)$ *is as follows:*

$$\Pr(AB)=\begin{cases} 1 & if \quad L \le p < L_{xy} \\ \Pr(AB)_1 = 1-\dfrac{[p-L_{xy}]^2}{2xy} & if \quad L_{xy} \le p < p_{Min} \\ \Pr(AB)_2 = \dfrac{U_{xy}+p_{Max}-2p}{2x} & if \quad p_{Min} \le p \le p_{Max} \\ \Pr(AB)_3 = \dfrac{[U_{xy}-p]^2}{2xy} & if \quad p_{Max} < p \le U_{xy} \\ 0 & if \quad U_{xy} < p \le U \end{cases} \tag{6}$$

, where $\begin{cases} x = a(1+K\lambda) \\ y = b(1-|\lambda|) \end{cases}$, $\begin{cases} L_{xy} = L+(N-1)b\lambda \\ U_{xy} = U+(1-N)b\lambda \end{cases}$, $\begin{cases} p_{Min} = l_A + u_B - N\lambda b \\ p_{Max} = l_B + u_A + N\lambda b \end{cases}$, and $N = \begin{cases} 1 & if\, \lambda > 0 \\ 0 & otherwise \end{cases}$.

Note that if $\lambda = -1$ and $K = 1$ then the purchasing probability $\Pr(AB)_2$ is not well-defined. It can be easily verified that we always have the following relation between the boundary limits in (6):

$$L \le L_{xy} \le p_{Min} \le \overline{p} \le p_{Max} \le U_{xy} \le U$$

, where $\overline{p} = (U+L)/2 = (p_{Min}+p_{Max})/2$. As figure 2 shows, L_{xy} (U_{xy}) is a linearly decreasing (increasing) function of λ when $\lambda < 0$ and then it remains fixed at L (U) when $0 \le \lambda$. In contrast, p_{Min} (p_{Max}) is fixed when $\lambda < 0$ and then it is a linearly decreasing (increasing) function of λ when $0 \le \lambda$ so that at $\lambda = 1$ it becomes L (U). This behavior of boundary limits implies that we can have different regions specifying the value of the purchasing probability, as it can be seen in figure 2. In special case of $\lambda = 1$, purchasing probability is simplified to only 1 relation since in this special case we have $p_{Min} = L$ and $p_{Max} = U$.

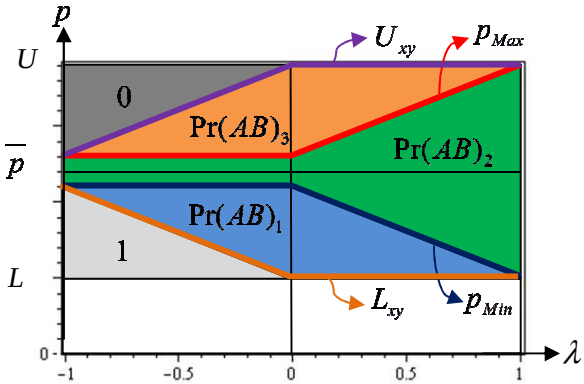

Figure 2 – Different regions of purchasing probability in feasible space of (λ, p)

Figure 3 shows the behavior of purchasing probability in special cases of $\lambda = -1, 0, +1$. As intuitively expected, at any level of product correlation, purchasing probability of a bundle reduces when bundle price increases ($\partial \Pr(AB) / \partial p \leq 0$ for any λ). However, depending on the sign of product correlation, we observe different behaviors, specifically, for positively correlated products we have $0 < \Pr(AB) < 1$ when $L_{xy} < p < U_{xy}$. For negatively correlated products, however, purchasing probability is zero (one) if the bundle price is more (less) than U_{xy} (L_{xy}). For bundle prices higher than U_{xy}, a purchasing probability of zero intuitively makes sense since products are negatively correlated and customers are less willing to purchase substitute products at relatively high prices. However, justification of a purchasing probability of one at lower prices for substitutable products is not straightforward at the first glance. Yet, such behavior can be explained by the fact that when the bundle price is low enough (less than L_{xy}), customers' willingness to acquire either product *A* or *B* can justify the payment for the whole bundle.

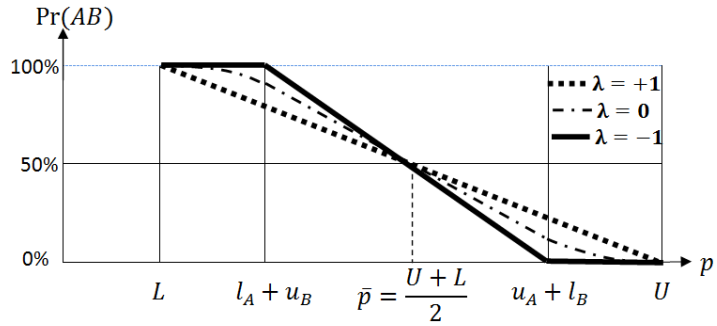

Figure 3 – Behavior of purchasing probability of $\Pr(AB)$ **vs. p, in special cases of** λ **=-1, 0, 1**

Since higher values of λ (more complementary products) could imply that the bundle is more attractive to the customer, at the first glance, one may expect that purchasing probability should an increasing function of λ. However, as the following corollary shows, this is not necessarily true.

Corollary 1: *For prices higher (less) than* $\bar{p}$*, purchasing probability is decreasing (increasing) in* λ*;* $\frac{\partial \Pr(AB)}{\partial \lambda} < 0$ ($\frac{\partial \Pr(AB)}{\partial \lambda} > 0$). Moreover, *when* $p = \bar{p}$*, purchasing probability is independent of* λ*;* $\frac{\partial \Pr(AB)}{\partial \lambda} = 0$.

The above corollary suggests that in order to achieve a higher purchasing probability, a more negative correlation of reservation prices are preferred at lower levels of bundle price ($p < \bar{p}$), while at higher levels of bundle price ($p > \bar{p}$), the preference of correlation is reversed. While the behavior of purchasing probability for $p > \bar{p}$ is intuitively expected, justification for $p < \bar{p}$ relies on the same fact that stated earlier for having purchasing probability of 1 for prices less than L_{xy}. That is, when $p < \bar{p}$, purchasing probability of more substitutable products is higher due to higher willingness to pay for either *A* or *B*. The special bundle price of $p = \bar{p}$ makes purchasing probabilities independent of λ, as stated by the following corollary.

Corollary 2: $\Pr(AB)\big|_{p=\bar{p}} = 1/2$, *which is independent of* λ.

Based on the derived purchasing probability, in the following sections, we analyze the cases of risk neutral and risk averse retailers.

4. Analysis of Risk Neutral Decision Makers

We can use purchasing probability (6) to find the optimal bundle price which maximizes the expected profit.

<u>Proposition 2</u>: *The unique bundle price which maximizes the expected profit is as follows:*

$$p^* = \begin{cases} p_1^* = \dfrac{2L_{xy} + c + \sqrt{(L_{xy} - c)^2 + 6xy}}{3} & if \quad c < c_{Min} \\ p_2^* = \dfrac{2c + U_{xy} + p_{Max}}{4} & if \quad c_{Min} \le c \le c_{Max} \\ p_3^* = \dfrac{2c + U_{xy}}{3} & if \quad c_{Max} < c \le U_{xy} \end{cases} \tag{7}$$

, *where* $c_{Min} = \dfrac{4p_{Min} - p_{Max} - U_{xy}}{2}$ and $c_{Max} = \dfrac{3p_{Max} - U_{xy}}{2}$.

Figure 4 shows the three different regions of the optimal bundle price in the feasible region of (λ, c). Each region of p_i^* is corresponding to the region of $\Pr(AB)_i$ in figure 2, where *i*=1, 2, 3. Specifically, c_{Max} is equal to U_{xy} at $\lambda = -1$ and it is a decreasing function in the negative correlation zone and an increasing function in the positive correlation zone, whose value becomes *U* at $\lambda = +1$. As such, c_{Max} has a local minimum at $\lambda = 0$. With respect to c_{Min}, it is always a decreasing function of λ, whose starting point at $\lambda = -1$ and ending at $\lambda = +1$ are both below L_{xy}. It can be easily verified that c_{Min} is zero at $\lambda_0 = [2(L-a)+3b]/5b$, which is less than one if *U<2L*.

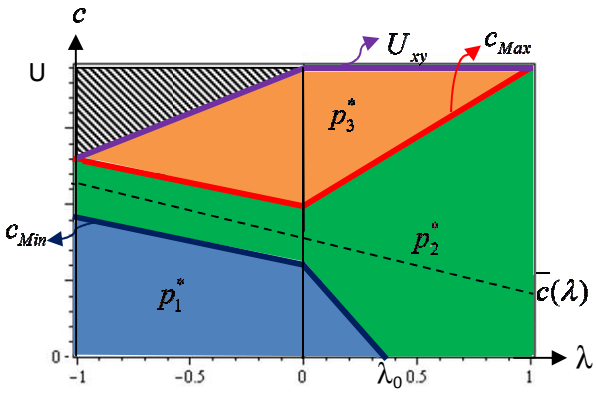

Figure 4 – The three regions of p^* in feasible region of (λ, c)

In figure 4, $\bar{c}(\lambda)$ is the bundle cost which results in $p^* = \bar{p}$. The following corollary elaborates on this property of $\bar{c}(\lambda)$.

Corollary 3: $p^* \le \bar{p} \Leftrightarrow c \le \bar{c}(\lambda)$, *where* $\bar{c}(\lambda) = \bar{p} - \frac{x}{2}$ *(x is defined in proposition 1).*

Later, we will see that this corollary has a significant managerial implication (see the discussion which follows corollaries 5 and 8). The following corollary characterizes the optimal expected profit.

Corollary 4: *The unique optimal expected profit and corresponding variance are, respectively:*

$$E[\pi(p^*)] = \begin{cases} \dfrac{[2xy - (p_1^* - L_{xy})^2](p_1^* - c)}{2xy} & if \quad c < c_{Min} \\ \dfrac{(2(a+L) + b(1+\lambda) - 2c)^2}{16x} & if \quad c_{Min} \le c \le c_{Max} \\ \dfrac{2(U_{xy} - c)^3}{27xy} & if \quad c_{Max} < c \le U_{xy} \end{cases} \tag{8}$$

As expected, for any λ, higher unit bundle costs (c) lead to higher optimal bundle prices and lower optimal expected profits ($\partial p^* / \partial c > 0$ and $\partial E[\pi(p^*)] / \partial c < 0$). The following corollary characterizes the behavior of expected profit with respect to λ.

Corollary 5: *When* $c_{Min} \le c$ *, we have:* $0 \le \frac{\partial E[\pi(p^*)]}{\partial \lambda} \Leftrightarrow c \le \overline{c}(\lambda)$.

The above corollary is highly important as it offers managerial insights on products selection. It shows how the preferred level of correlation between the reservation prices of the two products depends on unit bundle cost. When $c \ge c_{Min}$, optimal expected profit is decreasing (increasing) in λ if bundle cost is above (below) $\overline{c}(\lambda)$. In other words, the worst operating points happens at $\overline{c}(\lambda)$, where expected profit is at its minimum. The following corollary elaborates on this fact from the perspective of risk neutral decision makers.

Corollary 6: *When* $c \ge c_{Min}$, $Min(E[\pi(p^*)]) = \frac{x}{4}$ *which happens at* $c = \overline{c}(\lambda)$.

As such, by increasing or decreasing products correlations, risk neutral decision makers prefer to keep distance from $\frac{x}{4}$ as much as possible. For $c < c_{Min}$, due to the complexity of the relations, it is not feasible to analytically characterize the behavior of optimal expected profit with respect to changes in product correlation. However, numerical observations suggest that lowest optimal expected profit happens at $\lambda = 0$. The critical line of $\overline{c}(\lambda)$ is reemphasized in the next section when risk is considered.

5. Analysis of Risk Averse Retailer

The previous section characterized the optimal parameters for a risk neutral retailer, i.e. a retailer who seeks to maximize the expected profit regardless of the involved risk. To characterize the optimal solution for a risk-averse retailer, we use an MV approach, i.e.;

$$\begin{aligned} &Maximize \quad E[\pi(p)] \\ &subject\ to: \quad V[\pi(p)] < V_{\max} \end{aligned} \tag{9}$$

, where $V_{\max}$ is the acceptable level of variance (the retailer's risk tolerance). The following proposition describes the behavior of the optimal price under an MV approach. Let p^{MV} denote the optimal bundle price under an MV decision criteria.

<u>Proposition 3</u>: *Unique optimal price p^{MV} under MV decision criteria (9) has the following property:*

$$\textit{If } \mathrm{V}[\pi(p^*)]<\mathrm{V}_{Max} \text{ then } p^{MV} = p^* \text{ else } p^{MV} = \arg\max_p(\mathrm{V}[\pi(p)]=\mathrm{V}_{Max}) < p^*. \tag{10}$$

This behavior is resulted from the fact that the price that maximizes the expected profit is always smaller than the price that maximizes the profit variance.

<u>Proposition 4</u>: *Proposition 3 is valid for general cases where reservation prices follow any probability distribution with a non-decreasing hazard function.*

Note that the above proposition holds for almost all the commonly used probability distributions. To apply the above two propositions, variance of profit at p^* is needed. The following corollary is based upon (5) and (7).

Corollary 7: *The profit variance at the optimal bundle prices is:*

$$V[\pi(p^*)] = \begin{cases} \dfrac{(p^*-L_{xy})^2(2xy-(p^*-L_{xy})^2)(p^*-c)^2}{4x^2y^2} & \textit{if } c < c_{Min} \\ \dfrac{(2(a+L)+b(1+\lambda)-2c)^3(2(2x+c-a-L)-b(1+\lambda))}{2^8x^2} & \textit{if } c_{Min} \le c \le c_{Max} \\ \dfrac{2(U_{xy}-c)^3(9xy-2(U_{xy}-c)^2)}{3^6x^2y^2} & \textit{if } c_{Max} < c \le U_{xy} \end{cases} \tag{11}$$

Through the rest of this section, we characterize the behavior of profit variance and its sensitivity to product correlation.

Profit variance depends on unit cost via *e*. As such, similar to expected profit, it is a decreasing function of unit cost. Considering the fact that e is independent of λ, as intuitively expected, variance of profit is always an increasing function of λ, as stated by the following corollary.

<u>Proposition 5</u>: *Variance of profit is always increasing in* λ, except when $p = \overline{p}$ at which $\frac{\partial V[\pi(p)]}{\partial \lambda} = 0$ and $V[\pi(\overline{p})] = e^2 / 4$.

Since a risk averse decision maker prefers lower variances, such a decision maker should prefer lower product correlation from this perspective. Similar to expected profit, at bundle price of $\overline{p}$, variance is also independent of λ. Note that based on corollary 6 risk neutral decision makers try to keep distance from the conditions which lead to the minimum expected profit, $Min(E[\pi(p^*)]) = x / 4$, by either reducing or increasing product correlation. However, based on the above proposition, risk averse decision makers always prefer to choose lower product correlation since it results in lower variances as well as higher expected profit (avoiding the minimum expected profit).

As stated in the introduction, another risk measurement is the coefficient of variation. The following corollary elaborates on this risk measurement.

Corollary 8*: The square of the coefficient of variation is*: $COV^2 = \frac{V[\pi]}{E[\pi]^2} = \frac{1}{\Pr(AB)} - 1$, *which is always increasing in the bundle price.*

At $p = \overline{p}$ (or equivalently, $\Pr(AB) = \frac{1}{2}$), the coefficient of variation is 1. By moving toward smaller bundle prices COV reduces to amounts less than 1. In this range of bundle price, lower λ is preferred based on corollary 1. However, for bundle prices greater than $\overline{p}$, COV will be

larger than 1 representing a highly risky and chaotic performance. In this range of bundle price, COV is amplified at higher levels of λ. The managerial implication of the above corollary along with corollary 3 can be significant. Specifically, if the unit cost is less than $\overline{c}(\lambda)$, then lower products correlation yields both higher expected profit and lower variance of profit. Also, as stated before, in this region COV is less than one. Otherwise, if the unit cost is more than $\overline{c}(\lambda)$, then we will have a highly risky situation.

To provide more managerial insights, we introduce the notion of domination as follows. We say scenario X is dominant over scenario Y, which is shown by X ↘ Y, if X has equal or higher expected profit and lower profit variance. Obviously, X ↘ Y is a sufficient condition to have COV(X) < COV(Y), where COV(*i*) denotes the coefficient of variation of scenario *i*. The following corollary describes regions in figure 4 where the notion of domination is applicable to different values of λ.

Corollary 9: *The following dominance relations always hold:*

(a) In the region of (λ, c) *below* $\overline{c}(\lambda)$*, we have:* $\lambda_1 \square \lambda_2$ *for any* $\lambda_1 < \lambda_2$.

(b) In the region of (λ, c) *above* $\overline{c}(\lambda)$*, there is no domination for any* $\lambda_1 \neq \lambda_2$.

In case (b), there is no domination since for any $\lambda_1 < \lambda_2$ *we have* $V[\pi]\big|_{\lambda_1} < V[\pi]\big|_{\lambda 2}$ *and* $E[\pi]\big|_{\lambda_1} < E[\pi]\big|_{\lambda 2}$. Hence, an MV trade-off should be made if combination of (λ, c) falls above $\overline{c}(\lambda)$. Therefore, $\overline{c}(\lambda)$ (which is corresponding to $p^* = \overline{p}$) is a turning point for pure bundling policy.

6. Numerical Examples

In this section, we present some numerical examples to illustrate findings of the previous section. To assess product bundling from a risk perspective, we investigate the impact of unit bundle cost, products correlation, and K on optimal expected profit, associated variance, and the coefficient of variation. For products correlation, we consider three scenarios of highly negatively correlated ($\lambda = -0.9$), independent ($\lambda = 0$), and highly positively correlated ($\lambda = 0.9$) scenarios. Similarly, we consider three levels of $K = 0.1$, $K = 0.5$, and $K = 0.9$. To gain managerial insights, across all three levels of K, we keep the range of reservation prices of product *A* constant at $l_A = 20$, $u_A = 100$ (*a*=80). We also keep the middle point of reservation prices of product *B* constant at 60, same as the middle point of the reservation prices of product *A*. K can be considered as the level of heterogeneity in the reservation price uncertainty. A value of K which is close to 1 represents a situation in which the two products have almost the same level of reservation price uncertainty. When K is very small, one of the products (product *A*) has a much more reservation price uncertainty than the other product (product *B*). This might happen when the retailer bundles a product with established demand (low reservation price uncertainty) with a new product (high reservation price uncertainty). More specifically, the low level of heterogeneity ($K = 0.1$) is corresponding to $l_B = 56$ and $u_B = 64$ (*b*=8), the medium level of heterogeneity ($K = 0.5$) is corresponding to $l_B = 40$ and $u_B = 80$ (*b*=40), and finally the high level of heterogeneity ($K = 0.9$) is corresponding to $l_B = 24$ and $u_B = 96$ (*b*=72). The unit bundle cost varies between 0 and 120, across the above combinations. In the following, we first examine the expected profit of a risk neutral decision maker (E^*), then we switch to the risk averse decision

maker for which we look at the variance (V^*) and the coefficient of variation (COV^*) at the optimal price.

Figure 5 shows the optimal expected profit at different levels of products correlation and products heterogeneity, while unit bundle cost is varying. As expected, optimal expected profit is decreasing in bundle cost. However, at the lower levels of unit bundle cost, negative correlation is preferred over positive correlation and this preference is reversed at the higher levels of unit bundle cost. Furthermore, the impact of product correlation increases while products heterogeneity escalates. In summary, risk neutral decision makers should have more tendencies to negatively correlated products specially when heterogeneity is high and unit bundle cost is rather low.

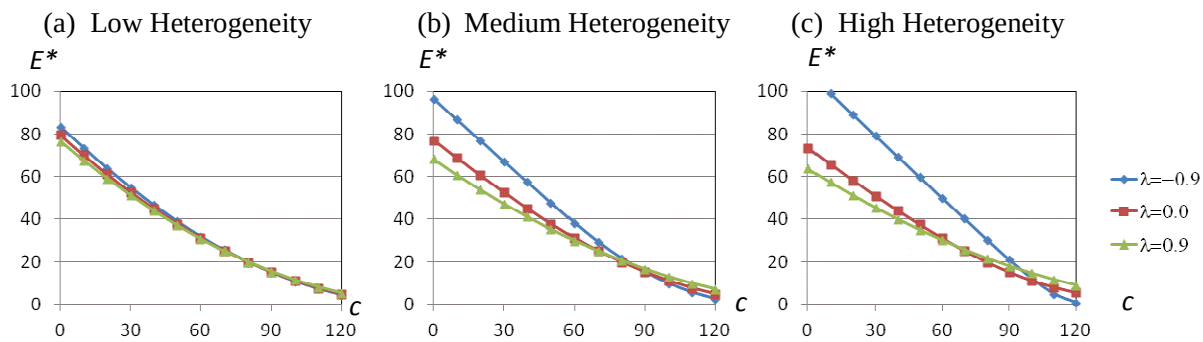

Figure 5 – Optimal expected profit (E^*) vs. unit bundle cost (c) at different levels of product heterogeneity (K) and correlation (λ)

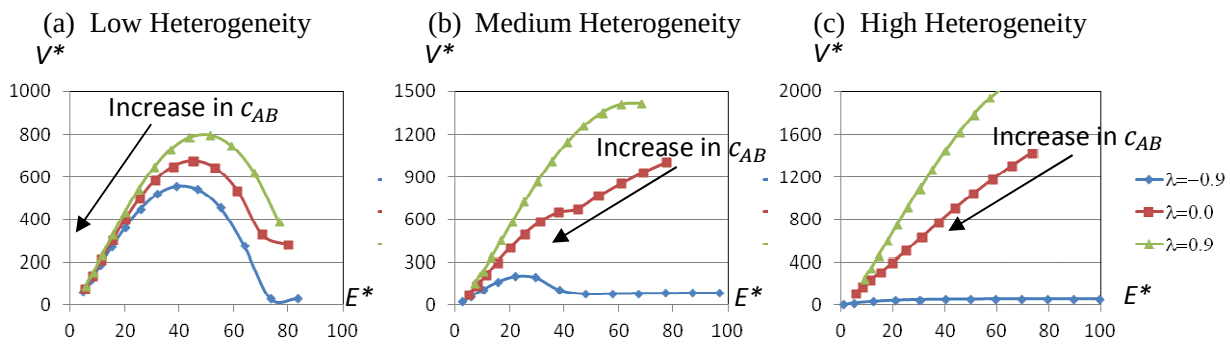

Figure 6 – Variance at the optimal expected profit (V^*) vs. optimal expected profit, at different levels of unit bundle cost (c), product heterogeneity (K) and correlation (λ)

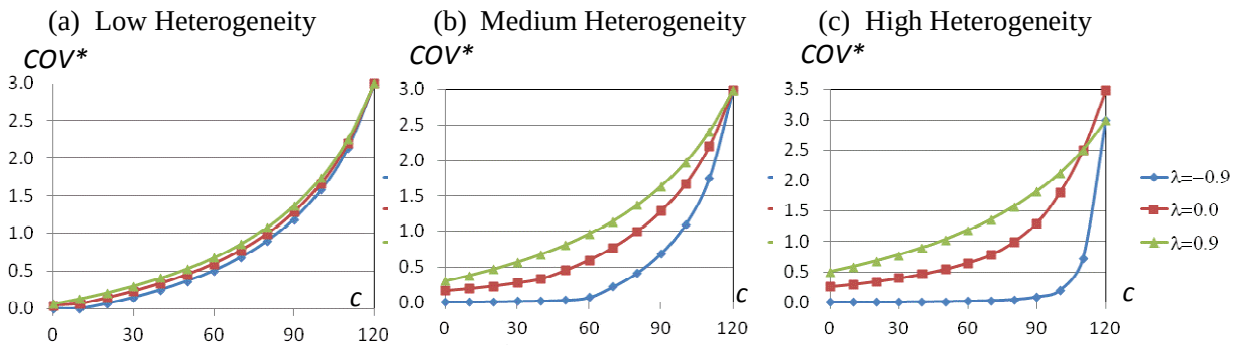

Figure 7 – Coefficient of variation at the optimal expected profit (COV^*) vs. unit bundle cost (c), at different levels of product heterogeneity (K) and correlation (λ)

Figure 6 shows the behavior of profit variance which should be considered by risk averse decision makers. As opposed to the risk neutral decision makers, we can see that risk averse decision makers are generally quite sensitive to product correlation and product heterogeneity. Specifically, as opposed to very high range of unit bundle costs where there is convergence independent of product correlation and product heterogeneity, divergence is amplified by reducing unit bundle cost. Furthermore, low heterogeneity reveals a non-monotonic behavior such that in the medium unit bundle cost could possibly violate the risk threshold. Over medium or high level of product correlation and product heterogeneity increase of expected profit due to lower unit bundle cost is simultaneously happening with sharp increase of variance. The highly negative product correlation, interestingly, offers advantage of increase of expected profit due to lower unit bundle cost while variance remaining flat. That is, sensitivity to unit bundle cost reduces especially for more negatively correlated products. As a managerial implication for information goods, selection of more negatively correlated information goods is highly critical in medium to high product heterogeneity.

To gain more profound understanding on the impact of risk consideration, figure 7 shows changes in coefficient of variation of profit while unit bundle cost is varying. As indicated in the

previous section, it is observed that beyond some critical unit bundle costs, COV increases to values greater than 1; which represents a highly risky situation. Such critical unit bundle costs are increasing functions of both products correlation and products heterogeneity. Also, while COV is less sensitive to products correlation at lower level of product heterogeneity, higher product heterogeneity improves attractiveness of more negatively correlated products.

5. Conclusions

In this study, we portrayed a new perspective of risk consideration in bundling problems. Specifically, we looked at profit variability via MV approach and COV. We derived explicit relations for purchasing probabilities, optimal expected profits, and variances of profit with most general case of uniformly distributed reservation prices and product correlation varying continuously from perfectly negatively correlated ($\lambda = -1$) to perfectly positively correlated ($\lambda = +1$) products.

We showed that there is a turning point for bundle price, $\overline{p}$, before (after) which purchasing probability and expected profit is decreasing (increasing) in product correlation. We also showed that if bundle price is greater than the turning point, then coefficient of variation will be greater than one; representing a highly volatile profit and risky situations.

We also showed that the optimal price for risk averse decision maker is always less than or equal to the optimal price of a risk neutral decision maker ($p^{MV} \leq p^{*}$) We proved this result for reservation prices with any general probability distribution whose hazard function is a non-decreasing; which holds for almost all the commonly used probability distributions. Such a result revealed an important managerial consequence. Specifically, if the variance of profit at p^{*} is less than the acceptable level of variance ($V_{\max}$) then optimal bundle price for a risk averse decision

maker under MV approach, p^{MV}, will be the same as the optimal price of a risk neutral decision maker; $p^{MV} = p^{*}$. Otherwise, the optimal bundle price of a risk-averse decision maker is always smaller than that of a risk neutral decision maker.

We also introduced the notion of domination, which was applied in the context of product selection; to choose the proper correlation level between the two products (λ). Specifically, we observed that if bundle price is less than $\overline{p}$ then smaller product correlation are preferred as it yields simultaneously higher expected profit and smaller variance of profit. However, if bundle price is greater than $\overline{p}$ then higher product correlation yields higher expected profit but at the cost of higher variance of profit. In other words, bundle price of greater than $\overline{p}$ is so costly that coefficient of variation will be greater than one.

Through numerical examples, we illustrated the role of product heterogeneity. We observed that as opposed to risk neutral decision maker who should prefer more negatively correlated products (except at very high level of unit bundle cost), a risk averse decision maker at low products heterogeneity and binding maximum risk level would prefer more positively correlated products. We also observed that high product heterogeneity increases the role of products correlations.

Our work can be extended from different perspectives. First, bundling policy could be extended to no bundling and mixed-bundling to compare performance of different bundling policies from risk perspective. Our research can also be extended to consider non-homogenous markets which consist of different segments. Our model was limited to a monopoly environment and considering other market structures such as duopoly or oligopoly could be other extensions. Investigating the role of the number of products in a bundle could be another direction for further

research. Finally, we applied MV formulation for risk assessment. Considering other approaches such as value at risk or utility function could be another direction for extending this study.

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Appendix

We provide proof of propositions, in this section. Proof of corollaries can be done by employing algebraic operations such as differentiation, simplification, etc., which is left to reader. Yet, we highlight critical points in proof of only corollaries having some complexity in their proofs.

<u>Proof of Proposition 1:</u> Consider first: $0 \le \lambda \le 1$. We initially calculate the probabilities for the most general case of independent reservation prices. Then, we calculate the case of correlated reservation prices from these probabilities. Let $r_X \sim U(l_X, u_X)$ and $r_Y \sim U(l_Y, u_Y)$ be independent. Also, $x = u_X - l_X$ and $y = u_Y - l_Y$. Given $K \le 1$, we have $l_A + u_B \le l_B + u_A$ and purchasing probability will be:

$$\Pr(r_X + r_Y > p) = \begin{cases} 1 & if \quad p \le l_X + l_Y \\ 1 - \dfrac{[p-(l_X+l_Y)]^2}{2xy} & if \quad l_X + l_Y \le p < l_X + u_Y \\ \dfrac{(u_Y + l_Y) + 2(u_X - p)}{2x} & if \quad l_X + u_Y \le p < l_Y + u_X \\ \dfrac{[(u_X + u_Y) - p]^2}{2xy} & if \quad l_Y + u_X \le p < u_X + u_Y \\ 0 & if \quad u_X + u_Y \le p \end{cases}$$

Now, $\Pr(AB) = \Pr(r_A + r_B > p) = \Pr(r_A + \lambda\left[l_B + K(r_A - l_A)\right] + (1-\lambda)\delta > p)$

$= \Pr(r_A + \lambda\left[l_B + K(r_A - l_A)\right] + (1-\lambda)\delta > p)$.

Let's assume: $r_X = r_A + \lambda\left[l_B + K(r_A - l_A)\right] and \ r_Y = (1-\lambda)\delta$. Therefore, we will have:

$$\begin{cases} l_X = l_A + \lambda l_B \\ u_X = u_A + \lambda l_B + \lambda K a \end{cases} \Rightarrow \quad x = (1+\lambda K)a \ \text{ and } \begin{cases} l_Y = (1-\lambda) l_B \\ u_Y = (1-\lambda) u_B \end{cases} \Rightarrow \quad y = (1-\lambda)b$$

$\Pr(AB) = \Pr(r_A + r_B > p) = \Pr(r_X + r_Y > p)$

For $-1 \le \lambda < 0$, using (1): $\Pr(AB) = \Pr(r_A + r_B > p) = \Pr(r_A + \lambda\left[u_B - K(r_A - l_A)\right] + (1-\lambda)\delta > p)$

Let's assume that $r_X = r_A + \lambda\left[u_B - K(r_A - l_A)\right]$ and $r_Y = (1-\lambda)\delta$. Therefore, we will have:

$$\begin{cases} l_X = l_A + \lambda u_B \\ u_X = u_A + \lambda l_B \end{cases} \Rightarrow \quad x = a - \lambda b \ \text{ and } \begin{cases} l_Y = (1-\lambda) l_B \\ u_Y = (1-\lambda) u_B \end{cases} \Rightarrow \quad y = (1-\lambda)b$$

$\Pr(AB) = \Pr(r_A + r_B > p) = \Pr(r_X + r_Y > p)$

<u>Proof of Proposition 2:</u> By substituting probabilities of proposition 1 (and 4) into (3) (and (9)), after differentiation and simplification, this proposition is proved. Note that the second derivative is also negative; which is indicating that optimal prices are at maximum points.

Proof of Proposition 3: Based on (3) and (4), respectively, we have: $E(\pi_i) = e\Pr(AB)$ &

$Var(\pi_i) = e^2 \Pr(AB)\left[1-\Pr(AB)\right] = E(\pi_i)e(1-\Pr(AB)) \Rightarrow \frac{\partial Var(\pi_i)}{\partial p} = \frac{\partial E(\pi_i)}{\partial p}e(1-\Pr(AB))$

$+E(\pi_i)(1-\Pr(AB)) - E(\pi_i)e\frac{\partial \Pr(AB)}{\partial p}$. Since $\frac{\partial E(\pi_i)}{\partial p} > 0$ (for $p < p^*$) and $\frac{\partial \Pr(AB)}{\partial p} < 0$

(for any λ, stated before corollary 1) $\Rightarrow \frac{\partial Var(\pi_i)}{\partial p} > 0$ for $p < p^*$.

Proof of Proposition 4: Let $f_{AB}(.)$ and $F_{AB}(.)$ be the probability density function and cumulative distribution function of $r_{AB}(= r_A + r_B)$, respectively. The profit from each customer can be written as:

$$\pi = \begin{cases} e & if \quad r_A + r_B \geq p \quad with \quad probability \quad of \quad \Pr(AB) = 1 - F_{AB}(p) \\ 0 & otherwise \end{cases},$$

The expected and variance of profit from each customer can then be written as:

$$E[\pi] = e\Pr(AB) = e\left(1 - F_{AB}(p)\right), \text{ and}$$

$$V[\pi] = e^2 \Pr(AB)\left[1-\Pr(AB)\right] = E[\pi]e\left[1-\Pr(AB)\right] = E[\pi]eF_{AB}(p).$$

To find the optimal prices which maximize the expected profit, we proceed as follows.

$$\frac{\partial E[\pi]}{\partial p} = \left(1 - F_{AB}(p)\right) - (p-c)f_{AB}(p) = 0 \Rightarrow \; p^* - c = \frac{1 - F_{AB}(p^*)}{f_{AB}(p^*)}$$

The left hand side of the above equation is an increasing function of p^* with a negative y-intercept. The right hand side of this equation is the inverse of *hazard function*. The hazard function of most famous distribution functions (including: Normal, Exponential, Gamma, Poisson, and Uniform) is non-decreasing, which makes its inverse a non-increasing function. The right hand side of the equation has a positive y-intercept. As a result, this equation has a unique solution, p^*. It is easy to verify that at p^* the sign of the derivative of the expected profit

changes from positive to negative. Therefore, p^* is the unique maximizer of the expected profit (considering the reasonable assumption of a non-decreasing hazard function).

$$\pi_{AB}^* = E[\pi]_{p^*} = (p^* - c)^2 f_{AB}(p^*)$$

For the variance of profit we have:

$$\frac{\partial V[\pi]}{\partial p} = \frac{\partial E[\pi]}{\partial p} eF_{AB}(p) + E[\pi]F_{AB}(p) + E[\pi]ef_{AB}(p).$$

We can see that for any bundle price smaller than the bundle price which maximizes the expected profit, the variance of profit is increasing. This means that the bundle price which maximizes the expected profit is smaller than the bundle price which maximizes the profit variance. In other words we have:

$$p \le p^* \quad \Rightarrow \quad \frac{\partial E[\pi]}{\partial p} \ge 0 \quad \Rightarrow \quad \frac{\partial V[\pi]}{\partial p} \ge 0.$$

<u>Proof of Proposition 5:</u> $\frac{\partial V[\pi]}{\partial \lambda} = e^2 \frac{\partial(\Pr(AB)(1-\Pr(AB))}{\partial \lambda} = e^2 \frac{\partial \Pr(AB)}{\partial \lambda}(1-2\Pr(AB))$, based on (5). Based on corollary 1 and figure 3, we have $\frac{\partial \Pr(AB)}{\partial \lambda} < 0 \; and \; \frac{1}{2} < \Pr(AB) \;\; if \;\; p < \overline{p}$.

Similarly, $\frac{\partial \Pr(AB)}{\partial \lambda} > 0 \; and \; \frac{1}{2} > \Pr(AB) \;\; if \;\; p > \overline{p}$. Thus, right hand side of $\frac{\partial V[\pi]}{\partial \lambda}$ is positive in either case. And per corollary 1, $\frac{\partial \Pr(AB)}{\partial \lambda} = 0 \;\; if \;\; p = \overline{p}$ $\frac{\partial Pr(AB)}{\partial \lambda} = 0$ if $p = \overline{p}$. This concludes proof of proposition 5.