Spring May, 1965

Measurement of transmission parameters of porous sound-absorbing materials. Part 1. Measurement Techniques in Terms of the Electrical Analogues

Edmund Ramsay Wigan, *Technical Consultant to Muirheads Instruments*

Part 1. Measurement Techniques in Terms of the Electrical Analogues

Measurement of transmission parameters of porous sound-absorbing materials

by E. R. Wigan

To help the reader to grasp unusual features of the test-techniques adopted this paper is broken into two parts: in Part 1 the electrical analogues of the acoustical systems of Part 2 will be employed. In the second part the test-gear and its usage will be dealt with and related published work discussed.

ADOPTION OF the novel "pseudo-impedance" artifice is partly responsible for complicating the measurements; although a little tedious to the user it makes measurements possible over a wide frequency-range without involving any "tuning" operation, so that the sample of material can be held in a short, rigid tube of fixed length. The electrical analogues that form the basis of this part of the paper are justified by the findings of Part 2 where it will be shown that sound pressure is attenuated in passing into the thickness of the test-material in much the same way that voltage is attenuated in travelling along an electrical transmission-line. The final stages of Part 2 describe how the primary acoustical transmission constants are derived; Part 1 carries the analysis up to this same point but in terms of the electrical analogue.

Occasionally it will be necessary to anticipate later findings but only to keep the discussion within proper bounds and in general this part will treat the behaviour of transmission lines by direct analogy as follows:

Alternating sound pressure	=	Alternating p.d.
Alternating volume-flow of air	=	Alternating current.
Acoustical impedance	=	Electrical impedance

Measurement of the Propagation Constant $\gamma=(\alpha+j\beta)$

Fig. 1 includes the transmission-line which is to play the part of a sample of sound-absorbing material, and shows the test-conditions that will be applied to it:

(*a*) an open circuit at the line output;

(*b*) a terminal load equal to Zo, the characteristic transmission impedance of the line;

(*c*) a known capacitive load ($C1$, $C2$ etc.).

In each case the input/output voltage-ratio, (note the inversion) is to be measured.

We define these ratios as follows:

$$E/E_a=/N\infty\underline{/\theta}$$
$$E/E_b=/T\underline{/\eta} \quad (1)$$
with $C=C1$, $(E/Ec)_1=/N1/\theta1$
$C=C2$ etc. $(E/Ec)_2=/N2\underline{/\theta2}$ . . etc.

In order to obtain the arguments of the vector quantities listed a phase-sensitive device must be used(*) to measure the ratios. To avoid typographical errors in what follows it is convenient to write $N\infty$, T, $N1$, $N2$, etc, without reference to argument, but it has to be borne in mind that these quantities, however symbolised, have to be treated as vectors.

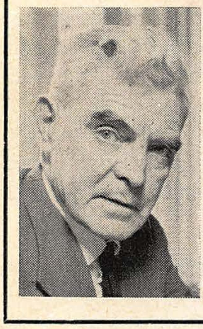
The propagation constant $\gamma=(\alpha+j\beta)$ is derived from $N\infty$ and T as follows:

From conventional transmission theory the relation between $N\infty$ and T is known to be

$$2(N\infty)=(T+1/T) \quad (2)$$

This vector equation is solved graphically by the nomograph Fig. 2. Notice that the graph can be "entered" with either $N\infty$ known to derive T, or with T known to derive $N\infty$. The pattern shown is repeated in other quadrants of the complex plane by reflection in the X and Y axes, angles θ and η (both measured positively anti-clockwise) increasing throughout each quadrant, in each of which they start and

(*) Measurements reported in Part 2 were made by the a.c. potentiometer of Ref 2.



E. R. Wigan, who is at present technical consultant to Muirhead & Co. Ltd., recently retired from the BBC Research Department. Of some 40 years with the BBC, M.o.S. (SRDE), and industry, he spent nearly half in research and has filed patents on a variety of instruments in the fields of mechanics, acoustics, electronics and optics. He has written papers on novel aspects of network theory and psycho-acoustics.

finish at equal values. This makes it easy to apply Fig. 2 when θ and η occasionally slightly exceed 90°, as is the case in Part 2.

The diagram is rapidly constructed by carrying out graphically the instructions of Equation (2), $/T/\underline{\eta}$ being added vectorially to $/1/T/\underline{-\eta}$; to protect it, when in constant use,

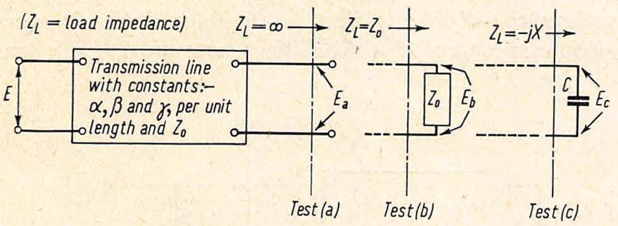


Fig. 1

from the points of dividers it should be mounted on strong card. The reader should take note that since Zo is at this stage unknown the voltage-ratio E/Eb is not a *measured* quantity but one derived from the primary ratio-measurement E/Ea by reliance upon conventional transmission theory. In the electrical circuit considered here reliance is justified so we can proceed, but in the acoustical case reliance is justified only when the sample of absorbing material has the perfectly homogeneous structure that is assumed in the development of conventional transmission theory.

Returning to the test-condition (b): it can be seen that this test-condition is the same as measuring the voltage ratio T in a line of infinite length with the measuring points separated by the length l of the short line actually tested. Thus we may apply the theory of infinite lines and derive

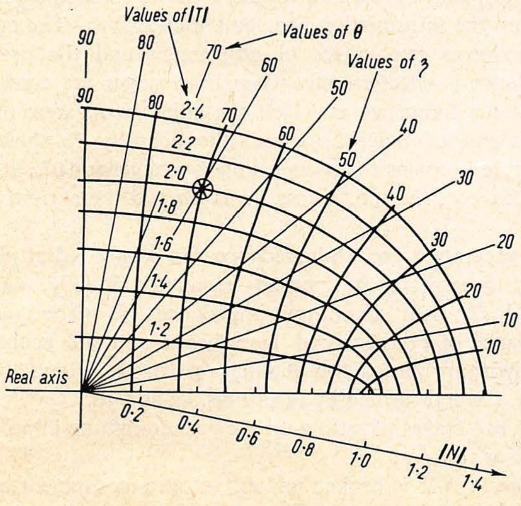


Fig. 2

γ, α and β from T. (Note that the quoted quantities are *intrinsic* constants so allowance has to be made for line-length l).

From Equation (1) we have, by definition:

$$T=/T/\underline{\eta}$$

and from general theory:

$$l\,.\,\alpha = \mathrm{Log}_e\,(/T/)$$

or more concisely $\qquad = \mathrm{Ln.}\ /T/$

while $\qquad l\,.\,\beta\,(_{radians}) = (\eta)_{Deg}/(57{\cdot}4) \qquad (3)$

From α and β we construct γ using the conventional definition:

$$(\gamma)=(\alpha+j\beta) \qquad (4)$$

Much later we shall need $(\gamma)^2$, which is

In Polar form: $(\gamma)^2 = /(\alpha^2+\beta^2)/2\ .\ \underline{(\text{Arc. Tan.}\ \frac{\beta}{\alpha})} \qquad (5)$

or Cartesian: $\qquad = \alpha^2-\beta^2+2j(\alpha\beta) \qquad (6)$

Study of Fig. 2 will show that there are obvious limitations to this process of deriving α and β from $N\infty$: for instance unless $/T/$ is at least 1·1 or 1·2, neither α nor β can be derived accurately. Thus the transmission-line must be "long" enough for attenuation to exceed 1 to 2 dB and/or phase-shift to exceed 20–30 deg. (Thus in acoustic tests, for instance, we shall find it difficult to derive α or β in soft, poorly attenuating materials when the frequency is below about 300 c/s, and even if the maximum sample-length (3 in.) is available; on the other hand, tests on samples of dense materials only 1 in. thick are feasible at less than 150 c/s.)

In most acoustical materials α, β and γ rapidly increase with frequency and to simulate this the electrical systems now to be discussed will be credited with the same property.

The Measurement of Zo

(*Note to the critical reader:* In purely electrical systems Zo can be measured in half-a-dozen ways but since they cannot be copied with acoustical test-gear they are not mentioned here. Although the scheme below cannot be copied either, it is given detailed attention because the basic theory associated with it leads on to the second scheme which is acoustically viable.)

Measurement of Zo without "pseudo-impedances"

Reference will be made to Fig. 1 and the associated equations in Section (2). In test-condition (c) a term $X1$ will be introduced to represent the impedance of the load capacitance $C=C1$; similarly $X2$ when $C=C2$; again the voltage-ratio $(E/Ec)_1$ will apply to the case that $C=C1$, and so on.

Consider $Z2$, the (unknown) output impedance of the line when E is held constant—as in Fig. (1)—as one of the Thévenin parameters, and Ea as the other. Then we may consider Eb as the p.d. across the load-impedance Zo applied to a circuit of e.m.f. Ea, $(=E/N\infty)$, and internal impedance $Z2$.

Whence

$$Ea/Eb=(1+Z2/Zo) \qquad (7)$$

and

$$(Ea/Ec)_1=(1+Z2/(-jX1) \qquad (8)$$

We have also from Equation (1):

$$Ea=E/N\infty$$
$$Eb=E/T \qquad (9)$$
$$(Ec)_1=E/N1,\ (Ec)_2=E/N2 \text{ and so on}$$

From (7–8–9):

$$Z2=(Ea/Eb-1)\,(Zo)=(\,(Ea/Ec)_1-1)\,(-jX1); \qquad (10)$$
$$Zo/(-jX1)=(N1-N\infty)/(T-N\infty) \qquad (11)$$

whence

(It will be seen that the unknown $Z2$ vanishes) quation (11) is solved by measurements on a Fig. 3 in which arbitrary values of $N\infty$, T and ows have been assumed:

$$N\infty = /0{\cdot}82\underline{/59{\cdot}°0}$$

...via Fig. 2:

$$T = /2{\cdot}0\underline{/70{\cdot}°0} \text{ (the point marked (*) in the Figure)}$$

Arbitrarily we make

$$N1 = /1{\cdot}0\underline{/106{\cdot}°0}$$

and make

$$(-jX1) = /1000\underline{/-90{\cdot}°0} \text{ (ohms)}$$

From Fig. 3, or by resolution of vectors, we deduce the vectors

$$(N1 - N\infty) = /0{\cdot}75\underline{/+160{\cdot}°0}$$
$$(T - N\infty) = /1{\cdot}21\underline{/+78{\cdot}°0}$$

From these data, and making use of Equation (11), it is concluded that:

$$/Zo\underline{/z} = /620\underline{/-8.°0} \text{ at the frequency of test.}$$

Provided the diagram Fig. 3 is drawn on Polar graph paper it is possible with dividers and parallel-rulers to derive both the Modulus $/Zo/$ and the argument ($\underline{/z}$) from the diagram without being involved in resolution of vectors. It will be seen that $\underline{/z}$, the *negative* argument of Zo can be read from Fig. 3 by erecting a perpendicular (dotted) on vector ($N1$-$N\infty$).

It is important to notice that vector (T-$N\infty$), in Equation (11), is related to the *inverse* of Zo in the same way that ($N1$-$N\infty$) is related to the *inverse* of $X1$. In fact these vectors both represent admittances; it is this that accounts for the appearance of $\underline{/z}$, the argument of vector (T-$N\infty$) in the diagram, as a *positive* (anticlockwise) angle, whereas calculation has derived a *negative* argument for Zo—inversion of course reverses the sign.

Following the same line of thought it is easily understood that the difference-vectors $N1$–$N\infty$, $N2$–$N\infty$, $N3$–$N\infty$ will be proportional to the load capacitances $C1$, $C2$ and $C3$. Consequently, by making $C2=2(C1)$, $C3=3(C1)$ the diagram provides a valuable check on the measurements, for the intervals along the line $N3$–$N\infty$ will then all be equal; thus a more precise estimate of $N1$–$N\infty$ is found by dividing the interval $N3$–$N\infty$ by 3, and similarly with any other capacitance-ratio.

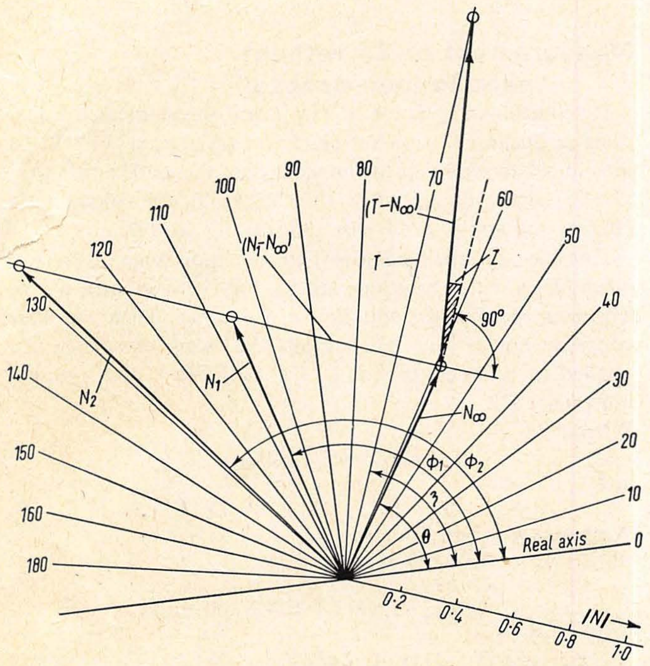


Fig. 3

For reasons fully discussed in Part 2 it is difficult to simulate the above operations acoustically; the "pseudo-impedance" described in the next section overcomes the difficulties.

Measurement of Zo with the aid of "pseudo-impedance"

One of the acoustical difficulties just mentioned is the necessarily finite input impedance of the probe-microphone used to observe the open-circuit pressure-ratio, corresponding to E/Ea in Fig. 1, so here we shall deal with the generation of its analogue—a pseudo-infinite voltmeter impedance—before going on to the second problem of measuring Zo by the same artifice.

Pseudo-infinite voltmeter impedance.

The meter D in Fig. 4a is imagined as having a small resistance r in order to simulate the finite probe microphone input-impedance. It will be shown how the p.d. eD across

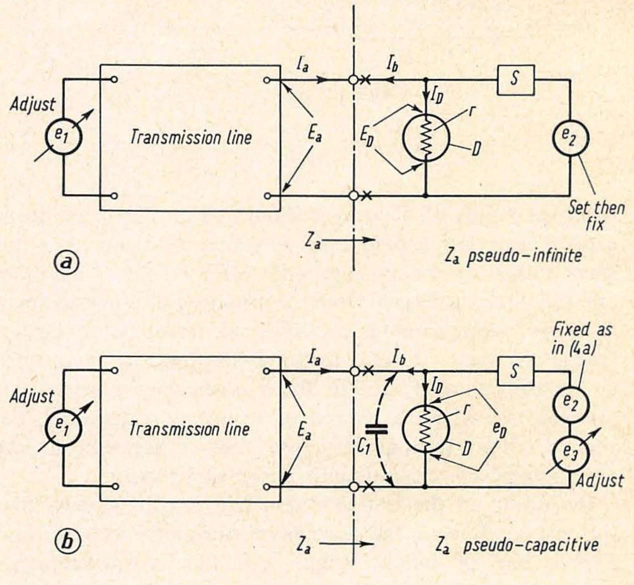


Fig. 4

this meter can be used to measure the open-circuit voltage Ea from the transmission-line, and thence $N\infty$. The operation involves two stages of adjustment and the pseudo-impedance is effective only when both stages are complete, and at the frequency at which the adjustments were made. At the end of Stage 2 (below) the quantity Za shown in the figure becomes the pseudo-impedance generated. At that stage $Za=\infty$, the p.d. across Za is Ea and the current $Ia=$ zero.

To generate a pseudo-impedance two separate, but phase-locked generators are needed—simulated in Fig. 4a, by $e1$ and $e2$ with an output-impedance S. (The output impedance of $e1$ is ignored here because in the acoustical case there are means for eliminating it). It follows that E in Fig. 1 is represented by $e1$ in Figs. 4a and 4b.

The two stages of setting up the pseudo-infinite impedance Za are as follows:

Stage 1: X-X is broken and $e2$ set at a convenient value and fixed.

The meter voltage eD and the ratio $eD/e2$ are noted.

Stage 2: *X-X* is closed and $e1$ adjusted *until* $eD/e2$ *is restored* in both modulus and argument.

Then, since $Ia=Ib=$zero, $Za=\infty$ and $e1/eD=N\infty$

Q.E.F.

(Notice that neither r nor S affects Za.)

On detailed analysis it will be found that at the close of Stage 2 the meter-current is being entirely supplied by generator $e2$; in consequence both Ia and Ib are zero and Za is effectively infinite.

A point of no particular importance here—but which will arise in the acoustical case later—is that at the outset of Stage 1 the output voltage Ea which will result at the close of Stage 2 is already settled—in fact the voltage eD chosen in Stage 1 later becomes equal to Ea.

In the acoustical case Ea corresponds to the output sound-pressure, and this must be chosen so that the whole test-system shall be no more than adequately loaded, for if pressure were excessive, both the tested material and the sound-generators might be operating in a non-linear regime. The initial choice of eD in Stage 1 can make sure this will not occur.

Having now established $N\infty$ with the aid of the low impedance voltmeter we can proceed to carry through the measurement of Zo using the same meter to apply pseudo-capacitive loads to the transmission-line.

Generation of Pseudo-capacitance.

For this operation (see Fig. 4b) a third generator $e3$ is needed, like $e2$ phase-locked to $e1$. (In practice $e3$ is made a part of $e2$).

Having carried out Stages 1 and 2 above to obtain $N\infty$, we have to make Za represent the reactance $(-jX1)$ used earlier:—

We proceed from Stage 2 *without disturbing* $e2$ *or* $e1$:

Stage 3. Break *X-X*. Temporarily connect a known capacitance $C1$ across meter D and set $e3$ to restore $eD/e2$ to the value found at Stage 1. Note the modulus and argument of $e3$.

Stage 4. Disconnect $C1$ and close *X-X*. Readjust $e1$ to restore the value of $eD/e2$. Measure $(e1/Ea)1$.

Stage 5. Reverse $e3$. Adjust $e1$ to restore $eD/e2$. Measure $(e1/Ea)2$.

Then $(e1/Ea)2=N1$, since $Za=-jX1$.

(Quantity $(e1/Ea)1$ is of use only indirectly—see below.)

These operations have effect as follows:

At the end of Stage 3 the current flowing into capacitor $C1$ is provided solely by $e3$; so when $C1$ is removed, and Stage 4 completed, the capacitor-current is forced to flow *from* $e3$ *as* Ib. From the point of view of the transmission-line this is equivalent to a current flowing in the direction $(-Ia)$. Thus the pseudo-impedance Za becomes $+jX1$ at the end of Stage 4.

To convert Za into $-jX1$ the reversal at Stage 5 is required, followed by the re-setting of $(ED\ e2)$; thereafter $(e1/Ea)2=N1$, as required.

As will be seen these operations put the current Ia strictly under control of $e3$; thus it is possible to *simulate any multiple of* $C1$ once the preliminary setting of $e3$ has been established. For instance, if $e3$, at the end of Stage 3 is multiplied by k, the following stages will lead to a pseudo-capacitance $(k.C1)$.

In Part 2 we shall take full advantage of this facility to generate acoustical capacitances that are quite unattainable by direct methods for unless both capacitance and frequency are small, direct representation of capacitance by a closed air-volume is confused by phase-errors—a matter more fully dealt with in Part 2.

Use will be found later for the ratio $(e1/Ea)_1$, established at the end of Stage 4, in easing acoustical testing conditions.

By means such as these, values of $N\infty$, γ, and Zo can be measured over a substantial frequency-range; the next step is to deduce the values of the primary constants R, C and L and their disposition within the equivalent network that represents unit-length of the tested transmission-line.

Deductions to be made from N∞

Since $N\infty$ is the open-circuit voltage-ratio of the line represented, temporarily in Fig. 5 as a simple *Tee*-network, $N\infty$ at any chosen frequency is given by

$$N\infty=E/Ea=(1+P/Q)$$

Consequently, from a Polar plot of $N\infty$ it can be deduced whether any of the equivalent networks illustrated in the figure represent the performance of the tested line.

It will be understood that the overall performance of the line cannot be represented at all frequencies by such simple diagrams, although they could apply precisely to very short lengths of line, or to the whole line at very low frequencies where attenuation was small.

At the outset of the acoustical investigation such diagrams brought to light the extreme improbability of structures such as (5·2a), or even (5·3a). (The latter is conventionally

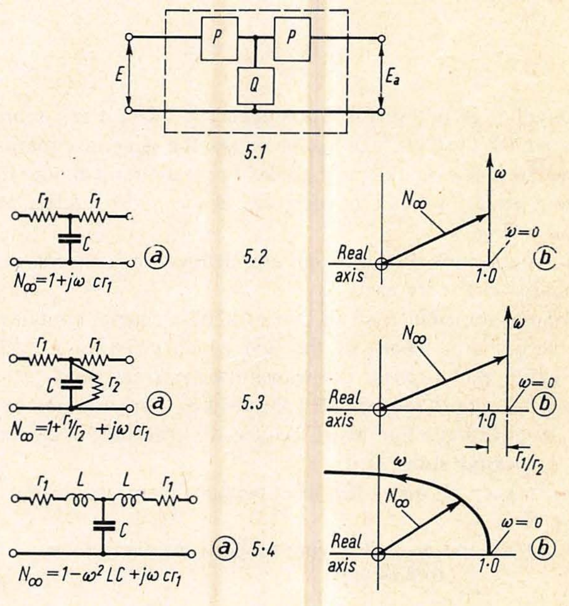


Fig. 5

assumed in most theoretical approaches to acoustical transmission-line problems). Plots of acoustical measurements of $N\infty$ seemed to agree most closely with Fig. 5.4b; to discover how closely it was necessary to measure Zo as well, as just described, and then to apply the method of analysis described in the final sections of this article.

Introductory Note on the use of Circuit Models

When the measurement of γ and Zo are not accurate the data, when plotted against frequency, fall irregularly about a smooth curve drawn among them. Where this curve has a rapid change of curvature, or the plotted data are widely

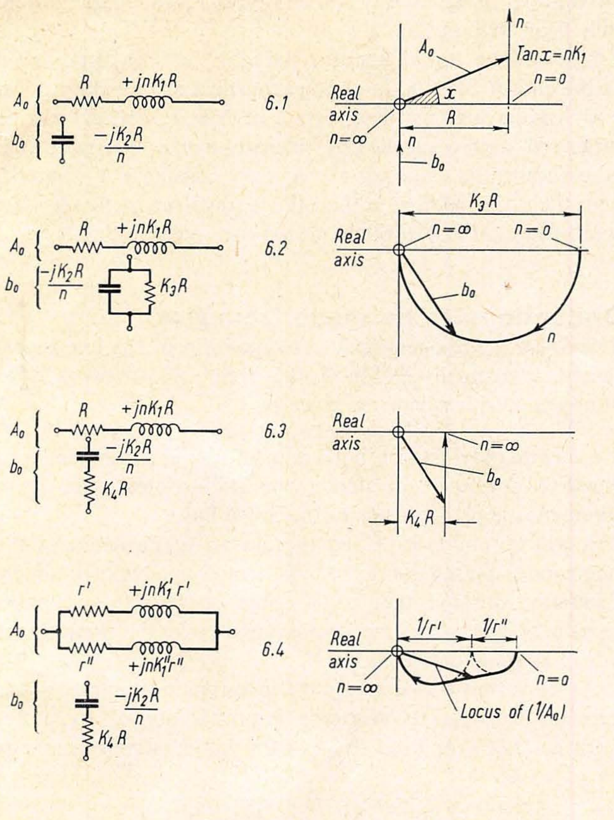


Fig. 6

scattered, it is not possible to make the curve truly representative of the "average" data unless the general equation of the curve is available as a guide. From a circuit model the guiding equations can be derived. Some such models are shown in Fig. 6.

The quantities *Ao* and *Bo* are defined in conventional transmission theory as:

Ao= Total impedance of all "series-like" elements contained within unit length of the line which generate a back-e.m.f. that *opposes* the transmitted current; and

Bo= Total admittance of all "shunt-like" elements per unit length of the line which reduce the transmitted current *by-passing* some of it.

Here, to keep all quantities in impedance form, we define: $bo = 1/Bo$.

Using *Ao* and *bo* and relying on conventional theory:

$$Ao/bo = (\gamma)^2 \qquad (12)$$

and

$$Ao \,.\, bo = (Zo)^2 \qquad (13)$$

Whence:

$$Ao = Zo\ (\gamma) \qquad (14)$$

$$bo = Zo/(\gamma) \qquad (15)$$

From the last two equations the values of *Ao* and *bo*, at each frequency, can be calculated from the already measured values of γ and *Zo*. The model (Fig. 6) which best fits the deduced *Ao* and *bo* data is then used as a basis of further analysis. Here we shall not involve ourselves in making a choice but merely set out some data which have been tailored to fit Model (6·1) which is typical of any conventional electrical transmission line. Some of the other models are known to apply, at least approximately, to acoustical systems that will be examined in Part 2.

It will be understood that the model is no more than a formal way of representing the true situation, the electrical symbols merely indicating the general form of the algebraic terms which express the quantities *Ao* and *bo*, The loci of each of these impedances are drawn in the figures.

The *K*-quantities are non-dimensional and *n* is the frequency in kc/s.

Deriving the Primary Constants from a Model

Because the reader has already been warned that the following data will be based on model 6·1 he might suppose that the associated analysis will resolve itself into an arid mathematical excercise. In fact it will bring out important relationships between parameters that will prove of great value in Part 2 and it will show that although the first analytical step involves *Ao* and *bo* only, much confirmatory information comes from the original γ and *Zo* data.

Because we use here data that have been tailored to fit model 6·1 there will be no "measurement errors" to contend with, and we shall defer to Part 2 the treatment used in practice to allow for them; here it will be assumed that columns 2 and 3 of Table I represent the measurements; from these the other columns are derived by direct vector multiplication and division.

TABLE I

(Data for Analysis)

n (Freq: kc/s)	γ (numeric)	Zo (ohms)	Ao (ohms)	bo (ohms)
0·1	$0{\cdot}89/\underline{46{\cdot}5^\circ}$	$11{\cdot}2/\overline{43{\cdot}6^\circ}$	$9{\cdot}94/\underline{3{\cdot}0^\circ}$	$125{\cdot}0/\overline{90{\cdot}2^\circ}$
0·2	$1{\cdot}27/\underline{47{\cdot}6}$	$7{\cdot}91/\overline{42{\cdot}2}$	$10{\cdot}02/\underline{5{\cdot}4}$	$62{\cdot}2/\overline{89{\cdot}8}$
0·5	$2{\cdot}03/\underline{51{\cdot}9}$	$5{\cdot}07/\overline{38{\cdot}0}$	$10{\cdot}28/\underline{13{\cdot}9}$	$25{\cdot}0/\overline{89{\cdot}9}$
1·0	$3{\cdot}00/\underline{58{\cdot}2}$	$3{\cdot}74/\overline{31{\cdot}75}$	$11{\cdot}20/\underline{26{\cdot}45}$	$12{\cdot}46/\overline{89{\cdot}95}$
2·0	$4{\cdot}76/\underline{67{\cdot}0}$	$2{\cdot}98/\overline{22{\cdot}5}$	$14{\cdot}2/\underline{44{\cdot}5}$	$6{\cdot}23/\overline{89{\cdot}5}$

Arguments: Positive: $/\underline{\quad}$ Negative: $/\overline{\quad}$

From each column of this table relations between parameters *K*1, *K*2 and *R* can be deduced when the loci of the tabulated data are plotted in the complex plane.

Thus from Fig. 7.1 which is the locus of *Ao*:

$$Ao = R\,(1 + jk1n)$$

the locus gives

$$R = 10\Omega, \text{ very closely;}$$

Also from the figure $K1 = 0{\cdot}5$, which can be derived otherwise by plotting the tangent of the argument in column 3, and taking the slope.

From direct inspection of column 5 it is clear that the *bo*-structure is almost purely capacitive—departures from an

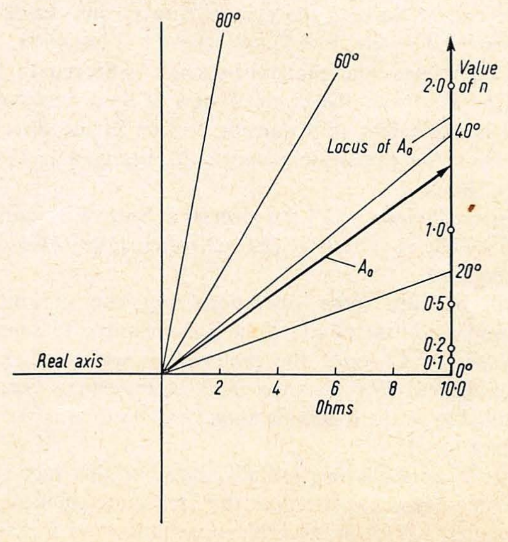


Fig. 7·1

argument of 90° are purely accidental here, but in a practical case would call for very close inspection as they might indicate that the proper circuit model was 6·2 or 6·3.

However, the pure capacitance is supported when the quantity $n(/bo/)$ is calculated and found to fluctuate only very slightly about the average 1·25.

Although the analysis could be counted complete at this stage with

$$R=10\Omega,$$
$$K1=0{\cdot}5,$$

and

$$K2=0{\cdot}125,$$

the additional loci of Figs. (7.2) and (7.3) reinforce the evidence.

These make use of the formulations:

$$(Zo)^2=Ao\,.\,bo=R^2\,(1+jK1n)\,.\,(-j\,K2/n)$$

whence

$$n(Zo)^2=R^2\,(K1\,.\,K2\,.\,n^2-jK2).$$

And

$$(\gamma)^2=Ao/bo=(1+jK1n)(+j\,n/K2)$$

whence

$$(\gamma)^2/n=n(K1/K2)+j\,(1/K2).$$

From the figures we deduce at once, using the imaginary parts:

$$K2\,.\,R^2=12{\cdot}5$$

and

$$1/K2=8{\cdot}0.$$

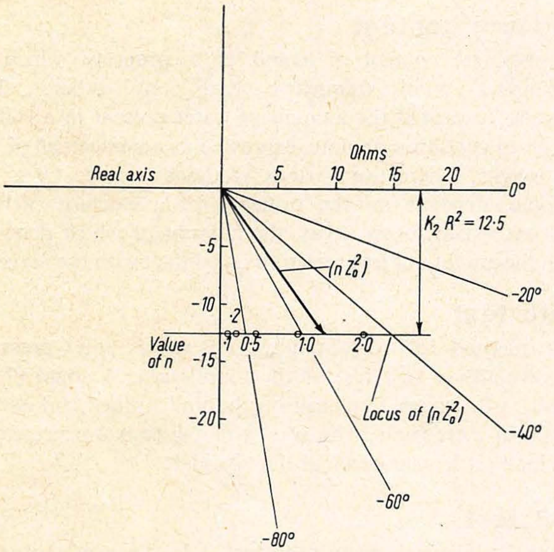


Fig. 7·2

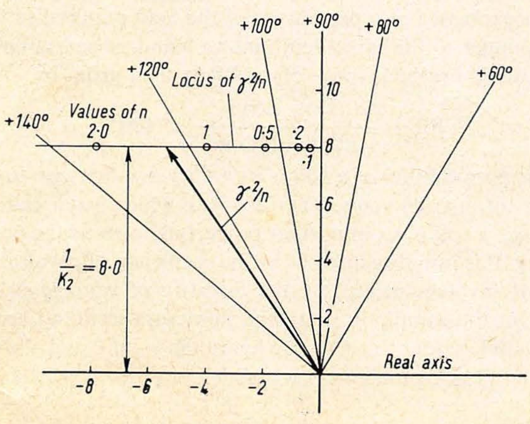


Fig. 7·3

But from the real parts, allowing for n and n^2, we have estimates of $K1.K2$ and $K1/K2$ also.

Thus it can be seen that the parameters can be inter-checked most efficiently should there by any doubt about the correct choice of the circuit model, or because the data themselves are corrupted by measurement-errors. Such a case will be treated

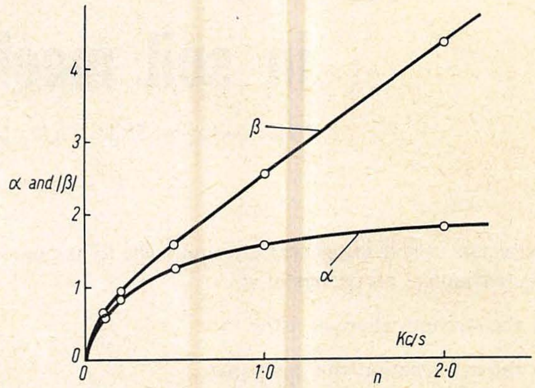


Fig. 8·1

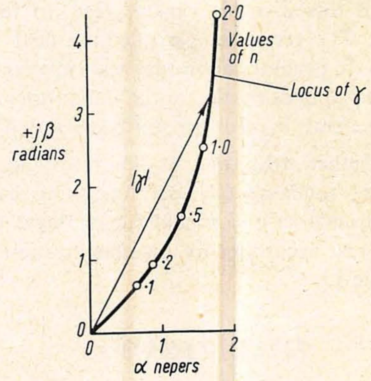


Fig. 8·2

in detail in Part 2, where most careful "smoothing" of corrupt data is needed. In such cases even the α, β and γ data come under suspicion and loci such as Figs. 8.1 and 8.2 are called for: these have been constructed from the data of Table 1.

Conclusion

This part of the paper has examined the mathematical theory that will justify the various acoustical operations that are the subject of Part 2. The mathematics having been disposed of it will be possible in Part 2 to deal with acoustical measurements without having to divert attention from them to explain the underlying theory.

Even so, in order to keep the discussion within bounds, it will be necessary to limit acoustical tests almost entirely to measurements on a single sample, namely 2 in. of Therbloc—a material commonly used for heat-insulation as well as sound absorption and, which, because of its mechanical stability, is an excellent subject for investigation. The reader can obtain from Ref. 1 an idea of the ambit of the discussion to be presented in Part 2.

References.

[1] E. R. Wigan; Letter to *Nature*, July 6, 1963, p. 59.

[2] E. R. Wigan; "A self checking Cartesian A. C. Potentiometer for use in the 100—10,000 c/s Range". (In the Press).