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A Self-Checking Cartesian A/C Potentiometer for Use in the 100c/s to 10 kc/s Range

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A Self-checking Cartesian A.C. Potentiometer for Use in the 100c/s to 10kc/s Range

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A new form of Cartesian a.c. potentiometer, based upon a set of CR networks is described, and the theory, design and applications are discussed in terms of a model instrument.

By a self-checking routine the performance parameters of the model are calibrated in absolute terms. Subsequently the precision of the calibration is demonstrated by using the model to determine vector voltage ratios in some circuits of critical performance.

The frequency range runs in 100c/s steps from 100c/s to 10kc/s, and by an artifice, interpolations within this step are possible. A frequency-bridge forms a part of the Cartesian network and serves to calibrate the associated driving oscillator.

Although not precision built, the model is capable of 0·5 per cent measurement accuracy, which can be improved to 0·1 per cent by adopting a replication routine.

(Voir page 492 pour le résumé en français: Zusammenfassung in deutscher Sprache auf Seite 499)

THIS a.c. potentiometer measures, in Cartesian co-ordinates, the vector ratio between pairs of alternating p.d.'s without taking any current from the tested circuit; this has the advantage that, once the argument of the ratio is known as well as the modulus, methods of circuit analysis become possible which may be more rapid, and are always more revealing than conventional techniques based on voltage magnitude alone.

Measurements are made in terms of the readings of X and Y slide-wires that carry currents of substantially equal magnitude and quadrature phase. In the potentiometer circuit—of which Fig. 1 is a simplification—capacitance and resistance are used as the controlling elements; as a result the potentiometer is largely insensitive to stray magnetic fields.

The circuit can be treated as a passive 'bridge' and is operated in much the same way by bringing the detector D to a series of 'null balances'. Because the 'null' can be established so precisely ratio measurements can be made with a precision (circa 0·5 per cent) impossible with a pointer instrument. With the experimental model arranged as in Fig. 2 tests can be made at frequencies between 100c/s and 10kc/s, this range being covered either in 100c/s preset steps, or by a slight modification of test-procedure, in much smaller increments.

The following description is based on the network of Fig. 1 which is electrically equivalent to Fig. 2.

The two slide-wires SW_1 and SW_2 have linearly divided scales with centre zeros and equal positive and negative markings (150 divisions). As they are effectively connected in series by the transformer T_3, the slide-wire p.d.'s are additive. Voltage vectors in the tested network which lie anywhere in the four quadrants of the complex plane can therefore be dealt with.

In making a measurement the 'potential leads' P_1 and P_2 are bridged across the unknown p.d. (e_1) appearing in the test-object Z; the X and Y dials are operated to bring D to zero (thus ensuring that no current flows out of Z) and, assuming that the a.c. potentiometer needs no calibration corrections, the unknown p.d. is registered as $(X_1 + jY_1)$.

When a second p.d. (e_2) has been registered similarly as $(X_2 + jY_2)$ the vector ratio e_2/e_1 is known to be

$$e_2/e_1 = (X_2 + jY_2)/(X_1 + jY_1)$$

Although tiresome to handle in this form, the ratio can be reduced to a single term in X and Y by arranging that

$$X_1 = 100 \text{ and } Y_1 = 0$$

For this the phase-shifter in Fig. 1 is adjusted so that the p.d. (e_1) in the denominator of the ratio is adjusted to align precisely with the potentiometer dial setting (100 + j0·0). This being done the ratio e_2/e_1 is read direct from the dials as

$$e_2/e_1 = (1/100) . (X_2 + jY_2) \quad \ldots\ldots\ldots\ (1)$$

Equation (1) is true only if the slide-wire currents I_2 and I_1 have equal magnitude and quadrature phase. The actual ratio will be expressed by $I_2/I_1 = (\alpha + j\beta)$ in which the factor α should ideally be zero, and β unity. In practice α will be finite and β will not be unity, and to be realistic equation (1) must be re-written

$$e_2/e_1 = (1/100) . (X_2 + \alpha Y_2 + j\beta Y_2) \quad \ldots\ldots\ (2)$$

The exactitude of the ratio measurement therefore depends not only on the relatively simple matter of scaling the X and Y dials (purely linearly), but also on measure-

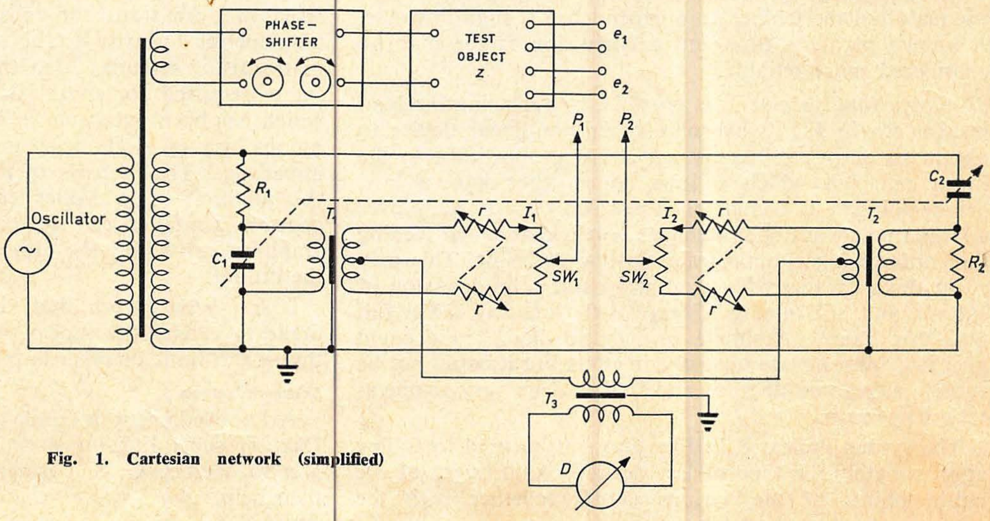

Fig. 1. Cartesian network (simplified)

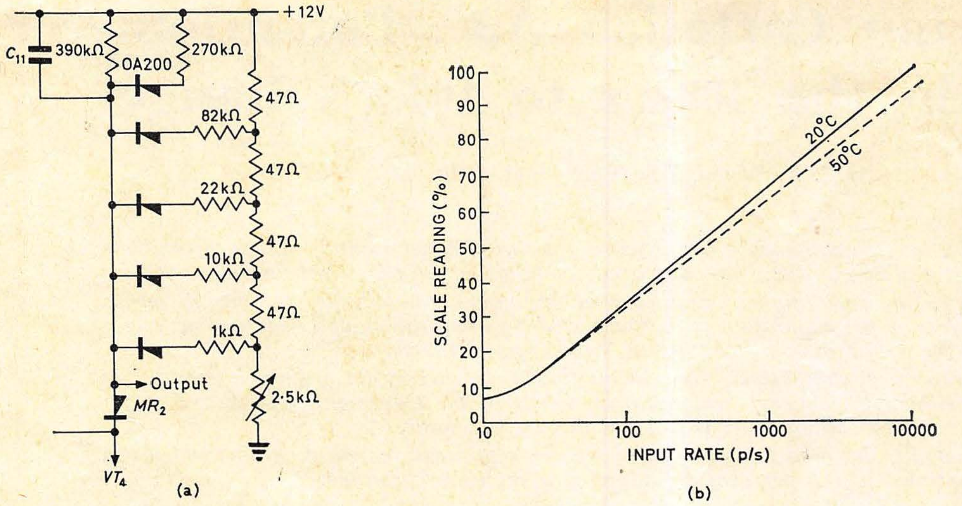

Fig. 6. (a) Circuit for logarithmic law
(b) Law with above circuit

Modifications

Although this unit has been constructed to a specification there are several interesting extensions.

(a) *Time-constant increase*: The polyester capacitors chosen for the integrating times are by no means large, being a total of 5μF and this total could be increased if necessary to give a time of 100sec or more.

(b) *Time-constant reduction*: Although the full unit cannot be used for a time-constant lower than 0·25sec without major redesign it is possible for oscillographs to monitor across C_{11}. This necessitates reducing its value and disconnecting R_{60}. It can be useful for sweeping techniques with a high count rate, i.e. a time-constant of 0·02sec (with C_{11} about 4μF) showing the pulse shape derived from, for example, a radioisotope-labelled blood sample as it passes a detector.

(c) *Range extension*: To extend the ranges downwards to say 1p/s is feasible by increasing the monostable capacitor. It is necessary, however, to increase C_{11} and the integrating time also.

It is also possible to use the ratemeter with a maximum count rate of 100kp/s. The monostable width has to be reduced to 100nsec. This is possible with C_2 in Fig. 2 about 100pF. The input pulse edge needs to be sharp and C_1 and C_2, Fig. 4, need to be reduced.

The switching time of the constant charge pump is also adequate but the temperature dependence is slightly worse. A known paralysis time and a correction factor may be considered more reliable.

(d) *Logarithmic scale*: It is possible, by replacing the load resistor R_{13} in Fig. 2 by some other non-linear device to obtain an output voltage which is not proportional to the input rate but which follows some other known law. Where the rate is completely unknown or may vary over a large range while the ratemeter is left unattended feeding a recorder a logarithmic scale may be desirable. This may cover three decades: for example, a full scale deflexion of 10kp/s, but still giving a large deflexion, say 0·3 × full scale for 100p/s. A simple germanium diode chain could give this approximate logarithmic effect but it could not be stretched to three decades and it would be very temperature dependent.

The system indicated in Fig. 6(a) can be used to follow approximately the type of law desired. Adjustment of the cut-in points of the various diodes together with the parallel loads added can give an outline of the required curve. The greater the number of diode networks used the more faithfully the curve will be reproduced. A second advantage is that the integrating time will automatically be increased at the lower rates as the time-constant of C_{11} and the effective load resistor is increased.

Naturally a great consistent accuracy could not be expected of this arrangement but experimental results are shown in Fig. 6(b).

(e) *D.C. Amplifier:* It is possible that in the near future this amplifier could be partially replaced by a circuit comprising field effect transistors. At the present time the drift would still be much worse than that experienced with the chopper amplifier unless very expensive units are used. New developments and price reductions, however, may soon reverse this position.

Specification

Several modules have been built. The following is a typical specification.

Dimensions	9in × 3½in × 3½in excluding meter
Rate Ranges	10; 40; 100; 400; 1 000; 4 000; 10 000p/s
Time-Constant Ranges	0·25, 1, 2·5, 10, 25, 50sec

Paralysis time error < 1·5 per cent
Linearity: Better than 0·2 per cent full scale
Zero drift < 0·01 per cent/°C } 10 to 60°C
Signal drift < 0·05 per cent/°C } 10 to 60°C
Supply stability 1 per cent for 4 per cent supply variations—Stabilized supply used
Chopper ripple at output < 1 per cent of full scale peak-to-peak.

Conclusions

A random pulse ratemeter has been described combining two new features. The more usual diode pump circuit is replaced by a transistor constant charge pump having a far superior linearity at the small voltage levels involved in transistor circuits. Also the relatively large time-constants required for useful integration present a problem which has been overcome in a relatively drift-free manner by the use of a chopper amplifier having a high input impedance. The increase of the effective time-constant by the addition of a Miller capacitor illustrates how the chopper amplifier can be considered as any other d.c. amplifier providing the ripple is adequately removed at the output.

It has been shown that various improvements can be made to extend the ranges, in particular to 100kp/s and that logarithmic or other laws are obtainable.

Acknowledgments

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ment of α and β. These parameters are measured by the self-checking procedure dealt with later when the model instrument is calibrated. It is shown that in the model α has a maximum value of $0{\cdot}27 \times 10^{-2}$ and β varies irregularly from 0·955 to 0·995 over the frequency range 100c/s to 10kc/s.

The model was not built to precise limits because, no matter how big the calibration corrections of the instrument might be, the self-checking procedure would always lead to precise measurement. A simple precision test is illustrated by Fig. 3(a) where the plotted points fall very closely indeed on the theoretical circle. To obtain this precision, however, both α and β had to be allowed for, and it will be appreciated that tedious computation could be avoided if α were made very small and β very nearly unity; this could have been approached with precision-grade components.

In the next section the various factors controlling α and β are dealt with, α getting smaller as the Q-factor of the controlling capacitors improves, and β depending on the capacitance values selected; in the model cheap industrial grade capacitors were used and adjusted to 1 per cent. The controlling feature of the design, however, is the circuit arrangement now to be dealt with.

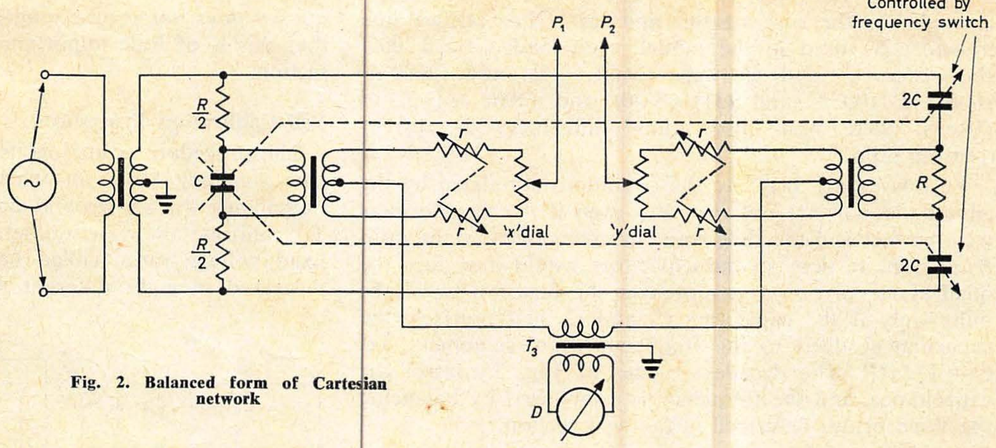


Fig. 2. Balanced form of Cartesian network

Theory of the Cartesian Network

Even with ideal circuit components the desired equality and quadrature between slide-wire currents depends on the following circuit theorem[1].

When two electrical elements, of impedance A and B, are connected in series across a generator of e.m.f. E, and internal impedance zero, a third element Q, connected in turn across A and B, will be found to be carrying currents I_A and I_B, respectively, which bear the ratio

$$I_A / I_B = A / B$$

this ratio being completely unaffected by the nature of Q.

The skeleton diagram, Fig. 4, applies this theorem to the a.c. potentiometer. Notice that the A, B and Q elements of the theorem have been duplicated so that the 'switching operation' suggested in the theorem has been avoided. Then, provided the duplication of the elements is perfect,

$$A_1 = A_2;\ B_1 = B_2;\ Q_1 = Q_2$$

leading to

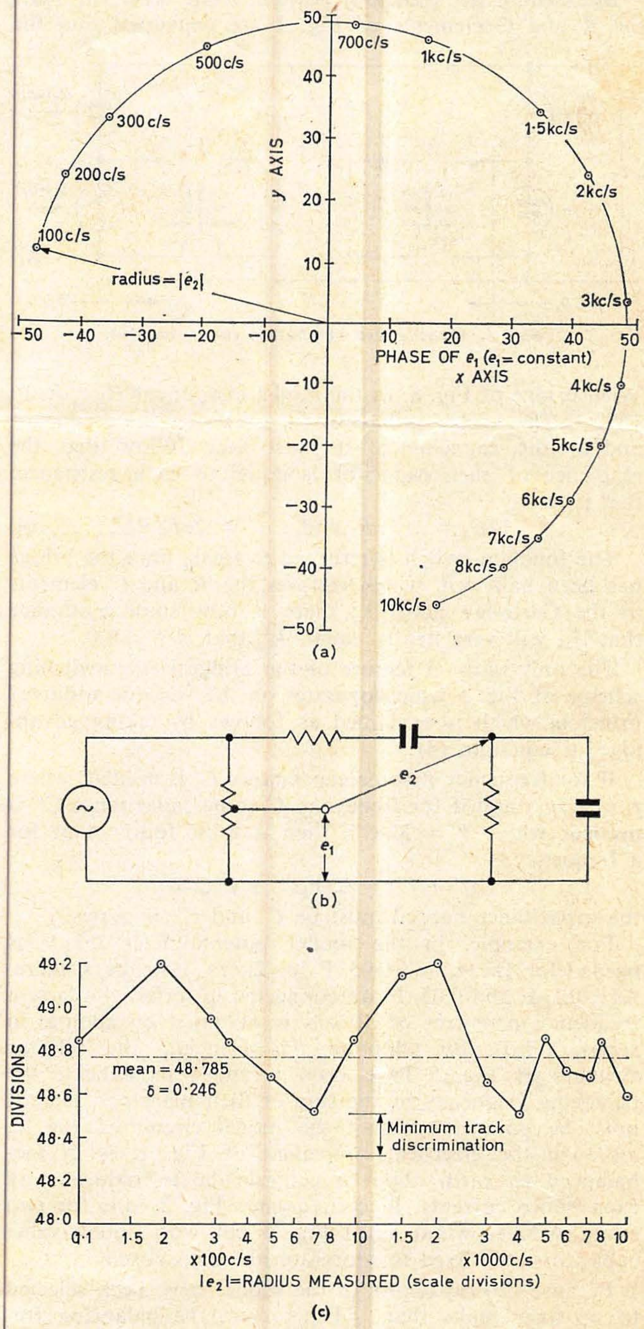
$$B/A = I_{p1}/I_{p2}, \quad \text{(the ratio of transformer input currents).}$$

The first two equalities are met by choosing $R_1 = R_2$ and $C_1 = C_2$; it is also relatively easy to make $Q_1 = Q_2$ at all frequencies provided the slide-wire resistances, SW_1 and SW_2, are equal and the transformers are of identical construction. Notice that there is no need for transformers of perfect performance, but that it is necessary that at any working frequency their imperfections shall be alike; it will then follow that their current-transfer ratios will be approximately the same also. Finally if the transformers are similar the slide-wire current will have very nearly the same ratio as the currents I_{p1} and I_{p2} so that

$$I_1 / I_2 = B / A \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In fact the transformers will never be exactly alike; for

Fig. 3. (below) (a) Simple precision test (b) RC circuit used (c) Plot of deviation



example, in the case of the unity-ratio Post Office line transformers used in the model, each loaded by 3 200Ω slide-wires, Q_1 and Q_2 were (2660 + j45) and (2600 + j44·0) at 100c/s and (3110 + j0) and (3060 + j4·0) at 10kc/s; there were also minor differences in current transfer ratio.

To this extent therefore the conditions stipulated by the circuit theorem are not met. But even if the transformers were perfect and the slide-wire currents strictly in the ratio B/A, imperfections in the capacitors would introduce the quadrature correction α; moreover the factor β would be unity only if the capacitors C_1 and C_2 had precisely the capacitance suited to the frequency of measurement, i.e. $C = 1/2\pi fR$. The decade switches of Fig. 5 control the capacitance, and the frequency is established by balancing the Wien bridge described in the next section.

The Frequency Bridge

By turning a 'function' switch from MEAS to FREQ the R and C elements of Fig. 1 are converted into the Wien bridge of Fig. 6. In the model instrument $R_1 = R_2 = 1592\Omega$. The bridge is balanced by varying the frequency and/or the capacitance. It must then follow that the reactance of each capacitor is equal to each resistance; that is to say

$$1/C_1 = 2\pi f \,.\, R_1 \text{ and } 1/C_2 = 2\pi f \,.\, R_2 \quad \ldots\ldots \quad (4)$$

The function switch is returned to MEAS, once the bridge has been balanced, which restores the R and C elements to the Cartesian network; there is now some assurance that $|I_1|$ will very nearly equal $|I_2|$, that is $\beta \simeq 1{\cdot}0$.

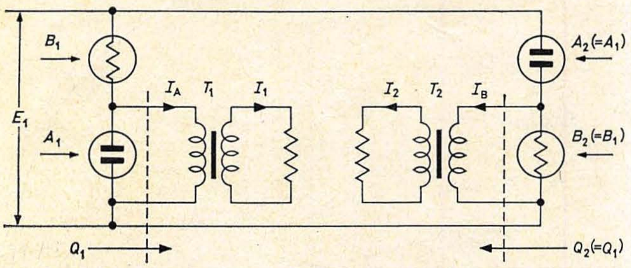


Fig. 4. Skeleton net (illustrating circuit theories)

The only unusual feature of the bridge is the switching scheme of Fig. 5 which operates on the 'decade-additive' principle which is explained as follows by taking advantage of equation (4):

If for frequency f' some capacitance C' is needed; where $f' = K/C'$; and if for frequency f'' some capacitance C'' is needed; where $f'' = K/C''$; then it must follow that for a frequency:

$$(f' + f'') = K(1/C' + 1/C''),$$

the capacitance needed must be C' and C'' in series.

For example, in the model potentiometer 0·1μF is needed for 1kc/s, and 0·05μF for 2kc/s. For 3kc/s, therefore, 0·1μF and 0·05μF are connected in series; similarly a frequency-increment of 100c/s is obtained by adding, in series, 1·0μF. By adopting this principle the selector switches of Fig. 5 have been arranged to indicate the balancing frequency by the sum of their markings. Here it must be pointed out that the model circuit of Fig. 2, although the electrical equivalent of Fig. 1, is in fact balanced to earth about a central line to reduce stray capacitance currents. In consequence Fig. 2 calls for two extra switches wired as in Fig. 5 but with double-value capacitors; the fixed resistors are also redisposed.

Because the capacitors in the model have been selected to no finer limits than ±1 per cent, the balancing frequency does not precisely agree with the switch-markings; that this is of little importance will be shown in the next section.

Self-Calibration Procedure[3]

The procedure is analogous to the method of getting a true weighing from an inaccurate chemical balance by exchanging the known and unknown weights on the pans. To calibrate the potentiometer at a chosen frequency a fixed voltage ratio, called here the 'reference ratio', is measured in eight different ways in terms of certain X

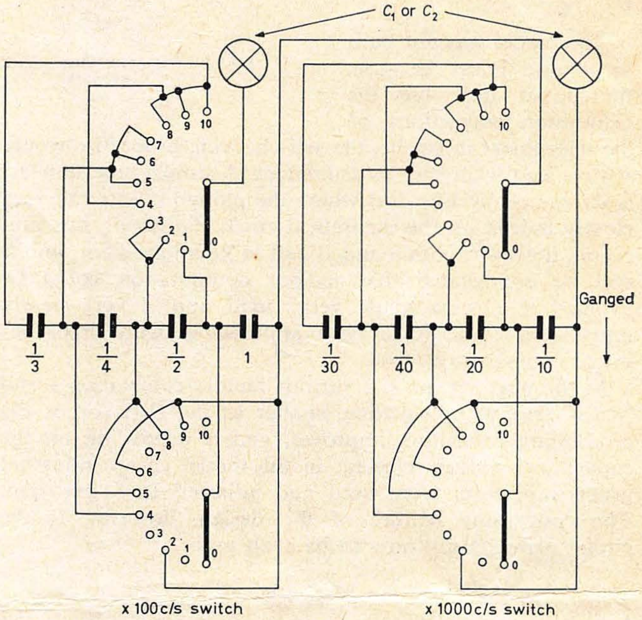


Fig. 5. Decade capacitor switching
(Used with $R_1 = R_2 = 1592\Omega$ in Fig. 1)

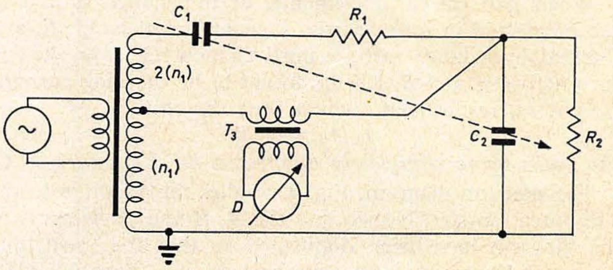


Fig. 6. Frequency bridge

and Y dial readings. Readings have to be assumed subject to unknown corrections, but when they are all 'averaged' the true measure of the reference ratio is derived and, by inference, all the potentiometer corrections are derived as well. In this way the quantities α and β are established together with the scale zeros on each dial without having to make reference to any standard quantity. This test-routine will be called the 'eight-point test'. (Attention is drawn to Appendix B which deals with a simpler three-point test applicable when the scale zeros are known to be correct).

The reference ratio is provided by a series combination of R and C connected at the output of the phase-shifter; R and C are chosen so that the ratio e_C/e_R approximates 1·0, 90° at the frequency at which the test is made. The approximation can be very crude and a modulus between 0·8 and 1·0 and an argument of 80° to 90° are acceptable.

Fig. 7(a) illustrates the first two (x_1y_1 and x_2y_2) of the eight successive measurements of the ratio e_C/e_R; all eight are set out in Fig. 7(b) and in Table 1. The phase-shifter is used to rotate the reference voltages so that they lie in each of the quadrants in turn. At the outset of each ratio measurement one or other of the voltages e_C or e_R is aligned with ± 100 divisions-mark on the X or Y axis of the potentiometer as shown in column 2 of Table 1; the X and Y co-ordinates of the non-aligned vector constitute the test-data, x_1, y_1; x_2, y_2; etc. The measurements in column 4 will be used to demonstrate the analysis.

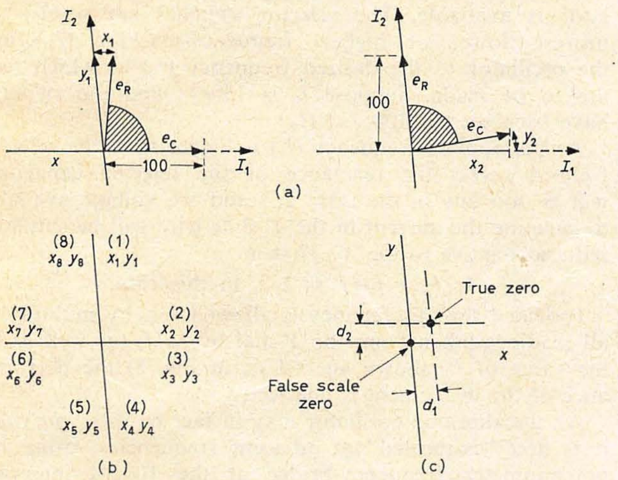


Fig. 7. (a) Calibration example (b) Test location (c) Location of electrical centres

TABLE 1

(1) TEST SERIAL NUMBER	(2) ALIGNMENT OF VOLTAGE e_C		(3) CO-ORDINATES OF VOLTAGE e_R		(4) DEMONSTRATION DATA AS COL. 3 (SIGNS AS MEASURED)	
	X	Y	X	Y	X	Y
1	+100	0	X_1	Y_1	+2·6	+107·6
3*	0	−100	X_3	Y_3	+97·0	+1·6
5*	−100	0	X_5	Y_5	−3·15	−106·0
7	0	+100	X_7	Y_7	−96·6	−1·75
	ALIGNMENT OF VOLTAGE e_R		CO-ORDINATES OF VOLTAGE e_C			
4	+100	0	X_4	Y_4	−2·0	−103·3
6*	0	−100	X_6	Y_6	−94·6	−2·35
8*	−100	0	X_8	Y_8	+1·3	−103·1
2	0	+100	X_2	Y_2	+93·05	+2·75

In Col. 4 the smaller of each pair of figures are designated as P and the larger Q. Thus $P_1 = +2{\cdot}6$; $Q_2 = +93{\cdot}05$.

First the P-data appearing in tests 3, 5, 6 and 8, and marked by an asterisk, have to be reversed in sign; thus P_3 becomes $-1{\cdot}6$ and P_5 becomes $+3{\cdot}15$ and so on. Then it can be shown that

From data of Col. 4

$$\alpha = (1/800) \, . \, [(P_3 + P_4 + P_7 + P_8) - (P_1 + P_2 + P_5 + P_6)]$$

$$\alpha = -2{\cdot}19 \times 10^{-2}$$

and the cotangent (γ) of the argument of the reference ratio is

$$\gamma = (1/800) \, . \, (P_1 + P_2 + \ldots + P_7 + P_8) \qquad \gamma = 0{\cdot}525 \times 10^{-2}$$

The location of the electrical centre of each slide-wire, defined by the (positive) quantities d_1 and d_2 in Fig. 7(c), is deduced:

$$d_1 = (1/8) \, . \, [(P_1 + P_2 + P_3 + P_4) - (P_5 + P_6 + P_7 + P_8)]$$

$$d_1 = +0{\cdot}04 \text{ divs}$$

$$d_2 = (1/8) \, . \, [(P_1 + P_2 + P_7 + P_8) - (P_3 + P_4 + P_5 + P_6)]$$

$$d_2 = +0{\cdot}05 \text{ divs}$$

(To make use of the corrections d_1 and d_2 in practice, they have to be reversed in sign and added to the test-data under observation.)

The values of d_1 and d_2 just deduced are very small because the zero-adjusters r in Fig. 1 were set in advance*. Because of the finite number of turns of resistance wire in the slide-wires, the slide-wire voltage changes in steps of 0·2 divisions, but as the average of eight individual measurements is being dealt with, differences as small as 0·05 divisions are significant; on the other hand the 0·05 intervals shown in the data recorded in column 4 have, of course, been derived by visual estimation between these steps, and are subject to different forms of error, some mechanical and others visual.

Some useful checks on the P-data can be made before dealing with the Q-data. If all measurements were made with absolute precision, P_1 would equal P_2 (see the two vector diagrams in Fig. 7(a)); moreover the same sort of equality must exist in each of the other quadrants of Fig. 7(b) provided the signs of P are adjusted as shown in column 1 of the Table. It follows that as a first check the adjusted P-data should fall into sequential pairs of the same sign; as a second check, if the measurements have been made carefully the magnitude discrepancy should not exceed $2 \times 0{\cdot}2$ divs. The quantity 0·2 appears twice because the measurement is concerned with a ratio, and the alignment setting (column 2 of Table 1) is just as likely to be in error as the P-data.

Although P_5 and P_6 in the Table differ by 0·80 divs, all other pairs are in order. A re-test always corrects the faulty data but in this case this single error will have negligible influence on the average and has been ignored.

Analysis of the Q-data is simplified if the quantity $|Q| - 100$ is dealt with: then only small quantities are involved. Writing this difference as Q' there are eight values, Q_1', Q_2', etc. up to Q_8', and these again fall into sequential pairs of equal magnitude (but here they have opposite sign). Tolerance $2 \times 0{\cdot}2$ divs again applies and it is seen that this is exceeded in the case of $Q_1' = +7{\cdot}6$ and $Q_2' = -6{\cdot}95$. The difference between other pairs being less than 0·4, one may ignore the single discrepancy and deduce

$$F = \frac{\text{(Voltage drop per scale-division on the } Y\text{-dial)}}{\text{(Voltage drop per scale-division on the } X\text{-dial)}}$$

$$= \text{'Scale Factor'} = \sqrt{[\alpha^2 + \beta^2]}$$

Deduced from Col. 4 Table 1

$$= 1 + (1/800) \, [(Q_2' + Q_3' + Q_6' + Q_7') - (Q_1' + Q_4' + Q_5' + Q_8')]$$

$$F = 0{\cdot}9526$$

(note that since α is very much smaller than β, $F \simeq \beta$ very closely)

and $e_C/e_R =$

$$1 + (1/800) \, . \, [(Q_1' + Q_3' + Q_5' + Q_7') - (Q_2' + Q_4' + Q_6' + Q_8')]$$

$$e_C/e_R = 1{\cdot}0172$$

The 'reference' voltage ratio e_C/e_R used in this demonstration was obtained from a good class resistor and a mica capacitor (which accounts for the very small value of γ) and the frequency used was 800c/s. (The reader may notice a discrepancy between the α and β given above and those plotted in Fig. 8. The explanation is that, owing to

** Both dials set to '0'; P_1 and P_2 clipped together; r-r adjusted for null balance at D.*

an error in the capacitance used in the frequency-bridge when set to 800c/s, the frequency used for the tests of Fig. 8 was in fact 770c/s which resolves the discrepancy: the reactance of the capacitor C_1 would be higher in that case, and the slide-wire p.d. also higher, in the ratio 800/770; hence β_{800} will exceed β_{770} in the reverse proportion.)

Usage

General

Some applications of the instrument will now be dealt with.

It is true that the 'phase-sensitive voltmeter' covers some of the same ground, but the null-balance technique of the a.c. potentiometer gives an inherent reading accuracy much higher than the pointer instrument. Again, as a ratio-measuring device, the potentiometer is self-calibrating and (see below) it can be voltage-calibrated if required.

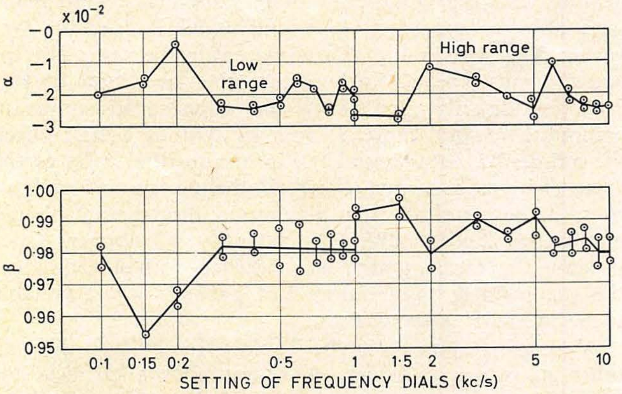


Fig. 8. α **and** β **by three-point test**
(average of $\alpha_1\ \alpha_2$ *and* $\beta_1\ \beta_2$*)*

(a) *Voltage Calibration*

It is a simple matter to include, somewhere within the tested network, a calibrated voltmeter; then in the course of the potentiometer ratio measurements the potential leads are connected to the meter terminals, thus yielding a calibration of the dials in voltage. This confers on the instrument the properties of a phase-sensitive voltmeter of effectively infinite input impedance, high reading accuracy and effectively isolated from ground.

(b) *Side-Wire Voltage Available* (E_s)

(Note: the total voltage (E_s) on the slide-wires consists of $+E_s/2$ and $-E_s/2$ with a central zero.)

Let E_1 be the p.d. delivered from the oscillator to the input terminals of the potentiometer network. Then, assuming unity-ratio slide-wire transformers (as in the model net) and no transformer losses:

$$|E_1/E_s|^2 = 1 + (1 + 1/q)^2;$$

($q = Q/R$, the ratio of the slide-wire transformer input impedance, when loaded, to the reference resistance R.)

In the model $q \simeq 2{\cdot}0$ whence, ideally,

$$|E_s| = (0{\cdot}555)\,E_1$$

Of this half, $(0{\cdot}277)\,E_1$, is available as the maximum positive or negative slide-wire voltage. In practice the transformer power-losses reduce this figure to a minimum of about $(0{\cdot}200)\,E_1$, near 200c/s, or $(0{\cdot}22)\,E_1$ at higher frequencies.

Unless there is a specific demand for higher voltages, the one-to-one slide-wire transformer ratio has advantages: higher ratios, though giving higher voltages, increase the capacitance to ground (or shield) and may affect measurement accuracy (see Conclusion).

When, in spite of this, the transformer ratio is increased the ratio should 'match' the slide-wire resistance to $R/\sqrt{2}$, not to the reference resistance R; this is because the reference capacitor C influences the impedance of the effective 'source' from which the slide-wire transformer is supplied.

(c) *Artifice for Making Tests at Other than 100c/s Intervals*

(This technique was used to obtain the data in Fig. 8 at 150c/s.)

Tests at frequencies not marked on the frequency-selector switches can be made, provided a calibrated oscillator is available. The selector switches are set to the nearest (lower or higher) frequency-marking (f_m) and the oscillator to the desired frequency (f_o) at which tests are to be made. Suppose f_o is 150c/s and the switches have been set to '100c/s' (f_m).

As the applied frequency (f_o) is higher than the selected frequency (f_m) the reactance of the selected capacitors will be too low in the ratio 2/3 and the voltage available to generate the current in the Y slide-wire will be similarly reduced relative to the X. That is

$$I_Y/I_X = f_m/f_o = 2/3, \text{ in this case.}$$

Provided this discrepancy is allowed for, by multiplying all readings taken from the Y-dial by f_m/f_o (as well as by the value of β, shown against f_m in Fig. 8) the performance of the instrument is unaffected.

An uncalibrated oscillator may in fact be used, provided it is first 'calibrated' at adjacent frequencies, using the potentiometer frequency-bridge at the 100c/s intervals, graphical interpolation being made within the interval. (It is assumed here that the frequency bridge is accurate—unlike that used in the model.)

(d) *Measuring Amplifier Gain and Linearity*

By comparing some fraction of the amplifier output voltage with the input, the instrument can measure, to well within 1 per cent, changes of gain due to changes of amplifier input level. To change the amplifier input level the voltage fed in common to the amplifier and potentiometer is altered, the potentiometer having been adjusted to balance the amplifier gain prior to the change of input; then the subsequent alteration of dial setting is observed, and thence the gain-change deduced.

It is to be noted that, arising from the basic circuit theorem, any change in non-linearity in the two slide-wire transformers resulting from the altered signal level will have no effect so long as the transformers have similar parameters. Moreover, as distinct from conventional methods of gain-measurement, the null-balance technique is quite uninfluenced by any distortion terms which may be generated in the amplifier (see Conclusion).

(e) *Measurement in the Presence of D.C.*

As no part of the slide-wire circuit is grounded, blocking capacitors in the potential leads will allow alternating voltages to be measured in valve or transistor amplifiers without upsetting the d.c. conditions.

(f) *Impedance Measurement In Situ*

Provided there is, in series with some branch of the tested network, a circuit element of known impedance, the input current to the branch can be measured and compared (in modulus and argument) with the input voltage to derive the input impedance of the branch.

By this means the conditions operating in, for instance, an active network can be explored without disturbing the power-supplies; a specific application is to the output stage of a feedback amplifier.

In passive networks also (see (a) below) the various local impedances can be observed in this way without disturbing the tested system.

The technique can be extended to 'pseudo-impedances'[4]. By this means an effectively zero impedance can be simulated and the true short-circuit current measured in phase and magnitude—an application of special interest to transformer designers.

Demonstrations of Precision

(a) *RC Transmission-Line ('Kelvin Cable')*

The voltage-vectors measured (at 200c/s) in all parts of a five-section artificial line are illustrated in Fig. 9(a). The subsidiary figures demonstrate the precision achieved after the α and β corrections of Fig. 8 have been made.

As an example of (f) above, the 'mid-series' and 'mid-shunt' impedances of each section of the line were deduced using the p.d. measured across the series element of each T-section to derive the current. The current vectors so derived yielded the data plotted in Fig. 9(b) which confirms the agreement required by theory between voltage and current attenuation in a symmetrical iterative network such as this.

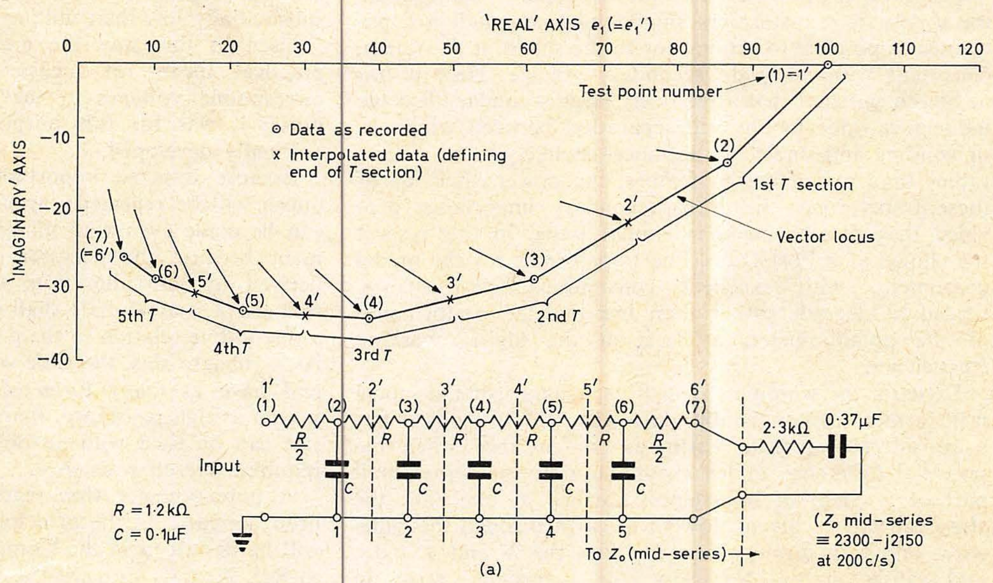
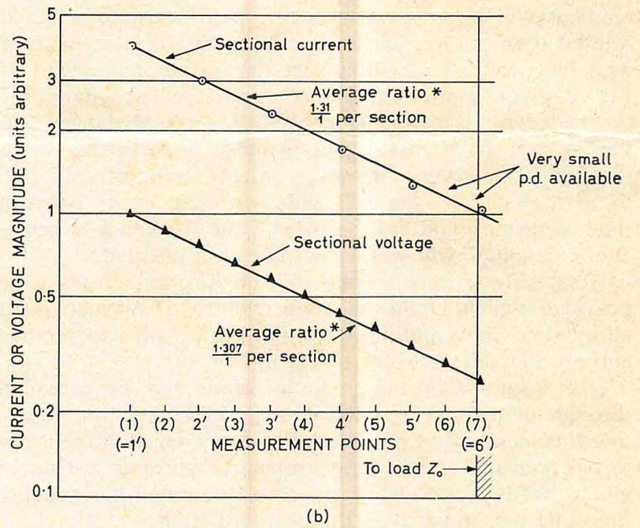
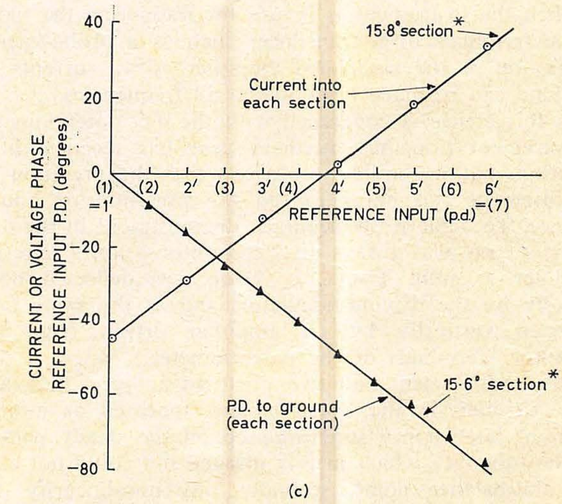


Fig. 9 (a). Network and test points. (b) Current and voltage attenuation (* *Theory predicts 1·320/1*) (c) Current and voltage phase-shift (* *Theory predicts 15·8°/section*)





(b) *Precision Over the Full Frequency-Range; a Circle Diagram*

In the simple *RC* network of Fig. 3(b) it can be arranged that the voltage vector e_2 lies on a circle when the input frequency is varied from 100c/s to 10kc/s, provided that at each frequency the voltage e_1 is 'normalized' in modulus and argument.

The data plotted in Fig. 3(a) were measured by the potentiometer with e_1 normalized to $X_1 = 50$ and $Y_1 =$ zero; (later the data were adjusted very slightly to bring the centre of the circle to the centre of the diagram).

The scale zero errors were removed by adjusting the controls *r* of Fig. 2 and the potentiometer data corrected for the values of α and β shown in Fig. 8 before plotting. To derive the radius precisely, the corrected co-ordinates (X_2', Y_2') of each point were squared, added and square-rooted.

The mean radius is 48·78 scale-divisions with roughly equal maximum and minimum deviations of 0·4 divisions, and standard deviation 0·25 divisions (see Fig. 3(c)).

The irregularity ($\pm 2 \times 0{\cdot}2$ divisions in Fig. 3(c)) in the measured radius is in line with the $\pm 0{\cdot}2$ divisions uncertainty in slide-wide p.d. for this quantity appears twice during the measurement—once when e_1 is normalized, and once when X_2 and Y_2 are measured. Defects in individual measurements of the radius, however, can be seen to approach $\pm 2 \times 0{\cdot}2$ divisions only twice, and are in general very much smaller. The maximum possible defect would have been $\sqrt{2}$ times greater if it had happened that both X' and Y' suffered discrepancies of the same sign simultaneously; the lower value found suggests that some degree of random addition of such errors must have taken place.

Indirectly this demonstration proves the reliability of the α and β data of Fig. 8, for there is no repetition in Fig. 3(b) of the irregularities which are so obvious in Fig. 8.

Conclusion

It has been shown that provided the instrument is self-checked and the α and β corrections derived and made use of, the reliability of the data is set largely by the discrete steps in which the p.d. can be picked off the slide-wires.

From time to time other factors, not dealt with above, have to be taken into account; for example, the unwanted currents flowing from the tested network to ground through the transformer winding-to-earth capacitance. There are so many ways in which these can cause minor errors that it is simplest to give general instructions only for avoiding them.

The most obvious is that the secondary windings of the slide-wire transformers should have as small a capacitance as possible to ground (or to the shield, if provided), consistent with adequate secondary voltage. Here it has to be remembered that a relatively large secondary leakage inductance—due to physical separation between windings, or winding and shield, for instance—is likely to be a good rather than a bad design feature, the power-efficiency of these transformers being of secondary importance provided they can be made in similar pairs. In this respect the choice of a Post Office line transformer for the model instrument, with excellent conventional performance, tended to degrade rather than improve the performance of the potentiometer, at least at the higher working frequencies.

Even so, the winding to earth capacitance of the model transformers causes trouble only when the test object has a considerable p.d. to earth, as for instance when the potential difference under examination forms some small part of a chain of impedances earthed at one end; the stray capacitive current flowing to earth through the slide-wires and transformers then modifies the X and Y p.d.s. and causes errors. Naturally, the percentage error in readings will be greatest when the p.d. measured is much smaller than the interfering signal, but it will also increase with frequency. Tests show that the 'balanced' circuit of Fig. 2 gives good immunity because the capacitance of the slide-wire transformer secondaries are 'balanced' to their screen. (It is found no advantage to earth the screen physically, however.) However, when an interfering p.d. to earth equal to the full slide-wire p.d. exists between the measuring point and earth, and the frequency is below 1kc/s, readings will not be significantly affected.

If separate terminals from the transformer screens were provided which could be connected to a Wagner earth adjusted to suit conditions it might be possible to remove entirely the stray current to earth.

It is found that the potential leads can be extended through quite large resistances (e.g. 10kΩ) without upsetting the accuracy of p.d. measurement, even at 10kc/s, but this is true only when stray capacitive currents are negligible; that is to say, when the tested potential has roughly the same p.d. to earth as the potential leads.

When the instrument is in use for measuring the p.d.'s at the terminals of a transducer such as a probe-microphone—or of the associated amplifier—stray currents of this kind can be made negligible at all frequencies.

As this article is concerned with the a.c. potentiometer measurement principle, auxiliary gear has received little attention, but it should be obvious that the precision of measurement can never exceed the precision of 'null' balance. To achieve the accuracy demonstrated in the diagrams a c.r.o. was always used and on occasion a selective amplifier as well. The c.r.o. beam was deflected horizontally by the driving oscillator, and at the same time deflected vertically by an amplifier driven from the 'detector' terminals of the potentiometer.

This arrangement had two great advantages; first, any noise or hum in the tested system appeared as a non-coherent interference superimposed on the steady pattern on the tube-face which merely masked but could not alter the null-balance point; secondly, any non-linearity distortion in the driving oscillator did not degrade the null balance, a characteristically looped figure appearing which closed to a curved line when balance was reached; at the same time the extremities of the loop fell on a horizontal line. In general it happened that the closing of the loop depended on one slide wire setting and the 'tilt' of the diagram on the other; thus 'convergent' adjustments were achieved rapidly even when the signal-to-noise ratio was substantially less than unity.

Used in this way the a.c. potentiometer is the only practical means of measuring single-frequency probe-microphone voltages in the presence of ambient room noise; it was for this purpose that the instrument was originally developed.

Because it is so important, a statement made at the outset will be repeated: accurate Cartesian measurements can be made even with an imperfectly constructed instrument because the necessary corrections can always be derived by self-calibration, the basic requirement being that the X and Y dials shall bear a calibration which has a one-to-one relation to the p.d.'s picked off the slide-wires. As a rider to this, the slide-wires should be evenly wound and have as many turns of wire as possible: for this reason a high resistance, many-turn, high-resistance, rheostat can be used with advantage if shunted down to the required overall resistance.

A potentiometer that reads in polar co-ordinates has been designed on the principles set out in this article and will be described in the Complete Specification subsequent to Ref. 2.

Acknowledgment

Most of the work reported here was done at the BBC Laboratories at Kingswood Warren and acknowledgment is made for permission to publish it; the basic theoretical work of Ref. 3 was done at Signals Research and Development Establishment, Christchurch, while under the Ministry of Supply.

Appendix

(A) Phase-Shifting Arrangements

Without a subsidiary phase-shifter potentiometer measurement of voltage ratios is so cumbersome and open to so many errors as to be impracticable. On the other hand, with the assistance of a phase-shifter one of the voltages can be represented by a simple number ($X_1 + \mathrm{j}0$) and the ratio can then be read off directly from the dials in Cartesian co-ordinates as $(X_2 + \mathrm{j}Y_2)/X_1$. (See Introduction.)

A very simple one will serve, for the requirements are elementary. No phase or magnitude calibration is needed; the performance need not be consistent over the working frequency range; finally, the input and output impedances may be allowed to vary while phase adjustments are being made. The design merely has to provide for smooth and continuous adjustment of phase and magnitude over the required range. The ideal arrangement has 360° coverage, or alternatively slightly more than 180°, with a reversing switch at the output to complete the circle. A simple RC assembly can meet such conditions.

In Fig. 1 the phase-shifter is required to shift the phase and magnitude of one of the voltages (e_1, e_2, etc) in the test-object Z, relative to the slide-wire p.d.'s. Now as this is a purely relative quantity the performance of the associated potentiometer cannot be affected by variations of the input impedance of the phase-shifter. (Imagine, for example, that the input terminals of the phase-shifter are shunted by some low impedance; the potentiometer, being in parallel, will be affected similarly and tested and testing p.d.'s will shrink equally; consequently the potentiometer null-balance will not be disturbed.) It follows that changes of the input impedance of the phase-shifter, arising during adjustments will be permissible. Put in other words, the phase-shifter can be designed as though it were supplied from a source of zero internal impedance—this markedly simplifies the circuit.

Variations of the output impedance during adjustment merely make its use slightly more tedious.

Finally, the phase-shifter network can have an overall voltage loss of the order of 1/5. This arises because the highest possible voltage ever required from the phase-shifter will be that due to both X and Y dials set at maximum, and in the main text (USAGE: GENERAL (b)) it is shown that E_s is of the order 1/5 of the input voltage (E_1) supplied to the potentiometer. As E_1 is also the input of the phase-shifter this implies that it need never supply more than $E_1/5$.

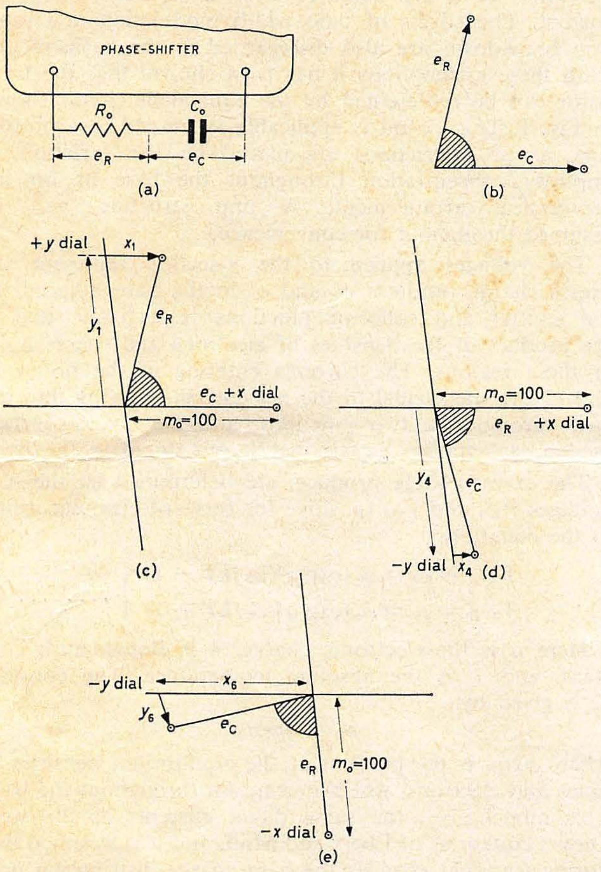


Fig. 10. Three point calibration

It may be objected that, when the performance of the test-object (e.g. an amplifier operating very near to overload) is conditioned by the absolute magnitude of the voltage delivered to it by the phase-shifter, the adjustments of voltage-loss through the phase-shifter in the course of setting up the null-balance of the potentiometer will upset the testing condition. This is true. But, by altering the voltage supplied to the assembly from the testing oscillator, the voltages in all parts of the assembly can be adjusted simultaneously without upsetting any balance conditions which may exist. Thus after a first null balance has been established, the input signal is set to produce in the test-object the required operating conditions; only residual adjustment of the X and Y dials will then be needed, and very often the phase-shifter does not have to be altered at all.

(B) RAPID METHOD OF CALIBRATION

Instead of carrying through the eight-point test of the main text, which may take 10 to 15min, the following 'three-point' test can be substituted provided the scale-zeros are corrected.

As the test will give two separate estimates of α and β, which act as checks, and takes only 2 to 3min it can be used to prove that the instrument is in working order. The two values of α and β will never precisely agree because of the limited voltage-discrimination of the slide-wires but the average may be taken as representative.

Unlike the eight-point test which, in order to validate the 'averaging' procedure, requires that e_C/e_R (see Fig. 10(b)) should be approximately equal to $1{\cdot}0, 90°$, this three-point test is valid for any value whatever of e_C/e_R. For purely practical reasons a ratio near to the ideal is preferred as it helps to reveal, at a glance, any gross errors of measurement procedure.

As in Fig. 7, the X and Y axes of the potentiometer have been shown set obliquely, but exaggerated, to show the effect of a negative value of α (see example below).

The calibration is carried out exactly as in the eight-point test (see Table 1) but only three of the eight tests there listed are made: Nos. 1, 4 and 6, which can be seen to correspond to Figs. 10(c), 10(d) and 10(e). It should be pointed out that the 'alignment' figure $M_o = 100$ used in Fig. 10 is not essential; any round number will do, but 100 makes calculation of α and β very simple. The following equations are true for any value of M_o.

Theory

It can be shown that if the X and Y slide-wires carry currents I_1 and I_2, respectively related by

$$I_2/I_1 = (\alpha + j\beta)$$

then, using tests (1) and (4) of Table 1:

$$\alpha_1 = -(1/2)\,.\,(X_1/Y_1 + X_4/Y_4) \quad \text{(B1)}$$

$$\text{and } (\alpha_1^2 + \beta_1^2) = -(M_o)^2/(Y_1\,.\,Y_4) \simeq (\beta_1)^2 \quad \text{(B2)}$$

Or, using tests (4) and (6) of Table 1:

$$\alpha_2 = -(1/2)\,.\,(X_4 + Y_6)/Y_4 \quad \text{(B3)}$$

$$\text{and } (\alpha_2^2 + \beta_2^2) = X_6/Y_4 \simeq (\beta_2)^2 \quad \text{(B4)}$$

It is important to notice that the X and Y data read off the dials have to be inserted into these equations without the changes of sign called for in Table 1. It follows that X_6, Y_4 and Y_6 are essentially negative, and as Y_1 is essentially positive, equation (B2) is viable.

Example Test made at 8kc/s for use in Fig. 8:

Potentiometer data

$M_o = 100$ $X_1 = +3{\cdot}9$ $Y_1 = +104{\cdot}6$
$X_4 = -1{\cdot}0$ $Y_4 = -98{\cdot}85$ $X_6 = -96{\cdot}75$
$Y_6 = -3{\cdot}5$

Whence: from equations (B1) and (B2)

$\alpha_1 = -2{\cdot}43 \times 10^{-2}$ $\beta_1 = 0{\cdot}981$

and from equations (B3) and (B4)

$\alpha_2 = -2{\cdot}26 \times 10^{-2}$ $\beta_2 = 0{\cdot}989$

which yields average values:

$\alpha_{av} = -2{\cdot}345 \times 10^{-2}$ $\beta_{av} = 0{\cdot}985$

Notes

The negative sign of α implies that, as in the figures, the top right quadrant of the Cartesian axes exceeds 90°. The negative sign of X_4 disagrees with the positive sign suggested in Fig. 10(d) because the reference voltages used in the test example above were much more closely in quadrature than in Fig. 10(b).

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A Large-Signal Transistor Analogue

By G. G. Bloodworth, M.A., D.U.S., A.M.I.E.R.E., and D. N. Axford*, M.A., M.Sc.

An analogue has been constructed which imitates the behaviour of the transistor model which is normally assumed in circuit design, except that the effective cut-off frequency is scaled down. The electrical behaviour of the base region is represented by a linear passive network, and the necessary non-linear voltage transformations at the junctions are generated from the characteristics of diodes. The analogue can be used to demonstrate the significance of the various parameters of a transistor and to predict its performance in circuit applications.

(Voir page 493 pour le résumé en français: Zusammenfassung in deutscher Sprache auf Seite 500)

THE behaviour of any real transistor is so complex that it cannot be represented exactly by an analogue, nor, indeed, described completely by a finite number of measured parameters. It is, however, possible to identify certain characteristics, for example the current gain β, which are essentially constant over certain ranges of operating conditions, and others with a variation which is almost exponential over wide ranges. Thus an acceptable model is obtained, supported by semiconductor theory and described by equations involving parameters which are constant, and therefore usefully measurable.

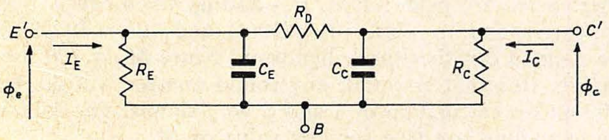

Fig. 1. The linear base region model

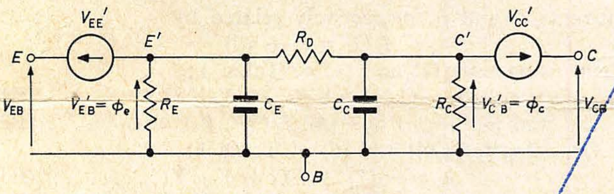

Fig. 2. The intrinsic transistor model

From such a model, transistor behaviour can be predicted either by solving the equations, or by building an analogue. Both methods of attack become progressively easier as the model is simplified, but each simplification involves restricting the operating range or accepting increased uncertainty in prediction. Eventually, theoretical analysis becomes so easy that no analogue need be considered. Thus, if the transistor is to amplify signals with very restricted amplitude and bandwidth, it can be represented simply by four complex numbers, for example the admittance parameters appropriate to the configuration.

Between these extremes there is a model which is accurate enough for many applications, but involves equations which are non-linear, and therefore not easy to deal with. For this model an analogue has been constructed which is relatively easy to use, and is adjustable so that it can represent a wide range of transistors. The analogue can be used to demonstrate the meaning and effect of each of the characteristics of a transistor, and also to predict its circuit performance.

The Transistor Model

The basic assumption made is that the processes of charge transfer and recombination within the base are linear, and that the emission efficiencies of the junctions are independent of current. This assumption implies operation at a low level or over a restricted range of current. The effects of base width modulation and junction breakdown are also disregarded in the basic model. With these assumptions it has been shown[1] that the transistor can be represented by the equivalent circuit shown in Fig. 1. This circuit is applicable to graded-base transistors, as no restrictions are placed on the variation of impurity concentration throughout the base or on the geometrical arrangement. A pnp structure will be assumed throughout for convenience.

The voltages applied to the π-section represent the excess charge products ϕ_e and ϕ_c in the base adjacent to the emitter and collector junctions respectively, that is the product of the densities of electrons and excess holes in these regions. The currents entering at the nodes E' and C' are then equal to the net currents flowing into the base through the two junctions, and the charge carried by the capacitances equals the charge stored in the base.

The excess charge products are determined by the bias voltages V_{EB} and V_{CB} (positive for forward bias) according to the equations

$$\left.\begin{aligned} V_{E'B} &= \phi_e = \phi_o(\exp qV_{EB}/kT - 1) \\ V_{C'B} &= \phi_c = \phi_o(\exp qV_{CB}/kT - 1) \end{aligned}\right\} \quad \ldots\ldots\ldots (1)$$

Here q is the electronic charge, k is Boltzmann's constant, and T is the absolute temperature. The constant ϕ_o is given by:

$$\phi_o = qp_Nn_N$$

where p_Nn_N is the product of the equilibrium densities of holes and electrons, which is constant throughout the base. This model gives the same direct currents as the well-known equations of Ebers and Moll, and it is assumed that during transient changes the base charge distribution does not lag appreciably behind the junction voltages. The grounded-base current gain in the normal direction is given by

$$\alpha_N = \alpha_{oN}/(1 + jf/f_{1N})$$

where $\alpha_{oN} = R_E/(R_E + R_D)$

and the cut-off frequency is given by

$$f_{1N} = (R_E + R_D)/2\pi R_E R_D C_E = 1/2\pi\alpha_{oN}R_D C_E \quad \ldots\ (2)$$

For inverse operation, R_C and C_C replace R_E and C_E and there are corresponding expressions for α_{oI} and f_{1I}.

For any particular transistor, the ratios between the values of the five components are determined by the four quantities α_{oN}, α_{oI}, f_{1N}, f_{1I}. The actual values are determined[1] by the voltage ϕ_o, which is chosen arbitrarily, together with the transistor current for any one set of bias conditions, for example the collector saturation current I_{CBO}.

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