Summer June, 1960

Single-Control Element Wien Bridge

Edmund Ramsay Wigan, *BBC Research Department*

Single-Control Element Wien Bridge

WITH VARIABLE L, C OR R

By E. R. Wigan, B.Sc., A.M.I.E.E.*

SUMMARY. *A great variety of apparatus uses the Wien bridge as the basis of design; the modified version described here has the same field of application and has the added advantage that the performance of the network is determined by one control-element only, as distinct from the two which are essential to the Wien bridge.*
Various alternative forms of the network are described, some of which contain resistance and inductance; moreover, it is shown that the network may be controlled either by means of a variable resistor or by a variable inductor or by a variable capacitor.
Besides being useful as frequency bridges, these networks have application in oscillators and selective amplifiers.

The circuits which will be discussed are variants of the Wien bridge which is shown in Fig. 1; they use a single variable element P (see Fig. 2) to adjust the frequency at which the bridge balances—the Wien bridge can be balanced only by varying two of the circuit elements. Like the Wien bridge, the modified version is used with a pair of 'ratio-arms' A and B which can be set up once and for all and do not have to be altered when P is varied to bring the circuit to balance at a new frequency

Although the behaviour of the new and old circuits will be discussed, for convenience, in terms of a frequency-bridge as indicated in the figures, the primary application of the new circuits is to oscillators and selective amplifiers. For these, the 'detector' terminals are short-circuited and an amplifier replaces the ratio-arms. To maintain oscillation the amplifier must have a voltage gain which is precisely the inverse of the voltage loss-ratio $B/(A+B)$. The constancy of the ratio-arms of the bridge therefore reappears as constancy of amplifier-gain—a most valuable feature when the oscillator is converted to a selective amplifier, since the 'sharpness' of tuning remains independent of the frequency selected.

The range of variation of the selected frequency ($f' = \omega'/2\pi$) depends upon how much of R_1 and R_2 is embraced by the resistance P. If the whole of R_1 is embraced (and with it a large part of R_2), the range of frequency is a maximum. Like the Wien bridge the new circuit can be arranged so that, by changing the fixed elements X_1 and X_2, the selected frequency can be modified; for example, multiplied by 10 or some other factor.

These remarks apply not only to Type-1 networks (Fig. 2) but also to Type-2 (Fig. 6). The special feature of the latter is that an additional fixed resistance r_2 is connected in the shunt branch, and the series branch is also extended. When these extensions are properly proportioned, the upper frequency limit can be increased if need be so that it reaches infinity either when $P = 0$, or even when P is finite.

In some interesting arrangements of the basic form, the resistances in these networks are replaced by capacitors and vice versa, the element P becoming a variable capacitor or inductor. Again, the reactances X may be inductive, with P resistive.

In one form of the network the variable resistance P is replaced by a varistor; this allows the frequency selected by the circuit to be controlled remotely. As this paper is concerned only with the theory of operation, applications such as these will not be dealt with. A few are described in the relevant Patent Specification No. 587,714 (1945), but currently-available varistors would improve their performance.

General Equations of the Type 1 Network

We shall deal in the main text only with structures similar to Fig. 2. The modifications to the basic equations which arise when alternative forms of network are used are dealt with in the Appendix.

When P is infinite, the circuit of Fig. 2 is identical with that of Fig. 1. We shall use the suffix ∞ to indicate the frequency f_∞ which is selected when $P = \infty$; later the suffix 0 will be used to indicate the frequency corresponding to $P = 0$; and any intermediate frequency will be marked as f'. This, it will be under-

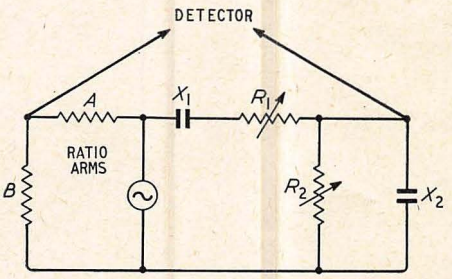


Fig. 1. *Conventional Wien bridge*

* *Research Dept., British Broadcasting Corporation*

stood, is not the *generalized* frequency, but the frequency at which the bridge will be balanced; i.e., the frequency at which the output voltage (e_2) is in phase with the voltage (e_1) applied at the input of the L-network X_1, R_1, X_2 and R_2.

It is easily shown that when $P = \infty$, the bridge is balanced when

$$\omega_\infty = 2\pi f_\infty = \sqrt{\frac{N}{M}} \cdot \frac{1}{C_2R_2} = \sqrt{\frac{1}{C_1C_2R_1R_2}} \quad .. \quad (1)$$

and $\quad 1/L = 1 + M + N \quad .. \quad .. \quad .. \quad (2)$

where L is the ratio $B/(A + B)$
M is the ratio R_1/R_2
N is the ratio X_1/X_2

It is the objective of the design to produce a network which will balance, no matter what the setting of P and, since infinity is one such setting, it follows that, *for all other values of* P the network must still balance when L is given by Equ. (2).

It will be noticed that since Equ. (1) contains the ratio of N and M, and Equ. (2) their sum, the quantity L can be modified without altering ω_∞, and vice versa.

The other extreme value of P is zero, and the network must be designed to permit this value also. The corresponding selected frequency will be written $f_0 = \omega_0/2\pi$.

It is shown in the Appendix that any frequency f' lying between the limits f_0 and f_∞ can be achieved, and that the ratio L will remain unchanged if the resistance P is so connected to R_1 and R_2 that the ratio y/a has a specific value d determined by the fixed elements in the circuits, thus:

$$\left(\frac{1}{L} - 1\right) = (M + N) = d + \left(\frac{N}{1 + d}\right); \quad .. \quad (3a)$$

whence $2d = (M+N-1)+\sqrt{(M+N-1)^2+4M} \quad (3b)$

Notice that d fixes the permissible *ratio* of y and a, *not* their magnitudes, so an infinity of circuit arrangements can be made which satisfy equation (3); they will all have the same value of f_∞, but the range between f_0 and f_∞ will be different in each case.

What this range will be can be determined either from the equation:

$$X_1 \,.\, X_2 = R_1 \,.\, R_2 - \frac{R_2y^2 + R_2a^2}{P + a + y} \quad .. \quad .. \quad (4)$$

or, alternatively, writing

$$K = 1 + \frac{P}{a + y}, \quad .. \quad .. \quad .. \quad .. \quad (5)$$

$$\text{from } \frac{X_1X_2}{R_1R_2} = 1 - \frac{y}{R_1}\left[1 - \frac{N}{(1 + d)^2}\right]\left[\frac{1}{K}\right] \quad .. \quad (6)$$

(This is derived in the Appendix.)

Now Equ. (1) is equivalent to $(X_1.X_2)_\infty = R_1 \,.\, R_2$ the subscript implying that the reactances are taken for $\omega' = \omega_\infty$; therefore Equ. (6) reduces to:

$$1 - \left(\frac{\omega_\infty}{\omega'}\right)^2 = \frac{y}{R_1}\left[1 - \frac{N}{(1 + d)^2}\right]\left[\frac{1}{K}\right] \quad .. \quad (7)$$

which gives the value of the selected frequency f' in terms of f_∞.

The extreme range of frequency-variation (f_∞/f_0) is given by substituting f_0 for f', and inserting the value

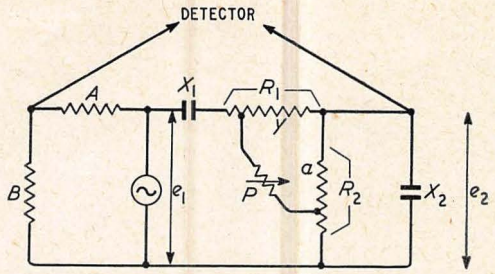


Fig. 2. *Modified Wien bridge; Type* 1 *network*

$P = 0$ in Equ. (5). This makes K unity, thus

$$1 - \left(\frac{\omega_\infty}{\omega_0}\right)^2 = \frac{y}{R_1}\left[1 - \frac{N}{(1 + d)^2}\right] \quad .. \quad .. \quad (8)$$

Since ω_0 is always $>\omega_\infty$ the greatest frequency range is obtained when (y/R_1) is a maximum, and when $N/(1 + d)^2$ approaches unity. The first condition is met by making $y = R_1$, but the other needs special consideration.

Design Considerations—Type 1 Networks

Equ. (3) implies a quite complicated relationship between L, M, N and d; it is illustrated in Fig. 3. The Y-axis of this figure is scaled in units of $[1 - N/(1 + d)^2]$ — the last term of Equ. (8).

For conciseness we write

$$1 - \frac{N}{(1 + d)^2} = F \quad .. \quad .. \quad .. \quad .. \quad (9)$$

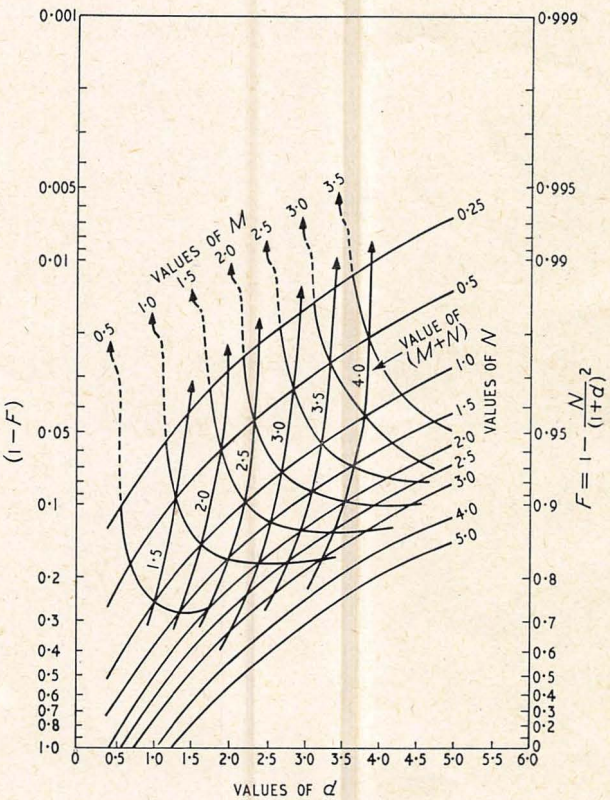


Fig. 3. *Illustrates the complicated relationship that exists between L, M, N and d. Plot of* $\left(\frac{1}{L} - 1\right) = (M + N) = \left(d + \frac{N}{1 + d}\right)$.

Fig. 3 is drawn in an unusual way, with an inverted log-scale because the important features of the diagram lie in the region in which F approaches unity; i.e., when $N/(1+d)^2$ is very small. Before discussing the graph, Equs. (7) and (8) will be recast with the aid of Equ. (9); notice that ω' and ω_0 now appear above the line

$$\frac{\omega'}{\omega_\infty} = \left[1 - \left(\frac{y}{R_1}\right)\left(\frac{F}{K}\right)\right]^{-\frac{1}{2}} = \left(1 - \frac{G}{K}\right)^{-\frac{1}{2}} \quad .. \quad (10)$$

$$\frac{\omega_0}{\omega_\infty} = \left(1 - \frac{y}{R_1} . F\right)^{-\frac{1}{2}} = (1 - G)^{-\frac{1}{2}} \quad .. \quad .. \quad (11)$$

where $G = F . (y/R_1)$ .. .. (12)

The first of these equations shows how, once the ratio y/R_1 has been chosen, the frequency f' varies in terms of K and therefore in terms of P. Everything depends upon the value which is given to G. This is shown most concisely by Fig. 4, but Fig. 5 gives a better idea of the relationship between f' and the resistance P.

It will be seen that values of G which exceed unity are shown in these figures; these cannot be achieved with the Type 1 network, but apply to the Type 2 discussed later.

To see how far it is possible to raise the maximum frequency f_0 in a Type 1 network, consider the implications of Equs (9) and (11) in the light of Figs. 3 and 4: f' cannot reach infinity (see Fig. 4) unless G reaches at least unity. Now from Equ. (12) this means that F must reach unity, for we can if we wish set y/R_1 at unity. Fig. 3 shows that although F can approach very near to unity and, in fact, approach near enough to unity to provide a very extended frequency range for the network, this can be done only at the price of combining small values of N with a considerable loss of voltage in passing through the network ($M + N$ large).

In the case of the network being employed as part of an oscillator the need to reduce N leads to difficulties. For instance, it implies a large capacitance C_1 in the series-arm of the network and, since P is connected directly to C_1 when $y = R_1$, the input impedance of the network will change violently when P is varied, a state of affairs which makes the oscillator design difficult. Moreover, the use of very low values of N leads to a lack of frequency-discrimination in the circuit; that is to say, there is a reduced rate of change of phase of the output relative to the input voltage when the frequency varies; this again is a bad feature in an oscillator.

Taking everything into account, it is found that in Type 1 networks the upper limit of frequency is given by $f_0/f_\infty < 3{\cdot}0$ approximately.

The discussion above has been entirely concerned with the extension of the frequency-range of the networks; the *reduction* of the range is easily dealt with by reducing y/R_1.

Here it should be pointed out that although

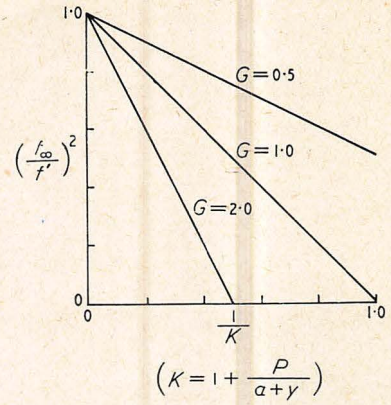


Fig. 4. *Indicates how the frequency f' varies in terms of K, and therefore P, and shows the variation with three values of G*

y alone has been mentioned throughout this discussion, y can never be changed without a proportional change in a, for y/a is fixed by d, and this by Equ. (3). One other general feature of these networks, whether of Type 1 or Type 2, is best dealt with at this point; it concerns the possible values of d.

From Equ. (3) we find, on re-arrangement, that

$$M = d - \frac{Nd}{1+d} \quad .. \quad .. \quad .. \quad .. \quad (13a)$$

whence $M < d$ or $R_2/R_1 > a/y$

and therefore $\frac{y}{R_1} > \frac{a}{R_2}$ .. .. (13b)

Equ. (13b) indicates that the fraction of R_2 which can be embraced by P must always be *less* than the fraction of R_1 which is embraced by P, so in the extreme case when $y/R_1 = 1$, $a/R_2 < 1$, implying that no conformation of the networks is possible which will bring the lower end of P to earth potential. This limitation seriously complicates any attempt to substitute a varistor for P, controlled by externally applied currents.

The Type 2 Network

The obvious difference between the Type 2 network shown in Fig. 6, and the Type 1 of Fig. 2, is the addi-

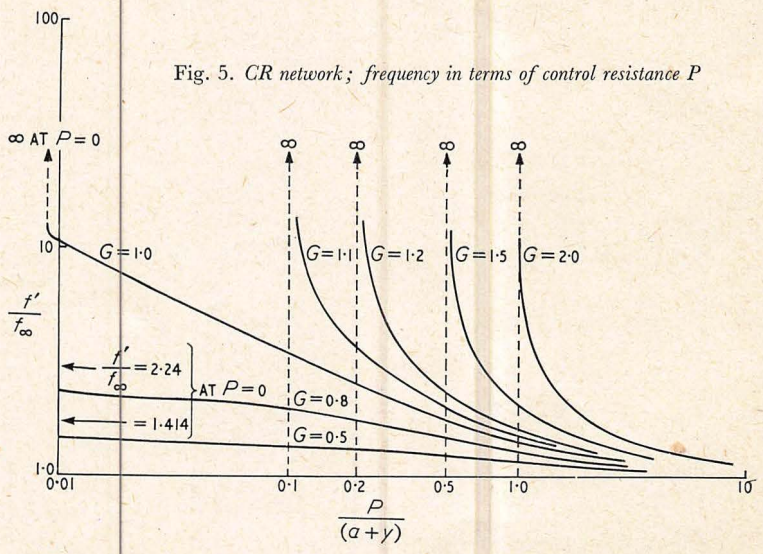


Fig. 5. *CR network; frequency in terms of control resistance P*

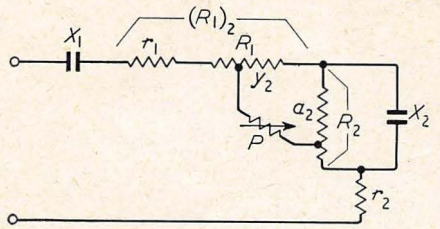


Fig. 6. *Type 2 network; a modified Type 1 network with the addition of r_2 in the shunt arm and an increased resistance in the series arm*

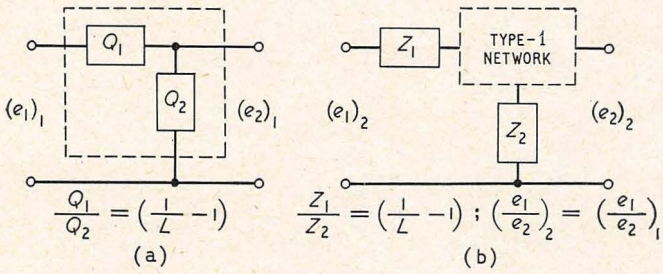


Fig. 7. *(a) The Type 1 network and (b) the derived Type 2 network*

tion of the resistance r_2 in the shunt arm, but there is a second difference which is hidden, this is an increase in the resistance in the series arm. This modification is most easily understood if the explanation follows the line of thought which led to its discovery; it was as follows.

Although there is no theoretical upper limit to the frequency f_0 attainable with Type 1 networks, there is a limit in practice because of the difficulty of making N small enough. In fact, however, it is not N which controls the frequency range but the term G [Equ. (11)], for to raise ω_0/ω_∞ it is necessary to increase G. At first sight it seems that G can never be greater than F, for it is evident from Fig. 2 that y cannot exceed R_1 and, therefore, the greatest value y/R_1 can have is unity. Suppose however that the circuit could be modified so that y *could* exceed R_1, then F could be made much less than unity (reducing the difficulties over the quantity N) and the product $F(y/R_1) = G$ could be made equal to or perhaps even greater than unity. Then f_0 could be made infinite, or it might even be possible to make f' infinite before P reached zero. The addition of resistances r_1 and r_2 in the series and shunt arms of the Type 1 network achieves all this and converts it from Type 1 to Type 2.

Consider first a case in which the additional resistances have no effect upon the network performance. Let Fig. 7(a) represent a certain Type 1 network (the variable element P may have any value whatever) then, so far as the input and output voltages $(e_1)_1$ and $(e_2)_1$ are concerned, two impedances Q_1 and Q_2 can be defined which take the place of the complete network at any frequency f'. Obviously $(e_1)_1/(e_2)_1 = 1 + Q_1/Q_2 = (1/L)$; or $Q_1/Q_2 = (1/L - 1)$.

Now let an impedance Z_1 be connected in series with Q_1 and an impedance Z_2 in series with Q_2; then, if $Z_1/Z_2 = (1/L - 1)$, the additional elements will have no effect whatever upon the voltage ratio and

$$(e_1/e_2)_1 = (e_1/e_2)_2$$

Notice that the quantities Z_1 and Z_2 are limited only as to ratio. It is convenient, in the case under examination, to make Z_1 and Z_2 resistive but, as will be seen in the Appendix, situations can arise in which they have to be made reactive. In Fig. 6 the elements r_1 and r_2 replace the elements z_1 and z_2 of Fig. 7(b).

The performance of a Type 2 network can be predicted (so far as the selected frequencies f' are concerned) by any of the equations given above for Type 1 networks with the single exception that the quantity y/R_1 appearing in the earlier equations can now exceed unity. The same rules as before define the value of d so that, in Fig. 6, d is given by the ratio $(y_2)/(a_2)$ and, as y_2 grows, a_2 must grow also.

When the Type 2 network has been derived step by step, in this way, from the originating Type 1 network it is quite easy to separate out the quantity R_1 from the resistance $(R_1)_2$ in the series arm of the network; but, when the originating network is not known, the primary equations have to be rewritten.

Adding suffices (1) and (2) to indicate the network configurations to which the terms apply we have

$$\left(\frac{1}{L} - 1\right)_1 = \left(\frac{1}{L} - 1\right)_2 \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

therefore

$$\left(\frac{1}{L} - 1\right)_2 = M_1 + N_1$$

also

$$\left(\frac{1}{L} - 1\right)_2 = \frac{M_2 + N_2}{1 + M_2 A} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

Where $M_2 = \dfrac{(R_1)_2}{R_2}$; $N_2 = \dfrac{X_1}{X_2}$; $A = \dfrac{r_2}{(R_1)_2}$

Now (14) and (15) yield

$$M_1 = M_2 \frac{1 - NA}{1 + M_2 A} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

which implies that when a resistance r_2 is added to a Type 1 network in the process of constructing a Type 2, there is a limit beyond which A cannot be raised:—

$$A < \frac{1}{N} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

For Equ. (16) shows that it is only in a network in which M_1 is vanishingly small that NA can approach unity.

Now, although this seems to indicate a severe restriction upon the resistance r_2, this is not so, for Equ. (17) is in terms of A, not r_2, and A grows only

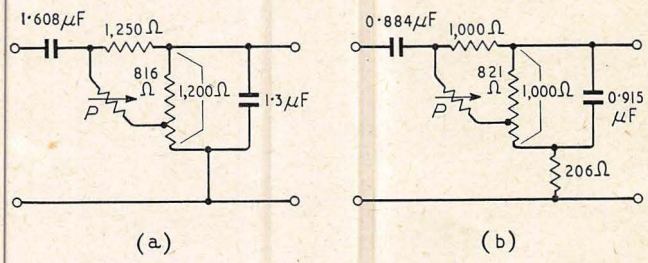


Fig. 8. *(a) Type 1 network with constants C_1, C_2, R_1 and R_2; and (b) Type 2 network which is used for the computation of the performance of a Type 2 network*

very slowly with r_2 since, by the definition given in Equ. (15) $A = r_2/[R_1 + (M + N) \cdot r_2]$.

In practice, unless remarkably small values of N are used, r_2 can be made several times R_1, but there is little application for such arrangements. Quite small values of r_2 will serve to raise G to unity (so that f' becomes infinite when $P = 0$) and no more than a very slight increase over this value is ever likely to be demanded.

On investigating the order of magnitude of r_2 likely to be required in practice, and writing r_2' as that value of r_2 which has to be added to a Type 1 network to raise f_0 to infinity ($G = 1$), we find

$$R_1/r_2' = \frac{(1+d)^2}{N} - 1(N + M)$$

or

$$\frac{R_1}{r_2'} = \left(1 + \frac{M}{N}\right)(1+d)^2 - (N + M) \quad .. \quad (18)$$

and replacing R_1 by $(R_1)_2$

$$\frac{1}{A'} = \frac{r_2'}{(R_1)_2} = \left(1 + \frac{M}{N}\right)(1+d)^2 \quad .. \quad .. \quad (19)$$

Equ. (19) is a help in checking the final composition of a Type 2 network, when the design is complete, but Equ. (18) is used in calculating the value of r_2 which has to be added to the prototype 1 network.

Performance of Type 1 and Type 2 Networks

The performance of both types of network is illustrated in Figs. 4 and 5, the former being a graphical statement of Equ. (10)

Inverting Equ. (10) we have

$$\left(\frac{f_\infty}{f'}\right)^2 = 1 - \frac{G}{K} \quad .. \quad .. \quad .. \quad .. \quad (20)$$

where $K = 1 + \dfrac{P}{(a + y)}$

so that $1/K$ is zero when $P = \infty$

and $1/K$ is unity when $P = 0$

All Type 1 networks have $G < 1{\cdot}0$

In Type 2 networks, G can have any value up to a maximum set by the choice of A which is made at the design stage.

We have already dealt with the choice of r_2' and A' which will lead to f' becoming infinite when $P = 0$; i.e., to $f_0 = \infty$; we now need a method of choosing G so that f' shall become infinite when P has a chosen finite value P'; that is to say we need to know, given the value of $P'/(a + y)$ [see Fig. 5], what value of G will be needed so that f' then becomes infinite.

Now the intercept on the X-axis of Fig. 4 occurs when $f' = \infty$ and calling this intercept $1/K'$

$$K' = G \quad .. \quad .. \quad .. \quad .. \quad .. \quad (21)$$

also

$$K' = 1 + \frac{P'}{(a + y)}$$

whence

$$\frac{P'}{(a + y)} = G - 1 \quad .. \quad .. \quad .. \quad .. \quad (22)$$

which is clearly demonstrated by Fig. 5, all the characteristics having a limit on the P-axis set by Equ. (22).

We will take two networks which have been assembled and tested, and these principles can then be demonstrated. Fig. 8(a) is a Type 1 network with the constants

$C_1 = 1{\cdot}608\ \mu F$ $\quad M_1 = 1{\cdot}043$

$C_2 = 1{\cdot}30\ \mu F$ $\quad N = 0{\cdot}809$

whence

$R_1 = 1{\cdot}25\ k\Omega$ $\quad (M+N) = 1{\cdot}852$

$R_2 = 1{\cdot}20\ k\Omega$ $\quad d = 1{\cdot}526$ from Equ. 3(b)

The correct value of a is $(1{\cdot}25/1{\cdot}526)\ k\Omega = 0{\cdot}819\ k\Omega$ (measured value $= 0{\cdot}816\ k\Omega$).

From Equ. (1) we have $f_\infty = 89{\cdot}75$ c/s
(measured value $= 89{\cdot}75$ c/s)

From Equ. (9) $\quad F = 0{\cdot}8737$

and, since $y = R_1$ $\quad G = 0{\cdot}8737$

And, finally, from Equ. (11) $f_0/f_\infty = 2{\cdot}81$
or $f_0 = 252$ c/s
(measured value $= 259$ c/s)

The frequency characteristics of this Type 1 network are illustrated in Fig. 9(a).

The Type 2 network of Fig. 8(b) is not related in any way to the Type 1 network; neither R_1, M_1, nor d is therefore available on sight. The analysis of this network is therefore an example of the calculation of the performance characteristics of a Type 2 network of which the prototype is unknown.

Since R_1 is not now in evidence, being contained in $(R_1)_2$, we have first to calculate R_1, and then deduce M_1; thence f_∞ and d can be calculated using Equs (1) and (2), and the full performance computed thereafter.

The immediately available constants are

$(R_1)_2 = 1{\cdot}00\ k\Omega$

$R_2 = 1{\cdot}00\ k\Omega$

$C_1 = 0{\cdot}884\ \mu F$

$c_2 = 0{\cdot}915\ \mu F$

$r_2 = 0{\cdot}206\ k\Omega$

whence $\begin{cases} M_2 = 1{\cdot}00 \\ N = 1{\cdot}035 \\ A = 0{\cdot}206 \end{cases}$

From Equ. (16) we derive $M_1 = 0{\cdot}652$, whence $R_1 = 0{\cdot}652\ k\Omega$, and $d = 1{\cdot}217$.

We are now in a position to calculate the frequency characteristics.

The primary quantities needed are

$$F = \frac{1 - N}{(1 + d)^2} = 0{\cdot}790$$

and $G = \left(\dfrac{y_2}{R_1}\right) \cdot F = 1{\cdot}212$

To be able to prepare Fig. 9(b) we need K' which, from Equ. (21), is $1/K' = 1/G = 0{\cdot}824$. The minimum frequency f_∞ is also required and, from Equ. 3(b), is $f_\infty = 219$ c/s. (Measured values were 218 c/s and $G = 1{\cdot}210$.)

It is important to remember that the curve of Fig. 9(b) represents only one of the infinite family of characteristics which the fixed elements of Fig. 8(b) are capable of producing; only the extreme member of this family is shown, corresponding to the whole of $(R_1)_2$ being embraced by P. Any of the curves in Figs. 4 and 5 marked with values of G less than 1·21 can be obtained merely by reducing the embrace of P. For example, to

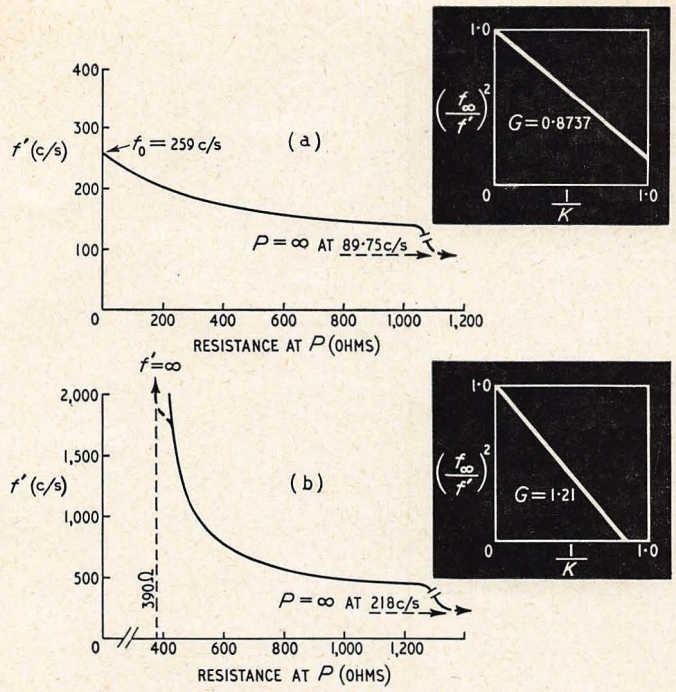


Fig. 9. *(a) The frequency characteristics of the Type 1 network illustrated in Fig. 8 (a); (b) represents one of the infinite family of characteristics which the elements of Fig. 8 (b) are capable of producing*

obtain the characteristic corresponding to $G = 1{\cdot}0$, we require: $1{\cdot}0 = G = (y_2/R_1)\,(F)$ and, since F will not be changed, $(= 0{\cdot}790)$, by altering the connections of P, we need $y_2 = 821\ \Omega$. The quantity d (which also remains unchanged) defines the associated value of a_2 so $a_2 = 676\ \Omega$.

The performance of the circuit under these conditions is shown by the curves marked $G = 1{\cdot}0$ in Figs. 4 and 5; to obtain any of the others y_2 and a_2 have to be altered in simple proportion to the required value of G.

Variants on the Basic Circuit

It is not proposed to deal with the design of the various alternative forms which the basic network may take; the differences in the frequency characteristics depend mainly upon whether P is resistive or reactive. Some circuits and their performance curves are illustrated in Fig. 10(a) — (d). (Circuit equations are summarized in the Appendix.)

In the case of the control element P being resistive, the curves take the shape of Fig. 10(a) when the fixed elements are capacitative, and of Fig. 10(d) when the fixed elements are inductive. It will be seen that the curves (for the same values of G) are mirror images in the two cases, when normalized and drawn to log-scale.

When the fixed elements are inductive, very low frequencies can be dealt with even though the inductive components may be small. Thus the 'infinities' of Fig. 5 are converted to 'zeros' by this transformation*

* It will be observed that the inductive element in the shunt arm of such networks could be made the primary winding of a transformer so that the secondary voltage might exceed the voltage input to the network. A single-valve oscillator using this idea is described in the cited Patent. The presence of the transformer makes it impossible to achieve extremely low frequencies, however, and a modification of the circuit analysis given here is called for; but the output/input voltage ratio L can still be kept constant in spite of this.

The effect of converting a CR network into an RL network (the symbols are written with the 'fixed' elements first) is to duplicate precisely the same characteristic of f' versus $[P/(a + y)]$; the quantity P is taken as the reactance of the variable element.

Finally, it is necessary to point out that networks of the general form described can be modified yet once more by adding fixed resistances to represent the losses in fixed reactive elements, without sacrificing the constant-loss principle (L = constant) of the ideal networks analysed. The presence of these resistances modifies slightly the values of f' computed on the ideal theory. It is for this reason that the LR networks are feasible, for all practical inductors will have relatively large resistance. The case of the RL network in which the variable-L contains resistance has not been analysed fully but it is suspected to fail to yield a constant value of L. Nevertheless, experiment proves that the performance of such a circuit is not far from that to be expected on the basis of ideal components.

Circuits in which the variable element is reactive, Figs. 10(b) and (c), depend upon equations similar to those set out above, but with M and N interchanged (see Appendix). Circuits such as (c) and (d) follow the equations but with f'/f_∞ replaced by f_∞/f' in addition to the interchange of M and N.

Practical Considerations

The values of f' can usually be predicted with high precision when the network is of Type 1, but the rapid variation of frequency with P in Type 2 networks makes it impracticable to predict their performance exactly, the main source of error lies in the correct setting of the points at which P should be connected to the network. Moreover, as P gets smaller and smaller, the effect of the ignored fixed loss-resistances of the circuit components becomes more and more marked and, in practice, the loss-ratio L will not remain constant if the computed values of y and a are used. However, if slight adjustments are made to y or a the value of L becomes invariant with f'; as called for.

When this has been done, it is possible to connect the network as the frequency-determining element of an oscillator, and to use it, with less than 'critical' reaction, as a selective amplifier. The degree of selectivity which can be achieved is remarkable and this selectivity can be retained when the circuit is 'tuned' by varying P, for the overall loop-gain remains constant since L remains constant.

In the oscillating condition, such a network has the advantage from the designer's point of view, that the duty required of the amplitude-controlling element, say, a varistor in the feed-back path, is the same at all frequencies; this has indirect value in making the oscillation frequency more nearly in accordance with the network constants and less dependent upon the properties of the gain-controlling network.

An important feature of all these networks is that the fixed elements may be changed, provided their ratio is not changed, without upsetting the circuit constants. Thus, by multiplying the fixed capacitances by 3·16 in Figs. 2 or 6, the frequency f' is divided by 3·16 for all settings of P. Now it is within the capacity of the Type 1

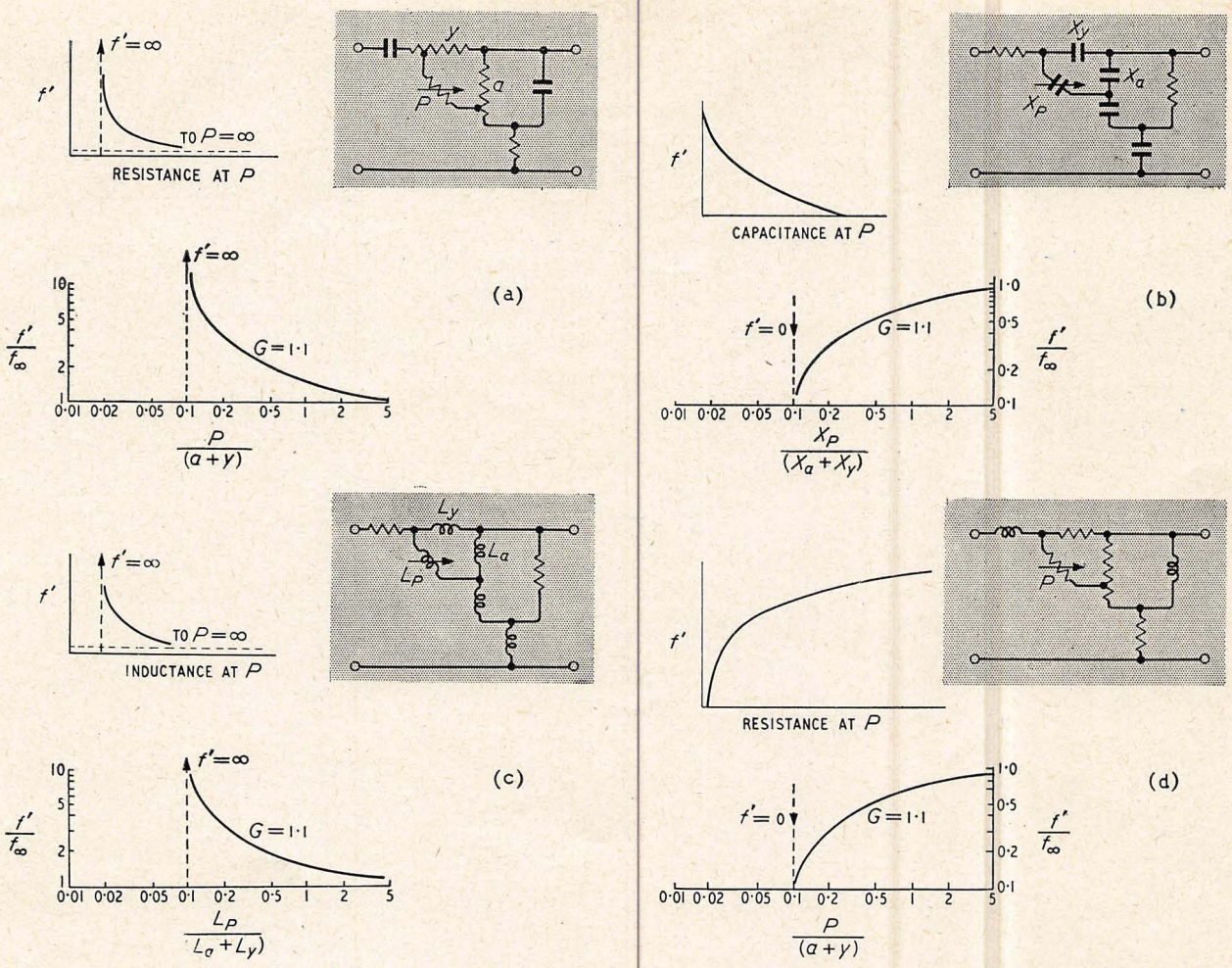


Fig. 10. *Some circuits and their performance curves*

network to provide a ratio of 3·16 between the minimum value of f_∞ of f' and the maximum f_0, so that by switching the capacitors, the frequency can be swept continuously with an overall frequency-coverage of 10: 1. The Type 2 network can give this coverage in a single structure, but not without involving a highly non-linear relationship between f' and P.

As a frequency-bridge, the networks are very satisfactory to use for, once the correct value of the 'ratio-arms' has been found, there is but one controlling element to be operated, and the 'balance' is always ideal.

It should be borne in mind that any combination whatever of elements R_1, R_2, C_1, C_2 and r_2 (or their equivalent in the inverted networks) will serve as the basis of a network with the properties set out here. Thus, it is quite unnecessary to go through the computations if it is not required to predict the circuit performance. Elements of the right order of magnitude can be assembled and the values of y and a found by trial in a few moments, preferably by setting the network up as a frequency-bridge. The loss-ratio L will then be strictly constant.

Acknowledgements

An analysis of the circuits which have been discussed was first made at the Signals Research and Development Establishment, Christchurch, and the writer is obliged to the Director for permission to use that material. In a Patent, No. 587,714 (filed in 1945), all the variants dealt with here, and a few others, were mentioned; however, the formulae which were presented were cumbersome and did not lend themselves readily to the design of networks which had to have specific properties.

The present paper corrects this. It arises out of work done recently at the BBC Engineering Research Department, Kingswood Warren, in which the special properties of these circuits have been exploited, and is published by the permission of the Director of Engineering, B.B.C.

APPENDIX

The Proportions of a Type 1 Network which lead to the Loss-Ratio L remaining Invariant with P

The network to be analysed is part of the circuit shown in Fig. 2 but repeated here for convenience in Fig. 11; it is of form *CR*; i.e., the fixed elements are capacitative and the variable element is resistive. The general form of the equations remains broadly unaltered in *RC*, *RL* and *LR* networks, but some of the symbols change places; details are given in the second half of this Appendix.

By making a delta-to-star transformation of the elements P, y and a of Fig. 11, the network is converted into the simple ladder net of

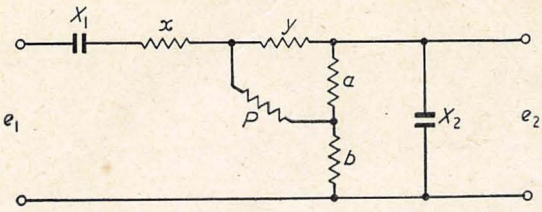


Fig. 11. *A prototype network*

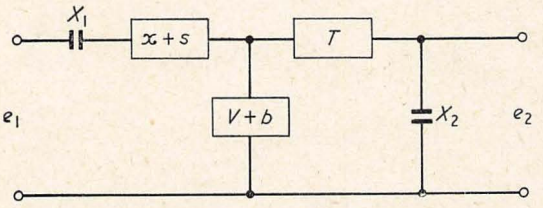


Fig. 12. *Produced from Fig.* 11 *by a delta-to-star transformation*

Fig. 12 where:—

$$\left.\begin{aligned} S &= \frac{Py}{P+a+y} \\ T &= \frac{ya}{P+a+y} \\ V &= \frac{Pa}{P+a+y} \end{aligned}\right\} \quad \ldots \quad (A.1)$$

After full analysis of this network, which will be omitted here, it is found that a primary parameter upon which the performance of the circuit depends is K where

$$K = 1 + \frac{P}{a+y} \qquad \text{(Note that } K = \infty \text{ when } P = \infty \text{ and } K = 1 \text{ when } P = 0\text{)}$$

Substituting in Equs. (A.1) we obtain:—

$$\left.\begin{aligned} S &= \left(1 - \frac{1}{k}\right).y \\ T &= \frac{ya}{(a+y)}.K \\ V &= a\left(1 - \frac{1}{k}\right) \end{aligned}\right\} \quad \ldots \quad (A.2)$$

A second parameter which controls the circuit performance proves to be d where $d = y/a$, whence, substituting in Equs. (A.2)

$$\left.\begin{aligned} S &= y.(1 - 1/K) \\ T &= (y/(1+d)\,(1/K)) \\ V &= (y/d).(1 - 1/K) \end{aligned}\right\} \quad \ldots \quad (A.3)$$

The input/output voltage ratio e_1/e_2 can now be written down on inspection* of Fig. 12. Making the substitutions of Equs (A.3), and on manipulation

$$\frac{e_1}{e_2} = \frac{1}{L} = 1 + \frac{X_1}{X_2} + \frac{x+s}{V+b} + \frac{TX_1}{(V+b)X_2} + j\left[\frac{T}{X_2} + \frac{\left(\frac{T}{X_2}\right)(x+s)}{V+b} - \frac{X_1}{(V+b) + \frac{(x+s)}{X_2}}\right] \quad \ldots \quad (A.4)$$

Now the conditions which we are searching for have to lead to e_1 and e_2 being in phase; this will occur at the specific value f' of the frequency which will reduce the imaginary part of Equ. (A.4) to zero. Whence, at the critical frequency f':—

$$X_1.X_2 = T(V + b + x + S) + (x + S)(V + b) \quad \ldots \quad (A.5)$$

* See *Wireless Engineer*, March 1948, Vol. 25, Correspondence, "Network Mnemonic", p. 97. The four elements of the net are given the symbolic reference numbers, reading from left to right (input to output) (1), (2), (3), (4); thus, in Fig. 12, (1) represents $(X_1 + x + S)$. Then it can be shown that

$$e_1/e_2 = \text{(unity)} + \frac{(1)}{(4)} + \frac{(1)}{(2)} + \frac{(3)}{(4)} + \frac{(1).(3)}{(2).(4)};$$

a series which can be memorized as the sum of three terms increasing by $\frac{1}{4}$ together with the product of the last two. Unity is added.

Since the imaginary part is zero the real part must give the value of $1/L$.

On making the substitutions of Equ. (A.3) we find:

$$e_1/e_2 = 1/L = 1 + N + \frac{(KR_1 - ad)}{(KR_2 - a)} + N.\frac{\frac{ad}{K(1+d)}}{\left(R_2 - \frac{a}{K}\right)} \quad \ldots \quad (A.6)$$

$$\text{and } X_1X_2 = M.R_2^2 - \left(\frac{aR_2}{K}\right).\frac{(M + d^2)}{(1+d)} \quad \ldots \quad (A.7)$$

where $M = R_1/R_2$

$N = X_1/X_2$

Now, if the network is to yield the same value of L for all values of P (which means for all values of K between unity and infinity), we have, substituting $P = \infty$, $K = \infty$, in Equ. (A.6)

$$\frac{1}{L} = 1 + M + N \quad \ldots \quad (A.8)$$

It is necessary that the substituting of $P = 0$, $K =$ unity must lead also to this same value of L and, on making the substitution of Equ. (A.6), and with some manipulation, we obtain

$$M = d - \frac{Nd}{1+d} \quad \ldots \quad (A.9)$$

which is equivalent to the alternative form of Equ. (3a) of the text:

$$\left(\frac{1}{L} - 1\right) = (M + N) = \frac{d+N}{1+d} \quad \ldots \quad (A.10)$$

Thus once M and N are chosen in the building of the network, the value of d is fixed once and for all and this defines the ratio but not the magnitudes of y and a which can be used; an infinity of choices are available all of them giving networks in which L is invariant with P, the value of $1/L$ being $(1 + M + N)$.

If f_∞ is written as the frequency corresponding to $P = \infty$, and f' the frequency corresponding to any value of P other than zero (for which f_0 is reserved) we find on inserting the appropriate value of K in Equ. (A.7) that

$$\left(\frac{f_\infty}{f'}\right)^2 = 1 - \frac{a}{R_1}.\frac{1}{K}.\frac{(M+d)^2}{1+d} \quad \ldots \quad (A.11)$$

which can be transformed into the more convenient form

$$\left(\frac{f_\infty}{f'}\right)^2 = 1 - \frac{y}{R_1}.\frac{1}{K}.\left(1 - \frac{N}{(1+d)^2}\right) \quad \ldots \quad (A.12)$$

which has been set out in Equ. (7) in the main text, and upon which the frequency characteristics of Figs 4 and 5 depend, as explained in an earlier section (Design Considerations—Type 1 Networks).

Performance Equations for RC, RL and LR Networks

An analysis similar to that made for the *CR* networks shows that equations of the same general form apply to each of the alternative conformations of the primary network. The full analysis is straightforward but tedious and will not be given here; instead the appropriate modifications to the equations given in the main text will be indicated.

The LR Network

In this case there is a change of sign of the imaginary term in Equ. (A.4). Equ. (A.5) is found still to apply because it depends upon the imaginary term in Equ. (A.4) becoming zero.

However, although the form of the equations has not changed, the nature of the terms X_1 and X_2 has altered for they increase instead of decrease with frequency, and as a result the mirror-image curves of Fig. 10 are produced.

CIRCUITS IN WHICH THE P-ELEMENT IS REACTIVE

The RL Network

Although the equation which replaces Equ. (A.4) is now quite different, it is found on reduction to lead to a series of subsidiary equations which are identical with those derived for the *CR* network, but with M and N interchanged, that is to say, M is the ratio of reactive elements and N the ratio of resistances.

At the same time the appearance of a reactive term at P leads to changes in the equations which tend to invert the frequency ratios (f'/f_∞, f_0/f_∞ and the like). However, the fact that the reactance is positive re-inverts these ratios so that finally the frequency-characteristics take the same shape as for the prototype *CR* network.

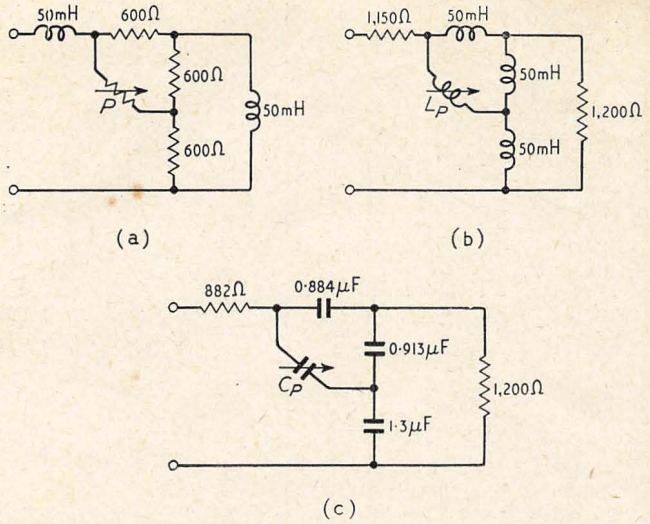


Fig. 13. *Three circuits that were set up and their performances compared with the calculated quantities*

The RC Network

As in the *RL* network above, the appearance of a reactive element at *P* inverts the frequency ratios and, in this case, there is no change of sign of the reactance to re-invert them. The mirror-image diagram therefore appears.

The symbols *M* and *N* also change places as in the *RL* network.

SUMMARY OF MODIFICATIONS TO CIRCUIT PERFORMANCE

General rules can be traced in Fig. 11 as follows

Changes of sign of the reactive element tends to invert the characteristic.

Use of a reactive element at *P* has the same effect; this means that if the reactive element is positive there is a re-inversion.

The symbols *M* and *N* change places if the *P*-element is reactive.

To fulfil these rules, the characteristics have to be drawn in terms of the reactance of the quantities *P*, *a* and *y*, where applicable, as indicated in the figures.

SOME EXAMPLES

Three circuits, see Figs. 13, (a), (b) and (c), were set up and their performance was compared with the calculated quantities as follows:—

An LR Network: see Fig. 13 (*a*)

$M = 0{\cdot}5 = 600/1200\ (\Omega)$
$N = 1{\cdot}0 = 50/50\ (\text{mH})$
$d = 1{\cdot}0$
whence
$F = 0{\cdot}75$ and (since $y/R_1 = 1{\cdot}0$), G also $= 0{\cdot}75$
$f_\infty = 2{\cdot}7$ kc/s (2·729 measured)
$f_0 = 1{\cdot}35$ kc/s (1·381 measured)

An RL network: see Fig. 13 (*b*)

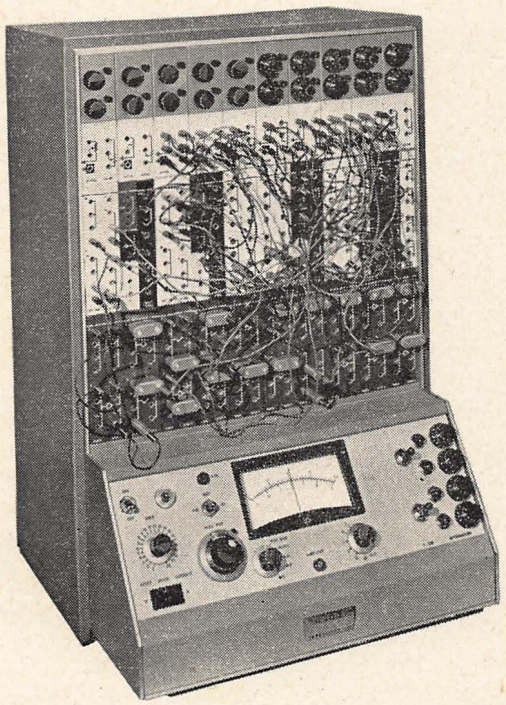
$M = 0{\cdot}5 = 50/100\ (\text{mH})$
$N = 0{\cdot}959 = 1150/1200\ (\Omega)$
$d = 1{\cdot}0$ (0·973 calculated)
whence
$F = 0{\cdot}75$ and (since $y/R_1 = 1{\cdot}0$), G also $= 0{\cdot}75$
$f_\infty = 2{\cdot}767$ kc/s, $f_0 = 5{\cdot}674$ kc/s (These are measured values; the calculated ratio f_0/f_∞ was 2·0)

An RC network: see Fig. 13 (*c*)

$M = 0{\cdot}607 = 0{\cdot}536/0{\cdot}884\ (\mu\text{F})$ . . (0·536 μF is the capacitance of 0·913 in series with 1·3 μF)
$N = 0{\cdot}735 = 882/1200\ (\Omega)$
$d = 0{\cdot}966$ (1·031 measured)
whence
$F = G = 0{\cdot}6745$
$f_\infty = 226$ c/s (225 measured) . . . . . C_P = zero
$f_0 = 97{\cdot}6$ c/s (99·95 measured) . . . . . C_P = infinite

In all these cases the bridge ratio *L* remained constant to within a very close margin indeed except in the case of the *RL* network of Fig. 13 (b); the cause of this was probably the inevitable resistance contained in the inductors used as the *P*-element.

The deficiency in the quantity *d* above may have been due to the loss-resistance of one of the capacitors in the circuit. Its presence did not interfere with the constancy of the ratio *L* however.



NEW PORTABLE ANALOGUE COMPUTER

Electronic Associates Ltd., a subsidiary of Electronic Associates Inc., U.S.A., are now marketing in England a new fully-transistorized portable analogue computer PACE TR-10, which has an accuracy of 0.1%. It is 16 in. wide × 24 in. high × 15 in. deep and weighs 93 lb. with a full complement of plug-in units. Besides its linear operational elements (which can carry out addition, integration and multiplication by a constant), it possesses a range of non-linear elements, including quarter square multipliers, function generators, relay comparators, etc. These elements can be directly plugged into a chassis where they can easily be replaced by different units should the need arise.

The Standard Basic PACE TR-10 is capable of solving up to two second-order differential equations, plus associated linear algebraic computations. It is the smallest model available and it comprises one pre-wired cabinet, one reference system, one power supply, five dual coefficient setting potentiometers, five dual operational amplifiers, two dual integrator networks, one dual tie-point panel, one service shelf, one reference panel, one patch cord set, one multiple block, one resistor set and one diode unit.

External read-out equipment for use with the TRIO includes as X-Y plotter Model 1100E and a two-channel strip chart recorder.

The plotter provides an accurate recording of any two problem variables and may also be used as a time-based recorder of function generator. The strip-chart recorder is for use in dynamic problems where a permanent record of any two simultaneously-recorded outputs against time is required.