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Anexo: El Desempeño como Litigante de la FNE
Una Mirada Cuantitativa

Diego G. Pardow

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ANNEX

1 A Simple Model of Litigation

Consider the litigation as a contest between a plaintiff ($\Pi$) and a defendant ($\Delta$). The problem at issue is whether the defendant has committed an offense. Should the defendant be found guilty, she will have to pay a sum of $R$ dollars to the plaintiff.\footnote{The variable $R$ is treated as exogenous in what follows, assuming that is potentially large enough to encourage the filing of a lawsuit. Substantively, it will be understood as a fine whenever the plaintiff is a public agency, or some amount of damages in the case of private litigants. As most models that start with a civil dispute, it could be extended to deal with other forms of punishment. See, Shavell (1985, 1987).} Because the litigation is conceived as a redistributitional conflict game, the same value of $R$ is a cash inflow to the plaintiff and a cash outflow for the defendant.\footnote{The structure of the payoffs is conventional on the literature. It should be noted, however, that it yields the same result when compared with the more intuitive structure in which the plaintiff starts bearing a negative harm and the defendant a positive amount of ill-gotten gains. For discussion, see Cooter and Rubinfeld (1989) 1072–1074.} The plaintiff, however, faces some agency costs such that in case of winning the litigation she would only receive a fraction $\delta \in (0, 1)$ of the amount of money at issue.\footnote{Certainly, in every litigation both parties have to deal with asymmetric information and agency costs. From this perspective, the real assumption is that the costs of the plaintiff are higher and that the parameter $\delta$ represents the difference against the plaintiff.} The recovery is thus represented by $\delta R$, whereas $(1 - \delta) R$ is a monetary measure of the agency costs.\footnote{In the classical terminology of Jensen and Meckling, the expression $(1 - \delta) R$ seems to fit nicely in the category of “residual loss”. It should be noted, however, that it is also meant to encompass the “monitoring expenditures by the principal”, as well as the “bonding expenditures by the agent”. See, Jensen and Meckling (1976), 308–310.} Assume further that there are two kinds of plaintiffs: public agencies whose legal standing is granted by law ($A$), and private law firms that need to obtain the consent of the victims in order to become their representative ($F$).

When the plaintiff is a public agency, the justification for assuming that the plaintiff faces higher agency costs than the defendant comes from the fact that bureaucracies are not only driven by monetary incentives, but by several other factors including general deterrence, policy issues and the like.\footnote{In turn, such a multiplicity of goals causes higher agency costs because the relationship between effort and output becomes harder to observe (Rose-Ackerman, 1986). For discussion, see McAfee, Mialon and Mialon (2008); Martini and Rovesti (2004).} Regarding private plaintiffs, the assumption is grounded on the idea that litigation may require joining several claims, leading the representation of a class of victims, or otherwise expending efforts in coordinating a large number of individuals. From this perspective, in the case of a public body the agency costs are something constant that is embodied in their structure. On the contrary, for a private firm the extent of the problem is going to be variable, and is going to depend on the type of cases undertaken.\footnote{For further elaboration on the difference between the costs from aggregating multiple principals, and}
As it is depicted in the Figure 1, the litigation unfolds on several stages. At the beginning of each stage, both parties decide simultaneously whether to engage in the fight or not.\footnote{Certainly, the rules of evidence often prescribe a sequential structure in which the plaintiff moves first. The assumption of simultaneity, however, is far more common in the literature (Talley, 2012, [...]).}

Engaging in the fight during any given round implies delivering the amount of effort $E > 0$. This variable is intended to summarize the efforts that litigants expend when producing and presenting evidence to the court. Hence, the strategy $E_{H}$ represents the amount of incriminating evidence presented by the plaintiff, whereas $E_{\Delta}$ represents the amount of exculpatory evidence that the defendant chooses to offer in her own defense. For simplicity, the amount of evidence presented on each round is held constant during the entire sequence. On the other hand, the price of producing and presenting a unit of evidence is assumed to be $C > 0$, which is also held constant throughout the game. The litigation costs are the same for both parties, and they both have complete information on this regard.\footnote{Both modeling assumptions are meant to keep the focus on the problem of agency costs. For an example of how to approach a litigation game with asymmetric costs and incomplete information, see Bernardo, Talley and Welch (2000).}

Accordingly, in each round the parties face the discrete choice of simply playing “in” or “out”, stop or keep going.\footnote{It is assumed that no bargaining occurs in this model. For a review of how the possibility of reaching a settlement affects the behavior of the parties in a similar framework, see Bourjade, Rey and Seabright (2009).}

We focus our attention in one particular moment of the game in which there are $n$ rounds that already happened and $m$ future rounds that may occur. If both parties decide to concede in the same round, the game ends up in a “tie”, this is, each of the parties has presented the same amount of evidence and shares an equal chance of winning the case.\footnote{This tie-breaker rule is actually a particular case in which the parties has the same burden of proof. A different configuration of the rules of evidence may lead to different solutions. See, note [...], infra.}

On the contrary, if only one of the parties concede, the other party is entitled to keep with the fighting. The continuation of the game is summarized as a choice of $k$ additional rounds in which the non-stopping party will be presenting evidence without the opposition of the stopping party. Finally, if both parties choose to fight, the litigation moves to the next round. This structure continues until one of the parties surrenders or the game reaches a final round.\footnote{Working with a finite number of rounds is justified on both practical and methodological grounds. On the one hand, real litigation has a finite number of rounds and the litigants should be able to reasonable foresee when the last round is going to happen. On the other hand, the main advantage of following the sequence of timing games is that the marginal benefit from presenting evidence decreases in the number of rounds. This, coupled with the assumption of keeping the costs constant throughout the sequence, ensures that the game always has a finite number of rounds (Simon, 1987). In any case, a game with infinite rounds generally yields the same solution than a single-round game with continuous effort (Laraki, Solan and Vieille, 2005). See also, Appendix, infra, p. 28.} Should the parties continue with the fighting throughout those inherent to the agent’s decision-making process, see Williamson (1985) [...].
the last round, they would also end up in a tie—this is, a situation in which each party has presented the same amount of evidence.

**Figure 1: Extensive Form of the Game**

This decision tree depicts the extensive form of the game during the $n$ round. The solid lines represent alternative courses of action and the dashed line the information set of the defendant. The payoffs for both parties appear at the bottom of the tree.

Presenting evidence involves a discrete choice. If both parties choose the same course of action, whether it is fight or concede, the amount of incriminatory evidence will be equal to the amount of exculpatory evidence. In contrast, when the parties take divergent choices, the fighting litigant will shift the scale in her favor by $k$ units of effort. In other words, the end-round effort ensures a tie while giving a chance to take the lead. The cost of such strategic advantage is the quantity $kC$. It should be noted, however, that contrasting with simple timing games the chances of winning for the leading party will generally be smaller than the unity. Unless one of the parties decides to concede on the very first round, there will be some evidence from each side introducing uncertainty on the outcome. Particularly, the model adopts a common functional form in which the plaintiff’s success probability is defined as (Hirshleifer, 1995):

$$p(\cdot) \equiv p(E_{\Pi}, E_{\Delta}) = \frac{(n + m + k_{\Pi})E_{\Pi}}{(n + m + k_{\Pi})E_{\Pi} + (n + m + k_{\Delta})E_{\Delta}}$$

(1)

The probability of conviction is thus formalized as a scale that weights the relative effort delivered by the plaintiff. In other words, is the fraction of incriminatory evidence
presented in the different stages of the trial, divided by the total amount of evidence. It is worth noticing that the function $p(\cdot)$ is increasing in $E_{\Pi}$ and decreasing in $E_{\Delta}$. This is, the chances of a ruling against the defendant increase with more incriminatory evidence, while decreasing with more exculpatory evidence. Nevertheless, because the function is summing a series of fixed efforts and the fixed efforts are delivered round-by-round, the marginal change triggered by an additional unit of evidence decreases monotonically as the number of round increases.\(^{12}\) As it was highlighted above, the evidence presented early on the litigation is assumed to have greater persuasion power than the evidence presented on subsequent stages.

2 Equilibrium Behavior

Given the fundamentals of the game, a unique, pure-strategy equilibrium emerges from noncooperative play. The equilibrium concept in what follows focuses on the last round in which both parties would present evidence, developing a stationary equilibrium that would work for any given round. The analysis begins with the easiest case in which $\delta = 1$, this is, when the agency costs are immaterial and both parties have the same amount of resources. The case in which the plaintiff has a strategic disadvantage will be discussed next.

We proceed backwards, evaluating the “additional” rounds in which the surviving party continues presenting evidence without the opposition of the conceding party. Because one of the parties is excluded from exerting any influence on the outcome, this portion of the game responds to a decision-theoretic environment. In other words, the surviving party has to perform a cost-benefit analysis on the issue of presenting additional evidence and solve a simple optimization problem. Particularly, the variable that would be subject to optimization is $k$, the number of “additional” rounds. In the case of the plaintiff, her choice can be formalized as follows:

$$\max_{k_{n} \geq 0} \left[ \frac{n + k}{2n + k} \delta R - (n + k)C \right]$$

(2)

Similarly, the defendant’s choice can be written as,

$$\max_{k_{\Delta} \geq 0} \left[ \frac{n}{2n + k} (-R) - (n + k)C \right]$$

(3)

If we exclude for now the parameter $\delta$, the following first-order condition characterizes

\(^{12}\)Under this framework, the marginal effect from presenting evidence is positive for the plaintiff and negative for the defendant. For discussion, see Hirshleifer and Osborne (2001), 173–181; Daughety and Reinganum (2012), [...]. In contrast, for both litigants the additional rounds of litigation have a negative effect on their ability to exert influence over the probability of conviction.
the best response of both litigants in this portion of the game,\textsuperscript{13}

\[
\frac{nR}{(2n + k)^2} = C
\]  

\textbf{Figure 2: Optimal number of “additional”} \textit{k-rounds}

\[
\text{Number of } n\text{-rounds}
\]

\[
\begin{array}{c|c|c}
\text{Number of } k\text{-rounds} & R/C = 100 & R/C = 80 \\
1 & R/C = 60 & R/C = 40 \\
0 & R/C = 20 & \text{ } \\
\end{array}
\]

This figure shows the relationship between the number of “additional” \textit{k-rounds}, “regular” \textit{n rounds}, and the cost-benefit ratio. The solid line represents the concave shape of the optimal amount of uncontested evidence, and the dots the discrete constraint imposed by the floor function.

The expression on the left-side of the equality represents the marginal benefit from presenting evidence during the continuation of the game, whereas the right-side denotes its marginal cost. Once we take into account the discrete constraint of our levels of effort, and consider that the best response for a rational litigant is to keep with the fighting until the marginal benefit equates the marginal cost, the optimal number of “additional”

\textsuperscript{13}Additionally, sufficiency is satisfied for each of the parties by the strict convexity of the probability of conviction \( p(\cdot) \) in both \( E_{\Pi} \) and \( E_{\Delta} \).
rounds would be:

\[ k^*_i = \left\lfloor \sqrt{\frac{nR}{C}} - 2n \right\rfloor \quad \text{for } i = \{\Pi, \Delta\} \]  

(5)

The right-hand of the equation above depends on two components: the number of “past” \( n \) rounds, and the cost-benefit ratio \( R/C \). Nevertheless, is the first variable the one dominating the behavior of our optimal \( k \). Because \( n \) appears on the inside of the root, as well as on the outside—as a part of the subtraction, the optimal amount of uncontested evidence follows a concave shape. In turn, considering that the lowest possible number of rounds is zero, by construction, \( R/C \) defines the range in which the optimal \( k \) is a positive number. In other words, this ratio determines for how long a rational litigant would choose to keep with the fighting, once the other party has already surrendered.

These ideas are depicted in the Figure 2, where the optimal number of “additional” \( k \) rounds is decreasing as the length of the litigation increases. Nevertheless, such a decrease is not monotonic. At the beginning of the game the amount of evidence presented by both parties is relatively low, which implies that the marginal benefit from additional fighting is going to be large—at least for a couple of rounds. During that interval the optimal \( k \) is going to be increasing in \( n \). As the litigation moves forward, however, the court will be more and more saturated of evidence. This process continues until the marginal benefit from further evidence is smaller than the marginal cost, and thus the optimal amount of “additional” rounds goes to zero for the remainder of the game.

We can now turn our attention to the \( n \)-portion of the game, this is, the rounds in which both parties are still influencing the outcome of the game. Although here we are moving in a game-theoretic environment, we can continue exploiting the symmetry of the payoffs of the parties. Conceiving the litigation as a redistributinal mechanism with symmetric costs implies that, at any stage of the game, both the plaintiff and the defendant should have exactly the same willingness to fight depending on the particular values of \( R \) and \( C \). Continuing with the notation from Figure 1, let \( \alpha \) denotes the probability that the plaintiff chooses to fight in the current round, and \( \beta \) the probability that the defendant takes the same course of action. Under this scenario, a mixed-strategy equilibrium would exist when \( \alpha \) and \( \beta \) are well-defined probabilities. Formally, this would be the case, if, and only if, the following condition holds:

\[ \alpha = \beta = \frac{k^*_i (\mu - R)}{\mu} \in (0, 1) \quad \text{where, } \mu \equiv 2C(k^*_i + 2n) \]  

(6)

Interestingly, this condition is never met under the current set up. First, from the denominator of the expression above we know that our mixed equilibrium is undefined whenever our optimal \( k \) is zero. The reason is that at this point the dynamics of the litigation will change substantially. If the optimal choice of our only remaining litigant is
not presenting further evidence, then the game moves from a last-stopper rule to a first-stopper rule. The first-stopper version of our game, in turn, has a unique pure-strategy equilibrium denoted by:  

\[
 s = \left\lfloor \frac{R + 2C}{4C} \right\rfloor 
\]  

(7)

The equilibrium above captures the idea that a rational litigant would only present evidence if there is a net gain from doing so, this is, when the payoff from fighting is larger than payoff from conceding (Cooter and Rubinfeld, 1989). Under this framework, however, each party makes her choice with complete independence of what the other party is doing. In other words, both parties will simultaneously fight or concede, depending exclusively on the relative values of \( R \) and \( C \). Accordingly, in equilibrium each of the parties has an equal chance of winning the case because both of them would be presenting the same amount of evidence.

Let us consider now whether there is a mixed-strategy equilibrium when the optimal \( k \) is greater than zero, this is, when the relationship among our variables is such that our surviving litigant would indeed present evidence during the “additional” rounds of litigation. Here, the problem is on the numerator of Equation (6), and particularly, in

\[
 U_{n+1}^{\delta}(fight|\Delta = fight) > 0 \implies \frac{R}{2} - (n + 1)C > \frac{nR}{2n + 1} - nC \implies \frac{R}{4n + 2} > C
\]

And,

\[
 U_{n+1}^{\delta}(fight|\Delta = not fight) \implies \frac{(n + 1)R}{2n + 1} - (n + 1)C > \frac{R}{2} - nC \implies \frac{R}{4n + 2} > C
\]

Likewise, for the defendant,

\[
 U_{n+1}^{\delta}(fight|\Pi = fight) > 0 \implies -\frac{R}{2} - (n + 1)C > \frac{(n + 1)R}{2n + 1} - nC \implies \frac{R}{4n + 2} > C
\]

And,

\[
 U_{n+1}^{\delta}(fight|\Pi = not fight) \implies -\frac{nR}{2n + 1} - (n + 1)C > \frac{R}{2} - nC \implies \frac{R}{4n + 2} > C
\]

The expression on the left-hand side of the inequality represents the marginal benefit from presenting evidence, whereas \( C \) denotes the marginal cost. Once we take into account the discrete constraint of our levels of effort, and consider that best response for a rational litigant is keeping with the fighting until the marginal benefit equates the marginal cost, the Equation (7) shows the number of rounds that we should observe in equilibrium. As before, sufficiency is satisfied by the strict convexity of the probability of conviction \( p(\cdot) \) in both \( E_\Pi \) and \( E_\Delta \).

\(^{15}\)In absolute terms, however, a larger recovery involves more fighting. For instance, if the amount at issue is less than twice the litigation costs, then there will be no litigation. Above that threshold there will be litigation, and the length of the fight will depend on how large is the difference between \( R \) and \( C \). The litigation thus follows the shape of a step function, in which the parties are sustaining the same outcome in equilibrium, until the cost-benefit ratio is large enough to finance another round of fighting.
the subtraction \((\mu - R)\). Recall that \(\mu\) is an expression of total litigation costs. On the other hand, we know that the optimal \(k\) is greater than zero only in a region that is characterized for (i) being at the beginning of the game where \(n\) is a small number, (ii) have an extension determined by how large is the fraction \(R/C\).

\[\frac{R}{C} = 100\]
\[\frac{R}{C} = 20\]
\[\frac{R}{C} = 40\]
\[\frac{R}{C} = 60\]
\[\frac{R}{C} = 80\]

This figure shows that \(\alpha\) and \(\beta\) are never well-defined probabilities. The solid line represents the concave shape of the optimal amount of uncontested evidence, as well as the different values that \(\alpha\) and \(\beta\) could take. The dots show the discrete constraint imposed by the floor function.

Notice that both features are related with low litigation costs, either because the marginal cost is relatively small, or because the number of rounds in which the parties have fought is also small. In other words, the same conditions that make \(k\) a positive number will simultaneously push \((\mu - R)\) towards the realm of the negative numbers. The Figure 3 conveys this idea, showing the behavior \(\alpha\), \(\beta\) and \(k\). The former two increase as the litigation moves forward, while at the same time the latter one is decreasing. As a result, the conditions for a mixed-strategy equilibrium are never met. For both parties, the unique pure-strategy equilibrium is to continue presenting evidence until the marginal
benefit of doing so equates the marginal cost.\footnote{Notice that the point in which marginal benefit equals marginal cost is going to occur, by construction, earlier for the “additional” rounds than for the “regular” rounds. In other words, in we define $n$ as the largest “regular” round in which Equation (4) holds—or equivalently, as the smallest “regular” round in which $k = 0$, then we have that $n < \hat{n}$, because,$$\frac{R - 4C + \sqrt{R(R - 8C)}}{8C} < \frac{R - 2C}{4C}$$\footnotetext{On the contrary, with incomplete information the parties would have doubts about whether to stop first or last. This skepticism is what enables the mixed-strategy equilibrium developed in Bernardo, Talley and Welch (2000).}}

From a substantive point of view, a key feature of the model is that the marginal benefits from presenting evidence are decreasing, but the marginal cost remains the same throughout the entire sequence. Under these conditions, at the beginning of the game the benefits from fighting in the “additional” rounds will be so large that there would be no reason to justify a mixed approach. On the contrary, as the litigation moves forward, the benefits from fighting in the “additional” rounds will become so small that the game will move to a first-stopper rule with a simple, pure-strategy equilibrium.

We finally shift to the case in with the agency costs are large enough for changing the behavior of the parties. Particularly, consider the strategic disadvantage of the plaintiff whenever $(1 - d)R \geq C$, this is, when the difference in resources created by the agency costs is larger than the fixed cost of producing evidence. In this case, the defendant would be able to finance at least one more round of fighting than the plaintiff. In turn, because there is complete information about the payoffs of the parties, it is common knowledge that the plaintiff will have less resources to finance the production of evidence, and that this difference will be large enough to force him to stop earlier in the litigation.\footnote{On the contrary, with incomplete information the parties would have doubts about whether to stop first or last. This skepticism is what enables the mixed-strategy equilibrium developed in Bernardo, Talley and Welch (2000).}

Under these conditions, the defendant’s best response is to fight always.

On the contrary, the plaintiff’s decision can be framed as a choice between conceding now or conceding later. In order to formalize this choice, recall that there are $n$ “past” rounds in the game, as well as $m$ “future” rounds and $k$ “additional” rounds. Because we already know that the plaintiff will concede earlier than the defendant, the goal is to find out when is this going to happen. In other words, the goal is find out under which conditions $m = 0$ is the optimal choice for the plaintiff. Considering that we are now solving for a particular value of $m$, let us reformulate the defendant’s decision regarding $k$, this is, the “additional” rounds and the amount of uncontested evidence as:

$$\max_{k \geq 0} \left[ \frac{n + m}{2n + 2m + k} (R) - (n + m + k)C \right]$$

Which, in turn, allows to rewrite the first-order condition of Equation (4) as,

$$\frac{(n + m)R}{2n + 2m + k)^2} = C \tag{9}$$
And, the optimal amount of “additional’ rounds of Equation (5) as,

\[
* k = \left\lfloor \frac{\sqrt{CR(m+n)} - 2C(n+m)}{C} \right\rfloor
\] (10)

Because the plaintiff is simply deciding between conceding now or conceding later, her choice can also be formalized as maximization of the number of rounds in which would be rational to continue with the fighting. This is,

\[
\max_{m \geq 0} \left[ \frac{n + m}{2n + 2m + k} \delta R - (n + m)C \right]
\] (11)

Which allows to rewrite the first-order condition as,

\[
\frac{k\delta R}{(2n + 2m + k)^2} = C
\] (12)

And the optimal amount of “future” rounds as,

\[
* m = \left\lfloor \frac{\delta^2 R - C(2\delta + 1)^2 n}{C(2\delta + 1)^2} \right\rfloor
\] (13)

Notice that the plaintiff’s willingness to present further evidence depends on the amount of evidence that is already in the pile. In other words, the optimal amount of “future” rounds depends on the number of “past” rounds that have already been fought by the parties. We can use this relationship to gain leverage on the underlying problem, and define the plaintiff’s stopping time considering the number of \(n\) rounds under which the optimal choice for \(m\) is zero. This is, the point of the game in which it is in the plaintiff’s best interest to concede. Formally, such condition can be expressed as,

\[
* n = \left\lfloor \frac{\delta^2 R}{C(2\delta + 1)^2} \right\rfloor
\] (14)

Across the different variations of the game, the effect of additional evidence is always decreasing as the litigation moves forward, and that the optimal number of rounds depends on the some sort of cost-benefit ratio. Moreover, even when the defendant has the strategic advantage of presenting a last set of evidence before the game ends, its effect on the probability of conviction depends on the length of the litigation. Because the marginal benefit from fighting is always decreasing for both parties, the pattern of the game can be summarized as follows: the longer the litigation, the lower will be the persuasive power of the last piece of evidence and the closer will be the plaintiff from reaching a tie.\(^{18}\)

\(^{18}\)This tie-breaker rule is actually a particular case in which both parties have the same burden of proof. A pro-defendant presumption reduces the number of rounds in which the parties would fight and would ultimately deter the plaintiff from filing a suit. On the contrary, a pro-plaintiff presumption increases the length of the litigation, until the weight of the evidence goes so strongly against her that the defendant would be the one stopping first and the roles in the game would be reversed. See, Bernardo, Talley and Welch (2000).
Although the probability of conviction follows the shape of a step-function—due to the discrete constraint imposed over the optimal choices of \( n \), \( m \) and \( k \), the extension of the gaps is going to decrease as the litigation moves forward.\(^{19}\) The reason is that the distance between the actual probability of conviction and the tie-breaker rule is decreasing monotonically in \( n \), \( m \) and \( k \).\(^{20}\)

### 3 Simulated Performance of the Litigants

The remaining of this section discusses further assumptions related with the factors determining how large are the agency costs born by the plaintiff, as well as with the empirical distribution of the recovery. The intuition is that public and private enforcers face a substantially different type of agency costs. This, coupled with a workable distribution about the main characteristics of the cases subject to enforcement, will allow us to fully specify the model and simulate the performance of the litigants. Therefore, the goal is evaluating what are the strategic consequences of distinguishing between public and private enforcers in light of their structural differences, and particularly, how the changes in the type of agency cost causes a change in the performance of the litigant.

Consider first the parameter \( \delta \). For simplicity, assume that there are only two kinds of offenses: those where small units of damage are dispersed among a large number of victims, and those where a relatively large damage is concentrated within a small number of victims. In the field of antitrust enforcement, for instance, a collusion of prices creates a small harm in a large class of consumers, whereas abusive dominance creates a large harm in small number of firms (e.g. Posner, 1976). Regarding public enforcement (\( A \)), the agency costs denoted by \( \delta^A \) are only going to depend on its bureaucratic structure, being thus independent from the sort of offense that is subject to enforcement. In contrast, the agency costs of private law firms (\( F \)) follows the characteristics of the offense: they will be high when the number of victims is large (\( \delta^F_H \)), and they will be low when the number of victims is small (\( \delta^F_L \)).\(^{21}\) Finally, assume further that the relationship between

\(^{19}\)Recall that what we are analyzing is the probability of conviction in equilibrium, so if there is no suit—and, accordingly, no evidence whatsoever—then the probability of winning the case is zero. In turn, whenever the evidence is outmatched 2:1, then there is a one-third chance of winning. The process continues, such that a 3:2 ratio represents a two-fifths chance, a 4:3 ratio represents a three-sevenths chance, and so on. With infinite resources the plaintiff could indeed reach a tie, making that the symmetric and asymmetric games behave equivalently in the extreme. Formally, \( \lim_{R \to \infty} p(\cdot) = 1/2 \)

\(^{20}\)In turn, piecewise differentiation shows that the agency costs reduce the ability of the plaintiff to litigate effectively. The larger the agency costs—and lower the fraction \( \delta \), the larger the expected recovery that would be required to finance another round of litigation. In other words, in our model the parameter \( \delta \) is translated into a strategic disadvantage, such that larger agency costs means an earlier stopping time, a smaller number of incriminatory evidence and a lower probability of conviction.

\(^{21}\)Enforcement agencies are regularly entrusted with stringent powers to prosecute both kind of offenses. This includes, among other things, an statutory release from obtaining the consent of the victims and from coordinating their potentially divergent interests. In contrast, private law firms are required to follow a rather complicated process in order to obtain legal standing for suing in a case of collusion,
the different types of agency costs is $\delta^F_H < \delta^A < \delta^F_L$.\footnote{The justification for this ordering is, once again, the well-known intuition that private enforcers have a comparative advantage in cases where the victim and the offender are clearly identified, while public enforcers are the ones having the edge in cases where such relationship is not clear. See, Landes and Posner (1975).}

Let us turn now to $R$, the amount of money at stake. Following a usual trend in the literature, this variable is assumed to be following a log-normal distribution.\footnote{See, e.g. Eisenberg et al. (2001); Sunstein et al. (2003). The asymmetry comes from the fact that the distribution is bounded on the low end at zero—awards cannot be negative— but effectively unbounded at the upper end. As a result, data on damage awards often have a strong positive skew: most of the data are located at the left-hand side, but there is a long right tail representing the small number of very large awards.} We are also going to assume that the size of the award and the type of case are independent.\footnote{Certainly, one may argue that a joinder of several claims should lead to a larger recovery. However, in class actions — and generally in representative litigation, the size of the individual harm of each victim tend to be rather small. On the contrary, with respect to abusive dominance, predatory pricing and other offenses that involve a limited number of victims, the individual damages may nonetheless reach huge amounts of money (i.e. a fraction of the annual sales of a company, the revenues from losing a market share). Considering that the relationship between the size of the award and the type of case is not self-evident, and that formalizing such relationship would aggregate several layers of complexity to the model, treating both variables as independent seems to be a reasonable simplification.}

Our universe of cases can be thus fully described through the following two elements: (i) the size of the harm, which follows a log normal distribution; and, (ii) the number of victims, which follows a categorical distribution. Under these conditions, we can effectively simulate how the different agency costs faced by public and private enforcers influence their performance as litigants.

The simulation starts taking several random samples of cases and implementing our model of litigation in two different environments. The first environment will simulate the performance of a public body with a constant agency cost of $\delta^A$, whereas the second one will simulate the performance of private plaintiffs having agency costs that alternate between the values $\delta^F_H$ and $\delta^F_L$. With the purpose of moving the simulation closer to what we might observe in the real world, a trial was ran for every case in each sample. In the trial the probability in equilibrium was realized, simulating whether our ruling was in favor of the plaintiff or the defendant according with the amount of “evidence” presented. Finally, using this dataset of synthetic cases, the relationship between the probability of a ruling for the plaintiff and the number of hearings was estimated with a probit regression for each sample.

The results are shown in Figure 4. The orange dots on the margins are our synthetic cases, with the ones on the top representing the convictions, and the ones on the bottom the acquittals. The solid lines are showing the values estimated by the probit regression model. The maroon line depicts the simulated performance of public enforcers and the violet one the simulated performance of private plaintiffs. Notice that the slope of the lines is always positive, indicating that both public and private enforcers are
monotonically increasing their effectiveness as the length of the litigation also increases. This is the first general intuition arising from the model: whenever one of the litigants has a moderate strategic disadvantage, her chances of success are directly proportional to the length of the trial.\(^\text{25}\)

**Figure 4: Simulated Performance of the Litigants.**

This figure shows the result of a probit regression analysis when performed over our synthetic data. The parameters of the simulation are as follows: (i) \( R \sim \ln \mathcal{N}(7,1) \); and, (ii) \( \delta^F = 0.1, \delta^A = 0.5, \delta^H = 0.9 \).

Second, the line for private plaintiffs has a higher slope than the one for public agencies, but the issue is the other way around regarding the intercept. The idea here is that, because their agency costs are independent of the number of victims, the performance

\(^{25}\)Provided that \( R \) can be made big enough, the result holds for any \( \delta < 0 \). In contrast, the meaning of “moderate” regarding the tie-breaker rule is significantly more involved. A presumption that goes strongly against the plaintiff would prevent her from filing a suit. On the contrary, a presumption strongly in favor of the plaintiff reverses the roles of the litigants, with the defendant stopping first and approaching the tie-breaker rule from above.
of public enforcers should be rather consistent across our different type of cases. On the contrary, due to the variable nature of their agency costs, private firms are going to have both a high-performance region in the subset of cases with a limited number of victims, as well as a low-performance region in the subset of cases where such number is large. Therefore, the different type of agency costs at play would be causing two different profiles of litigant: whereas the public agencies are characterized by its consistency, private plaintiffs are notorious by its fluctuating performance.

4 Continuous Effort

One of the main assumptions of the model developed in the body of the paper is that presenting evidence in any given round involves a discrete choice. From a methodological point of view, the justification for this assumption is that it enables that the model works with a finite number of rounds. In turn, having a finite number of rounds is what enables that the persuasive power of the evidence decreases as the litigation moves forward. This annex provides further justification for this assumption by discussing a model in which the parties are allowed to choose freely their level of effort.

Figure A-1: Extensive Form of the Continuous Game

\[
p = p(E_\Pi, E_\Delta) \equiv \frac{E_\Pi}{E_\Pi + E_\Delta}
\]

This decision tree depicts the extensive form of the continuous variation of the game. The perpendicular lines represent the choice of the parties modeled in a continuous range, whereas the dashed line depicts the information set of the defendant. The payoffs for both parties appear at the bottom of the tree.

Consider a model similar to the one developed in the body of the paper, but with the variation that the parties present all the evidence simultaneously in a single-round game.
As before, the court decides whether the defendant is liable after observing the effort of the parties and in light of the available evidence. We begin considering that the plaintiff’s choice involves maximizing the amount of effort devoted to the litigation,

\[
\max_{E_{II} \geq 0} [p(\cdot)R - C E_{II}]
\]

Which give us the plaintiff’s best response,

\[
\frac{R E_{\Delta}}{(E_{II} + E_{\Delta})^2} = C
\]

And the corresponding optimal level of effort,

\[
E_{II}^* = \sqrt{\frac{R E_{\Delta}}{C} - E_{\Delta}}
\]

Regarding the defendant, she also maximizes the amount of evidence presented during the trial considering her expected payoff,

\[
\max_{E_{\Delta} \geq 0} [p(\cdot)(-R) - C E_{\Delta}]
\]

Which, gives us the defendant’s best response,

\[
\frac{R E_{II}}{(E_{II} + E_{\Delta})^2} = C
\]

And the corresponding optimal level of effort,

\[
E_{\Delta}^* = \sqrt{\frac{R E_{II}}{C} - E_{II}}
\]

Because the optimal levels of effort of the parties are interrelated, such that what each the parties is going to do depends on what the other one is doing, we have a system of equation that solves:

\[
E_{II}^* = E_{\Delta}^* = \frac{R}{4C}
\]

Let us now consider how this solution compares with the one that was developed for the discrete case in the body of the paper. Recall that whenever the agency costs of the plaintiff are similar to the ones of the defendant, the stopping time of the game is represented by Equation (7). We can conceive how a infinite game would look like multiplying the number of rounds \(n\) by the arbitrary quantity \(t\), and, in order to maintain the unitary cost, also divide the cost \(C\) by the same arbitrary quantity \(t\). If we evaluate what happens with our equilibrium when the \(t\) approaches to infinity, we have that

\[
\lim_{t \to \infty} \left[ \frac{tR + 2C}{tAC} \right] = \frac{R}{4C}
\]

In sum, the continuous variation of the game yields the same equilibrium than the variation with infinite rounds.
References


