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Don Fullerton, *University of Illinois at Urbana-Champaign*
Daniel Karney, *Ohio University*

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Multiple pollutants, co-benefits, and suboptimal environmental policies[☆]

Don Fullerton[a,*], Daniel H. Karney[b]

[a] University of Illinois at Urbana-Champaign and NBER, United States
[b] Ohio University, United States

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ABSTRACT

In our analytical general equilibrium model, polluting inputs can be substitutes or complements. We study a tax increase on one pollutant where the other faces a tax or permit policy. Our solutions highlight key parameters and welfare effects with gains from abatement plus positive or negative co-benefits from other pollutants in the covered and uncovered sectors. We demonstrate several ways taxes and permits differ. First, the change in taxed pollutant depends on whether the other pollutant faces a tax or permit policy. Also, only with a tax on the other pollutant can a co-benefit arise. The sign of co-benefits depends on the sign of cross-price elasticities and on whether the other pollutant's price is above or below marginal damages. Finally, the other pollutant's tax or permit policy also affects emissions in the uncovered sector (leakage). In a numerical illustration of carbon tax in U.S. electricity, we calculate emissions of CO_2 and SO_2 in both sectors. For plausible parameters, co-benefits are larger than direct

Introduction

In a multiple pollutant setting, the first-best can be achieved when each pollutant faces a tax or permit price that reflects its marginal environmental damage (Hanley et al., 2007, pp. 138–149). Not all pollutants are regulated, however, and even regulated ones likely face suboptimal policy. Thus, multiple pollutants create complications for regulators: tightening rules on one pollutant can affect emissions of other pollutants. Policymakers who adopt a new carbon policy may not be able to adjust each regulation on other types of pollution, especially where different laws and jurisdictions govern different pollutants. In fact, studies of a particular regulation may include “ancillary” co-benefits from reducing other pollutants (Burtraw et al., 2003, Groosman et al., 2011, Kolstad et al., 2014).

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[*] Corresponding author.
E-mail addresses: dfullert@illinios.edu (D. Fullerton), karney@ohio.edu (D.H. Karney).

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To consider the general problem of multiple pollutants, our simple analytical general equilibrium model has two sectors with competitive markets and constant returns to scale production functions. Our standard assumptions include full information, perfect factor mobility, and certainty, but a less standard assumption is that each sector has three inputs: a clean input and two kinds of pollution. With three inputs, any pair can be complements or substitutes. We refer to the clean input as labor, but it could represent labor, human capital, physical capital, or a composite of all clean inputs. For concreteness, our primary numerical example has one sector for electricity generation and another sector for the rest of the economy (including transportation), where both sectors use inputs of labor, sulfur dioxide (SO_2), and carbon dioxide (CO_2). Both pollutants in both sectors may face existing suboptimal policies, and then we consider a small increase in the tax on only one pollutant in one "covered" sector (e.g., carbon tax only in electricity generation). All equations are differentiated to linearize the model and to solve for the effects of that small policy change on all prices, quantities, and economic welfare.

The point of the general model is that raising a tax on one pollutant might increase or decrease the other pollutant. Thus, the model can encompass the example of Sigman (1996) who studies chlorinated solvents used for metal cleaning and degreasing; she finds that raised disposal costs for liquid chemical wastes leads to more air emissions. The empirical literature has many such examples.[1] Regarding our specific numerical example for electricity generation, Färe et al. (2012) find that nitrogen oxides (NO_X) and sulfur dioxide (SO_2) are substitutes in production, but Agee et al. (2014) argue that CO_2 and SO_2 could be substitutes *or* complements.[2] If they were substitutes, then a tax on CO_2 could lead to more SO_2, but the U.S. EPA assumes that a carbon tax reduces use of coal and therefore both CO_2 and SO_2 (i.e. complements). In this case, a carbon tax can have large positive co-benefits if it reduces damages from SO_2.

Another complication, however, is that SO_2 from power plants has been limited in the U.S. by a fixed number of permits under the Acid Rain Program (as studied by e.g., Schmalensee et al., 1998, Burtraw et al., 1998, Carlson et al. 2000). In this case, if a carbon tax changes demand for SO_2, then permit prices may rise or fall depending on the degree of carbon and sulfur complementarity in production. But, if permits fix the quantity of SO_2, then the carbon tax has *no* co-benefit from reducing sulfur emissions.

For these reasons, our general model allows each pollutant to face a pre-existing tax or permit price. Thus, we analyze four combinations. The case where both face a tax provides important baseline results showing the importance of cross-price substitution elasticities. We solve explicitly for the tax-tax and tax-permit scenarios, but the model is symmetric, so the permit-tax and permit-permit scenarios are analogous. Interestingly, we only need to consider the two pollutant policies in a "covered" sector (e.g., electricity generation), even though the other sector may also emit both types of pollution.

While some features of our model have appeared before, we obtain new results by combining all four of the following. First, we model analytically the general case where two pollutants can be complements or substitutes in production.[3]

Second, all pollutants need not be controlled by the same type of policy. While one pollutant might be subject to a tax, another is restricted by permits. Therefore, we use a framework that can analyze all combinations of tax or permit policies and allow for a relatively easy comparison of policy scenarios available to regulators.[4]

Third, these policies are likely not optimal; the price per unit of pollution does not equal marginal environmental damage. A pollutant's policy may not be optimal for at least three reasons: technical limitations and information constraints may preclude correct estimation of social costs and benefits; political concerns may prevent adoption of a first-best policy; and, a pollution tax would reflect conditions at the time of enactment rather than current conditions. Furthermore, multiple pollutants – even from a single source – may not have a single regulator using a comprehensive approach. We address situations where one regulator chooses a policy given regulations on other pollutants.[5]

Fourth, a pollution tax or permit system is unlikely to cover all sectors. The newly proposed Clean Power Plan applies only to the electricity generating sector, for example, just as did the Acid Rain Program of SO_2 permits. A carbon tax might be able to cover more than just power plants, but it cannot cover all carbon emissions from small industry and residential sources (Metcalf and Weisbach, 2009). If it does not cover the entire economy, then a rise in the carbon price in the covered sector may have multiple second-best effects, as carbon emissions shift to uncovered sectors (i.e. carbon leakage).[6]

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[1] Greenstone (2003) finds no evidence for iron and steel that the Clean Air Act increased water or ground pollution, but Gibson (2015) looks at a wider set of industries and finds that it increased water pollution. Gamper-Rabindran (2006) finds that off-site recycling is a substitute for chemical waste disposal. Ren et al. (2011) find that reducing CO_2 by the use of biofuel can increase nitrogen runoff from farms.

[2] In response to SO_2 controls, the switch to low-sulfur coal with lower heat rate could increase CO_2 per kilowatt hour. Also, desulfurization equipment uses electric power that requires burning more coal and may generate added CO_2 emissions from the chemical reactions that capture SO_2. If the response is to shut down dirty plants, then effects on CO_2 depend on whether new plants use coal or natural gas. They find (p.81) that the "marginal effect on CO_2 emissions from reducing SO_2 is negative" (i.e. complements).

[3] For examples of other models with multiple pollutants that could be complements or substitutes in production, see Moslener and Requate (2007), Holland (2012b), Ren et al. (2011), and Agee et al. (2014).

[4] Ambec and Coria (2013) also analyze a mix of taxes and permits when pollutants can be substitutes or complements, using a "prices v. quantities" approach of Weitzman (1974). They rank welfare outcomes of policy mixes. Our paper differs first by using a general equilibrium approach and second by assuming perfect certainty. Third, because they solve for optimal combinations of policy, they do not consider co-benefits. We consider *sub*-optimal policy, so a change in carbon tax may affect other pollutants that are not priced optimally. Then the change in that other pollutant can provide positive or negative co-benefits.

[5] Moslener and Requate (2007) derive optimal abatement strategies in a dynamic multi-pollutant model. We limit our analysis to welfare effects of small changes from a suboptimal equilibrium, because many studies already consider first-best and second-best optimal policy with other distortions.

[6] Baylis et al. (2014) analyze and discuss the carbon leakage issue in greater detail. In addition, Holland (2012a) and Karp (2013) provide recent, analytical models of carbon leakage.

This combination of features in our analytical model allow us to show four ways that pollution taxes and permits differ, even with perfect certainty. First, for a tax increase on one pollutant, its quantity change depends on whether the other pollutant faces a tax or permit policy. At coal-fired power plants that face an increased carbon tax, even the carbon abatement itself depends on whether SO_2 is taxed or permitted. Second, a raised carbon tax can increase or decrease SO_2 emissions that face a tax, creating co-benefits that affect welfare, but not when SO_2 faces a binding permit policy. Third, reductions in SO_2 emissions may *reduce* welfare if that pollutant is already over-regulated in the covered sector with a tax above its marginal environmental damage. Fourth, whether that other pollutant in the covered sector faces a tax or permit policy also affects emissions of both carbon and sulfur in the uncovered sector, with additional impacts on welfare.

To illustrate these points, our numerical exercise uses data from 2007, one of the last years with binding U.S. regulation of SO_2 emissions under the Acid Rain Program. In the tax-tax scenario, a 10% increase in carbon tax reduces CO_2 by 4.3% and SO_2 by 3.7%. However, the same carbon tax in the tax-permit scenario results in a much smaller decrease in CO_2 emissions and zero change in SO_2. Despite the same tax increase in both scenarios, welfare gains in the tax-tax scenario are almost twice the gains in the tax-permit scenario. Only in the tax-tax case can carbon policy yield co-benefits from SO_2 reduction, and this co-benefit slightly exceeds the primary benefit under plausible parameters.

We note several important caveats. First, we emphasize that our simple analytical model cannot provide detailed or realistic predictions of effects of actual policy proposals. Instead, we intend only to study conceptual issues regarding interactions between sectors and between pollutants in general, when a tax on one can affect both pollutants and welfare in ways that depends on key parameters and on existing suboptimal policy. We insert values for parameters into our formulas only to illustrate the size of effects we study, not to compete with other detailed models.[7]

Second, for tractability, we collapse important dynamic effects into a simple static model. Carbon emissions cause complex future damages, but Nordhaus (2014) and others calculate the present value of costs from one ton of emissions today. Those future damages negatively affect the utility of infinitely lived individuals or of a dynasty where current generations care about their children. Also for tractability, we cannot capture the way SO_2 damages vary by location, but we discuss likely effects. We assume both pollutants damages are separable in utility, but we discuss all of these caveats below.

Third, while we study tax policy and permit prices, we do not solve for effects of non-price policies like mandates or energy efficiency standards. Such standards can be studied in analytical general equilibrium models, but doing so here would add much complexity to this analysis. We point to Fullerton and Heutel (2010) and Goulder et al. (2016) for analytical general equilibrium models that incorporate mandates and compare them to price policies. To comply with a mandated reduction in emissions per unit of output, for example, firms can reduce emissions or increase output (or some of each). Thus it is equivalent to the combination of a tax on emissions with all revenue used to subsidize output. These issues are indeed interesting but are left to other papers.

Fourth, other papers study effects of a large policy change such as the introduction of an optimal carbon tax (e.g. $40/ton). For several reasons, however, we consider only small changes. First, our linearization is strictly valid only for small changes, and we can avoid the complexities of using two methodologies in one paper. Second, these small changes capture the fact that actual policy reform is most often incremental: policymakers make compromises and try out new policy ideas while avoiding major shocks to the economy. That is, the immediate adoption of a large carbon tax is unlikely. Third, our approach can still be used to analyze a large carbon tax, by considering a small change to a large pre-existing tax. Indeed, the U.S. already has restrictions on carbon, voluntary carbon markets, energy efficiency standards, and a gasoline tax. Those restrictions are represented in our simplified model by a pre-existing carbon tax. Finally, our small changes are conceptually useful to see all directions of change for each variable.

Section 2 below presents our basic model with multiple pollutants. Section 3 provides closed-form, analytical solutions for changes in endogenous variables, given an exogenous change in policy. Section 4 outlines our welfare analysis. Section 5 identifies plausible parameter values to calibrate the model. Section 6 uses those values for numerical results, and it conducts sensitivity analysis. Section 7 briefly concludes.

Model

As in other studies using linearization (e.g., Bovenberg and de Mooij, 1994; Fullerton and Metcalf, 2001), we assume perfect competition, full information, mobile factors, many identical agents, certainty, lump-sum transfers, costless enforcement of policies, and perfect mixing of pollutants (i.e. no “hot-spots”). While both sectors face a positive price for carbon emissions from various existing energy policies, we model effects of tightening carbon policy in a covered sector that does not apply to the uncovered sector. We compare only long-run equilibria and do not consider adjustment costs. This static model does include the key features necessary to answer our primary question: How does sub-optimal policy toward multiple pollutants affect firm decisions and total welfare?

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[7] For example, it may be interesting that results from our numerical illustrations closely track results from the large computer simulation model of Burtraw et al. (2014) when using similar parameter values, but the similarity of results is only a coincidence. We solve analytically for general equilibrium effects in a highly aggregated model, while they study many details of the electricity sector in a partial equilibrium model.

Initial setup

The covered sector produces output Y by a constant return to scale (CRTS) production function $Y = Y(L_Y, C_Y, S_Y)$, where L_Y is a productive resource called labor that could be a composite of all non-polluting factors (labor, capital, land, and technology). Firms pay a market-clearing price (p_L) for the composite labor input and emit both carbon (C_Y) and sulfur (S_Y).[8] In sector Y, carbon and sulfur each face a tax or permit price, depending on prevailing regulation, so the firm pays a price p_{CY} when emitting carbon and p_{SY} when emitting sulfur. The government returns all revenue from taxes or permit sales via lump-sum transfer to households.[9] Because of CRTS, the scale of production is irrelevant, so we treat all quantities as amounts per household.

The other good is X, produced in the uncovered sector by the CRTS function $X = X(L_X, C_X, S_X)$ using labor (L_X), carbon (C_X), and sulfur (S_X).[10] Firms in sector X also pay p_L per unit of labor. If this sector does not face explicit carbon or sulfur policies, it does face implicit prices (p_{CX} and p_{SX}) from other policies such as a gasoline tax, BTU tax, or fuel-efficiency standards. For ease of exposition, we make all prices explicit.

The only binding resource constraint in this economy is $L \equiv L_Y + L_X$. Here, a fixed total amount of labor is perfectly mobile between sectors X and Y, so leisure does not enter utility. In both sectors, all inputs are necessary for production and exhibit diminishing marginal returns. These conditions – along with regularity conditions on the consumer side – guarantee an interior solution.

To maximize a generic utility function $U = U(X, Y; C, S)$, individuals can choose their own consumption of goods X and Y, but they cannot choose aggregate carbon emissions $C = C_Y + C_X$ or sulfur emissions $S = S_Y + S_X$. They do not care about those emissions *per se*, but they do care about effects on climate and air quality that are functions of emissions.[11] Mathematically, those functions can be subsumed within the utility function, so utility is a negative function of C and of S. Also, although climate and air quality changes could affect demands for private commodities in various ways, we assume for tractability here that those negative effects are separable entries in utility.[12]

Finally, p_X is the market clearing price for good X, and p_Y is the analogous price for good Y. The household maximization problem is:

$$\max_{\{X,Y\}} U(X, Y; C, S) \quad s.t. \quad p_L L + R \geq p_X X + p_Y Y$$

where R is the lump-sum rebate of revenue from the government, viewed as fixed by the consumer but calculated as $R \equiv \left(p_{CY}C_Y + p_{SY}S_Y\right) + \left(p_{CX}C_X + p_{SX}S_X\right)$.

Log-Linearization

Totally differentiate the resource constraint $L \equiv L_Y + L_X$ to get:

$$0 = \alpha_X \hat{L}_X + \alpha_Y \hat{L}_Y \tag{1}$$

where $\alpha_X \equiv L_X/L$ is the share of labor in production of good X, and $\alpha_Y \equiv L_Y/L$ is the share of labor in Y ($\alpha_X + \alpha_Y = 1$). We use the "hat" notation to denote a proportional change in any variable (e.g. $\hat{L}_X \equiv dL_X/L_X$).

Totally differentiate the production functions to show how final output changes when firms adjust input quantities:

$$\hat{X} = \theta_{XL}\hat{L}_X + \theta_{XC}\hat{C}_X + \theta_{XS}\hat{S}_X \tag{2}$$

$$\hat{Y} = \theta_{YL}\hat{L}_Y + \theta_{YC}\hat{C}_Y + \theta_{YS}\hat{S}_Y \tag{3}$$

where θ_{gi} is the factor share of income for input i in the production of good g (e.g., $\theta_{XL} \equiv p_L L_X / p_X X$). Thus, $\theta_{XL} + \theta_{XC} + \theta_{XS} = 1$, and $\theta_{YL} + \theta_{YC} + \theta_{YS} = 1$.

With competition, the zero profit conditions are $p_X X = p_L L_X + p_{CX}C_X + p_{SX}S_X$ and $p_Y Y = p_L L_Y + p_{CY}C_Y + p_{SY}S_Y$. Totally differentiate these equations and use the profit maximizing first-order conditions to get:

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[8] Pollutant S could be generic "smoke" that accounts for all non-carbon pollution. Also, since firms are identical, trades are irrelevant, and the permit system is equivalent to non-tradable quotas.

[9] The permit revenue collected and returned lump-sum depends on initial permit allocation. Revenue is maximized when all permits are auctioned, but a permit policy could raise no revenue if permits are handed-out for free to firms. Our model is agnostic about this choice, because (1) many identical households get these funds either as tax rebate or as profits of the firms they own, and (2) the opportunity cost to firms of SO_2 emissions does not depend on the initial allocation but only on the prevailing market price for permits.

[10] A special case is where X is perfectly clean ($C_X = S_X = 0$), eliminating leakage from the model.

[11] Effects of C include heat waves, lost biodiversity, sea level rise, lost production, and increased frequency and severity of droughts, storms, and floods. Effects of S include acid rain and reduced air quality.

[12] These assumptions would be interesting to investigate but beyond the scope of this paper (and already studied elsewhere). With non-separable utility, changes in pollution can further affect demands (Carbone and Smith, 2008). With consumer heterogeneity, policies have distributional effects (Bento, 2013). With firm heterogeneity, tradable permits would be more efficient than firm-specific quotas. With locational heterogeneity, different sources would have differential damages (Muller and Mendelsohn, 2009).

$$\hat{p}_X + \hat{X} = \theta_{XL}\left(\hat{p}_L + \hat{L}_X\right) + \theta_{XC}\left(\hat{p}_{CX} + \hat{C}_X\right) + \theta_{XS}\left(\hat{p}_{SX} + \hat{S}_X\right) \tag{4}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YL}\left(\hat{p}_L + \hat{L}_Y\right) + \theta_{YC}\left(\hat{p}_{CY} + \hat{C}_Y\right) + \theta_{YS}\left(\hat{p}_{SY} + \hat{S}_Y\right). \tag{5}$$

Since pollution is separable in utility, we can use σ_U for the elasticity of substitution in utility between X and Y. Differentiation yields changes in demand behavior from a shift in output prices:

$$\hat{X} - \hat{Y} = \sigma_U\left(\hat{p}_Y - \hat{p}_X\right). \tag{6}$$

To handle three inputs in sector Y, we follow Allen (1938), as in Mieszkowski (1972).[13] Define e_{ij} as the Allen-elasticity of substitution between input i and input j. That is, e_{ij} measures the effect on the quantity of i from a change in the price of input j, holding all other input prices constant. As shown in Appendix A, the relative input factor responses in sector Y are given by:

$$\hat{L}_Y - \hat{S}_Y = (e_{LL} - e_{SL})\theta_{YL}\hat{p}_L + (e_{LC} - e_{SC})\theta_{YC}\hat{p}_{CY} + (e_{LS} - e_{SS})\theta_{YS}\hat{p}_{SY} \tag{7}$$

$$\hat{C}_Y - \hat{S}_Y = (e_{CL} - e_{SL})\theta_{YL}\hat{p}_L + (e_{CC} - e_{SC})\theta_{YC}\hat{p}_{CY} + (e_{CS} - e_{SS})\theta_{YS}\hat{p}_{SY}. \tag{8}$$

If e_{ij} is positive, then the inputs are substitutes; if it is negative, they are complements. Each input is a complement to itself ($e_{ii} \le 0, \forall\, i$). The Allen-elasticities are symmetric, $e_{ij} = e_{ji}$, and at most one of the three cross-price elasticities can be negative.[14]

Our base-case parameterization uses U.S. EPA's assumption that these pollutants are complements, meaning that $e_{SC} = e_{CS} < 0$, but we show the sensitivity of results to this assumption because Agee et al. (2014) argue that CO_2 and SO_2 could be substitutes *or* complements (see footnote 2). Also, the cross-price elasticity can be positive for other pollutants; Färe et al. (2012) find nitrogen oxide and SO_2 emissions are substitutes.

Sector X also has three inputs, and the model allows for arbitrary substitution among them (as in sector Y). We define L as numeraire ($\hat{p}_L = 0$), however, and no regulatory change in the uncovered sector means that its carbon and sulfur prices remain constant relative to that numeraire ($\hat{p}_{CX} = \hat{p}_{SX} = 0$). Facing no change to any relative input prices, the uncovered sector does not choose to alter relative use of inputs; it only grows or shrinks proportionately. Therefore, instead of representing substitution in that sector with two complex equations similar to (7) and (8), we can simply use:

$$\hat{C}_X = \hat{L}_X, \tag{9}$$

and

$$\hat{S}_X = \hat{L}_X. \tag{10}$$

Equivalently, these equations could be written with a full set of Allen elasticities for sector X, but no change in relative input prices implies that those elasticities all drop out.

These ten linear equations are solved below for the equilibrium impacts of an exogenous shock, $\hat{p}_{CY}$, a small change in the covered sector's price of carbon.[15]

This model can be extended, but adding any one feature adds disproportionately to complexity of the model and solutions. For example, the model would be more realistic if the covered good Y (e.g. electricity) were an intermediate input to production of X. We undertake this extension in a new working paper about leakage (Baylis et al., 2017) where we show: (a) how to model this feature; (b) that it raises the number of simultaneous equations from 10 to 14; and (c) that it somewhat mutes the differential effect of climate policy, since some cost of the policy is "transmitted" via the intermediate input to the other sector. However, it does not change our qualitative results discussed below.

Analytical solutions for a change in carbon policy

Eqs. (1)–(10) are the linear system for general equilibrium effects of a small policy change. Begin by applying the numeraire condition $\left(\hat{p}_L = 0\right)$ and the assumption that pollution prices in sector X are fixed $\left(\hat{p}_{CX} = \hat{p}_{SX} = 0\right)$ to all remaining

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[13] Fullerton and Heutel (2007) similarly model relationships among labor, capital, and a single pollutant.

[14] A profit-maximizing firm conforms to $a_{iL} + a_{iC} + a_{iS} = 0$, where a_{ij} is the partial elasticity of substitution in production (related to the Allen-elasticity of substitution by $a_{ij} = \theta_{Yj}e_{ij}$).

[15] These equations cannot exactly consider the introduction of a new carbon tax, because the initial p_{CY} cannot be zero in the denominator of $\hat{p}_{CY} = dp_{CY}/p_{CY}$. The initial tax could be very small, however.

Table 1
Policy scenario matrix, for a change in carbon policy ($\hat{p}_{CX} = \hat{p}_{SX} = 0$).

	Sulfur tax	Sulfur permit
Carbon Tax	$\hat{p}_{CY} > 0$ $\hat{p}_{SY} = 0$ (Tax-Tax)	$\hat{p}_{CY} > 0$ $\hat{S}_Y = 0$ (Tax-Permit)
Carbon Permit	$\hat{C}_Y < 0$ $\hat{p}_{SY} = 0$ (Permit-Tax)	$\hat{C}_Y < 0$ $\hat{S}_Y = 0$ (Permit-Permit)

equations. Next, simplify Eq. (4) and compare it to Eq. (2) to show that $\hat{p}_X = 0$. Thus, good X acts as an equivalent numeraire, because all of its inputs have unchanged relative prices. Eq. (4) becomes redundant to Eq. (2), leaving nine equations.

The two pollutants in sector Y are regulated, where either C_Y or S_Y can face an environmental tax or permit policy. In a tax-tax scenario, the carbon tax exogenously rises $\left(\hat{p}_{CY} > 0\right)$, while the sulfur tax remains constant $\left(\hat{p}_{SY} = 0\right)$, leaving the carbon quantity $\left(\hat{C}_Y\right)$ and sulfur quantity $\left(\hat{S}_Y\right)$ to vary endogenously. In contrast, a tax-permit scenario means that the carbon tax rises while the sulfur quantity remains unchanged $\left(\hat{S}_Y = 0\right)$, so markets adjust the carbon quantity $\left(\hat{C}_Y\right)$ and sulfur price $\left(\hat{p}_{SY}\right)$. Thus, among the four potential policy variables in the set $\left\{\hat{p}_{CY}, \hat{C}_Y, \hat{p}_{SY}, \hat{S}_Y\right\}$, two will be specified as exogenous changes, while the other two remain endogenous. The other seven unknown variables are $\left\{\hat{X}, \hat{L}_X, \hat{C}_X, \hat{S}_X, \hat{Y}, \hat{L}_Y, \hat{p}_Y\right\}$, so each policy scenario yields a linear system with nine equations and nine unknowns.

Table 1 categorizes the four scenarios, with two pollutants and two policy regimes for each. However, we explicitly analyze only two scenarios: the tax-tax case and tax-permit case (with permits for S_Y). The model's symmetry means that these two cases implicitly also solve the remaining two scenarios (permit-tax and permit-permit cases).[16] This symmetry highlights the fact that the type of policy on sulfur fundamentally determines how the covered sector reacts to the new restriction on carbon, regardless of whether that carbon restriction is a tax or permit policy.

Tax-tax scenario

The exogenous change in the tax-tax scenario is $\hat{p}_{CY} > 0$. The change in the price of good Y is $\hat{p}_Y = \theta_{YC}\hat{p}_{CY}$ (by substituting Eqs. (3) into (5) and cancelling terms). Thus, the price of Y always rises relative to the price of X (since $\hat{p}_X = 0$). Further algebra reveals the change in output of good Y (given the sulfur tax, p_{SY}):

$$\hat{Y}\Big|_{p_{SY}} = \alpha_X\left[\gamma_C e_{CC} + \gamma_S e_{SC} - \sigma_U\right]\theta_{YC}\hat{p}_{CY} \tag{11}$$

where $\gamma_C \equiv \left(\frac{\alpha_Y}{\alpha_X}\frac{\theta_{YC}}{\theta_{YL}}\right) > 0$, and $\gamma_S \equiv \left(\frac{\alpha_Y}{\alpha_X}\frac{\theta_{YS}}{\theta_{YL}}\right) > 0$. Perhaps surprisingly, the added carbon tax in Y might raise output.[17] We interpret the three terms in the brackets in Eq. (11) when $\hat{p}_{CY} > 0$. First, $\gamma_C e_{CC} \le 0$ reflects an own-price effect on carbon use from an increase in carbon price. All inputs have a positive marginal product, so less carbon means less Y. In the third term, when the carbon tax increases p_Y, then $\sigma_U > 0$ means that consumers shift demand away from good Y. Thus, the sign of $\hat{Y}$ depends on the second term. If the two pollutants are complements $\left(\gamma_S e_{SC} < 0\right)$, then the higher price of carbon reduces sulfur. Less of this input would also reduce output Y, which then unambiguously falls. This term has the opposite sign when the two pollutants are substitutes, so then the change in Y is ambiguous.

Alternatively, Eq. (11) can be rewritten as:

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[16] For the first column of Table 1 with a sulfur tax, the carbon tax case is functionally equivalent to the box below it with carbon permits. Given a sulfur tax, a 1% higher carbon tax that leads to a 2% change in carbon is equivalent to a 2% change in carbon permits (leading to 1% higher price). Similarly, for the column with sulfur permits, effects of a carbon tax are equivalent to effects of a carbon permit.

[17] Output always falls in the simpler model of Baylis et al. (2014), but the carbon tax can raise output in a model with three inputs and particular complementarities (Fullerton and Heutel, 2007).

$$\hat{Y}\Big|_{p_{SY}} = -\,[\alpha_X\sigma_U + \alpha_Y e_{LC}]\theta_{YC}\hat{p}_{CY}. \tag{11'}$$

This equation shows that when carbon and *labor* are substitutes $(e_{LC} > 0)$, then Y must fall.[18] We explicitly show those two forms for $\hat{Y}$ because Eq. (11) highlights the cross-pollutant elasticity e_{SC}, while Eq. (11') provides a more compact closed-form solution with fewer parameters.

Next, we solve for the change in the covered sector's carbon emissions:[19]

$$\hat{C}_Y\Big|_{p_{SY}} = \left[\underbrace{-(\alpha_X\sigma_U + \alpha_Y e_{LC})}_{\substack{\text{Output}\\ \text{Effect}}} + \underbrace{e_{CC}}_{\substack{\text{Substitution}\\ \text{Effect}}}\right]\theta_{YC}\hat{p}_{CY}. \tag{12}$$

The second term in this equation is the substitution effect. It is always negative, because $e_{CC} \le 0$; the higher carbon tax induces firms to substitute away from carbon. The first term is called the output effect because it equals $\hat{Y}$ from Eq. (11'). In general its sign is ambiguous, but only in very unusual cases would a carbon tax increase output in the covered sector. Therefore, C_Y likely falls.[20]

Next, we report the change in sulfur (S_Y):

$$\hat{S}_Y\Big|_{p_{SY}} = [-(\alpha_X\sigma_U + \alpha_Y e_{LC}) + e_{SC}]\theta_{YC}\hat{p}_{CY} \equiv A\hat{p}_{CY}. \tag{13}$$

Notice the definition of A, used below. Eq. (13) has a form similar to (12) where the output effect is the same and is usually negative. The substitution effect in (12) has a clear sign (because $e_{CC} < 0$), but the substitution effect in (13) is e_{SC}. If the two pollutants are substitutes $(e_{SC} > 0)$, then the carbon tax has a positive effect on sulfur. Since the output effect is usually negative, however, the net effect on sulfur would still be ambiguous. If the two pollutants are complements (as in EPA assumptions used below), then the carbon tax reduces sulfur in both terms. Next, we find $\hat{L}_Y$, which is similarly ambiguous.[21]

Next, we look at pollution leakage, defined as the change in pollution in the other sector. Recall that relative prices of inputs in sector X do not change, so firms choose to adjust all inputs in equal proportion $\left(\hat{L}_X = \hat{C}_X = \hat{S}_X\right)$. Therefore, solving for the proportional change of one input, such as labor, yields the proportional change in the other inputs. The changes for labor and pollution in sector X are:

$$\hat{L}_X\Big|_{p_{SY}} = \hat{C}_X\Big|_{p_{SY}} = \hat{S}_X\Big|_{p_{SY}} = \hat{X}\Big|_{p_{SY}} = -\,(\alpha_Y/\alpha_X)\hat{L}_Y = \alpha_Y[\sigma_U - e_{LC}]\theta_{YC}\hat{p}_{CY}. \tag{14}$$

With CRTS production, proportional changes in all inputs means the same proportional change in output $\left(\hat{X}\right)$. The closed-form solution in Eq. (14) is quite simple and offers the possibility that carbon and sulfur leakage are negative; that is, tighter policy in the covered sector may reduce both types of pollution in the other sector. Baylis et al. (2014) derive a similar result in a simple model with only carbon emissions. In that model, the higher price of covered sector output Y induces households to substitute into X (a positive effect on leakage), but the higher price of carbon input in Y induces firms to substitute into more use of labor, drawing labor away from X (a negative effect on leakage). Appendix B formalizes results on leakage and then uses those results to show and to discuss the change in total emissions from both pollutants.

In summary, these closed-form solutions show how the signs of endogenous outcomes are determined by cross-price

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[18] This $e_{LC} > 0$ does not imply $e_{SC} < 0$, but $e_{SC} < 0$ does imply $e_{LC} > 0$ (since the properties of Allen-elasticities guarantee $0 = \theta_{YL}e_{LC} + \theta_{YC}e_{CC} + \theta_{YS}e_{SC}$ and $e_{CC} < 0$). Therefore, to guarantee $\hat{Y} < 0$, carbon and labor being substitutes is a more general condition than the pollutants being complements. Also, it might be easier to determine empirically whether C and L are substitutes, since L is often well measured.

[19] Eq. (12) can be inverted to yield the solution to the permit-tax scenario (a tighter carbon permit policy, while sulfur is subject to a tax): $\hat{p}_{CY}|p_{SY} = \left[\frac{1}{-(\alpha_X\sigma_U + \alpha_Y e_{LC}) + e_{CC}}\right]\left[\frac{\hat{C}_Y}{\theta_{YC}}\right]$.

[20] From (11'), output Y rises when C_Y and L_Y are complements ($e_{LC} < 0$) and $|\alpha_Y e_{LC}| > |\alpha_X\sigma_U|$. But from (12), carbon still falls if $|e_{CC}| > |\alpha_X\sigma_U + \alpha_Y e_{LC}|$. In our numerical example, α_X is much larger than α_Y, so output of Y always falls. Since p_Y/p_X rises, Y can only rise if X rises more, which is only possible if real income rises (e.g. if the increase in p_{CY} reduces distortions from initial $p_{CY} < p_{CX}$).

[21] Given fixed p_{SY}, the change in labor is $\hat{L}_Y = -\,\alpha_X[\sigma_U - e_{LC}]\theta_{YC}\hat{p}_{CY}$. Then $\sigma_U > e_{LC}$ guarantees $\hat{L}_Y < 0$ (because consumers substitute away from Y). Since $\sigma_U > 0$, this equation says that L_Y must fall, unless labor and carbon are more substitutable in production than the two goods are substitutable in utility.

elasticities that need to be estimated. An increase in the carbon tax reduces carbon emissions in the taxed sector (except in the unusual case where that output rises). The same carbon tax may or may not raise sulfur emissions when the pollutants are substitutes, but it reduces SO_2 when the pollutants are complements.

Do these results require a general equilibrium model? Perhaps similar results could be obtained in a simpler model of the covered sector with substitutability of clean and dirty inputs to production, and that model could be linked to another model of the other sector to calculate leakage. But our general equilibrium model can handle multiple relative price changes for two outputs and three inputs simultaneously, in a way that captures the effects of each market on the others. It ensures all required consistency conditions: the sum of both sectors' input demands must fit within economy-wide resource constraints, and both sectors' production must fit within household budgets.

Tax-permit scenario

If the carbon tax is raised while sulfur faces a permit policy, then $\hat{p}_{CY} > 0$ and $\hat{S}_Y = 0$. The change in price of Y depends on the cost of all three inputs, where labor is numeraire, so $\hat{p}_Y = \theta_{YS}\hat{p}_{SY} + \theta_{YC}\hat{p}_{CY}$. Thus, we first solve for the endogenous price of sulfur:

$$\hat{p}_{SY}\Big|_{S_Y} = -\left[\frac{-(\alpha_X\sigma_U + \alpha_Y e_{LC}) + e_{SC}}{-(\alpha_X\sigma_U + \alpha_Y e_{LS}) + e_{SS}}\right]\frac{\theta_{YC}}{\theta_{YS}}\hat{p}_{CY} = -\frac{A}{D}\hat{p}_{CY} \tag{15}$$

where A is the coefficient from (13) in the tax-tax scenario.[22] The sign of this numerator is ambiguous, as before, but now the sign of the denominator is also unknown. To avoid technicalities, a footnote describes why we can assume the sign of D is negative.[23] Here, we focus on intuition about the numerator – for any two pollutants. If they are substitutes ($e_{SC} > 0$), then a tax on C can raise demand for S and raise its price ($\hat{p}_{SY} > 0$). If so, then costs of *both* polluting inputs rise, and $\hat{p}_Y > 0$. If the pollutants are complements ($e_{SC} < 0$), then the signs of A and $\hat{p}_{SY}$ are likely negative: the carbon tax raises the cost of one input, but the reduced demand for sulfur reduces the cost of the other polluting input.

Next, solving the closed-form solution for $\hat{p}_Y$ yields:

$$\hat{p}_Y\Big|_{S_Y} = \left(\frac{\theta_{YC}D - \theta_{YS}A}{D}\right)\hat{p}_{CY} \tag{16}$$

where the sign of the denominator is assumed negative. If the pollutants are complements ($e_{SC} < 0$), as just described, then the carbon tax reduces the price of the *other* polluting input, so production costs might rise or fall. If they are substitutes, the carbon tax raises demand for sulfur and the price of sulfur, so the cost of production clearly rises ($\hat{p}_Y > 0$).

Recursively, the change in output is given $\hat{Y} = \theta_{YL}\hat{L}_Y + \theta_{YC}\hat{C}_Y$, which is simply the weighted-sum of the changes in the two inputs (since the tax-permit scenario fixes the quantity of sulfur). Solving for the closed-form solution yields:

$$\hat{Y}\Big|_{S_Y} = -\left(\frac{e_{SC}\theta_{YC}D - e_{SS}\theta_{YS}A}{D}\right)\hat{p}_{CY} \tag{17}$$

where (17) looks similar to (16), but Allen elasticities enter each numerator term to switch from a price solution to a quantity solution. Also, Eq. (17) has a negative sign, unlike (16), because price and quantity generally move in opposite directions.

Next, insert $\hat{S}_Y = 0$ into (3) and solve for $\hat{C}_Y = \frac{1}{\theta_{YC}}\left[\hat{Y} - \theta_{YL}\hat{L}_Y\right]$, an expression that decomposes the change in carbon into output and substitution effects. Here, due to the fixed sulfur content, the substitution effect is just the scaled change in the clean labor input. If we replace the endogenous outcomes ($\hat{Y}$ and $\hat{L}_Y$), the closed-form solution is:

$$\hat{C}_Y\Big|_{S_Y} = -\left(\frac{(e_{SC} - e_{CC})\theta_{YC}D - (e_{SS} - e_{SC})\theta_{YS}A}{D}\right)\hat{p}_{CY}. \tag{18}$$

This equation is similar to (17), but it has two more elasticities in the numerator. We would certainly expect the quantity of carbon to fall when the carbon tax increases, but Eq. (18) in general has an ambiguous sign. However, the two pairs of elasticities in the numerator can help explain the ambiguity. The pair $(e_{SC} - e_{CC})$ measures the relative changes in S and C

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[22] The denominator (D) is the coefficient on the change in sulfur from an alternate tax-tax scenario where the sulfur tax changes while the carbon tax is held constant, as given by:

$$\hat{S}_Y|p_{CY} = \left[-(\alpha_X\sigma_U + \alpha_Y e_{LS}) + e_{SS}\right]\theta_{YS}\hat{p}_{SY} \equiv D\hat{p}_{SY}.$$

This equation merely switches the roles of C_Y and S_Y in Eq. (12).

[23] While $-e_{LS}$ could be positive in D, both $-\alpha_X\sigma_U$ and e_{SS} must be negative. Thus, D is *likely* negative, but sufficiently flexible demand for sulfur in e_{SS} would guarantee it. Closed-form solutions for other outcomes in this scenario have the same denominator as in Eq. (15), so we assume $D < 0$.

given a carbon price change. Most plausible parameters suggest that the own-price effect is larger than cross-price effects, so $(e_{SC} - e_{CC}) > 0$ and the first term in the numerator is negative (reducing carbon in Y).

Interestingly, a comparison of Eqs. (12) and (18) shows that whether sulfur faces a tax or a permit policy clearly changes how the carbon tax affects the quantity of carbon emissions. When the increased carbon tax affects sulfur quantities in the tax-tax case, pollutant complementarity means that those changes in sulfur quantities have their own feedback effects on the desired quantity of carbon input.[24]

A similar procedure is used to find $\hat{L}_Y$. Then, since $\hat{L}_X = -(\alpha_Y/\alpha_X)\hat{L}_Y$, and all inputs to X change proportionately, we have:

$$\hat{L}_X\Big|_{S_Y} = \hat{C}_X\Big|_{S_Y} = \hat{S}_X\Big|_{S_Y} = \hat{X}\Big|_{S_Y} = \left(\frac{\alpha_Y}{\alpha_X}\right)\left(\frac{(e_{SC} - e_{LC})\theta_{YC}D - (e_{SS} - e_{LS})\theta_{YS}A}{D}\right)\hat{p}_{CY}. \tag{19}$$

This leakage expression (for $\hat{C}_X$) is significantly more complicated than leakage in Eq. (14) for the tax-tax scenario. All else equal, however, we expect to find larger leakage in the tax-tax scenario because the fixed input of S in the tax-permit setting means fewer adjustments generally for a given carbon tax change. Our numerical results later in Table 4 confirm this intuition. For similar reasons, we expect total carbon emissions to fall by more in the tax-tax scenario. Overall, the tax-permit case yields no simple results for carbon emissions: C_Y probably falls, but C_X may rise or fall. However, $\hat{S}_Y = 0$ in this scenario, and therefore the change in total sulfur emissions is simply given by Eq. (19). Also, while the possibility of "sulfur leakage" (a change in S_X) is a distinct feature of this model, its relevance in the numerical section is limited ($0.1–$1.3 million).

Overall, compared to the tax-tax scenario, we find that the tax-permit scenario has more ambiguous outcomes that depend on parameter values. As in the tax-tax scenario, however, we can still decompose input changes $\hat{C}_Y$ and $\hat{L}_Y$ into output and substitution effects. In the end, we still highlight the intermediate conclusion that the cross-price elasticities need to be estimated. As shown in the next section, welfare implications depend on the sign of each pollution change and on whether that pollutant is over- or under-regulated in the initial equilibrium.

Welfare changes

In general, changes in welfare depend on changes in all pollutants from all sectors. If a new policy targets one pollutant in one sector, it may have general equilibrium effects that increase or decrease other pollutants in the same sector or in other sectors. For concreteness, we look at CO_2 policy in one sector that may affect SO_2 emissions in that sector, or both CO_2 and SO_2 from the other sector. Since SO_2 has environmental damages, one might think that its reduction would raise welfare, but we now show that SO_2 abatement might reduce welfare – if it is already over-regulated.

Define λ as the marginal utility of income, and μ_C as the marginal environmental damage (MED) from a unit of carbon, $\mu_C \equiv -\left(\partial U/\partial C\right)/\lambda$. Also, define μ_S as the MED from a unit of sulfur, $\mu_S \equiv -\left(\partial U/\partial S\right)/\lambda$. Appendix C shows that the welfare change is:

$$\frac{dU}{\lambda I} = (p_{CX} - \mu_C)\frac{C_X}{I}\hat{C}_X + (p_{CY} - \mu_C)\frac{C_Y}{I}\hat{C}_Y + (p_{SX} - \mu_S)\frac{S_X}{I}\hat{S}_X + (p_{SY} - \mu_S)\frac{S_Y}{I}\hat{S}_Y. \tag{20}$$

The left-hand side is the dollar value of the change in utility $\left(dU/\lambda\right)$, divided by national income (I). Thus, it represents the percentage change in welfare (as in Bovenberg and de Mooij, 1994, or Fullerton and Metcalf, 2001). The right-hand side is the sum of gains or losses from changes in each pollutant. If each $p_C = \mu_C$ and $p_S = \mu_S$, then this equation shows that the right-hand side is zero, so $dU = 0$. Such policies are first best, as no change in any pollutant can increase welfare further. In our static framework, the MED values capture the present value of all future damages (to a dynasty or infinitely lived decision-maker). However, the literature finds that the discount rate assumptions have a large impact on the MED from carbon (Nordhaus, 2014; Interagency Working Group on Social Cost of Carbon IWGSCC, United States Government, 2015).

Consider the case where carbon is initially under-priced relative to its MED, with $p_{CX} < p_{CY} < \mu_C$. Suppose all pollutants face taxes, but policymakers increase the carbon tax in Y ($\hat{p}_{CY} > 0$), holding other taxes constant $\left(\hat{p}_{SY} = \hat{p}_{CX} = \hat{p}_{SX} = 0\right)$. Any reduction in C_Y then raises welfare through the second term above. Yet this carbon tax may have a negative effect on welfare through the first term if the uncovered sector increases use of carbon $\left(\hat{C}_X > 0\right)$. Thus, the sign of the change in welfare is ambiguous even when just considering carbon emissions. Moreover, the carbon tax may change SO_2 in either sector.

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[24] Eq. (18) can be inverted to yield the solution for the permit-permit scenario (tighter carbon permit policy, while sulfur also faces a permit policy): $\hat{p}_{CY}|S_Y = -\left[\frac{D}{(e_{SC} - e_{CC})\theta_{YC}D - (e_{SS} - e_{SC})\theta_{YS}A}\right]\hat{C}_Y.$

Table 2
Benchmark levels.

Symbol	Value	Units	Source
Panel A: GDP by sector and revenue			
I	13,807.5	\$ billions	U.S. BEA (2009)
$p_Y Y$	379.7	\$ billions	U.S. EIA (2009)
$p_X X$	13,427.8	\$ billions	Authors' calculation
R	58.1	\$ billions	Authors' assumption
Panel B: Covered sector carbon			
C_Y	2,397.2	MM tons	U.S EPA (2009a)
p_{CY}	15.0	\$/metric ton	Hassett et al. (2009)
$p_{CY} C_Y$	36.0	\$ billions	Authors' calculation
Panel C: Covered sector sulfur			
S_Y	8,973.0	thousand tons	U.S. EPA (2009b)
p_{SY}	530.0	\$/ton	Bloomberg Data
$p_{SY} S_Y$	4.8	\$ billions	Authors' calculation
Panel D: Uncovered sector carbon			
C_X	3,338.6	MM tons	U.S EPA (2009a)
p_{CX}	5.0	\$/metric ton	Authors' assumption
$p_{CX} C_X$	16.7	\$ billions	Authors' calculation
Panel E: Uncovered sector sulfur			
S_X	2282.0	thousand tons	U.S. EPA (2009b)
p_{SX}	175.0	\$/ton	Authors' assumption
$p_{SX} S_X$	0.4	\$ billions	Authors' calculation
Panel F: Labor by sector and total			
L_Y	338.9	\$ billions	Authors' calculation
L_X	13,410.5	\$ billions	Authors' calculation
$L = L_X + L_Y$	13,749.4	\$ billions	Authors' calculation

Notes: (1) "MM tons" refers to millions of metric tons; (2) the equation for R is in *Initial setup*.

Suppose the two pollutants in sector Y are complements, as in our example below, so the carbon tax reduces the other pollutant ($\hat{S}_Y < 0$). Still, however, Eq. (20) shows that the effect on welfare is ambiguous. If sulfur is already priced at $p_{SY} = \mu_S$, then $\hat{S}_Y$ has no impact on welfare. If sulfur happens to be over-priced, then a carbon tax that reduces SO_2 emissions has a *negative* co-benefit, because the gain from reducing sulfur (μ_S) is more than offset by the cost of reducing sulfur (p_{SY}). Only if sulfur is under-priced does its abatement provide a positive co-benefit.[25] This same logic holds for changes in sulfur emissions from sector X; the third term on the right-hand side of Eq. (20) may yield a welfare increase or decrease from sulfur abatement in that sector depending on whether the pollutant is over-priced or under-priced relative to it MED.

Next, consider the case where SO_2 is controlled by binding permits ($\hat{S}_Y = 0$). Then only carbon and sulfur leakage in the uncovered sector can offset welfare gains from reducing C_Y. If p_{CY} is "near" the MED, then the welfare gain from reducing covered carbon can be offset by the welfare loss from leakage ($\hat{C}_X > 0$ or $\hat{S}_X > 0$). Recall that $\hat{C}_X = \hat{S}_X$ so both types of leakage must have the same sign. Leakage can be negative, however, where reduction of under-priced pollutants leads to gains. Also, even though the direction of leakage is the same for both pollutants in the uncovered sector, the signs of the two effects on welfare change can differ if one is over-regulated while the other is under-regulated. To evaluate final welfare changes requires values for all parameters.

Parameter values

This section provides parameter values for a numerical illustration that uses equations above to solve for endogenous outcomes and welfare. Analytical expressions in section 3 are complex, with some ambiguous signs, so this calculation can help evaluate both signs and magnitudes. The covered sector in this example is all of U.S. electricity generation, which emits both CO_2 and SO_2 and which can substitute away from carbon in the long run by switching from coal to natural gas, solar, or wind power. The uncovered sector is the rest of the economy (ROE), which also emits both CO_2 and SO_2.

We note that the price of sulfur permits under the Acid Rain Program has recently fallen to zero in the U.S., as the policy is no longer binding. But we wish to illustrate our model for positive pollution prices, so we calibrate the model to emission

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[25] Alternatively, suppose carbon and sulfur are substitutes and sulfur is overpriced ($p_{SY} > \mu_S$). In that case, strictly speaking, the carbon tax may have a positive co-benefit by increasing sulfur emissions.

Table 3
Primary parameter values.

Parameter	Value	Source
Panel A: Share parameters		
$\alpha_X \equiv L_X/L$	0.975	See Table 2
$\alpha_Y \equiv L_Y/L$	0.025	
$\theta_{YL} \equiv L_Y/(p_Y Y)$	0.893	
$\theta_{YC} \equiv (p_{CY}C_Y)/(p_Y Y)$	0.095	
$\theta_{YS} \equiv (p_{SY}S_Y)/(p_Y Y)$	0.013	
Panel B: Allen-elasticities (Partial Elasticities)		
e_{SC} (a_{SC}, a_{CS})	-3.6 $(-0.34, -0.05)$	Authors' calculation
e_{LS} (a_{LS}, a_{SL})	0.5 (0.01, 0.45)	Considine and Larson (2006)
e_{LC} (a_{LC}, a_{CL})	0.5 (0.05, 0.45)	Authors' assumption
e_{LL} (a_{LL})	-0.1 (-0.05)	Authors' calculations
e_{CC} (a_{CC})	-4.2 (-0.40)	
e_{SS} (a_{SS})	-8.4 (-0.11)	
Panel C: Other parameters		
σ_U	0.25	Ross (2008)
μ_C	39	Tol (2009)
μ_S	1510	Muller and Mendelsohn (2009)

Notes: Values subject to independent rounding.

data and economic data from 2007. Also, our simplified analytical model does not capture all possible effects of a carbon tax in general equilibrium, such as in a large computational model with explicit fuel markets. It is therefore not a “forecast” of actual effects from a carbon tax, but merely an illustration of analytical results above.

Benchmark levels and share parameters

Table 2 records all of our benchmark levels (used to obtain our primary parameters). First, the U.S. National Income and Product Accounts report 2007 GDP of $13,808 billion, which we use for national income, I (U.S. BEA, 2009).

Our model evaluates small changes from an initial equilibrium with a hypothetical carbon tax, so sales in the electricity sector must cover this extra cost. Thus, for $p_Y Y$, we obtain $380 billion by adding end-use electricity sales of $344 billion (U. S. EIA, 2009) plus $36 billion for the increased cost of electricity from our hypothetical initial carbon tax of $15 per metric ton (Hassett et al., 2009). The U.S. EPA's Greenhouse Gas Inventory (U.S EPA, 2009a) reports that electric generators emitted 2397 million metric tons (MMtons) of CO_2 in 2007 (constituting 42 percent of all U.S. domestic CO_2 combustion emissions that year). Thus, $p_{CY}C_Y$ equals $36 billion. The ROE emitted 3339 MMtons of CO_2 from fossil fuel combustion in 2007. We use $5 per metric ton of CO_2 as the carbon price in sector X, and so $p_{CX}C_X$ equals $16.7 billion.[26]

The National Emissions Inventory (U.S. EPA, 2009b) reports electric generators emitted 8973 thousand tons of SO_2 from fossil fuel combustion in 2007 (69.4 percent of all domestic SO_2 emissions). For the initial p_{SY}, we use the average of the 2007 vintage allowance price during that year, or $530 per ton.[27] Thus, $p_{SY}S_Y$ equals $4.8 billion. The ROE emitted 2282 thousand tons of SO_2 from fossil fuel combustion in 2007, and we use $175 per ton of SO_2 as its sulfur price, so $p_{SX}S_X$ is $0.40 billion.[28]

Since production $Y(L_Y, C_Y, S_Y)$ exhibits CRTS with zero profits, clean input costs $(p_L L_Y)$ can be identified by subtracting the costs of pollution from sales revenue. Then the normalization $p_L = 1$ determines L_Y. Since the electricity sector has $40.8 billion in emission costs, the remainder of $339 billion must be paid to the non-polluting input. A similar calculation for the rest of the economy determines L_X.

Next, these levels are converted into share parameter values for the log-linearized Eqs. (1)–(10), and hence in our solutions. Table 3 panel A shows that the U.S. electricity sector uses 2.5% of total clean inputs $(\alpha_Y = 0.025)$, and those clean

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[26] Sector X is all of the economy but electricity. The U.S. gasoline tax divided by CO_2 emissions is $5/ton, but some of that gasoline tax covers congestion or other externalities, not just carbon. Also, emissions in sector X face other implicit costs from energy efficiency and other regulations. In particular, a carbon tax in the electricity sector would raise the price of electricity used as an intermediate input, which is implicitly a small tax on carbon used in production of X. Finally, the Chicago Climate Exchange (CCX) was a market for voluntary carbon allowances, so the price on this exchange represents an opportunity cost that firms faced when emitting CO_2. In 2007, the average daily mid-price for 2007 allowances was $3.18 per metric ton. Rather than attempt a precise calculation, we just use $5/ton for p_{CX}.

[27] The permit price data are reported by Evolution Markets and retrieved via Bloomberg data terminal.

[28] The p_{SX} of $175 per ton is selected such that p_{XY}/p_{SY} is proportional to p_{CX}/p_{CY}.

inputs account for 89.3% of the inputs in sector $Y\left(\theta_{YC}=0.893\right)$. The share parameters for sector X (e.g. θ_{XC}) drop out of the closed-from solutions and thus are omitted.

Allen-elasticities

Eqs. (7) and (8) contain nine Allen-elasticities that determine input demand responses in the covered sector. However, the Allen-elasticities are symmetric ($e_{ij}=e_{ji}$). Also, knowing the cross-price Allen-elasticities $\left(e_{ij}\text{ for } i\neq j\right)$ and share parameters determines the own-price Allen-elasticities (see Appendix A). Thus, we focus on identifying the cross-price Allen-elasticities (e_{SC}, e_{LC}, e_{LS}), and we use them to determine the own-price elasticities (e_{LL}, e_{CC}, e_{SS}). All these are best interpreted as long-run elasticities.

A key question is whether the two pollutants are substitutes or complements. As Appendix D explains, we use the EPA's analysis of proposed carbon cap-and-trade legislation to determine our primary value, and we find $e_{SC}=-3.6$ (so carbon and sulfur are complements under EPA assumptions). Also, for labor and sulfur substitution, we round the 0.47 estimate of Considine and Larson (2006) to $e_{LS}=0.5$, and we assume that same value for the labor and carbon elasticity (e_{LC}). Remaining own-price terms are derived from those. Table 3 panel B reports all primary-case elasticity values.

Table 3 panel B also shows the partial elasticities between inputs (in parentheses), where $a_{ij}=\theta_{Yj}e_{ij}$ is the partial elasticity that reports the percent change of input i for a one percent change in price j (Stern, 2011). In general, the partial elasticities are not symmetric ($a_{ij}\neq a_{ji}$), so each row in panel B reports three values. The $e_{SC}=e_{CS}=-3.6$ Allen elasticity at first seems large in absolute value, but the corresponding partial elasticities are only $a_{SC}=-0.34$ and $a_{CS}=-0.05$ because the values for θ_{YC} and θ_{YS} are so small. Similarly, the own-price Allen elasticities for carbon and sulfur have very large absolute values, but the partial elasticities are only $a_{CC}=-0.40$ and $a_{SS}=-0.11$.

Other parameters

Table 3 panel C reports remaining parameters. For substitution in utility, we use $\sigma_U=0.25$, as employed by the EPA in their computational general equilibrium model for household substitution between energy and all other goods (Ross, 2008). This low elasticity of substitution is consistent with empirical estimates of a low price elasticity of demand for electricity. Tol (2009) surveys the literature and finds an average estimate for the MED of carbon dioxide (μ_C), which we convert to \$39 per metric ton for 2007. Muller and Mendelsohn (2009) calculate an average MED of sulfur (μ_S) that we convert to \$1510 per ton.[29] The fact that μ_S is \$1510/ton while p_{SY} is only \$530/ton suggests considerable scope for welfare improvements from coordinating carbon and sulfur policy. Here, we look only at changes to the carbon tax of \$15/ton.

Numerical results

Results with primary parameters

Our numerical results appear in Table 4. Column (1) reports results for the tax-tax policy scenario, where the carbon tax is increased by 10 percent, and an SO_2 tax remains constant (so $\hat{p}_{CY}$ is 0.10 and $\hat{p}_{SY}$ is zero by assumption; those chosen entries are in bold). As a result of this policy change with primary-case parameters, covered CO_2 emissions fall by 4.26 percent, and SO_2 emissions fall by 3.65 percent. As the carbon price increases, complementarity between pollutants $\left(e_{SC}=-3.6\right)$ leads to the decline in SO_2. Producers increase their use of the clean input, a result expected from the substitutability between carbon and labor $\left(e_{LC}=0.5\right)$. Next, the price of good Y increases by slightly less than one percent, and output of Y falls by 0.24 percent. Since production of X retains an unchanged ratio of inputs, the increase in $\hat{L}_Y$ means that $\hat{L}_X$, $\hat{C}_X$ and $\hat{S}_X$ must all fall (slightly).

Thus, the tax on CO_2 in sector Y leads to a decline in *both* carbon and sulfur emissions in *both* sectors. Intuition for the negative leakage result in Baylis et al. (2014) is simple: a carbon tax can induce firms to use more clean inputs per unit of output, and thus draw resources away from the other sector, which reduces the other sector's output and emissions. The same mechanism operates in this model, but for multiple pollutants. With no fixed total for carbon pollution, decreasing its use in one sector does not necessitate increasing its use in the other sector, and similarly for sulfur emissions.

The welfare gain in the tax-tax scenario for the primary case (Panel B, column 1) comes mostly from carbon reduction in the covered sector. Although carbon and sulfur are reduced by similar percentages (Panel A), welfare changes are proportional to the difference between the existing tax and MED. Also, the level of carbon emissions is measured in million metric

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[29] Tol (2009) reports a social cost of carbon of \$105 per metric ton (in 1995), a value we convert to CO_2 equivalent and inflate to 2007 dollars using the All Items CPI (Series ID# CUSR0000SA0). Muller and Mendelsohn (2009) report an SO_2 MED estimate in 2002 dollars, which we also inflate to 2007.

Table 4
Numerical results.

Variable/Policy	Scenario			
	Tax-Tax [Baseline] (1)	Tax-Permit [Match Tax] (2)	Tax-Permit [Match Carbon] (3)	Tax-Permit [Match Welfare] (4)
Panel A: Quantity and price percent changes				
$\hat{C}_Y$	−4.26	−2.63	**−4.26**	−4.81
$\hat{p}_{CY}$	**10.00**	**10.00**	16.17	18.30
$\hat{S}_Y$	−3.65	**0.00**	**0.00**	**0.00**
$\hat{p}_{SY}$	**0.00**	−33.62	−54.37	−61.51
$\hat{L}_Y$	0.23	0.13	0.21	0.23
$\hat{Y}$	−0.24	−0.13	−0.22	−0.25
$\hat{p}_Y$	0.95	0.53	0.85	0.96
$\hat{X} = \hat{L}_X = \hat{C}_X = \hat{S}_X$	−0.006	−0.003	−0.005	−0.006
Panel B: Welfare changes in $million [Primary Case]				
(MEDs: $\mu_C = 39.0$ and $\mu_S = 1510$)				
From $\hat{C}_Y$	2,444.8	1,511.8	2,444.8	2,765.9
From $\hat{S}_Y$	321.2	**0.0**	**0.0**	**0.0**
From $\hat{C}_X$	6.6	3.7	5.9	6.7
From $\hat{S}_X$	0.2	0.1	0.2	0.2
Total	2,772.8	1,515.6	2,450.9	**2,772.8**
Panel C: Welfare changes in $million [Alternative Case 1]				
(MEDs: $\mu_C = 19.5$ and $\mu_S = 1510$)				
From $\hat{C}_Y$	459.1	283.9	459.1	519.4
From $\hat{S}_Y$	321.2	**0.0**	**0.0**	**0.0**
From $\hat{C}_X$	2.8	1.6	2.5	2.9
From $\hat{S}_X$	0.2	0.1	0.2	0.2
Total	783.3	285.5	461.8	522.4[a]
Panel D: Welfare changes in $million [Alternative Case 2]				
(MEDs: $\mu_C = 39.0$ and $\mu_S = 10000$)				
From $\hat{C}_Y$	2444.8	1511.8	2444.8	2765.9
From $\hat{S}_Y$	3103.4	**0.0**	**0.0**	**0.0**
From $\hat{C}_X$	6.6	3.7	5.9	6.7
From $\hat{S}_X$	1.3	0.7	1.2	1.3
Total	5556.1	1516.2	2451.9	2773.9[b]

Notes: (1) Each cell is independently rounded. (2) Cells marked in bold indicate exogenously set variable.

[a] The carbon tax increase would need to be 27.47% instead of 18.30% to match welfare change in (1).

[b] The carbon tax increase would need to be 36.62% instead of 18.30% to match welfare change in (1), but the sulfur price in the covered sector falls by more than 100 percent in this model.

tons, while sulfur emissions are measured in thousands of short tons. Overall welfare improves by 0.02 percent in the tax-tax case, which is a utility gain worth $2.77 billion per year for this small increase an existing sub-optimal tax.

If the carbon MED were only half of the size measured by Tol (2009), as presented in alternate case 1 (Panel C), then the welfare gains fall by more than half relative to the primary case. The model's differentiated equations are linear, but Eq. (20) above shows that welfare changes are not proportional to damages μ. When the initial p_{CY} is $15/ton, and μ_C is cut in half (from 39.0 to 19.5), the remaining uncorrected distortion $\left(p_{CY} - \mu_C\right)$ falls from −24 to −4.5, a drop of over 81 percent.

Alternate case 2 in Panel D raises the sulfur MED significantly, from $1510 to $10,000/ton (while returning the carbon MED to the original $39.0/ton). The Muller and Mendelsohn (2009) estimate of the marginal damage from sulfur is $1310/ton for 2002 (which we scaled to 2007 to obtain the $1510/ton in Table 3), but Muller (2014) reports an emission-weighted estimate for the same year that is $15,906/ton, more than ten times larger. Similarly, Burtraw et al. (2014) use a sulfur MED that is more than $8000/ton, and a carbon MED near $40 per metric ton. In other words, this alternate case 2 employs parameters close to those of Burtraw et al. (2014). The overall welfare gain then doubles in the tax-tax scenario (to 0.04 percent, or $5.56 billion). The majority of this gain now comes from sulfur reductions in the covered sector (as in Burtraw et al., 2014).

These results are remarkably similar to those from the detailed "Haiku" electricity market model of Burtraw et al. (2014). For a hypothetical cap-and-trade policy in the U.S. electric power sector, they also find that the co-benefit from SO_2 reduction is slightly larger than the primary gain from CO_2 reduction.[30] This similarity is a coincidence, however, since the models are so different. Their partial-equilibrium computational model uses considerable detail in the energy sector to simulate a large carbon tax, while our simple analytical general-equilibrium model has only two sectors and two inputs to show effects of a small tax change. Interestingly, however, Woollacott and Wing (2015) use a large CGE model and also find that a carbon tax yields substantial co-benefits from reducing SO_2 emissions at U.S. power plants. Our model captures essential elements of the problem, but in any case, the simplicity and transparency of our algebraic solutions can provide more intuition than is available from a large computer model.

Table 4 column (2) shows the tax-permit scenario, where we match the carbon tax change of 10% from column (1) but hold sulfur permits fixed.[31] Here, the decrease in C_Y is approximately half that in the tax-tax case; as a result the welfare gain with the same primary-case parameters is also approximately half ($1.5 billion). The sulfur price falls significantly, by nearly 34 percent, and leakage is still slightly negative. With sulfur fixed in this tax-permit scenario, alternate cases 1 and 2 have welfare gains that are significantly less than half those of the tax-tax scenario. This column numerically demonstrates how effects of the carbon tax depend on whether sulfur faces a tax or permit policy.

Table 4 column (3) is another version of the tax-permit scenario, but where we match the change in sector *Y*'s carbon emissions to the tax-tax scenario. Here, the tax-permit scenario needs a larger increase in the carbon tax (16.2%) to reach the same 4.26% reduction in carbon emissions as in the tax-tax scenario. Interestingly, even with this higher tax rate, the tax-permit scenario has a smaller effect on output than in the tax-tax scenario (−0.22% vs. −0.24%). But the smaller output effect on emissions is offset by a larger substitution effect. Table 4 column (4) sets the carbon tax increase in the tax-permit scenario to match the welfare gain of the tax-tax scenario. We find that this tax rate increase would need to almost double (18.3% rather than 10%), and carbon emissions in the covered sector fall by 4.81 percent.

Our model employs a single value for sulfur damages, since it has no spatial dimension, but damages from SO_2 can differ by source (Muller and Mendelsohn, 2009). According to Burtraw et al. (2014), the MED for sulfur in the Eastern U.S. is almost four times that in the West. Thus, a change in carbon policy could affect welfare by spatial reallocation of sulfur emissions. Here, we use U.S EPA (2009c) to check if nationwide carbon pricing in the 2009 American Clean Energy and Security Act (ACES) would have caused a regional shift in sulfur emissions. We use those two sulfur MEDs from Burtraw et al. (2014), and weight them by the EPA's SO_2 emissions for states in each half of the country to get a weighted-average national MED, and then we weight them by EPA's estimate of sulfur emissions by region *with* ACES to get a new MED. We find that the weighted-average national MED for sulfur would have been only 1.3 percent lower in 2020 under ACES, which indicates that U.S. carbon policy does not lead to substantial regional reallocation of sulfur emissions in a way that affects welfare. This rough check suggests that heterogeneity of damages does not significantly affect our results.

Table 5 decomposes the input variables $\left(\hat{C}_Y, \hat{S}_Y, \hat{L}_Y\right)$ from Table 4 into output and substitution effects, using the analytical expressions in section 3. In column (1), for example, the carbon change in sector *Y* is −4.26%, with −0.24 percentage points coming from the output effect and −4.01 percentage points coming from the substitution effect (with independent rounding). The output effect for each cell in Table 5 is just the value of $\hat{Y}$ from the corresponding column in Table 4 (so the output effect is constant within each column). Since the decomposition is linear, the remainder must be the substitution effect. The ratio of the two effects can be quite informative. In the first panel, for $\hat{C}_Y$, the ratio of the substitution effect to the output effect is 16.54 in column (1), but it rises to 18.53 in the three tax-permit columns (2)–(4) because fixing the sulfur input keeps output from falling as much as in the tax-tax scenario. In the second panel, $\hat{S}_Y$ is −3.65% in the tax-tax scenario but is held to zero by the fixed number of permits in the tax-permit scenarios. With sulfur fixed by assumption, the output and substitution effects are undefined. In the third panel, labor is the largest input, and so its percentage changes are small. The ratio of effects is the same across all scenarios.

Sensitivity

Whether the two pollutants are substitutes or complements drives many of the numerical results. For this reason, we conduct a sensitivity analysis to vary e_{SC} from −4.5 to +4.5 along the horizontal axis (in three figures for the tax-tax scenario). When varying e_{SC}, we hold the input shares and the other cross-price Allen-elasticities constant (and satisfy $\theta_{Yi}e_{ii} + \theta_{Yj}e_{ij} + \theta_{Yk}e_{ik} = 0$ by letting own-price elasticities adjust). In each figure, one curve represents the primary-case assumption of $\sigma_U = 0.25$, while one curve has no output effect $(\sigma_U = 0)$, and the other has a larger output effect $(\sigma_U = 1)$.

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[30] Burtraw et al. (2014) simulate a change in the carbon price from zero to $18/ton, raising ($28 billion of revenue), while our analytical model can consider only a small 10% increase in the $15/ton carbon tax (raising about $2 billion). Our results are therefore smaller, but proportional to the results in their paper.

[31] In the tax-permit case, Eq. (15) finds $\hat{p}_{SY} = -(A/D)\hat{p}_{CY}$, implicitly defining composite parameters $A = [-(\alpha_X\sigma_U + \alpha_Y e_{LC}) + e_{SC}]\theta_{YC}$ and $D = [-(\alpha_X\sigma_U + \alpha_Y e_{LS}) + e_{SS}]\theta_{YS}$. With primary parameter values, we find that $A = -0.365$ and $D = -0.109$ (so the denominator is negative, as we argued it would be).

Table 5
Primary-case output and substitution effects (% change).

Variable/Policy	**Scenario name**			
	Tax-Tax [Baseline] $\left(\hat{Y} = -0.24\right)$ (1)	Tax-Permit [Match Tax] $\left(\hat{Y} = -0.13\right)$ (2)	Tax-Permit [Match Carbon] $\left(\hat{Y} = -0.26\right)$ (3)	Tax-Permit [Match welfare] $\left(\hat{Y} = -0.25\right)$ (4)
$\hat{C}_Y$	−4.26	−2.63	−4.26	−4.81
Output:	−0.24	−0.13	−0.22	−0.25
Substitution:	−4.01	−2.50	−4.04	−4.57
Ratio [Sub/Out]:	16.54	18.53	18.53	18.53
$\hat{S}_Y$	−3.65	**0.0**	**0.0**	**0.0**
Output:	−0.24	–	–	–
Substitution:	−3.41	–	–	–
Ratio [Sub/Out]:	14.05	–	–	–
$\hat{L}_Y$	0.23	0.13	0.21	0.23
Output:	−0.24	−0.13	−0.22	−0.25
Substitution:	0.47	0.26	0.43	0.48
Ratio [Sub/Out]:	−1.95	−1.95	−1.95	−1.95

Notes: (1) Each cell is independently rounded. (2) Cells marked in bold indicate exogenously set variable. (3) When sulfur is fixed by assumption, the dashes indicate that the output and substitution effects are undefined.

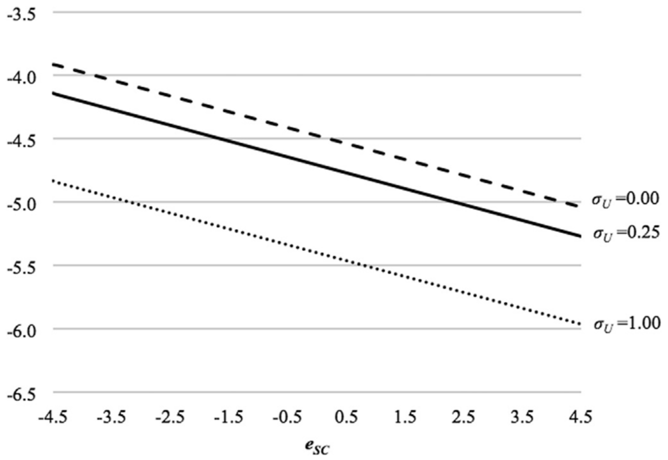

Fig. 1. Percent change in C_Y for varying e_{SC} and σ_U in the tax-tax scenario.

Fig. 1 shows the percent change in covered carbon (C_Y) for a 10 percent increase in the carbon tax, while the sulfur tax remains constant. From Eq. (12), we know that the substitution effect (e_{CC}) strongly influences this outcome, and thus we observe only small variations in $\hat{C}_Y$ for different values of e_{SC} or σ_U. The value of $\hat{C}_Y$ on the vertical axis only varies by about two percentage points, from −4.0 to −6.0 percent. A strong output effect ($\sigma_U = 1$) leads to more abatement, all else equal.

Next, Fig. 2 shows that changes in the sulfur quantity vary greatly across different values of e_{SC}. In fact, the sign and size of $\hat{S}_Y$ ranges from −5.0% to +4.0% as the pollutants become strong substitutes. The case with the strong output effect $\left(\sigma_U = 1\right)$ allows consumers to shift away from good Y, which reduces sulfur demand.

Welfare gains are always positive with the primary MED values, even when carbon and sulfur are substitutes ($e_{SC} > 0$), because benefits from carbon reduction exceed losses from extra sulfur emissions. To show where welfare may rise or fall, Fig. 3 uses Alternate Case 2 values for marginal environmental damages $\left(\mu_C = 39.0;\ \mu_S = 10000\right)$. It shows that welfare can indeed fall when tightening carbon policy if the pollutants are substitutes – because the other pollutant yields a negative co-benefit. When e_{SC} takes a large, positive value, then the added tax on one pollutant can increase emissions of the other

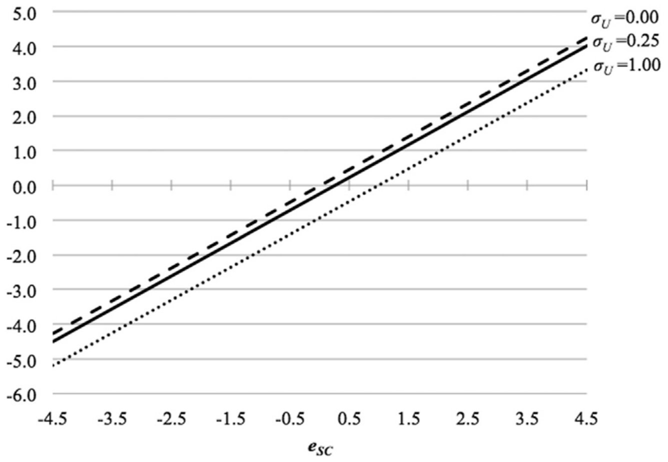

Fig. 2. Percent Change in S_Y for Varying e_{SC} and σ_U in the Tax-Tax Scenario.

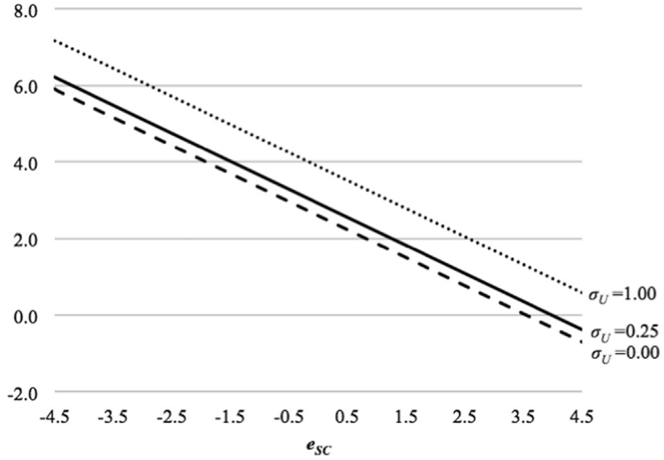

Fig. 3. Change in welfare in the tax-tax scenario with alternate case 2 values for MED (μ_C = 39.0 and μ_S = 10000, in $billions/year).

pollutant (as shown in Fig. 2), and the high damages reduce welfare. A strong output effect $\left(\sigma_U = 1\right)$ mitigates that danger to welfare, because falling output of Y reduces input demand for those other emissions. In summary, total benefits from the extra carbon tax can be positive or negative, depending on parameter values.

Conclusion

To show expressions for all price and quantity outcomes in general equilibrium, this paper builds a two-sector, two-pollutant, analytical model of a closed economy using standard assumptions of perfect competition, constant returns to scale, mobile factors, and perfect certainty. The two pollutants in the covered sector may be complements or substitutes, where either might be controlled by a tax or by permits. We find closed-form solutions that show equilibrium outcomes for any parameter values, and we provide intuition.

The paper highlights four important ways that pollution taxes and permit policies are not equivalent. First, the quantity change for a pollutant subject to a tax increase depends on whether the other pollutant in the covered sector faces a tax or permit policy. Second, only if that other pollutant faces a tax rather than a fixed number of permits can general equilibrium effects increase or decrease its quantity and impact overall welfare. Third, a decrease in sulfur can reduce welfare if it is already over-regulated. Fourth, the choice between a tax or permit policy on the other pollutant in the covered sector also affects both pollutants in the other, uncovered sector.

This setup allows us to identify three reasons that co-benefits might be positive or negative: (1) other pollutants in the covered sector might be complements or substitutes for the taxed pollutant; (2) those other pollutants might be under- or over-corrected already; and, (3) other pollutants might rise or fall in the other, uncovered sector.

The model is general enough to be used for any multiple pollutant problem within or across media. For an example, we conduct a numerical analysis of the U.S. electricity sector that emits carbon dioxide and sulfur dioxide from burning coal. Our most plausible parameters reflect the U.S. EPA assumption that CO_2 and SO_2 are complements in that sector, and so a tax on CO_2 provides co-benefits by reducing SO_2. The numerical exercise helps demonstrate the four ways that taxes and permits differ in a general equilibrium model with multiple pollutants and initially suboptimal policies.

Appendix A. Factor demand responses in sector *Y*

Here, we derive Eqs. (7) and (8), describing the input demand responses to changes in input prices. Define the input demand functions from cost minimization:

$$L_Y = L_Y(p_L, p_{CY}, p_{SY}, Y)$$
$$C_Y = C_Y(p_L, p_{CY}, p_{SY}, Y)$$
$$S_Y = S_Y(p_L, p_{CY}, p_{SY}, Y).$$

Next, totally differentiate each input demand function; divide through each equation (by L_Y, C_Y, and S_Y, respectively); and rearrange:

$$\frac{dL_Y}{L_Y} = \frac{\partial L_Y}{\partial p_L}\frac{p_L}{L_Y}\frac{dp_L}{p_L} + \frac{\partial L_Y}{\partial p_{CY}}\frac{p_{CY}}{L_Y}\frac{dp_{CY}}{p_{CY}} + \frac{\partial L_Y}{\partial p_{SY}}\frac{p_{SY}}{L_Y}\frac{dp_{SY}}{p_{SY}} + \frac{\partial L_Y}{\partial Y}\frac{Y}{L_Y}\frac{dY}{Y}$$
$$\frac{dC_Y}{C_Y} = \frac{\partial C_Y}{\partial p_L}\frac{p_L}{C_Y}\frac{dp_L}{p_L} + \frac{\partial C_Y}{\partial p_{CY}}\frac{p_{CY}}{C_Y}\frac{dp_{CY}}{p_{CY}} + \frac{\partial C_Y}{\partial p_{SY}}\frac{p_{SY}}{C_Y}\frac{dp_{SY}}{p_{SY}} + \frac{\partial C_Y}{\partial Y}\frac{Y}{C_Y}\frac{dY}{Y}$$
$$\frac{dS_Y}{S_Y} = \frac{\partial S_Y}{\partial p_L}\frac{p_L}{S_Y}\frac{dp_L}{p_L} + \frac{\partial S_Y}{\partial p_{CY}}\frac{p_{CY}}{S_Y}\frac{dp_{CY}}{p_{CY}} + \frac{\partial S_Y}{\partial p_{SY}}\frac{p_{SY}}{S_Y}\frac{dp_{SY}}{p_{SY}} + \frac{\partial S_Y}{\partial Y}\frac{Y}{S_Y}\frac{dY}{Y}.$$

Recall that constant returns to scale production implies that the input demand functions are homogenous of degree one in Y, so $\frac{\partial L_Y}{\partial Y}\frac{Y}{L_Y} = 1$, $\frac{\partial C_Y}{\partial Y}\frac{Y}{C_Y} = 1$, and $\frac{\partial S_Y}{\partial Y}\frac{Y}{S_Y} = 1$. Also, using the algebraic identity $a_{ij} = \theta_{Yj}e_{ij}$, where a_{ij} is the elasticity of demand for input i with respect to input price j, and rewriting using the "hat" notation, then:

$$\hat{L}_Y = e_{LL}\theta_{YL}\hat{p}_L + e_{LC}\theta_{YC}\hat{p}_{CY} + e_{LS}\theta_{YS}\hat{p}_{SY} + \hat{Y}$$
$$\hat{C}_Y = e_{CL}\theta_{YL}\hat{p}_L + e_{CC}\theta_{YC}\hat{p}_{CY} + e_{CS}\theta_{YS}\hat{p}_{SY} + \hat{Y}$$
$$\hat{S}_Y = e_{SL}\theta_{YL}\hat{p}_L + e_{SC}\theta_{YC}\hat{p}_{CY} + e_{SS}\theta_{YS}\hat{p}_{SY} + \hat{Y}.$$

Finally, because these equations are not independent, subtract the third from the first and second, respectively, to eliminate $\hat{Y}$, and rearrange using the symmetry of the Allen-elasticities $(e_{ij} = e_{ji})$ to yield Eqs. (7) and (8).

Appendix B. Pollution theorems for the tax-tax scenario

This appendix provides theorems on leakage and total pollution for the tax-tax scenario.

Theorem 1. (Leakage): *In the tax-tax case, where the carbon tax rises and the sulfur tax is fixed* ($\hat{p}_{CY} > 0, \hat{p}_{SY} = 0$), both carbon and sulfur emissions in the unregulated sector increase if and only if $\sigma_U > e_{LC}$ (so they both fall if and only if $e_{LC} > \sigma_U$ where $\sigma_U \geq 0$).

Proof. By inspection of Eq. (14).

Although the notation and interpretation is slightly different, the intuition underlying the result in Theorem 1 is similar to that in Baylis et al. (2014) as explained in the main text.

Recall emission totals are $C = C_X + C_Y$ and $S = S_X + S_Y$. Totally differentiate to get $\hat{C} = \beta_{CX}\hat{C}_X + \beta_{CY}\hat{C}_Y$ and $\hat{S} = \beta_{SX}\hat{S}_X + \beta_{SY}\hat{S}_Y$, where β_{pg} is the share of total pollutant p from sector g (e.g. $\beta_{CX} \equiv C_X/C$). Further algebra for the tax-tax scenario shows that *total* carbon and sulfur emission changes are:

$$\hat{C}\Big|_{p_{SY}} = \left[\alpha_Y(\sigma_U - e_{LC}) - \beta_{CY}(\sigma_U - e_{CC})\right]\theta_{YC}\hat{p}_{CY} \tag{B1}$$

$$\hat{S}\Big|_{p_{SY}} = \left[\alpha_Y(\sigma_U - e_{LC}) - \beta_{SY}(\sigma_U - e_{SC})\right]\theta_{YC}\hat{p}_{CY.} \tag{B2}$$

The signs of these changes are generally ambiguous, but each equation's first term is an indirect effect on the other sector, $\alpha_Y(\sigma_U - e_{LC})$. It increases emissions when leakage is positive (Theorem 1). The second term is a direct effect on the covered sector. In Eq. (B1), the direct effect of a carbon tax $[-\beta_{CY}(\sigma_U - e_{CC})]$ necessarily reduces carbon. If sector *Y* emits most carbon $\left(\beta_{CY} \approx 1\right)$, then this term dominates the total effect on carbon. In (B2), the sign of $[-\beta_{SY}(\sigma_U - e_{SC})]$ is ambiguous, but it is negative if the pollutants are complements $\left(e_{SC} < 0\right)$ or if they are substitutes with $\sigma_U > e_{SC} > 0$. Thus, we have:

Theorem 2. (Total Pollution): In the tax-tax case $(\hat{p}_{CY} > 0, \hat{p}_{SY} = 0)$, this model yields three results: (a) If leakage is negative $(\sigma_U < e_{LC})$, then total carbon emissions fall;

(b) *total carbon falls if and only if* $\beta_{CY}(\sigma_U - e_{CC}) > \alpha_Y(\sigma_U - e_{LC})$*; and*

(c) *total sulfur emissions fall if and only if* $\beta_{SY}(\sigma_U - e_{SC}) > \alpha_Y(\sigma_U - e_{LC})$.

Proof. By inspection of Eqs. (B1) and (B2).

Part (c) of Theorem 2 is interesting because the sign of $\beta_{CY}(\sigma_U - e_{SC})$ is ambiguous. Indeed, if $\beta_{CY}(\sigma_U - e_{SC}) < 0$, then it must be that $e_{SC} > 0$. Both $e_{SC} > 0$ and $e_{LC} > 0$ may hold simultaneously, however, in which case the necessary and sufficient condition in (c) may still hold. Total sulfur may fall, depending on parameters values.

Appendix C. Derivation of the welfare equation

Totally differentiate the utility function:

$$dU = \frac{\partial U}{\partial X}dX + \frac{\partial U}{\partial Y}dY + \frac{\partial U}{\partial C}dC + \frac{\partial U}{\partial S}dS_Y.$$

Insert the first-order conditions (FOCs), $\frac{\partial U}{\partial X} = \lambda p_X$ and $\frac{\partial U}{\partial Y} = \lambda p_Y$ from the household maximization problem, where λ is the Lagrange multiplier on the budget constraint (i.e. the marginal utility of income). Next, totally differentiate the production functions for *X* and *Y* and substitute the resulting *dX* and *dY* into the equation above to get:

$$dU = \lambda p_X\left(\frac{\partial X}{\partial L_X}dL_X + \frac{\partial X}{\partial C_X}dC_X + \frac{\partial X}{\partial S_X}dS_X\right) + \lambda p_Y\left(\frac{\partial Y}{\partial L_Y}dL_Y + \frac{\partial Y}{\partial C_Y}dC_Y + \frac{\partial Y}{\partial S_Y}dS_Y\right) + \frac{\partial U}{\partial C}dC + \frac{\partial U}{\partial S}dS.$$

Continuing, totally differentiate the resource constraint to find $dL_X = -\,dL_Y$ and substitute to eliminate dL_X. Next, substitute in the profit-maximizing FOCs from sector *X* (e.g.,$p_X\left(\partial X/\partial L_X\right) = p_L$) and sector *Y* (e.g., $p_Y\left(\partial Y/\partial S_Y\right) = p_{SY}$). Also, distribute p_X and p_Y across terms to find:

$$dU = \lambda\left(p_{CX}dC_X + p_{SX}dS_X\right) + \lambda\left(p_{CY}dC_Y + p_{SY}dS_Y\right) + \frac{\partial U}{\partial C}dC + \frac{\partial U}{\partial S}dS.$$

Note that the $p_L dL_Y$ terms cancel. Next, divide by λ, and substitute the expressions $dC = dC_X + dC_Y$, $dS = dS_X + dS_Y$, $\mu_C \equiv -\left(\partial U/\partial C\right)/\lambda$, and $\mu_S \equiv -\left(\partial U/\partial S\right)/\lambda$ into the previous equation to obtain:

$$\frac{dU}{\lambda} = p_{CX}dC_X + p_{SX}dS_X + p_{CY}dC_Y + p_{SY}dS_Y - \mu_C\left(dC_X + dC_Y\right) - \mu_S\left(dS_X + dS_Y\right)$$

where μ_C and μ_S are the marginal environmental damages from carbon and sulfur. Finally, divide through by total income *I*, and employ the “hat” notation, to get the welfare equation in the text.

Appendix D. Elasticity values

The U.S EPA (2009c) examines the American Clean Energy and Security Act of 2009 using many assumptions in a large complicated model to provide a long-run projection of future emission quantities and prices for both CO_2 and SO_2. Table D1 reports the emission price-quantity pairs from the EPA's analysis of HR 2454, where panel A is the projected business-as-usual scenario, and panel B makes projections for a simplified version of the proposed cap-and-trade legislation. We run a simple, linear regression to “estimate” e_{SC}, using these price and quantity projections from Table D1 Panel B as if they were data. This regression is not based on any observed behavior in response to price changes; rather, it is only meant to summarize all of the EPA assumptions in the form of our single e_{SC} parameter.

To proceed, recall that $a_{ij} = \theta_{Yj}e_{ij}$ is the partial elasticity that measures the percent change of input *i* for a one percent change in price *j* (Stern, 2011). We obtain estimates for a_{CS} and a_{SC} using a log-log setup in ordinary least squares while controlling for the other pollutant's price and output as required by the definition. The intercept is omitted because the EPA's modeling and projections are deterministic, without sampling error for the intercept to absorb. Our procedure here does not yield any statistical properties; it only summarizes the EPA's assumptions about the many unknown parameters and the chosen model structure. The estimate of a_{CS} is -0.031 and of a_{SC} is -0.443; these are then divided by θ_{YS} and θ_{YC},

Table D1
U.S. EPA's projected input-output and quantity-price pairs for SO_2 and CO_2 emissions in the electricity generating sector (2006 Dollars).

Year	Total Generation cost ($billions)	Total electricity output (TWh)	SO_2 quantity (1000 tons)	SO_2 price ($/ton)	CO_2 quantity (million metric tons)	CO_2 price ($/metric ton)
Panel A: Business-as-usual						
2012	123.8	4096	4277	283	2362	–
2015	131.5	4142	4005	255	2359	–
2020	150.9	4352	3833	308	2462	–
2025	165.7	4578	3691	415	2566	–
Panel B: HR 2454						
2012	120.7	4056	4627	130	2272	11.34
2015	121.1	3966	4119	117	2164	13.14
2020	126.5	3930	3818	142	2065	16.95
2025	138.1	4044	3523	191	2008	21.59

Note: The EPA's analysis only reports values for the four years included in the table. The sulfur dioxide quantity-price pairs come from the national Acid Rain Program's cap-and-trade regime. TWh is a terawatt-hour or 10^12 W-hours, a unit of energy.

respectively, yielding $e_{CS} = -2.45$ and $e_{SC} = -4.68$. Allen elasticities are symmetric, so we average those e_{CS} and e_{SC} to find –3.57, but we use the rounded value of -3.6 to avoid unwarranted claims of accuracy.

We also need cross-price elasticities relating the clean input (L_Y) to each pollutant (C_Y and S_Y). Considine and Larson (2006) find 0.47 for the long-run Morishima-elasticity of substitution between labor and sulfur (m_{LS}). Assuming for the moment that $e_{LC} = e_{LS}$, results in Blackorby and Russell (1989) can be used to relate the Morishima-elasticity of substitution $\left(m_{ij}\right)$ to the Allen-elasticity via the algebraic identity $m_{ij} = \theta_{Yi}\left(e_{ji} - e_{ii}\right)$. Then it can be shown for $Y(L_Y, C_Y, S_Y)$ that the cross-price Allen-elasticity e_{LS} also equals m_{LS}, when assuming $e_{LC} = e_{LS}$. Since m_{LS} was estimated to be 0.47, we use the rounded value of 0.5 for both e_{LC} and e_{LS} for our numerical example in the text.

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