Journal of Environmental Economics and Management **43,** 135–157 (2002)
doi:10.1006/jeem.2000.1169, available online at http://www.idealibrary.com on IDEAL®

Can Taxes on Cars and on Gasoline Mimic an Unavailable Tax on Emissions?[1]

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Received September 14, 1999; revised November 24, 1999; published online February 13, 2001

An emissions tax is efficient, but measurement of every car's emissions would be inaccurate and expensive. With identical consumers, we demonstrate the same efficiency for: an emissions tax; a gas tax that depends on fuel type, engine size, and pollution control equipment (*PCE*); a vehicle tax that depends on mileage; or a combination of uniform tax rates on gasoline and engine size with a subsidy to *PCE*. With heterogeneous consumers, efficiency can be obtained by a vehicle-specific gas tax or mileage-specific vehicle tax, but *not* by flat rates. We characterize second-best uniform tax rates on gasoline and on car characteristics. © 2001 Elsevier Science

Continued growth of cities, increases in vehicle-miles traveled, and Americans' renewed love for large vehicles contribute to increasing externalities from vehicle emissions, including worsened health, diminished visibility, and possible global warming. Technological advances in the measurement of car emissions renew hope that a tax can be levied directly on these emissions (Harrington *et al*. [23]). If so, individuals would reduce pollution efficiently (Pigou [34]). At least for the time being, however, the emissions taxes or permits that work well for stationary sources such as electric generating plants are not considered feasible for mobile sources of pollution. The technology is not available to measure the emissions of each vehicle in a way that is cost-effective and reliable, that is resistant to tampering by the vehicle's owner, and that satisfies legal restrictions against the search of a private vehicle. In this paper, we investigate alternative market incentives for the reduction of car pollution.

We focus on economic incentives because these kinds of policies "tend to be less costly than approaches which regulate the technology of the car or the fuel"

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0095-0696/01 $35.00

(Harrington *et al.* [23, p. 16]).[2] If a Pigovian tax were available, it would induce households to drive fewer miles, to buy fuel-efficient cars, to install pollution control equipment (*PCE*), to purchase cleaner fuel, to avoid cold start-ups, and perhaps to drive less aggressively.[3] Different households would choose different amounts of each abatement method. Thus, any efficient policy would need to induce the same behaviors. We investigate other policies that would influence people to drive fewer miles and to buy smaller cars, better pollution control equipment, and cleaner fuel.

To determine the form of these efficient policies, we model the household choice of miles, vehicle attributes, *PCE*, fuel cleanliness, and other goods. Initially, we clarify our basic framework in a model of homogeneous consumers. We then model heterogeneous consumers that differ by income and two taste parameters (one for miles and one for vehicle size). Using each model, we confirm the efficiency of the emissions tax, and we evaluate other policies that differ in terms of feasibility. For one case, we consider a tax on gasoline that can depend on the vehicle at the pump. Another alternative is a car tax that depends on the car's characteristics and mileage. Assuming those policies are not available either, we then derive conditions that characterize second-best optimal uniform tax rates on gas and on engine size, and we discuss how these rates depend on the joint distribution of tastes for miles and engine size.[4]

We wish to focus on the economic efficiency of alternative instruments in the face of heterogeneous choices about gasoline and car characteristics, but we wish to abstract from purely distributional issues. We therefore assume government can use individual-specific lump-sum taxes. We also ignore labor-leisure choices and assume government has no revenue constraint. Thus, rather than focus on Sandmo's second-best problem [36] of being unable to tax leisure, we focus on Sandmo's second-best problem [37] of being unable to tax emissions.[5]

Because vehicle emissions cannot be monitored at the source, Eskeland and Jimenez [14] analyze indirect instruments relating to cars and fuels. Eskeland [12] expands this analysis and builds a simple general equilibrium model with homoge-

[2] For an estimate of the cost savings from the use of incentive instruments rather than mandates, see Kling [28]. For a review of such studies, see Bohm and Russell [6]. Many researchers evaluate costs of current air pollution or the costs and benefits of abatement due to new vehicle and fuel technologies (Faiz *et al*. [15], Hall *et al*. [20], Kahn [25], Kazimi [27], Krupnick and Portney [30], Krupnick and Walls [31], Small and Kazimi [40]). Others focus on the effects of command and control (CAC) policies such as emissions requirements and Corporate Average Fuel Economy (CAFE) standards (Goldberg [19], Harrington [21], Kahn [26]).

[3] Because of cold start-up emissions, Burmich [8] finds that a 5-mile trip has almost three times the emissions per mile as a 20-mile trip at the same speed. Sierra Research [39] finds that a car driven aggressively has a carbon monoxide emissions rate that is almost 20 times higher than when driven normally.

[4] Optimal tax results are derived here analytically, using general functional forms. In a later paper, Fullerton and West [17] assign specific functional forms, use a large sample of households, simulate second-best policies, and compare welfare gains as a percentage of the gain from the ideal-but-unavailable Pigovian tax.

[5] See, also, Balcer [4] and Wijkander [43]. Comprehensive treatments of optimal taxation are in Auerbach [3] and Stern [41]. Sandmo [37] has no revenue requirement, but assumes that government can tax neither the dirty good nor leisure. Thus the second-best tax rates on other taxable goods depend on various cross-price elasticities. If *all* goods other than the dirty good are taxable, then Fullerton and Wolverton [17] show that the *first*-best allocation can still be attained with a tax on the endowment and a subsidy to all clean goods.

neous consumers. These papers explore optimal combinations of mandates and taxes that can mimic the unavailable emissions fee, with homogeneous consumers. Eskeland and Devarajan [13] proceed to discuss heterogeneity, and they show how combinations of policies can be used to approach the effect of a Pigovian tax. A key is whether emissions are as sensitive to a gas tax as to an emissions fee.

Other papers explore market incentives that could be used in place of the emissions tax.[6] Harrington *et al*. [22] consider the cost-effectiveness of a mandated vehicle inspection and maintenance (I/M) program compared to incentives, with uncertainty. The incentive is a fee that is based on the vehicle's emission *rate*, assuming miles are not observable. Thus, motorists can reduce their fee by repairing their vehicle, but not by driving less. Sevigny [38] incorporates the choice of miles with a second-best emissions tax, but this tax requires knowledge of each vehicle's average emissions per mile and the accurate measurement of miles traveled.[7]

Since our paper provides a general equilibrium model with heterogeneous consumers who can choose miles and car characteristics, it is most similar to the existing paper by Innes [24]. He also analyzes first-best and second-best combinations of feasible policy instruments. We state below where some of our results confirm those of Innes, but we also show how some other results differ in ways that can be attributed to assumptions employed in each model. Because of the recent increase in consumers' affinity for large vehicles, we focus on engine size as one important determinant of emissions.[8] Thus our model differs from Innes in three respects. First, we allow consumer heterogeneity in terms of two different taste parameters (for miles and for engine size). Second, we write explicit expressions for miles per gallon (MPG) and emissions per mile (EPM) that are functions of engine size and other car characteristics. Third, we derive closed-form solutions for first-best tax rates on gasoline, engine size, and PCE.

In both the homogeneous-consumer model and the heterogeneous-consumer model, we evaluate four kinds of policies. First, we confirm that a single rate of tax on all emissions would achieve a first-best allocation of resources. Assuming that is not feasible, we then find a closed-form solution for a first-best gasoline tax where the rate depends on the vehicle at the pump. This second policy may not be feasible either, if consumers can siphon gas from one vehicle to another, so we

[6] Plaut [35] compares instruments one at a time. Kohn [29] shows that any combination of a tax on emissions and subsidy to abatement are equivalent. For any such combination to be administered, however, emissions must be measurable. Train *et al*. [42] analyze "feebates," in which rebates are provided to vehicles with higher-than-average fuel efficiency and fees are levied on less efficient vehicles. These feebates are feasible incentives because fuel efficiency can be measured, but they are not perfectly efficient because they do not depend on miles driven.

[7] All of these schemes are imperfect. Emissions per mile (EPM) cannot be measured perfectly, because it depends on how the car is driven. Miles cannot be measured perfectly, because drivers can roll back the odometer. Harrington *et al*. [23] discuss remote sensing at a selection of locations as a good approximation, but some drivers may disproportionately miss or intentionally avoid those locations. Our schemes are not perfect either, as they miss some behaviors mentioned above (cold start-ups, aggressive driving).

[8] Current Clean Air Act regulations impose the same restrictions on all cars, so new-vehicle emissions vary by engine size only because of weaker regulations for trucks and sport utility vehicles (SUV). Light trucks and SUVs are "one out of every two family vehicles sold," and will be the "fastest growing source of global warming gases in the United States over the next decade" (Bradsher [7, p. 1]). In addition, actual subsequent emissions rates may vary by engine size, even within a vehicle class.

solve for a first-best vehicle tax that depends on characteristics of the vehicle and on mileage. This third policy is not enforceable if consumers can roll back their odometers. Last, we consider a three-part policy with a single rate of tax on gasoline, a single rate of tax per unit of engine size, and a single rate of subsidy to *PCE*. With homogeneous consumers this last policy achieves first best, but in the heterogeneous case the first best would require a different rate of tax for *each* consumer. Assuming such rates are not feasible, we solve for conditions that characterize the second-best uniform rates of tax on gasoline and engine size.

This model presents policymakers with complicated conditions for setting these second-best tax rates, so we then investigate easier approaches. In particular, we investigate the bias from the simple but erroneous assumption that consumers are homogeneous and that all drive the mean number of miles in the mean sized car (using the closed-form expressions for *first-best* tax rates, evaluated at mean miles and engine size). This bias depends on convexity of *MPG* and *EPM* as functions of engine size, and on the correlation in consumer preferences for miles and engine size. Preliminary evidence suggests that both *MPG* and *EPM* are convex, and that therefore the erroneous assumption of homogeneity would lead to a tax rate less than the desired second-best uniform tax on gasoline. On the other hand, preliminary evidence suggests that miles and engine size are negatively correlated and that therefore the assumption of homogeneity would lead to a tax rate that exceeds the desired second-best uniform tax on engine size.

I. HOMOGENEOUS CONSUMERS

In this section, we use a simple model of homogeneous consumers to set up our notation, to exposit the basic model, and to evaluate all four kinds of policies. In the spirit of Baumol and Oates [5], we assume perfect information, perfect competition, and no market failures other than a negative externality from emissions.[9] We also assume that each household gets its tax revenue back in a lump sum. This is a general equilibrium model, but a simple one where producer prices are fixed (and consumer prices vary with tax rates). The economy consists of n identical households, each of which owns one vehicle. Each vehicle is composed of some attributes that affect emissions (such as engine size, fuel efficiency, and *PCE*) and other attributes that do not affect emissions (such as leather seats or a sunroof). Households buy gasoline in order to drive miles, and they choose among grades of fuel-cleanliness.

Households gain utility from driving miles m, the "size" of the vehicle s, and other goods and services, x. Broadly interpreted, s represents any vehicle characteristic that gives households utility and that increases emissions per mile. More specifically, we can define s to be a measure of engine size such as cubic inches of displacement (CID). Also, consumers may gain or lose utility from pollution-control equipment, c, and per-gallon fuel cleanliness, f. PCE includes catalytic converters and other emissions-reducing equipment directly installed on a vehicle. In general, this c should reflect the condition as well as the amount of *PCE*. Fuel

[9] We ignore existing mandates in the theoretical model below, but we recognize that these mandates affect the estimated ways in which actual emissions per mile depend on engine size and other car characteristics. Thus, incentive policies may work because they encourage purchase of regulated cars.

cleanliness is an attribute of gasoline such as volatility or oxygenation.[10] As discussed more below, c or f may raise utility if the consumer enjoys providing cleaner air, or may reduce utility if the car's performance is affected. Moreover, additional c or f is more expensive. Vehicle characteristics not related to emissions are included in x. Finally, household utility is affected by aggregate auto emissions, E. Thus the household's utility function is[11]

$$u = u(m, s, c, f, x, E), \tag{1}$$

where u is continuous, differentiable, and strictly quasi-concave in its first five arguments. We also assume that the technology is homogeneous and convex, so all identical consumers choose the same point. We assume internal solutions initially but discuss corner solutions later.[12]

The emissions per mile (EPM) that a car discharges depends positively on size and negatively on PCE and the clean-fuel characteristic.[13] Thus $EPM = EPM(s, c, f)$. Since each of the n households drives m miles, aggregate emissions E can be calculated by $nmEPM(s, c, f)$. Next, fuel efficiency is measured in miles per gallon (MPG) and depends on engine size and the quantity of the clean-car good on the vehicle,[14] so $MPG = MPG(s, c)$. Cars with larger engines get lower gas mileage, so $MPG_s \equiv \partial MPG/\partial s$ is negative. The addition of a clean-car good such as a catalytic converter adds weight to a vehicle, diminishing fuel efficiency, and therefore MPG_c is also likely to be negative.[15]

Consumers do not purchase m directly, but through the combination they choose of gasoline (g), size (s), and the clean-car good (c). Gas demand is related to desired m by

$$g = \frac{m}{MPG(s, c)}. \tag{2}$$

[10] More volatile gasoline leads to more evaporative emissions. The addition of oxygenates to gasoline alters the stoichiometric air/fuel ratio. Provided the carburetor setting is unchanged, this alteration may reduce emissions of carbon monoxide (CO) and hydrocarbons (HC), but can also increase emissions of oxides of nitrogen (NOX). And, if the mixture becomes too lean (high air/fuel), HC emissions can increase due to misfiring (OECD [33]).

[11] Driver utility may also be affected by the age of the vehicle, and vintage is an important determinant of emissions. For simplicity, in this paper, we ignore vintage and thus the possibility of a subsidy to newness. Adding vintage is straightforward, and it yields a newness subsidy analogous to the size tax below (see Fullerton and West [17]).

[12] A referee points out that one could generalize this model by allowing incomes to vary while assuming that consumers have identical quasi-linear preferences (linear in the numeraire, x).

[13] Preliminary results indicate that the effect of size on EPM is important. We use data from the California Air Resources Board (CARB [9]) to estimate this relationship for three pollutants (CO, HC, NOX) using 342 cars and light-duty trucks of model years 1962 through 1995. Emissions per mile first decrease in CID, then increase, in convex U-shapes. This regression for CO is typical (standard errors in parentheses):

$$\underset{}{\ln\text{CO}} = \underset{(8.89)}{34.24} - \underset{(3.43)}{13.28}\ln\text{CID} + \underset{(.33)}{1.36}\ln\text{CID}^2, \qquad R^2 = .14,\ n = 342.$$

[14] Fuel efficiency may also be a function of the clean-fuel characteristic, f. Oxygenated fuel contains methyl tertiary butyl ether (MTBE) or ethanol, each with lower energy content per gallon than conventional gasoline. For simplicity, we do not incorporate f into MPG, but we provide an intuitive explanation of this effect when warranted.

[15] According to Dunleep [11], the addition of one cylinder decreases fuel efficiency by 3%. Also, the equipment mandated in U.S. tier 1 emissions regulations lowers fuel efficiency by 1%.

Consumers incorporate these relationships when they decide what size car and how much gasoline will maximize their utility in Eq. (1) above.

A. The Social Planner's Problem

The social planner maximizes utility of the representative household by choosing m, s, c, f, and x, recognizing that individual amounts affect aggregate emissions. The social planner is also constrained by exogenous prices and household income, y. Thus, the social planner maximizes utility in Eq. (1) subject to Eq. (2) and a resource constraint

$$u[m, s, c, f, x, nmEPM(s, c, f)] + \delta\left[y - \left(\frac{p_g + p_f f}{MPG(s, c)}\right)m - p_s s - p_c c - x\right] \tag{3}$$

with respect to m, s, c, f, and x. The price per gallon of gas without any clean characteristic is p_g, and the price per unit of the clean-fuel characteristic per gallon is p_f. The total price of a gallon of gasoline is $(p_g + p_f f)$, and the private cost of driving a mile is $(p_g + p_f f)/MPG(s, c)$. The price of s is p_s, which represents the price of adding a cubic inch of displacement to an engine. The price per unit of the clean-car good is p_c, and the price of x is normalized to one.

This maximization yields first-order conditions in Eqs. (4), shown in Table I. Subscripts on u, EPM, and MPG indicate partial derivatives ($u_m = \partial u/\partial m$), and δ is the marginal social value of income. The quantity in brackets in (4a) is the total implicit price of a mile, while the quantity in brackets in (4b) is the overall cost per unit of size, including the extra amount that must be paid for miles due to the lower MPG caused by the incremental unit of s. Similarly, the quantity in brackets in (4c) is the overall cost of PCE, including the extra amount that must be paid for miles due to the lower MPG. In (4d), the term in brackets is the overall cost per unit of the clean-fuel characteristic.

These first-order conditions say that the marginal social gain from driving another mile, or from an additional unit of s, c, f, or x, is equal to the marginal social cost of each. The u_E term on the left-hand sides of (4a)–(4d) reflects the effect on utility of the increment to aggregate emissions from driving an additional mile, increasing vehicle size, adding PCE, or cleaner gas.

B. The Household Problem

In contrast to the social planner, a household does not recognize that its own choices affect aggregate emissions.[16] However, it may face taxes or subsidies on its consumption of s, c, f, x, and g. If it were available, a tax on emissions would enter the budget constraint. The household optimization problem is to choose m,

[16] Technically, we could say that each household recognizes only its own contribution to aggregate emissions, but that effect becomes nil as n becomes large. The key distinction is that only the social planner recognizes how the individual's decision is multiplied by n to determine aggregate emissions.

TABLE I
First-Order Conditions with Homogeneous Consumers

Equations (4) from the Social Planner's Problem

$$u_m + u_E nEPM(s,c,f) = \delta\left[\frac{p_g + p_f f}{MPG(s,c)}\right] \tag{4a}$$

$$u_s + u_E nmEPM_s = \delta\left[p_s + m\left(\frac{-(p_g + p_f f)MPG_s}{MPG(s,c)^2}\right)\right] \tag{4b}$$

$$u_c + u_E nmEPM_c = \delta\left[p_c + m\left(\frac{-(p_g + p_f f)MPG_c}{MPG(s,c)^2}\right)\right] \tag{4c}$$

$$u_f + u_E nmEPM_f = \delta\left[\frac{p_f m}{MPG(s,c)}\right] \tag{4d}$$

$$u_x = \delta \tag{4e}$$

Equations (6) from the Household Problem

$$u_m = \lambda\left[\left(\frac{p_g + t_g + (p_f + t_f)f}{MPG(s,c)}\right) + t_e EPM(s,c,f)\right] \tag{6a}$$

$$u_s = \lambda\left[p_s + t_s + m\left(\frac{-(p_g + t_g + (p_f + t_f)f)MPG_s}{MPG(s,c)^2}\right) + t_e EPM_s m\right] \tag{6b}$$

$$u_c = \lambda\left[p_c + t_c + m\left(\frac{-(p_g + t_g + (p_f + t_f)f)MPG_c}{MPG(s,c)^2}\right) + t_e EPM_c m\right] \tag{6c}$$

$$u_f = \lambda\left[\frac{(p_f + t_f)m}{MPG(s,c)} + t_e EPM_c m\right] \tag{6d}$$

$$u_x = \lambda[1 + t_x] \tag{6e}$$

s, c, f, and x to maximize

$$u(m,s,c,f,x,E) + \lambda\left[y - \left(\frac{(p_g + t_g + (p_f + t_f)f)}{MPG(s,c)}\right)m - (p_s + t_s)s - (p_c + t_c)c - (1 + t_x)x - t_e EPM(s,c,f)m\right]. \tag{5}$$

In this budget constraint, y is income including the lump sum tax rebate, t_g is the tax per gallon of gas, t_f is the tax per unit of clean-fuel characteristic, t_s is the tax per unit of size, t_c is the tax per unit of PCE, t_x is the tax per unit of x, and t_e is the tax per unit of emissions.

The first-order conditions for this problem are Eqs. (6), also shown in Table I. Emissions can be made to enter the consumer problem implicitly through the pollution tax t_e. The price per mile would then include the emissions tax per mile. Similarly, the implicit prices of s, c, and f include the emissions tax associated with the change in emissions.

C. Solutions

1. *The Pigovian Tax.* The tax on emissions, t_e, provides the basic efficient policy against which alternatives can be compared. Suppose all other tax rates are set to zero ($t_g = t_f = t_s = t_c = t_x = 0$). In this case, (4e) and (6e) imply $\lambda = \delta$. To equate the first order conditions for miles, (4a) and (6a), replace $\lambda = \delta$ into (4a) and subtract it from (6a) to obtain

$$t_e = \frac{-u_E n}{\lambda} \equiv MED. \tag{7}$$

We define the right-hand side as "marginal environmental damages" (MED) per unit of emissions. It is the sum of all n households' disutilities from emissions, u_E, translated into money terms when divided by the marginal utility of income. This is the usual Pigovian tax, and it is greater than zero so long as $u_E < 0$. Using this value of t_e, and $\lambda = \delta$, then the first order condition (4b) matches (6b), (4c) matches (6c), and (4d) matches (6d).

Thus the Pigovian tax on emissions by itself induces households to make all the optimal choices about miles, car size, fuel, and pollution control equipment.

2. *A Complicated Gas Tax.* If the measurement of emissions were impossible, so that $t_e = 0$, we can find a different policy that attains the exact same efficient outcome. This policy is a complicated gas tax that depends upon fuel characteristics, f, and on vehicle characteristics, s and c. This tax is

$$t_g = \frac{-u_E n}{\lambda} EPM(s, c, f) MPG(s, c). \tag{8}$$

This tax represents the additional damage caused by an increase of one gallon of gas. It is the damage per unit of emissions (MED) times emissions per mile (EPM) times miles per gallon (MPG). If we substitute (8) into t_g in (5) before differentiation, then additional terms in the first order conditions involve derivatives of EPM and MPG with respect to s, c, and f. These first order conditions match the social optimum in (4). Thus, this policy attains the first best without a separate tax on size or subsidy to fuel cleanliness or PCE (Innes [24]).

A useful intuition is that emissions are determined by $nmEPM(s, c, f)$, where m is determined by g, so the optimality of an emissions tax can be replicated perfectly by any policy that can correctly influence every emission determinant (s, c, f, and g). In addition, each household must *know* how its gas tax is affected by its own choice of s, c, and f.

It seems reasonable for a gas tax to depend on the fuel characteristic, f. But to achieve the first best, the gas tax must also depend on characteristics of the vehicle at the pump (s and c). A gas tax that is fixed to reflect the average vehicle will not influence each household to modify its vehicle. However, the complicated gas tax would be costly to administer.[17]

3. *A Complicated Vehicle Tax*. If the gas tax cannot depend on characteristics of the vehicle, the efficient outcome can still be attained by a vehicle tax that depends on mileage. Suppose that the consumer's budget constraint in (5) is modified to subtract a tax t_v per vehicle (still assuming that each household owns one vehicle). Suppose all other tax rates are zero, and the vehicle tax is

$$t_v = \frac{-u_E n}{\lambda} mEPM(s,c,f). \tag{9}$$

To set this tax for each car, authorities must know the car's characteristics (s and c), and its mileage (m). If drivers know how their tax depends on these choices, they can be influenced in these choices.[18] Substitution of (9) into (5) before differentiation yields all the social planner's first order conditions in (4). Note, however, that this solution is essentially an emissions tax! Authorities know the car's characteristics and mileage, so (9) just calculates the car's emissions [$mEPM(s,c,f)$] and multiplies by the Pigovian tax rate [$-u_E n/\lambda$]. Like an emissions tax, this vehicle tax may not be feasible. It requires much information, and it can be circumvented by drivers who roll back their odometers.

4. *Separate Fixed Tax Rates*. We now suppose that none of the policies above are available, but that government can set separate tax rates on gasoline, engine size, and *PCE*. We assume that the gas tax can be made to depend on characteristics of the fuel, f, but not characteristics of the car. Similarly, neither the size tax nor PCE subsidy can depend on miles or other choices.

As it turns out, the gas tax looks exactly like (8) above, but it does not vary with s or c. In other words, the authorities must calculate the fixed rate of tax from (8) using the optimal values of s and c, but that rate does not vary with one's own choices. Then the size tax is

$$t_s = \frac{-u_E n}{\lambda} m\left[EPM_s + EPM(s,c,f)\frac{MPG_s}{MPG(s,c)}\right]. \tag{10}$$

The size tax has two components. The first term is $-u_E n/\lambda$ (that is, *MED*) times the change in emissions per mile from a change in size (EPM_s), times miles (m). This gives the direct damage caused by an increase of one unit of size. This term is positive as long as emissions affect utility negatively ($u_E < 0$) and size affects emissions positively ($EPM_s > 0$). The second term is an indirect effect from an

[17] "For example, a tamper-resistant computer code would likely be required on each automobile; similarly, gasoline pumps would have to be equipped to automatically tack the appropriate tax onto any gasoline that is dispensed to a particular automobile. Moreover, since a simple siphoning of gas will permit consumers to bypass taxes on high-emission vehicles, the scope for abuse, particularly among those high-emitting consumers who are arguably the most important targets of the tax, would be tremendous" (Innes [24, p. 226]).

[18] It would be hard for this vehicle tax to vary with f, to induce the right choice of fuel, but that one margin could be influenced separately by a subsidy to fuel producers to make cleaner fuel or a mandate to make cleaner fuel.

additional unit of size through its effect on fuel efficiency.[19] As long as $MPG_s < 0$, this term is negative and is thus a rebate. Specifically, it is a rebate of part of the gas tax in (8). Because an additional unit of size decreases fuel efficiency, the household knows that an increase in the size of its vehicle's engine will cost additional gas tax. Thus part of the external cost of size is already internalized by the gas tax.[20]

Because the two components of the size tax are opposite in sign, this theory does not predict the sign of t_s. Since the right-hand term before the brackets is positive, the sign of t_s is determined by the sign within the brackets. Thus the size tax is positive whenever

$$\frac{EPM_s}{EPM(s,c,f)} > \frac{-MPG_s}{MPG(s,c)}. \tag{11}$$

These two terms are proportional effects of size on emissions per mile (EPM) and on miles per gallon (MPG). When an additional unit of size brings about a larger percentage change in emissions per mile than in fuel efficiency, the size tax is positive. If fuel efficiency deteriorates proportionally more than emissions increase, then size is subsidized! In this latter case, the gasoline tax more than completely internalizes the impact of size on emissions. Empirical exploration of the relative effects in (11) will uncover the sign of the size tax. Then t_c is

$$t_c = \frac{-u_E n}{\lambda} EPM_c m + \frac{-u_E n}{\lambda} EPM(s,c,f) m \frac{MPG_c}{MPG(s,c)}. \tag{12}$$

This tax is perfectly analogous to the size tax. The first term is negative to reflect the effect on damages of an added unit of *PCE*. The second term is a rebate due to the effect that *PCE* has on fuel efficiency (already internalized by the gas tax). Since the second term is also negative, the sign of the clean-car tax is always negative. That is, t_c is necessarily a subsidy.[21]

All four policies in the homogeneous-consumer model induce households to make socially optimal trade-offs at the margin, so they are valid only for internal solutions. A more complete analysis is required to deal with corner solutions.[22] If households dislike pollution control equipment enough ($u_c \ll 0$), then the subsidy within the gas tax or in (12) may not induce them to buy any of it. In this case, the

[19] Both of these terms contain m, the "baseline" number of miles. Authorities calculate the fixed t_s from (10) using the optimum m, but this rate does not vary with the individual's own choice of miles. Of course, in this model with homogeneous consumers, all households drive the same type of vehicle the same number of miles per year, and the size tax is the same for everyone. When we introduce heterogeneity in Section II, the first-best solution requires that each household pay a size tax that reflects its own choice of miles.

[20] This rebate also appears in Innes' second-best vehicle tax, which equals the "predicted social costs of emissions, less the portion of these costs that are internalized by the gasoline tax" (Innes [24, p. 222]).

[21] If c measures the amount of *PCE* installed, this subsidy could be paid upon purchase of the vehicle. More generally, if c reflects the condition of the equipment as well as the amount, then this subsidy could reward testing, maintenance, and repair of *PCE*.

[22] We derived Kuhn–Tucker conditions from a model with non-negativity constraints on clean-car and clean-fuel characteristics. The results include Eqs. (8), (10), and (12) for internal solutions and an inequality for each corner solution. The additional intuition is minimal, however, so we just outline these results in the text.

corner solution with $c = 0$ is indeed part of the social optimum, even though the marginal conditions (6) are not satisfied. If households care nothing about this equipment, however, then a different problem arises: when $u_c = 0$, the right-hand side of first order condition (6c) must equal zero at the optimum. Since $t_e = 0$, the subsidy to *PCE* (either within the gas tax or t_c) can only induce consumers to buy any such equipment if it is equal to the *entire private cost of PCE*, including both the direct cost, p_c, and the extra fuel costs incurred due to the negative effect that *PCE* has on fuel efficiency. With a 100% subsidy, however, the choice of c is indeterminate. Thus, if $u_c = 0$, then incentives do not work. The optimal *PCE* can only be achieved by a mandate (as in Innes [24]).

The same analysis applies to the clean-fuel characteristic. When $u_f = 0$, the right-hand side of (6d) must equal zero at the optimum. For households to choose cleaner gas, the subsidy within the gas tax must equal the entire cost of the attribute, p_f.

We think that u_c and u_f are unlikely to be exactly zero.[23] In fact, these *marginal* utilities are likely to fall with the amount of c or f. Even if u_c is negative, a big enough subsidy can induce the household to buy more of this good, until u_c on the left-hand side of (6c) falls to the level of the (negative) private marginal cost on the right-hand side of (6c).

II. HETEROGENEOUS CONSUMERS

The tax rates derived in the previous section are uniform across all consumers. In this section, we introduce heterogeneity to see if and when the optimal tax rates need to differ among consumers. If the emissions tax t_e were available, we confirm that a single $t_e = MED$ would achieve the first-best social optimum. If not, then individual-specific tax rates on other emissions-related goods can still achieve the first-best. If policy is unable to apply individual-specific tax rates, then it cannot achieve the first-best. We then characterize the second-best uniform tax rates that best approximate the unavailable tax on emissions.

Whereas Innes [24] allows households to differ in terms of income and one taste parameter, we use the parameter α to represent the household's preference for miles and β to represent the preference for size of the car. Together with income, these parameters are jointly distributed according to the distribution function $h(\alpha, \beta, y)$ with positive support on $[\underline{\alpha}, \overline{\alpha}] \times [\underline{\beta}, \overline{\beta}] \times [\underline{y}, \overline{y}]$. The integral of this distribution over α, β, and y is the population, n. In a CES or Cobb–Douglas specification of utility, for example, α could be the weight on miles, β would be the weight on size, and $(1 - \alpha - \beta)$ would be the weight on x. Our analysis is not

[23] Individuals may get positive utility from using the latest technologies, or negative utility from inconvenience or decreased performance. Couton *et al.* [10] use data from France, where some vehicles do not have catalytic converters, to estimate a hedonic regression of vehicle price on vehicle characteristics. They find that a catalytic converter increases price by 8%. "This hedonic price was increasing significantly over the period, which would imply an increased demand of the equipment" [10, p. 437]. The OECD [33] finds that "oxygenated fuels perform better than hydrocarbon-only fuels; [t]hey give better antiknock performance" and "at high altitudes and in hot weather give . . . better handling performance compared with wholly hydrocarbon fuels" [33, p. 60]. Also, Marell *et al.* [32] find that environmental concern plays a role in the decision to replace an automobile.

limited to these special cases, however, and it is not limited by any particular relationship between α and β. Those who live far from their place of work have a high demand for miles (α), but they may prefer either a small car (for better gas mileage) or a large car (for comfort and safety). We show the importance of the *correlation* between α and β.

To focus on the issue of heterogeneity, we now ignore the clean-car and clean-fuel characteristics. Thus fuel efficiency and emissions per mile depend only on size, and each household generates $mEPM(s)$ units of emissions. Aggregate pollution is thus

$$E = \int_{\alpha} \int_{\beta} \int_{y} mEPM(s) h(\alpha, \beta, y) \, \partial\alpha \, \partial\beta \, \partial y, \tag{13}$$

where choices of m and s are individual-specific. A household's utility function is

$$U = u[m, s, x; \alpha, \beta] - \mu E, \tag{14}$$

where μ is the household's change in welfare from additional pollution ($\partial U/\partial E$). While we allow α and β to differ among households, to analyze different choices and abatement costs, we are not concerned with differential benefits from environmental protection (μ).[24]

A. The Social Planner's Problem

The social planner must maximize a measure of social welfare such as a weighted sum of n households' utilities. To set up this problem, we must specify weights that meet three criteria. First, we choose weights so that a dollar given to any household has the same effect on social welfare. To achieve this condition, we divide each household's utility by its own marginal utility of income (λ).[25] Second, when t_e is available, we want the maximization of our social welfare function to yield the solution of Pigou [34]. Since this solution is based on marginal conditions (such as marginal environmental damages) at the optimum, we use the values for λ that occur at the first-best social optimum (λ^*). Third, when first-best instruments are not available, we want to be able to find second-best uniform tax rates that maximize the same social welfare function. Therefore we use prices at the Pigovian equilibrium to evaluate λ^*, and we use those λ^* to get the weights ($1/\lambda^*$) for *all* subsequent evaluations of other policies. The result is a money-metric measure of social welfare.

[24] This change restricts utility to a quasi-linear form—linear in emissions. If we kept the more general form, then first order conditions and first-best tax rates below would include complicated-looking integrals over all disutilities, u_E, instead of just $n\mu$, but all else would remain the same.

[25] To avoid redistributions in the tax rate problem below, as in the homogeneous consumer model, we assume that each individual's tax revenues are returned in a lump sum to the same individual.

The social planner's problem is to maximize this welfare function subject to a resource constraint (the integral over all individual budget constraints),

$$\int_{\alpha}\int_{\beta}\int_{y}\left[\frac{u[m,s,x]}{\lambda^*} - \mu\int_{\alpha}\int_{\beta}\int_{y}[mEPM(s)]h(\alpha,\beta,y)\,\partial\alpha\,\partial\beta\,\partial y\right]h(\alpha,\beta,y)\,\partial\alpha\,\partial\beta\,\partial y \quad (15)$$
$$+\,\delta\left[\int_{\alpha}\int_{\beta}\int_{y}\left[y - \frac{p_g}{MPG(s)}m - p_s s - x\right]h(\alpha,\beta,y)\,\partial\alpha\,\partial\beta\,\partial y\right]$$

with respect to each consumer's m, s, and x (given their individual α, β, and y). Income plus tax rebates is y, and the marginal social value of income is δ. To maximize (15), we can ignore the outer integral to obtain the individual marginal conditions and then incorporate the impact an individual's choice of miles and size has on emissions by differentiating the aggregate emissions term with respect to the individual's m and s.

The resulting first-order conditions for household i are Eqs. (16), shown in the top of Table II. The first term in each equation represents the individual's money value of marginal utility from each good. The second term in (16a) represents the

TABLE II

First-Order Conditions with Heterogeneous Consumers

Equations (16) from the Social Planner's Problem[a]

$$\frac{\left(\frac{\partial u_i}{\partial m_i}\right)}{\lambda_i^*} - n\mu EPM(s_i) = \delta\left[\frac{p_g}{MPG(s_i)}\right] \quad (16a)$$

$$\frac{\left(\frac{\partial u_i}{\partial s_i}\right)}{\lambda_i^*} - n\mu m_i EPM_{si} = \delta\left[p_s + m_i\left(\frac{-p_g MPG_{si}}{MPG(s_i)^2}\right)\right] \quad (16b)$$

$$\frac{\left(\frac{\partial u_i}{\partial x_i}\right)}{\lambda_i^*} = \delta \quad (16c)$$

Equations (18) from the Household Problem[a]

$$\frac{\partial u_i}{\partial m_i} = \lambda_i^*\left[\left(\frac{p_g + t_g}{MPG(s_i)}\right) + t_e EPM(s_i)\right] \quad (18a)$$

$$\frac{\partial u_i}{\partial s_i} = \lambda_i^*\left[p_s + t_s + m_i\left(\frac{-(p_g + t_g)MPG_{si}}{MPG(s_i)^2}\right) + t_e EPM_{si} m_i\right] \quad (18b)$$

$$\frac{\partial u_i}{\partial x_i} = \lambda_i^*[1 + t_x] \quad (18c)$$

[a] These equations represent n first-order conditions, one for each individual i.

external cost of an additional mile driven by individual i. Similarly, the second term in (16b) represents the external cost of an additional unit of size purchased by individual i. The first-order conditions (16) say that the money-metric social marginal utility of each good equals the social marginal cost of that good. Also, looking at (16c), note that the left-hand side is individual i's change in utility from an additional unit of x, divided by the marginal utility of income. In other words, it is the dollar value of another unit of x (the price of x). Since the price of x equals one, (16c) says that the social marginal utility of income, δ, also equals one.

B. The Household Problem

In contrast to the social planner, a household does not recognize that its own emissions add to aggregate emissions. The household problem is to maximize

$$u_i(m_i, s_i, x_i) - \mu E + \lambda_i \left[y_i - \left(\frac{(p_g + t_g)}{MPG(s_i)} \right) m_i - (p_s + t_s) s_i - (1 + t_x) x_i - t_e EPM(s_i) m_i \right] \tag{17}$$

with respect to m_i, s_i, and x_i. The first-order conditions are shown in Eqs. (18) of Table II. These equations are heterogeneous counterparts to the first-order conditions (6) of Table I, but without the clean-car and clean-fuel characteristics. In addition, each consumer has a separate set of optimality conditions.

C. Solutions

1. *The Pigovian Tax.* To solve for a Pigovian tax, set all taxes except t_e equal to zero ($t_s = t_g = t_x = 0$). Then, using $\delta = 1$, (16c) and (18c) match each other. Also using $\delta = 1$, set (16a) and (18a) equal to each other. The household-specific variables drop out, leaving

$$t_e = \frac{n\mu}{\delta} \equiv MED. \tag{19}$$

Using $\delta = 1$ and this value of t_e, then (16b) and (18b) also match each other. Thus, given the weights we have chosen, a uniform Pigovian tax on emissions by itself induces all households, no matter their tastes for miles and size, to drive the optimal number of miles in the right-sized cars. Of course, policymakers do not necessarily weight households so that income to one is the same as income to another. In this paper, however, we weight households simply in a way that implies that the maximization of social welfare yields the standard Pigovian formula (19). This first-best uniform Pigovian tax can be used as a benchmark to identify other first-best policies, and more importantly, against which to evaluate other *second-best* policies.

2. *A Complicated Gas Tax.* As in the representative-agent model, when a Pigovian tax is not possible ($t_e = 0$), we can derive a first-best gas tax. In the heterogeneous-consumer model, this tax is

$$t_{gi} = n\mu EPM(s_i) MPG(s_i). \tag{20}$$

This formula is conceptually the same as in the homogeneous-consumer model (except that we dropped the c and f). As in that prior model, a tax rate per gallon of gasoline that depends on the individual's own choice of car characteristic (s_i) can optimally influence the determinants of emissions. The only difference here is that heterogeneous consumers then optimally choose different car sizes and mileage. Thus each pays a different rate per gallon.

3. *A Complicated Vehicle Tax*. Authorities might be able to impose a tax on each vehicle that depends on a direct measure of the emissions rate (EPM), or on the determinants of emissions (s_i), and multiply by a measure of mileage,

$$t_{vi} = n\mu EPM(s_i) \cdot m_i. \tag{21}$$

This formula again matches that of the homogeneous-consumer model, and it again achieves first best, but in this case the tax amount would differ among heterogeneous households. This policy would not achieve first best in a model with other less-measurable determinants of emissions (like cold start-ups and aggressive driving). Even the measure of miles is problematic, as annual odometer readings would provide incentive for individuals to roll back their odometers.[26]

4. *Separate Fixed Tax Rates*. Suppose now that the gas tax and size tax can be set at different rates for different consumers, but that they must be fixed for each consumer. The gas tax cannot vary directly with one's own choice of vehicle, and the vehicle tax cannot vary directly with miles driven. We use the first-order conditions in Table II to derive a gas tax that looks just like (20) and a size tax:

$$t_{si} = n\mu EPM_{si} m_i + \frac{n\mu EPM(s_i)\, MPG_{si} m_i}{MPG(s_i)}. \tag{22}$$

Authorities could use (20) and (22) together to fix each household's tax rates based on that household's own optimal size (s_i) and miles (m_i). These tax rates achieve first best, but the information requirements are enormous. Because these first-best rates must be individual-specific, any set of rates that are uniform across all consumers cannot achieve first best.

Thus, we find that heterogeneity matters. Suppose that the first three policies above are not feasible, and policy is limited to a single uniform rate of tax on gasoline and single uniform rate of tax on engine size (or other vehicle characteristic). This policy achieves first best in the homogeneous-consumer model, but not in the heterogeneous-consumer model. Moreover, a greater degree of heterogeneity means greater divergence from first best.

For these reasons, we now consider how to set the second-best uniform tax rates on gasoline and engine size. One possibility is that these rates could be calculated from (20) and (22) using the mean size and miles. How well these uniform tax rates would perform depends on the technological relationships $EPM(s)$ and $MPG(s)$ and on the relationship in preferences between size and miles. In the next section, we find conditions that characterize second-best uniform tax rates, and we compare them to the rates using means in (20) and (22).

[26] "Even if only a small proportion of consumers cheat in this way, those who cheat are likely to be those who drive the most, who therefore have the greatest incentive to cheat and who are arguably the most important targets of mileage taxation" (Innes [24, pp. 226–227]).

III. SECOND-BEST TAXES ON GASOLINE AND SIZE

To find the second-best tax rates, we must find the single (uniform) gas tax rate and size tax rate that maximize social welfare, taking as given households' demand behavior for miles, size, and other goods and services.[27] Assuming producer prices are fixed, this is equivalent to maximizing this weighted sum of indirect utilities,

$$\int_\alpha \int_\beta \int_y \left[\frac{V(t_s, t_g, t_x; y, \alpha, \beta)}{\lambda^*} - \mu E \right] h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y, \qquad (23)$$

with respect to t_s and t_g. As a normalization, the tax on x can be set to zero, as in the first-best scenario.[28] Using Roy's Identity, this maximization results in the first order conditions,

$$\int_\alpha \int_\beta \int_y \left[\frac{-\lambda s}{\lambda^*} - \mu \int_\alpha \int_\beta \int_y [A(t_s)] h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y \right] h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y = 0 \qquad (24a)$$

$$\int_\alpha \int_\beta \int_y \left[\frac{-\lambda g}{\lambda^*} - \mu \int_\alpha \int_\beta \int_y [A(t_g)] h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y \right] h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y = 0 \qquad (24b)$$

where (for $j = s, g$)[29]

$$A(t_j) \equiv gMPG(s)EPM_s \frac{\partial s}{\partial t_j} + gEPM(s)MPG_s \frac{\partial s}{\partial t_j} + EPM(s)MPG(s) \frac{\partial g}{\partial t_j}. \qquad (25)$$

The chosen quantities (s, g, and x) as well as the marginal utility of income (λ) are functions of α and β. In (24a), the first term in the integral ($-\lambda s/\lambda^*$) represents the change in welfare from a change in the size tax, holding aggregate emissions constant. The second term, involving $A(t_s)$, is the change in utility due to the effect that a size tax has on aggregate emissions.[30] Similarly, the first term in (24b) is the change in welfare from a change in the gas tax, holding aggregate emissions constant. The second term incorporates the change in welfare from the effect that the gas tax has on aggregate emissions.

[27] For the sake of clarity, here we consider linear second-best size tax rates. Perhaps policymakers could assess nonlinear size tax rates fairly easily. The use of nonlinear schedules could incorporate heterogeneity by accounting for convexity or concavity of $EPM(s)$ and $MPG(s)$, but not for the possible correlation between size and miles.

[28] Income, y, is exogenous, so a (lump-sum) tax on income t_y is equivalent to a tax on all commodities at the same rate; any set of (t_s, t_g, t_x) can be scaled up or down, with commensurate changes in t_y. Thus any one rate can be set to zero (see Fullerton [16]).

[29] Actually, each $A(t_j)$ also depends on the other tax rate (that is, both t_s and t_g), to the extent that $\partial s/\partial t_j$ or $\partial g/\partial t_j$ behaviors depend on the level of the other tax rate.

[30] When an emissions tax achieves the first-best, where $\lambda = \lambda^*$, then (24a) says that the cost to the taxpayer of an increase in t_s is the amount of s purchased, and that this marginal cost should be equal to the marginal benefits in terms of reduced emissions.

Thus the tax rates on size and gasoline should each be set so that the aggregate marginal gain in private welfare equals the aggregate marginal loss from the effect on emissions. As shown in the $A(t_j)$ term of each first order condition, the extent to which emissions are reduced depends on the degree of responsiveness of miles and size to taxes on size and gasoline. Thus second-best optimal tax rates on size and gasoline depend on the elasticities of demand for these goods. But the way in which changes in size affect emissions is through the technological relationships that size has with emissions per mile and fuel efficiency. The functions $EPM(s)$ and $MPG(s)$ are therefore major determinants of the second-best tax rates.

These first order conditions cannot be used to solve for the second-best uniform tax rates on size and gasoline. To find closed-form solutions we would have to specify the functional forms of $h(\alpha, \beta, y)$, $EPM(s)$, $MPG(s)$ and the demands for size, miles, and other goods and services. Still, these first order conditions can be used to shed some light on the nature of such taxes. Instead of trying to raise revenue efficiently (Sandmo [36]), these tax rates are trying to tax something that approximates emissions (Sandmo [37]). In particular, if consumers with a high preference for miles (high α) also happen to have a high preference for size (high β), then that correlation is likely to affect the second-best optimal t_g and t_s. In addition, preferences for size determine emissions through the relationship size has with MPG and EPM.

Furthermore, first-order conditions (24) do not provide clear guidance about how to set uniform tax rates in the face of heterogeneity. Closed-form solutions for the second-best tax rates are not available, but two alternatives come to mind. First, we can calculate the "expected value" or weighted average of the first-best individual-specific tax rates in Eqs. (20) and (22). These average rates might then be applied uniformly to all individuals. These rates are not the same as the second-best rates from (24), but at least they incorporate information about the distribution $h(\alpha, \beta, y)$ of heterogeneous individuals. Second, policymakers might simply ignore heterogeneity, and just use the economy-wide means for miles and size as if all individuals were the same. A comparison of these two alternatives will not tell us how near or far we are from true second best rates, but it will reveal something about the direction of the bias.[31]

[31] We recognize that comparing a rate evaluated at the averages with the "average of the rates" is not the same as comparing it with the second-best uniform rate. By inspection of (24), however, nonlinearity can be seen to affect second-best rates in the same direction as it affects average rates. Consider, for example, the effect of convexity on the second-best gas tax rate in (24b), using (25). If either $EPM(s)$ or $MPG(s)$ is convex, or both, then the third term in (25) increases with size at an increasing rate; households with large cars emit a disproportionately high amount. The effect of the gas tax on the gasoline consumption and emissions of these households is higher than for those with small cars. Households with larger cars thus have larger $A(t_g)$ terms, and larger social marginal costs of emissions in (24b). The second-best uniform gas tax rate increases to reflect these costs. As we shall see below, convexity has the same effect on the average of the individual-specific gas tax rates. The bias of rates evaluated at the averages is thus in the same direction relative to average rates as relative to second-best rates.

The average of all different gas tax rates in (20) is

$$\bar{t}_g = \frac{\int_\alpha \int_\beta \int_y n\mu EPM(s_i)MPG(s_i)h(\alpha, \beta, y)\,\partial\alpha\,\partial\beta\,\partial y}{\int_\alpha \int_\beta \int_y h(\alpha, \beta, y)\,\partial\alpha\,\partial\beta\,\partial y} \tag{26}$$
$$= \int_\alpha \int_\beta \int_y \mu EPM(s_i)MPG(s_i)h(\alpha, \beta, y)\,\partial\alpha\,\partial\beta\,\partial y.$$

We ask how this concept differs from the simple calculation of the gas tax rate for the person with average choices:

$$t_g(\bar{s}) = n\mu EPM(\bar{s})MPG(\bar{s}). \tag{27}$$

Specifically, we want to identify the circumstances under which the average of the gas tax in (26) is greater than the gas tax rate for the person with average choices in (27). We can thus discover the conditions under which uniform taxes based on average choices would likely fall short of attaining the second-best emissions reduction.

Since miles (m) do not appear in these two equations, whether (26) exceeds (27) does not depend on correlation between size and miles. It does depend on the characteristics of $EPM(s)$ and $MPG(s)$. Convexity of $EPM(s)$, for example, would mean that increases in size increase emissions per mile at an increasing rate. This would raise the weighted average using $EPM(s_i)$ in (26) relative to the tax rate using average size in (27). Convexity in $MPG(s)$ also raises (26) relative to (27). So, if either function is sufficiently convex (or if both are convex), then the use of average size to calculate the gas tax rate would result in a lower tax rate than (would understate) the second-best uniform tax rate. Conversely, if either $EPM(s)$ or $MPG(s)$ is sufficiently concave, then using the average value of size to calculate the gas tax rate would result in a higher rate than (would overstate) the second-best uniform tax rate.

To determine the likely magnitude of (26) relative to (27), we need estimates of the possible nonlinear effect of engine size on fuel efficiency and emissions. While it is widely known that fuel efficiency decreases with engine size, a literature search locates no statistical estimates of the nonlinear nature of this relationship. Nor could we find any estimates of the effect of size on emissions per mile.[32] For these reasons, we use the CARB data to estimate *EPM* and *MPG* as polynomial functions of engine size. These regressions omit other explanatory variables in order to capture the full effect of size, the only taxed characteristic. These very preliminary results suggest that *EPM* is increasing over most of the range of size

[32] Dunleep [11] provides only rough estimates of the effect of size on *MPG*. Kahn [26] examines the effect of size on emissions in parts per million rather than on emissions per mile.

and is convex, while *MPG* is decreasing and also convex.[33] Thus the use of average car size would likely *understate* the second-best optimal gas tax.[34]

Now consider the individual-specific size tax rate in (22). We want to reveal the circumstances under which the average of the size tax rate,

$$\begin{aligned}\bar{t}_s = &\int_\alpha \int_\beta \int_y \mu EPM_{si} m_i h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y \\ &+ \int_\alpha \int_\beta \int_y \frac{\mu EPM(s_i)\, MPG_{si} m_i}{MPG(s_i)} h(\alpha, \beta, y)\, \partial\alpha\, \partial\beta\, \partial y \end{aligned} \tag{28}$$

is larger than the size tax for the person with the average choices,

$$t_s(\bar{s}, \overline{m}) = n\mu EPM_{\bar{s}} \overline{m} + \frac{n\mu EPM(\bar{s})\, MPG_{\bar{s}} \overline{m}}{MPG(\bar{s})}. \tag{29}$$

Since both s and m appear in both equations, the difference between the average size tax rate (28) and the size tax rate using average miles and size (in (29)) depends both on whether preferences are correlated and on whether $EPM(s)$ or $MPG(s)$ is nonlinear. To focus on the effects of correlation, assume for now that these functions are linear (so EPM_s and MPG_s are constants). Correlation, then, does not affect the first term in either (28) or (29), but it does affect the second term (which is negative, since $MPG_s < 0$). Suppose that size and miles are negatively correlated, so $EPM(s)$ and m are negatively correlated. Then the subtracted second term in (28) is disproportionately smaller than in (29), which increases (28) relative to (29). Thus, to the extent that those who own larger cars drive proportionately fewer miles, then the use of the average person's size tax tends to *understate* the second-best size tax.

We use the 1994 Residential Transportation Energy Consumption Survey (RTECS) to find preliminary evidence of a very small but statistically significant negative correlation.[35]

Now consider the effects of nonlinearity, assuming no correlation. Regardless of the effects of convexity or concavity on the first terms in (28) and (29), we can show

[33] Typical results for regressions of *EPM* on size are shown in footnote 22. The following regression indicates that *MPG* is decreasing and also convex in engine size measured by CID (standard errors in parentheses):

$$\underset{}{MPG} = \underset{(.88)}{35.41} - \underset{(.0085)}{.106\text{CID}} + \underset{(.000017)}{.00012\text{CID}^2}, \qquad R^2 = .72,\ n = 342.$$

EPA data on MPG yield similar results.

[34] Fullerton and West [17] impose functional forms, use a large sample of households, and find that the value of the gas tax rate evaluated at the averages is indeed 25% smaller than the second-best rate.

[35] We use the RTECS because other sources lack data on annual miles or engine size. CARB data do not list annual mileage, while the Nationwide Personal Transportation Survey (NPTS) has miles but not engine size. The 1994 Consumer Expenditure Survey has multiple odometer readings that enable mileage to be calculated, and it has the number of cylinders, but not CID. Using vehicles in the RTECS, the correlation between CID and miles is −.0439, significant at the 2.6% level. Also, regression analysis indicates that miles and size may be nonlinearly related.

that effects on the second term are ambiguous. For example, if both functions are convex, then the average *EPM* is larger than the *EPM* evaluated at the average s, which increases the numerator of the second term of (28) relative to (29). But then the average *MPG* is larger than the *MPG* evaluated at the average s in the denominator. The net effect on the second term is ambiguous. A similar explanation applies to each possible combination of concavity and convexity.

Thus, the overall effect of nonlinearity on the size tax is ambiguous. Only if the effect of the negative correlation between size and miles is large enough to offset any opposing effect of convexity would these theoretical considerations suggest that the size tax based on average size and miles (in (29)) would likely understate the second-best uniform size tax.[36]

Do uniform tax rates calculated using the mean miles and size *approximate* the second-best tax rates? As the correlation between size and miles becomes more negative, or as the effect of convexity becomes larger, this approximation gets worse.[37] It appears unlikely that second-best uniform tax rates would be closely approximated by the rates based on the means. In order to maximize social welfare, we need a comprehensive empirical investigation of the technologies $EPM(s)$ and $MPG(s)$, the distribution $h(\alpha, \beta, y)$, and behavioral parameters.

IV. CONCLUSIONS

In a simple model, we duplicate the outcome from a tax on emissions by instead placing a complicated tax on gasoline. Because the rate depends on fuel cleanliness, engine size, and *PCE*, it can optimally affect choices of all these goods. For this first-best gas tax to be feasible, however, the attributes of each vehicle would have to be identifiable at the pump. If this gas tax is not feasible, then perhaps a complicated vehicle tax could depend upon vehicle characteristics and on miles driven. This policy also can attain the first-best outcome. In the case where none of those policies is feasible, we investigate the combination of a tax on gasoline that depends only on the cleanliness of the fuel, a flat rate of tax on engine size, and a flat rate of subsidy to *PCE*. In the homogeneous-consumer model, this combination still achieves first best.

We then build a model of heterogeneous consumers that differ by income, tastes for miles, and tastes for engine size. The same policies all can achieve first best in this model, also, but the three-part combination requires an individual-specific gas tax and an individual-specific size tax. If these tax rates must be uniform across heterogeneous households, then this combination does not achieve first best.

[36] Fullerton and West [17] find that the size tax evaluated at the averages is indeed smaller than the second-best rate. In fact, while latter is slightly positive, the former is slightly negative. The difference, however, is not large.

[37] In addition, consumers may differ in their elasticities of demand for miles, gas, and size. For example, if miles demand is more price-sensitive among owners of large, more-polluting cars, then a gasoline tax will have a larger impact on emissions than if consumers were homogeneous, since owners of dirtier cars would reduce miles by more than owners of cleaner cars.

Thus, heterogeneity matters for whether the first best can be achieved. It also matters for how to set the second-best rates. Suppose, for example, that authorities proceed on the simple but erroneous assumption that consumers are identical and so all drive the mean number of miles in the mean sized car. The use of these means in our individual-specific formulas will not achieve first best, but will achieve second best if preferences for size and miles are uncorrelated and both $EPM(s)$ and $MPG(s)$ are linear. However, if either of these technological relationships is convex, then the second-best uniform gas tax rate would exceed that simple calculation (using mean size and miles). In addition, if the taste for miles is negatively correlated with the taste for engine size, then the second-best uniform size tax would exceed the rate using means.

Thus an important avenue for future research is an empirical investigation of the degree of heterogeneity, the correlation of preferences for miles and engine size, and the technological relationships among vehicle attributes such as engine size, fuel efficiency, and emissions rates. In addition, the model could be extended to consider other vehicle characteristics. Vehicle age is an important determinant of emissions, because emissions standards have become increasingly stringent over time, because emissions-control equipment deteriorates over time, and because new technologies and lighter materials have become available. Thus policies that accelerate vehicle retirement might also reduce emissions in a cost-effective way. The theory in this paper could be extended to incorporate such policies.[38]

The model also could be extended to consider current mandates for the control of car pollution. We do not include any explicit vehicle-emissions standards in our model, but we recognize that existing standards affect the current relationships between vehicle size, vehicle age, and emissions per mile. Thus incentives that affect the choice of vehicle rely for their effectiveness on the existence of those standards.

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[38] Alberini *et al.* [1, 2] build a theoretical model of owners' car tenure and scrappage decisions, and they analyze the results from an experimental vehicle retirement program in Delaware. Innes [24] and Fullerton and West [17] also incorporate vintage choice into their models.

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