Liquidity Policies and Financial Fragility

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Abstract

This paper proposes an endogenous model of the formation of financial networks, where government and central bank policies that aim at enhancing market liquidity play a key role. Under these policies, large and less liquid investments become more profitable, but to finance them banks need to resort to the interbank market. This makes the structure of the financial network - and its associated exposure to shocks, i.e., fragility - to be dependent on liquidity policies chosen by the government and central bank. It is shown that, despite increasing the capitalization of the banking system, policies that enhance liquidity can make it more fragile.

Keywords: Financial networks; market liquidity; financial fragility.

JEL Classification: G21; G28.

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1 Introduction

The aftermath of the global financial crisis of 2007-2009 saw the implementation of government and central bank policies aimed at increasing the liquidity of markets, in particular quantitative easing programs (Fawley and Neely (2013)). If, as argued by Greenlaw et al. (2018), the effectiveness of these measures remains unclear, their future consequences onto real and financial markets are even less so. The literature on leverage cycles (e.g., Geanakoplos (2010)) shows that periods of financial distress tend to be preceded by credit booms and, if anything, one would expect a period of correction, or contraction, to follow one of excess liquidity.

As far as systemic risk is concerned, the likelihood that a contraction will turn into a full-fledged financial contagion episode depends, among other things, on the bilateral (credit) exposure of financial institutions - the structure of the financial network. This point was made in the seminal work of Allen and Gale (2000) and, more recently, by Acemoglu et al. (2015)), with far reaching implications. For instance, financial institutions are now charged capital requirements on the basis of the threat they pose to the stability of the financial system (Bank for International Settlements (2017)).

If market liquidity induced by government and central bank policies can lead to contractions whose magnitude depend on the structure of the financial network, then the questions that follow are: (i) how financial networks can be affected by government and central bank policies that target market liquidity? and (ii) what is the resulting impact on financial fragility, i.e., the likelihood of financial contagion in the network? The main contribution of the paper is to provide a model that addresses the first question and, based on it, numerical simulations are performed in order to form a view on the second one.

Conceptually, the financial network focus of the paper is intended to represent the interbank market. The argument that policies that enhance market liquidity can alter the connectivity of banks in the interbank market can be illustrated based on Figure 1, taken from Shin (2009). The figure depicts how financial intermediaries adjust their balance sheets when there is a change in the value of assets (e.g., due to an increase in market liquidity). Ceteris paribus, an increase
in the value of assets leads to an increase in the equity of financial intermediaries, creating some “spare capacity” in their balance sheets due to a decrease in leverage. To take advantage of it, financial intermediaries engage in new borrowing and lending and, if carried through the interbank market, the structure of the financial network will change. Indeed, evidence that financial intermediaries become more leveraged following an increase in the asset side of their balance sheets is presented in Adrian and Shin (2010).

There is also evidence to support the claim that interbank lending is positively associated with measures of market liquidity, as illustrated by Figure 2. On it, the normalized federal funds rate and the agency and government sponsored enterprises (GSE)-backed mortgage pools are taken as proxies for policies that affect market liquidity. The impact of these policies on asset prices is reflected on the S&P/Case-Shiller 20-City Composite Home Price Index. Confronted against these series, the volume of interbank loans of all commercial banks is negatively associated with the federal funds rates, and positively associated with government participation in the housing market. In other words, interbank lending is positively associated with measures of market liquidity. On the other hand, if liquidity in the housing market is to have any effect on the decision of banks to engage more
in interbank transactions, then banks should be at the same time investing more in real estate related assets: this is confirmed by the volume of mortgage debt outstanding in depository institutions.

The model developed is thought of as an economy divided into several regions, each of them with its own idiosyncrasies in terms of investment opportunities and consumers’ preferences. There is a representative bank in each region, responsible for taking deposits and making investments. Investments can be made in two types of long-term assets, namely large and small projects. Large projects command a higher payoff than small ones, but at the same time demand an extra level of initial capital that can be secured only through a loan taken from another bank. This is the mechanism that creates connections between banks.

Embedded in the framework is banks’ traditional maturity mismatch problem, i.e., the financing of long-term assets with short-term liabilities. Banks are assumed to be short of capital to service depositors, which forces them to sell a fraction of their assets - projects or loans - before they are ripe. This happens at a fire-sale cost: there is a penalty applied to any fraction of an asset sold before maturity. This cost is taken to be proportional to the size of the asset, making large projects to be sold at a higher discount - or, in other words, have a lower recovery rate - than small projects and loans. Government and central bank policies targeting market liquidity are modeled in reduced form as a mechanism that enhances market

Figure 2: Interbank loans and market liquidity.
liquidity, alleviating the fire-sale cost that banks face when they sell assets before maturity. In the framework proposed, a too-big-to-fail policy is tantamount to large projects having lower fire-sale costs compared to small projects.

The idiosyncrasies in terms of projects’ payoffs and preferences of depositors, combined with the fire-sale cost and the prevailing market liquidity (i.e., the policy chosen by the government or central bank), will determine the profitability of assets in each of the different regions of the economy. When banks “meet” in the process of investing the money obtained from depositors, this profitability will determine which banks are serving as lenders and which as borrowers (in case they decide to engage in interbank lending at all). The important point is that market liquidity and its associated policy can lead to interbank lending in circumstances where otherwise (i.e., in the complete absence of a policy that reduces fire-sale costs) it would be nonexistent. This is, ultimately, what makes the structure of the financial network dependent on the market liquidity policy.

Intuitively, one of the effects of enhanced marked liquidity is to make banks better capitalized. Concomitantly, though, this is accompanied by the creation of connections among banks, presumably establishing new channels for financial contagion. This trade-off between the banking system’s networth and the fragility of the associated financial network is assessed by the number of bank failures caused by shocks onto banks’ assets. Upon these shocks, projects turn out to pay only a fraction of the original promise, and the resulting losses will spread in the financial network according to its structure. The main result of the paper is to show that, indeed, an increase in its overall networth caused by policies that enhance market liquidity can increase the fragility of the banking system.

The paper is structured as follows: section 2 discusses the related literature; section 3 details the model; section 4 discusses the link formation process that originates the financial network; section 5 derives results related to the implications of market liquidity policies for the financial network structure; section 6 gives the balance sheet characterization of the banking system based on its associated financial network, and introduce the shocks to assets’ payoff that will be used to study financial fragility; section 8 presents the main result of the paper, i.e., the
trade-off between networth and financial fragility that arises through policies that enhance market liquidity; section 9 presents concluding remarks and venues for future research.

2 Related Literature

The paper studies how market liquidity policies impact financial networks, and the corresponding effect on financial fragility (i.e., exposure to systemic risk). The starting point is the seminal contribution of Allen and Gale (2000) which, however, does not consider how networks are formed and hence the role played by government and central bank policies in the decision of banks to become connected.

In Allen and Gale (2000), the insurance motive is the main driver of the creation of connections across banks. The same rationale is present in Freixas et al. (2000), where depositors face liquidity needs in different locations of the economy, resulting in the establishment of credit lines among banks that protect against an excessive withdrawal of funds. Brusco and Castiglionesi (2007) and Babus (2015) show how this liquidity coinsurance can lead to financial contagion in situations of excessive risk-taking by banks and low capital levels, with Zawadowski (2013) arguing that, in fact, banks significantly under-insure against counterpart risk by not taking into account network externalities caused by bank failures.

The links among banks that arise due to the insurance motive can be viewed as their response to a scenario where a bail-in is the only type of assistance available. This rationale, however, does not capture the effects of government and central bank policies that enhance market liquidity and, as such, is not explored in the paper. Instead, the focus is on connections among banks that are created not because of co-insurance but rather due to profitable investment opportunities, along the lines of Kiyotaki and Moore (1997). Other mechanisms that lead to connectivity among banks that are also not considered are diversification (Lagunoff and Schreft (2001)), asset commonality (Cifuentes et al. (2005); Allen et al. (2012)), and information-based models of contagion (Caballero and Simsek (2013); Pritsker (2013); Alvarez and Barlevy (2015)).
The motivation for the adoption of market liquidity policies that reduce fire-sale costs is based on Shleifer and Vishny (1992) and Gorton and Huang (2004), although neither explores the consequences on the structure of financial networks. The paper takes for granted the existence of market-liquidity policies, however does not engage in a discussion of the merits of it as opposed to alternative policies that can fulfill the same goals, as Diamond and Rajan (2012), Farhi and Tirole (2012) and Keister (2015).

The framework used for the study of the propagation of shocks and financial contagion is similar to that in Aldasoro and Angeloni (2015), based on input-output analysis. The main result of the paper stating that an increase in the networth (i.e., capitalization) of the banking system due to market-liquidity policies can lead to a higher exposure to systemic risk and hence financial fragility can also be interpreted through the lens of Acemoglu et al. (2015).

Although not providing a new methodology, the paper is also related to the literature on financial fragility and systemic risk. The shocks that might lead to contagion are similar to those studied in Allen and Gale (2000), i.e., perturbations of the network, or zero probability events. The adopted measure of financial fragility and corresponding exposure to systemic risk is simulation-based, more precisely the number of bank failures that result from negative shocks to banks’ assets as in Nier et al. (2007). Proper measures of systemic risk are proposed in Acharya et al. (2011), Huang et al. (2011) and Adrian and Brunnermeier (2016), to cite a few.

3 Model

Consider a 1-good ($), 3-period economy, $t = 0, 1, 2$, divided in $N$ of regions, $N = \{1, \ldots, N\}$. Every region $i \in N$ has a representative bank $B_i \in \mathbb{B}$, with $\mathbb{B} = \{B_1, \ldots, B_N\}$ representing the set of banks and, as explained in section 3.1, each of these banks has an initial endowment of $e_i > 0$. Every region $i$ has $N - 1$ continuums of depositors, $\mathbb{D}^i = \{D^i_1, \ldots, D^i_{N-1}\}$, each of them of unit mass. Depositors are endowed with $\$1$ and have Diamond-Dybvig preferences, i.e., they
face uncertainty regarding the time of consumption, which is represented as follows:

$$U_i(c_1, c_2) = \begin{cases} c_1, & \text{with probability } \omega_i, \\ c_2, & \text{with probability } 1 - \omega_i. \end{cases}$$ (1)

Any depositor from a continuum $D^i_j$ in an arbitrary region $i$ will, with probability $\omega_i$, face consumption needs at $t = 1$, denoted by $c_1$ (early depositor), whereas with probability $1 - \omega_i$ the consumption needs will arise only at $t = 2$, denoted by $c_2$ (late depositor). Uncertainty is resolved at $t = 1$, and the probability of consuming at either $t = 1$ or $t = 2$ varies across but not within regions (i.e., $\omega_i \neq \omega_j$ for any two regions $i$ and $j$, but the same for every depositor in the same region). Figure 3 illustrates the continuums of depositors for an arbitrary region $i$.

Figure 3: Region $i$ and its continuums of depositors.

A representative bank $B_i$ has available an infinite supply of two types of (long-term) assets to invest in, namely a large and a small project. These investment opportunities are characterized as follows: a large project requires an initial investment of $2$ and yields at $t = 2$ a payoff of $r^*_i$, whereas a small project demands an initial investment of $1$ and yields at $t = 2$ a payoff of $r_i$. By assumption, large projects yield a higher net-present value than small ones, which results in $r^*_i > 1 + r_i$. The cash flows of projects are represented in Figure 4.

Projects are available only to the representative bank of the respective region, i.e., bank $i \in N$ cannot invest in projects not located in region $i$. Moreover, projects can be partially sold before maturity at fire-sale prices as explained in section 3.2. Banks are allowed to borrow (long-term) from other banks, and have
also available a 1-period bond that pays zero interest rate. Depositors do not have access to either long-term projects or short-term bonds, and are thus forced to deposit their money at local banks.

3.1 Banks’ Meeting Protocol

The meeting protocol assumes that banks meet once, in pairwise fashion. This results in \( N - 1 \) rounds of meetings among a total of \( N \) banks at \( t = 0 \), the date when these meetings take place. Figure 5 illustrates one possible realization based on this protocol, for \( N = 4 \) - the double-sided arrows, or edges, represent all the possible scenarios in a meeting between banks, with an arrow from \( B_i \) to \( B_j \) denoting a $1 interbank loan from the former to the later.

Synchronized with the interaction process is the arrival of depositors at their respective local banks. Each continuum of depositors arrives sequentially and makes a deposit of $1, in a time fashion that matches the meetings between banks.

Based on the example of figure 5, at round 1 a mass of \( D_1^1 \) arrives at bank \( B_1 \) to

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As conveyed in the sequence, the order of the sequence on meetings in inconsequential for the results of the paper.
make deposits, and a mass of $D_1^2$ at bank $B_2$; at round 2, $D_2^1$ arrives at $B_1$, and $D_2^3$ at bank $B_3$; finally, at round 3, $D_3^1$ arrives at bank $B_1$, and $D_3^3$ at bank $B_4$. Therefore, one of the $N - 1$ continuums of depositors in each region will arrive at the respective local bank whenever that bank is meeting another one from a different region. Since the interaction process of banks is composed of $N-1$ rounds, by the end of $t = 0$ all the money from depositors will be at their respective banks.

In order to capture differences in capitalization, it is assumed that, at every round of interaction, banks receive an endowment of $e_i > 0$. As it will be shown later, this endowment will be mainly used to service early depositors, diminishing the fraction of long-term assets needed to be sold before maturity and thus leading better capitalized banks to incur lower fire-sale costs.

One assumption imposed on banks related to the meeting protocol is that, once a meeting has taken place, banks need to immediately decide how to allocate their resources, i.e., the $1 from depositors plus the endowment of $e_i$ that are available at every meeting. Thus, upon a meeting, the possibilities for each bank are:

(i) Invest in a small project, or;

(ii) Borrow to invest in a large project.

### 3.2 Maturity Mismatch and the Need for Liquidity

With assets’ payoffs accruing only at $t = 2$ and part of their liabilities maturing at $t = 1$ (i.e., early depositors), banks are exposed to a maturity mismatch problem. The endowment received by banks when they meet each other is not enough to cover withdrawals by early depositors, i.e., $e_i < \omega_i$. In this way, these depositors can only be served if banks sell a fraction of their assets before maturity, i.e., at $t = 1$. This comes at a cost since banks are assumed to fetch only a fire-sale price when negotiating assets before their expiry date, as follows:

2This amounts to myopic behavior as banks cannot keep the resources available at a specific meeting for the eventuality of a more profitable interaction with another bank in the future.

3The conditions for each of these outcomes is discussed in section 4.
(i) One unit of investment in a large project paying \( r^*_i \) at \( t = 2 \) is priced at \( \rho^* r^*_i \) at \( t = 1 \), with \( 0 < \rho^* < 1 \);

(ii) One unit of investment in a small project or loan paying \( r_i \) at \( t = 2 \) is priced at \( \rho r_i \) at \( t = 1 \), with \( 0 < \rho < 1 \).

Since loans are always the size of an investment in a small project (\$1), the fire-sale cost associated with them is taken to be the same as that for small projects, \( \rho \). Another assumption is that the fire-sale cost of large projects is higher than the one that applies for small projects and loans, the rationale being that larger investments are more complex and, therefore, more costly to be transacted:

\[
0 < \rho^* < \rho < 1.
\]  \hspace{1cm} (2)

In summary, large projects face a discount factor of \( \rho^* \), and small projects, \( \rho \). By way of reducing these discount factors, the market liquidity policy discussed next helps to alleviate fire-sale costs imposed on banks due to assets sold prematurely.

### 3.3 Market Liquidity Policy

The fire-sale cost faced by banks might prevent them from investing in large projects since small assets are less costly to be transacted before maturity. Compared to small projects, however, the large ones offer a higher net-present value, which makes them preferable from a maximization of wealth point of view. This is the motivation for a policy that enhances market liquidity, and the one studied here affects the fire-sale costs faced by banks in the following way:

(i) Large projects: one unit of investment in a project paying \( r^*_i \) at \( t = 2 \) is priced at \( [\rho^* + \gamma^* (1 - \rho^*)] r^*_i \) at \( t = 1 \);

(ii) Small projects and loans: one unit of investment in an asset paying \( r_i \) at \( t = 2 \) is priced at \( [\rho + \gamma (1 - \rho)] r_i \) at \( t = 1 \).

Thus, \( \gamma \) and \( \gamma^* \) are defined as the liquidity policy parameters for small and large projects, respectively. In the absence of the liquidity policy, i.e., \( \gamma^* = \gamma = 0 \),
the original discount factors apply, $\rho$ for small projects and $\rho^*$ for large ones. The opposite side of the spectrum is when $\gamma^* = \gamma = 1$, which eliminates fire-sale costs.

In order to capture the effects of what could be thought as a too-big-to-fail policy, it is assumed that large projects command more support from the liquidity policy than small assets, i.e., $\gamma^* > \gamma$. Despite such a policy, however, large investments are still assumed to be more costly to be sold than small ones, i.e.:

$$
\rho + \gamma (1 - \rho) > \rho^* + \gamma^* (1 - \rho^*) .
$$

The market liquidity policy, in conjunction with depositors’ preferences and bank’s endowment, will determine the best course of action for banks when they meet. The decision to stay in autarky (to rely only on depositors’ funds to invest in a local small project) or connected (borrow from another bank to invest in a local large project) will depend on how profitable either of these options become upon the interaction of the factors above, as explained in the next section.

## 4 The Interbank Loan Decision

The investment decision of banks following their meetings at $t = 0$ will give rise to a network structure, with edges representing a $1$ interbank loan. These connections, or links, will be established only on profitability grounds and, therefore, banks need to consider the costs and benefits associated with all investment opportunities available at each of their meetings with other banks. In what follows, it is considered a meeting of two arbitrary banks, $B_i$ and $B_j$, and how they evaluate each option from the menu of investments that arise once they have met. The profitability of each investment opportunity is determined by the budget constraints (BCs) that must be satisfied for each investment option available.

### 4.1 Investment in a Small Project (and Loans)

The decision of bank $B_i$ to invest in a small project entails remaining in autarky and, as such, will yield the same profit regardless of the counterpart it is matched
with in a meeting. The budget constraints at each date that an bank $B_i$ faces when investing in a small project are:

\[ 1 + b_i \leq 1 + e_i, \quad \text{(BC at } t = 0) \]

\[ b_i + \alpha^i_r r_i \left[ \rho + \gamma (1 - \rho) \right] \geq \omega_i, \quad \text{(BC at } t = 1) \]

\[ (1 - \alpha^i_r) r_i = (1 - \omega_i) + e_i + \pi_i, \quad \text{(BC at } t = 2) \]

where:

- $\pi_i$: profit of bank $i$ with an investment in a small project;
- $b_i$: investment in the short-term bond;
- $\alpha^i_r$: fraction of the small project to be liquidated at $t = 1$.

The budget constraint at $t = 0$ expresses that total expenses, i.e., investment in the small project, $\$1$, plus investment in the short-term bond, $b_i$, cannot exhaust the amount of total resources available, namely deposits, $\$1$, and the endowment, $e_i$. At $t = 1$, the revenue from the short-term bond, $b_i$, plus the proceeds from the liquidation of a fraction of the small project, $\alpha^i_r r_i \left[ \rho + \gamma (1 - \rho) \right]$, should suffice to service early depositors, $\omega_i$. Finally, at $t = 2$, the fraction not liquidated of the small project, $(1 - \alpha^i_r) r_i$, must allow the bank to meet the demand from late depositors, $1 - \omega_i$, plus the amount owed to equity holders, $e_i$. What is left from the payoff of the small project after paying late depositors and equity holders constitutes the profit of the bank, $\pi_i$.

Since bank $i$ does not want to (i) waste resources and (ii) sell a higher fraction of the small project than what is necessary, the budget constraints at $t = 0$ and $t = 1$ will be binding, allowing one to solve for $b_i$ and $\alpha^i_r$:

\[ b_i = e_i, \quad \text{(5)} \]

\[ \alpha^i_r = \frac{\omega_i - e_i}{r_i \left[ \rho + \gamma (1 - \rho) \right]}. \quad \text{(6)} \]
Substituting into the expression for bank $i$’s profit, $\pi_i$ becomes:

$$\pi_i = r_i - \left\{ (1 - \omega_i) + e_i + \frac{\omega_i - e_i}{\rho + \gamma (1 - \rho)} \right\}$$

$$\Leftrightarrow \pi_i = r_i - \pi_i,$$

(7)

where:

$$r_i := (1 - \omega_i) + e_i + \frac{\omega_i - e_i}{\rho + \gamma (1 - \rho)}.$$  

(8)

Bank $i$ would be willing to accept deposits that it could channel to a small project as long as $\pi_i \geq 0$, i.e., $r_i \geq \pi_i$, which is assumed for every $i \in N$. As a final remark, the investment in an interbank loan is such that both its payoff and the fire-sale costs associated with a sell-off are the same as those applicable to small projects. Therefore, interbank loans and small projects are perfect substitutes, implying that the budget constraints and profit from an investment in the former are the same as those for the later, as per the analysis above.

### 4.2 Investment in a Large Project

If bank $i$ is to invest in a large project, an interbank loan has to take place for $1$ to be borrowed from bank $j$. The budget constraints are then written as:

$$2 + b_i \leq 2 + e_i, \quad \text{(BC at } t = 0)$$

$$b_i + \alpha_i^i.\pi_i^* \left[ \rho^* + \gamma^* (1 - \rho^*) \right] \geq \omega_i, \quad \text{(BC at } t = 1)$$

$$\left(1 - \alpha_i^i \right) \pi_i^* = (1 - \omega_i) + e_i + y_{ij} + \pi_{ij}^*, \quad \text{(BC at } t = 2)$$

$$y_{ij} \geq r_j. \quad \text{(IR of the Lender)}$$

(9)

Differently from an investment in a small project, the budget constraint at $t = 0$ shows an extra $1$, representing the interbank loan. As a result, at $t = 2$ there is also an extra expense representing the repayment of the loan, i.e., principal
plus interest, denoted by $y_{ij}$, that bank $i$ owes to bank $j$. The cost of the loan will depend on how much bank $j$ will charge for it, which makes the profit of the bank in a large project to depend on the characteristics of the lender. This justifies the choice of notation for $\pi^*_{ij}$ to denote the profit of bank $i$ with an investment in a large project that is partially financed by a loan taken from bank $j$.

The individual rationality constraint in bank $i$’s problem, IR, represents the opportunity cost of bank $j$, which could use the $1$ obtained from depositors to invest in a small project rather than to lend it. Therefore, the compensation for the loan that bank $i$ needs to offer to bank $j$ should be as large as the payoff of the later with a small project, i.e., $y_{ij} \geq r_j$. Without loss of generality, the borrower is assumed to have all the bargaining power and, as such, offers the minimum compensation for the loan, at which the lender is indifferent between lending or not, resulting in $y_{ij} = r_j$.

Analogously to an investment in a small project, the budget constraints at $t = 0$ and $t = 1$ will bind, and taking into account the repayment of the interbank loan from bank $B_i$ to bank $B_j$ yields the following:

\begin{align*}
y_{ij} &= r_j, \\
b_i &= e_i, \\
\alpha^{i*}_{ir} &= \frac{\omega_i - e_i}{r^*_{i} \left[\rho^* + \gamma^* (1 - \rho^*)\right]},
\end{align*}

Bank $i$’s profit with a large project when borrowing from bank $j$, $\pi^*_{ij}$, is then:

\begin{equation}
\pi^*_{ij} = r^*_{i} - \left\{ (1 - \omega_i) + e_i + r_j + \frac{\omega_i - e_i}{\rho^* + \gamma^* (1 - \rho^*)} \right\}. \quad (13)
\end{equation}

4.3 Summary: the Decision to Form Links

By comparing the profits associated with each investment opportunity at the time of their meeting, banks $B_i$ and $B_j$ decide whether to (i) stay in autarky, i.e., channel the funds available to their respective small projects, or (ii) celebrate an interbank loan, allowing the borrower to invest in a large project and the lender, in
a loan. Figure 6 provides a summary: the blue dashed line represents the scenario of absence of interbank lending, with both banks choosing to channel their funds to a small project in their respective regions. The green dashed line corresponds to the case where bank \(B_i\) lends to bank \(B_j\), whereas the red one indicates the opposite, with both leading to an investment in a large project.

![Figure 6: Investment opportunities in a meeting of two arbitrary banks.](image)

### 5 Liquidity Policy and Network Structure

Having established the network formation process resulting from banks’ meetings and the corresponding interbank loan decisions, the question studied now is how market liquidity affects the structure of banks’ network. To this purpose, the networks that emerge in the absence of a liquidity policy, i.e., for \(\gamma^* = \gamma = 0\), are compared to those networks formed under policies that enhance market liquidity, i.e., for \(\gamma^* \geq \gamma > 0\). The first characteristic for comparison is the number of
Proposition 1. For any set of parameters, the introduction of a liquidity policy has a non-negative effect on the number of links (interbank loans) in the network.

Proposition 1 establishes that liquidity policies do not decrease the number of interbank loans among banks and, instead, can lead to the creation of new links in a network. The reason for this result is that policies that enhance market liquidity make less costly the premature liquidation of large projects, therefore increasing the incentives for banks to borrow from each other. It also follows from the proof of Proposition 1 that under a too-big-to-fail policy (TBTF) whereby the liquidity of large projects in enhanced in a more pronounced way relative to small ones (i.e., \( \gamma^* > \gamma > 0 \)), the incentives for the creation of interbank loans are even stronger.

There is, however, a less intuitive way by which bank’s network structure can be affected by liquidity policies. As the following proposition shows, enhanced liquidity might switch the role of borrowers and lenders that, in the absence of liquidity policies, would had established an interbank loan contract:

Proposition 2. Consider a network of banks formed in the absence of a liquidity policy. If in such a network two arbitrary banks establish an interbank loan, their roles might be reversed in the analogous network formed under enhanced liquidity.

To understand Proposition 2, consider a network formed in the absence of a liquidity policy, and that \( B_i \) borrows from \( B_j \) in such network. In this scenario, \( B_i \)’s profit with a large project (i.e., including the costs associated with the loan taken from \( B_j \)) is higher than with a small one. With the introduction of a liquidity policy, \( B_i \) still wants to borrow from \( B_j \), however the corresponding decrease in fire-sale costs might increase the profitability of large projects to a level where \( B_j \) is better positioned to bargain for a loan in case both banks want to borrow. In this situation, then, \( B_j \) will become the borrower and \( B_i \), the lender.

Propositions 1 and 2, therefore, show that liquidity policies play a pivotal role in the determination of the network structure that arises following banks’ meetings.

\(^4\)The proofs of all propositions are provided in the appendix.
The next question is how this structure is related to the capacity of banks to absorb shocks, and whether or not the network as a whole is more prone to financial contagion in the presence of liquidity policies. To answer these questions, it is necessary to first characterize the balance sheet of banks, which is done next.

6 Characterization of the Balance Sheet of Banks

The characterization of the balance sheet of banks informs how much networth they have available to absorb shocks. Since interbank loans are part of their debt, banks’ balance sheet will depend on the structure of the financial network that arises after their meetings. The network is described by the following matrix:

$$X = \begin{bmatrix} 0 & \chi_{12} & \cdots & \chi_{1N} \\ \chi_{21} & 0 & \cdots & \chi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{N1} & \chi_{N2} & \cdots & 0 \end{bmatrix}.$$ \hspace{1cm} (14)

In matrix (14), $\chi_{ij}$ is an indicator function such that $\chi_{ij} = 1$ if bank $B_i$ lends to $B_j$, and 0 otherwise. In addition to the adjacency matrix representing interbank loans, the other primitives of the model necessary to determine the balance sheet of banks are: fire-sale parameters for small and large projects, $\rho$ and $\rho^*$; liquidity policy, $\gamma$ and $\gamma^*$; payoff of large and small projects, $r_i^*$ and $r_i$, early depositors, $\omega_i$, and $B_i$’s endowment available per meeting, $e_i$, represented as follows:

$$r^* = \begin{bmatrix} r_1^* \\ r_2^* \\ \vdots \\ r_N^* \end{bmatrix}, \quad r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}.$$ \hspace{1cm} (15)

In regards to the asset side of the balance sheet, the first step is to calculate the quantity of each investment undertaken, i.e., large projects, small projects, and loans. For an arbitrary bank $B_i$, the row-sum of matrix (14), $n^i_L = \sum_{j \in N} \chi_{ij}$, provides its number of debtors (loans made), whereas the column-sum, $n_{i^*} = \sum_{i \in N} \chi_{ij}$, provides its number of creditors (loans received).
∑_{j∈N} \chi_{ji}, the number of creditors (large projects). Moreover, since in each of their \(N − 1\) meetings banks will be making an investment, the number of small projects they invest in is given by \(n^i_L = (N − 1) − n^i_L − n^i_r\).

To serve early depositors, banks liquidate a fraction of their assets before maturity, at \(t = 1\). That fraction is \(\alpha^i_t\) for investments in either loans or small projects, as in (6), and \(\alpha^i_{t∗}\) for investments in large projects, as in (12). Hence, on the balance sheet of \(B_i\) at \(t = 2\), there will be \(a^i_L = r_i (1 − \alpha^i_t) n^i_L\) worth of loans, \(a^i_{t∗} = r^∗_i (1 − \alpha^i_{t∗}) n^i_{t∗}\) of large projects, and \(a^i_r = r_i (1 − \alpha^i_t) n^i_r\) of small projects. The size of the asset side of \(B_i\)’s balance sheet is, therefore, \(a^i = a^i_L + a^i_{t∗} + a^i_r\).

In terms of liabilities, the balance sheet of banks is composed of the amount owed to late depositors, loans taken from other banks (debt) and networth (equity endowment plus profits). As for late depositors, since $1 is deposited at bank \(B_i\) in each of its \(N − 1\) meetings with other banks, and only a fraction \(1 − ω_i\) of these remains to be withdrawn at \(t = 2\), the amount owed to late depositors is \(l^i_ω = (N − 1) (1 − ω_i)\). Debt owed to other banks comes from interbank loans taken to finance large projects. As discussed in section 4.2, the interest on a interbank loan should cover the opportunity cost of lenders (i.e., the amount they would get by investing in a small project) and, therefore, debt owed by \(B_i\) is \(l^i_d = \sum_{j∈N} \chi_{ji} r^i_j\).

A bank’s networth, \(W^i\), is given by the sum of its equity, \(l^i_e\), and total profits, \(l^i_π\); or \(W^i = l^i_e + l^i_π\). Equity is given by \(l^i_e = (N − 1) e_i\), given a bank’s receipt of an endowment of \(e_i\) in each of the \(N − 1\) meetings with other banks. Profits associated with loans and small projects are the same, \(π_i\), and since there are \(n^i_L\) loans and \(n^i_r\) small projects, profit with investments in these two assets is \((n^i_L + n^i_r) π_i\). As for large projects, bank \(B_i\)’s profit when the investment is financed by an interbank loan taken from \(B_j\) is \(π^i_{ij}\), yielding profits from such investments of \(\Sigma_{j∈N} \chi_{ji} π^i_{ij}\). Total profits are then \(l^i_π = (n^i_L + n^i_r) π_i + \Sigma_{j∈N} \chi_{ji} π^i_{ij}\), completing the characterization of the liability side of a bank’s balance sheet, or \(l^i = l^i_ω + l^i_d + W^i\).

In this way, for any network formed in the presence or absence of liquidity policies, the balance sheet of every bank is characterized, with assets (investments in loans, small and large projects) and liabilities (debt to other banks and networth) being readily determined. Knowledge of the balance sheet of banks makes it pos-
sible to determine the impact on the financial network following negative shocks to projects’ payoffs as discussed in the next section.

7 Real Shocks and Financial Contagion

To understand the fragility of a financial network, it is necessary to study how it responds to shocks affecting banks’ assets. The shocks considered here are of the following type: if bank $B_i$ has made an investment in a large project that yields a payoff of $r_i^*$, a shock $\delta_i$ implies that the project’s payoff turns out to be only $r_i^* (1 - \delta_i)$. Similarly, if the investment was in a small project, upon a shock $\delta_i$ the project’s payoff becomes $r_i (1 - \delta_i)$ as opposed to an original promise of $r_i$. In this way, the shocks in the model originate from the real side of the economy. Moreover, it is important to highlight that the shocks are to be understood as probability zero events, or perturbations of the network, as in Allen and Gale (2000), and hence unhedgeable. The vectors of shocks to small and large projects are:

$$
\delta = \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_N
\end{bmatrix}, \quad \delta^* = \begin{bmatrix}
\delta_1^* \\
\delta_2^* \\
\vdots \\
\delta_N^*
\end{bmatrix}.
$$

The total loss a bank faces due to shocks is written as $\Delta^i = \Delta^i_L + \Delta^i_r + \Delta^i_r$, where $\Delta^i_L = a_i^r \delta_i$ corresponds to the loss on small projects, $\Delta^i_r = a_i^r \delta_i^*$ to the loss on large projects, and $\Delta^i_L$ to the loss on interbank loans. These last are due to financial contagion and, thus, endogenously determined: bank $B_i$’s loss on interbank loans, $\Delta^i_L$, depends on the bankruptcy status of its debtors, i.e. $B_j$ will repay the loan if its total losses are greater than its networth, $\Delta^j < W^j$, which in turn depends of its own debtors avoiding default, and so forth.

As an example of how financial contagion spreads in the network, assume that bank $B_i$ has lent to $B_j$, i.e., $\chi_{ij} = 1$, and that $\Delta^j - W^j > 0$ ($B_j$ is bankrupt). From the previous characterization of the balance sheet of banks, the debt of $B_j$ with other banks is $l^j_d$, which implies a (partial) default of $(\Delta^j - W^j) / l^j_d$ on its interbank...
loans. Bank $B_i$, which expected to receive a payment of $r_i$ for the fraction $\left(1-\alpha_i^r\right)$ of the loan to $B_j$ still in its balance sheet, ends up losing $r_i \left(1-\alpha_i^r\right) \left(\Delta^j - W^j\right)/l^j_d$. Therefore, contagion losses faced by $B_i$ are endogenously given by:

$$
\Delta^i_L = r_i \left(1-\alpha_i^r\right) \sum_{j \in N} \chi_{ij} \frac{\left(\Delta^j - W^j\right)^+}{l^j_d}.
$$

In matrix form, the system of equations that determines $\tilde{\Delta}^i := (\Delta^i - W^i)^+$ is:

$$
\tilde{\Delta} = \left(\tilde{X} \tilde{\Delta} + \Delta^r + \Delta_s - W\right)^+,
$$

where $\tilde{X} \tilde{\Delta}$ represents the vector of contagion losses, $\Delta^r$ and $\Delta_s$ the losses on large projects and small projects, respectively, and $W$ the networth of individual banks. The matrix $\tilde{X}$ is given by:

$$
\tilde{X} = \begin{bmatrix}
0 & \frac{r_1(1-\alpha_1^r)\chi_{12}}{l^1_d} & \cdots & \frac{r_1(1-\alpha_1^r)\chi_{1N}}{l^1_d} \\
\frac{r_2(1-\alpha_2^r)\chi_{21}}{l^2_d} & 0 & \cdots & \frac{r_2(1-\alpha_2^r)\chi_{2N}}{l^2_d} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{r_N(1-\alpha_N^r)\chi_{N1}}{l^N_d} & \frac{r_N(1-\alpha_N^r)\chi_{N2}}{l^N_d} & \cdots & 0
\end{bmatrix},
$$

which can be further decomposed as:

$$
\tilde{X} = \text{diag} \begin{bmatrix}
r_1 \left(1-\alpha_1^r\right) \\
r_2 \left(1-\alpha_2^r\right) \\
\vdots \\
r_N \left(1-\alpha_N^r\right)
\end{bmatrix} \times \begin{bmatrix}
0 & \chi_{12} & \cdots & \chi_{1N} \\
\chi_{21} & 0 & \cdots & \chi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\chi_{N1} & \chi_{N2} & \cdots & 0
\end{bmatrix} \times \text{diag} \begin{bmatrix}
\frac{1}{l^1_d} \\
\frac{1}{l^2_d} \\
\vdots \\
\frac{1}{l^N_d}
\end{bmatrix}.
$$

Matrix $\tilde{X}$, therefore, is simply a weighted version of $X$, the matrix representing the network structure. The max operator in expression (18) precludes a closed-form characterization of banks’ total losses, which would allow a more clear understanding of the role played by liquidity policies on financial contagion. However, if shocks are such that banks’ networth become negligible - a stress scenario by definition - the analysis is then simplified.

---

5The dimension of each of these vectors is $N \times 1$. 20
8 The Trade-Off Between Networth and Fragility

By decreasing fire-sale costs, policies that enhance market liquidity directly affect the price of assets and, thus, banks’ networth. Concomitantly, market liquidity adds incentives for banks to create interbank loans that enables specific types of investments (i.e., large projects), opening channels for financial contagion. The strongest of these two effects will determine whether or not liquidity policies make the banking system more fragile, to which the answer rests on the analysis of matrix equation (18) replicated below:

\[ \tilde{\Delta} = (\tilde{X}\tilde{\Delta} + \Delta_r^* + \Delta_r - W) + . \]  

(21)

The left-hand side of (21), \( \tilde{\Delta} \), represents the total losses that banks face following shocks to their assets, and the right-hand side shows that these losses will depend on \( W \), the networth in banks’ balance sheet. Thus, in order to explore the trade-off between fragility and networth, it is necessary to focus on liquidity policies that undoubtedly increase the capitalization of the banking system, defined as the sum of banks’ networth, i.e., \( \sum_{i \in N} W^i \). The conditions for such are:

**Proposition 3.** Let \( \pi^L := \min \{ \pi_1^L, \ldots, \pi_N^L \} \) and \( \pi := \max \{ \pi_1, \ldots, \pi_N \} \) denote the minimum and maximum profits with a small project among all the banks, in the presence and absence of a liquidity policy, respectively. If \( \pi^L > \pi \), then the liquidity policy increases the capitalization of the banking system, \( \sum_{i \in N} W^i \).

In turn, the measurement of the banking system’s fragility involves calculating the number of bank failures that would ensue upon negative shocks to banks’ assets (real shocks as discussed in the previous section). In practice, this entails conducting a stress test of the financial network formed after banks’ meetings in order to assess its robustness to shocks. For this purpose, in a network with \( N \) banks where bank \( B_i \) is hit by shocks of the type of those in (16), the set:

\[ D^i := \{ j \neq i | \Delta^j > W^j \} \]  

(22)

contains all the banks that become bankrupt as a result of \( B_i \)’s failure. Thus, the cardinality of this set, \( |D^i| \), provides the number of banks that fail specifically
due to financial contagion. For contagion to ensue, however, a necessary condition is that \( B_i \) is bankrupt itself and, therefore, whenever \( |D^i| \) is positive, the total number of bank failures is given by \( 1 + |D^i| \). Thus, the index:

\[
f^i := \begin{cases} 
1 + |D^i| & \text{if } D^i \neq \emptyset \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (23)

provides a measure of the exposure of the network to an eventual failure of bank \( B_i \). Aggregating the exposure of the financial network to each of its banks then provides a measure of its financial fragility, given by:

\[
f := \sum_{i \in N} f^i.
\]  \hspace{1cm} (24)

It is based on these measures of fragility that the following result holds:

**Proposition 4.** *Liquidity policies that increase the capitalization of the banking system can make it more fragile.*

The proof of the proposition is based on the following example, which consists of a network of 6 banks formed in an economy where fire-sale costs are represented by \( \rho^* = 0.05\rho \) (i.e., large projects are much less liquid than small ones). Moreover, assume that, in the presence of a liquidity policy, \( \gamma^* = 0.8 \) and \( \gamma = 0.3 \), whereas in the absence, \( \gamma^* = \gamma = 0 \). The remaining parameters are provided in Table 1 and, in conjunction, they satisfy the conditions of Proposition 3. As such, any liquidity policy that ought to be specified would lead to an increase in the capitalization of the system formed by the 6 banks. The networks formed in the presence and absence of the liquidity policy are displayed in Figure 7.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Large Project, ( r^* )</th>
<th>Small Project, ( r )</th>
<th>Early Depositors, ( \omega )</th>
<th>Endowment, ( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank 1</td>
<td>3.23</td>
<td>1.19</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Bank 2</td>
<td>3.00</td>
<td>1.01</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td>Bank 3</td>
<td>2.22</td>
<td>1.19</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>Bank 4</td>
<td>2.55</td>
<td>1.08</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Bank 5</td>
<td>2.97</td>
<td>1.00</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Bank 6</td>
<td>2.71</td>
<td>1.21</td>
<td>0.14</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Table 1: Parameters used for the generation of the networks in Figure 7.*
The impact of liquidity policies on financial fragility is tied to the magnitude of the shocks that affect projects’ payoffs on the balance sheet of banks. Thus, the robustness of the two networks can only be assessed through simulations, which in turn are conducted in the following way: for each bank, 1,000 simulations are performed where, in each of them, shocks $\delta$ and $\delta^*$ are imposed on bank’s large and small projects, respectively. The shocks are independently drawn from $U[0, 1]$ uniform distributions and, in order to isolate the exposure of the network to each of its individual components, imposed on banks one at a time.

For the two networks created in the presence and absence of liquidity policies, the index of fragility $f$ in (24) is then calculated. The first step is to count, across all the 1,000 simulations where a particular bank $B_i$ is affected by shocks, the number of times that the bank itself fails (direct failures), and the number of bank failures due to financial contagion (indirect failures). This leads to the index $f^i$ in
(23), where the exposure of the network to $B_i$ is measured. After performing the same exercise for all the banks and aggregating their individual $f^i$s, the fragility index associated with each of the two networks is obtained.

Table 2 presents the results from the simulations, with the number of direct bank failures denoted by 'Dir Fails', and indirect failures, 'Ind Fails'. Moreover, it also shows a few banks' characteristics implied by the structure of the two networks created: 'LR' is the leverage ratio; 'Links', the number of incoming arrows (or, equivalently, the amount borrowed by banks); 'Networth' corresponds to banks' networth as calculated in section 6.

Based on the simulation results, banks are ranked in descending order according to their index $f^i$. For example, in the network created under the presence of liquidity policy, bank 5 occupies the first position as the one that most exposes the network to fragility: across the 1,000 simulations performed, $B_5$ ended up bankrupt in 660 of them, and led to another 492 bank failures due to contagion. Thus, $f^5 = 1,152$, which is higher than the index of any other bank in the network. Aggregating the indices across the 6 banks for the two networks, the overall fragility index in (24) is $f = 5,159$ for the network created under the presence of liquidity policy, and $f = 4,192$ for the network created under its absence. Hence, the liquidity policy leads to a banking system that is more fragile.

As previously mentioned, the parameters in Table 1 satisfy the conditions of Proposition 3 and, as such, imply that the total networth of banks in the system under the presence of liquidity policy is higher than in the alternative one. Indeed, adding up banks’ networth leads to $W = \sum_{i \in N} W^i = 17.69$ for the network with the presence of liquidity policy, and to $W = 15.01$ for the network without it. Therefore, the liquidity policy leads to a better capitalized banking system that, nonetheless, is more fragile.

Apart from ranking the banks according to the fragility index $f^i$, Table 2 presents a few measures that, in conjunction, are often used as a proxy for the systemic importance of a bank. These are the networth, total debt (represented by the number of incoming links), and the leverage ratio, calculated as:
\[ LR_i := 1 - \frac{W^i}{a^i} = \frac{\widehat{l}_o^i + l_d^i}{\widehat{a}^i}. \] (25)

The most critical bank (i.e., the one with the highest index \( f^i \)) in the presence of liquidity policy is bank 5, which is (i) the second most capitalized bank, (ii) the second most indebted bank, and (iii) the bank with the third lowest leverage ratio. In the absence of liquidity policy, bank 2 assumes the role of most critical bank, featuring (i) the third lowest networth, (ii) the second highest amount of debt, and (iii) third highest leverage ratio.

It follows from the above that, firstly, liquidity policies as those considered here can significantly affect the exposure to fragility that different banks bring to the network and, secondly, the association of this measure of exposure and more traditional financial indicators is not linear: an analysis of the systemic importance of banks based solely on traditional financial indicators might be very misleading.

9 Concluding Remarks

This paper proposes a model of the endogenous formation of financial networks, where policies that enhance market liquidity play a key role in determining the structure of the banking system. Such policies lead to the formation of different networks, each with a distinct degree of financial fragility, i.e., exposure to bank failures after real sector shocks. Based on this, the relationship between liquidity policies and financial fragility was studied, and it was shown that, despite increasing the capitalization of the banking system (sum of banks’ networth), liquidity policies can lead to networks that are inherently more fragile.

The paper highlights the importance that knowledge of the structure of the network of banks might represent in the design of government and central bank policies. Not surprisingly, after the global financial crisis of 2007-2009 many central banks started to (i) collect data of the bilateral exposure of financial institutions, allowing a more detailed identification of financial networks, and (ii) perform stress
tests that explicitly take into account the structure of the financial system, providing a more precise measure of how it is exposed to systemic risk.

The model proposed does not address the impact caused on network structure by policies implemented during systemic events (e.g., bailouts). For instance, a specific bank that is perceived as too-big-to-fail might have a strategic advantage in the interbank market that, in turn, can lead to changes in the network structure similar to those explored in the paper. Moreover, a more detailed studied is required in order to establish whether the framework considered here can generate the typical core-periphery banking structure observed in reality (e.g., ). These are venues to be explored in future research.
References


Appendix (Not Intended for Publication)

The following is used in the proof of Proposition 1. Upon the meeting of two arbitrary banks $B_i$ and $B_j$, the question of whether or not an interbank loan will be created depends on the relation among the parameters, given by:

1. **$B_i$ wants to borrow from $B_j$, but not vice versa:** in this case, $B_i$ is better off investing in a large project, i.e., $\pi^*_ij > \pi_i$, but not bank $B_j$, or $\pi_j > \pi^*_ji$. From expressions (7) and (13), the following then holds:

\[
\frac{r^*_i - (r_i + r_j)}{\omega_i - e_i} > \frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]} > \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j}.
\]

(26)

2. **$B_j$ wants to borrow from $B_i$, but not vice versa:** conversely to the previous outcome, now $B_j$ is the one after an interbank loan, $\pi^*_ji > \pi_j$, although $B_i$ is not, $\pi_i > \pi^*_ij$:

\[
\frac{r^*_j - (r_j + r_i)}{\omega_j - e_j} > \frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]} > \frac{r^*_i - (r_i + r_j)}{\omega_i - e_i}.
\]

(27)

3. **Both $B_i$ and $B_j$ want to borrow:** In this case, $\pi^*_ij > \pi_i$ and $\pi^*_ji > \pi_j$, or:

\[
\min \left\{ \frac{r^*_i - (r_i + r_j)}{\omega_i - e_i}, \frac{r^*_j - (r_j + r_i)}{\omega_j - e_j} \right\} > \frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]}.
\]

(28)

In this scenario, the bank to which the loan would be more profitable has the bargaining power and, thus, ends up as the borrower. From (13), if $\pi^*_ij > \pi^*_ji$:

\[
[r^*_i - (1 - \omega_i) - e_i - r_j] - [r^*_j - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^*(1 - \rho^*)},
\]

then $B_i$ ends up borrowing from $B_j$. On the other hand, if $\pi^*_ji > \pi^*_ij$:
\[ r_j^* - (1 - \omega_j) - e_j - r_i) - [r_i^* - (1 - \omega_i) - e_i - r_j) > \frac{(\omega_j - e_j) - (\omega_i - e_i)}{\rho^* + \gamma^* (1 - \rho^*)}, \]

(30)

then it is \( B_j \) who borrows from \( B_i \).

4. **Neither \( B_i \) nor \( B_j \) wants to borrow:** in this case, both banks are better off investing in a local small project, i.e., \( \pi_i > \pi_i^* \) for \( B_i \) and \( \pi_j > \pi_j^* \) for \( B_j \), which is equivalently expressed by:

\[
\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \max \left\{ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \right\}.
\]

(31)

Apart from the conditions above, the following lemma is also used in the proof of Propositions 1 and 2:

**Lemma 5.** For any \( 1 > \rho > \rho^* > 0 \) and \( 1 > \gamma^* \geq \gamma \geq 0 \), the following is satisfied:

\[
\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.
\]

(32)

**Proof:** Consider the function \( h(\gamma, \gamma^*) \) defined by:

\[
h(\gamma, \gamma^*) := \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.
\]

(33)

For \( \gamma^* = \gamma \), it follows that:

\[
h(\gamma, \gamma) = \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]} := H(\gamma).
\]

(34)

Taking the derivative of \( H \) yields:

\[
H'(\gamma) = -\frac{(\rho - \rho^*) ([\rho^* + \gamma (1 - \rho^*)] + (1 - \gamma) (1 - \rho^*) [\rho + \gamma (1 - \rho)])/[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]^2}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma (1 - \rho^*)]^2} < 0,
\]

(35)
where the inequality following from $\rho > \rho^*$. For $\gamma \geq 0$, it follows, therefore, that $H(0) \geq H(\gamma)$, i.e.:

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]}.$$  \hfill (36)

For $\gamma^* \geq \gamma$, both:

$$\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]} \geq 1$$  \hfill (37)

and

$$\frac{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]} \leq 1,$$  \hfill (38)

hold and, therefore:

$$\frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma (1 - \rho^*)]} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]}.$$  \hfill (39)

Combining (36) with (39) leads to (32), i.e.:

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)][\rho^* + \gamma^* (1 - \rho^*)]}.$$  \hfill (40)

□

**Proof of Proposition 1**

With $N$ banks, consider the original network formed in the absence of liquidity policy, i.e., $\gamma = \gamma^* = 0$, and take two of those banks, $B_i$ and $B_j$. Without loss of generality, assume that:

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}.$$  \hfill (41)

The question to be addressed is then what the original network would be if, instead, liquidity policy was in place, i.e., $\gamma^* \geq \gamma > 0$. From Lemma 5:
\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}. \tag{42}
\]

If there is no interbank loan between \(B_i\) and \(B_j\) in the original network, then (31) and (41) imply that:

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}. \tag{43}
\]

In the presence of liquidity policy, it holds either that:

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \tag{44}
\]

which from (31) implies that banks are still better off without an interbank loan, or:

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_i^* - (r_i + r_j)}{(\omega_i - e_i)} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \tag{45}
\]

which from (26) implies that \(B_i\) borrows from \(B_j\), or:

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}, \tag{46}
\]

which from (28) implies that both banks want to borrow and, thus, an interbank loan will be created. Therefore, if in the original network \(B_i\) and \(B_j\) do not have interbank loan agreement, in the presence of liquidity policy they will either continue to not engage in a lending relationship or, conversely, will do so.

On the other hand, if in the original network \(B_i\) and \(B_j\) do have a loan agreement, then (31) and (41) imply that:

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{\rho - \rho^*}{\rho \rho^*}. \tag{47}
\]

In the presence of liquidity policy, it holds either that:
\[ r_i^* - (r_i + r_j) \geq \frac{\rho - \rho^*}{\omega_i - e_i} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}, \]  

(48)

which from (26) implies that \( B_i \) borrows from \( B_j \) and, thus, the interbank loan remains in place, or:

\[ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}, \]

(49)

which from (28) implies that both banks want to borrow and, regardless of who ends up doing so, an interbank loan between the two banks continue to exist.

Therefore, if in the original network \( B_i \) and \( B_j \) do not have a loan agreement, in the presence of liquidity policy they might create a link and, otherwise, the interbank loan continue to exists (although the borrower and the lender might reverse their roles). \(\square\)

**Proof of Proposition 2**

With \( N \) banks, consider the original network formed in the absence of liquidity policy, i.e., \( \gamma = \gamma^* = 0 \), and take two of those banks, \( B_i \) and \( B_j \). Without loss of generality, assume that:

\[ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}. \]  

(50)

Assume now that \( B_i \) borrows from \( B_j \) in the original network (i.e., with \( \gamma^* = \gamma = 0 \)). This implies, from (26) and (28), that either:

\[ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \]  

(51)

holds, or else that both:

\[ \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{\rho - \rho^*}{\rho \rho^*}. \]  

(52)
and

\[
[r_i^* - (1 - \omega_i) - e_i - r_j] - [r_j^* - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*}.
\]  

(53)

are verified. Considering first (51), Lemma 5 implies that, in the presence of liquidity policy:

\[
\frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.
\]  

(54)

Therefore, enhanced liquidity leads to either:

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}
\]  

(55)

or

\[
\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}.
\]  

(56)

If (55) is the case, from (26) it follows that \(B_i\) continues to borrow from \(B_j\). Conversely, if instead (56) holds, then \(B_i\) will keep borrowing from \(B_j\) as long as (29) is satisfied, i.e.:

\[
[r_i^* - (1 - \omega_i) - e_i - r_j] - [r_j^* - (1 - \omega_j) - e_j - r_i] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}.
\]  

(57)

However, one can show that, if:

\[
[r_i^* - r_j] - (r_j^* - r_i) \left\{ \frac{\rho^* + \gamma^* (1 - \rho^*)}{1 + [\rho^* + \gamma^* (1 - \rho^*)]} \right\} < (\omega_i - e_i) - (\omega_j - e_j) < (r_i^* - r_j^*) \left( \frac{\rho \rho^*}{\rho - \rho^*} \right).
\]  

(58)

then (50) is satisfied whereas (57) is not, which means that, in the absence of liquidity policy, \(B_i\) borrows from \(B_j\), whereas in its presence, \(B_i\) becomes the lender and \(B_j\), the borrower.
In the second case, i.e., if both (52) and (53) simultaneously hold, the fact that $\rho^* < \rho^* + \gamma^* (1 - \rho^*)$ implies that:

$$\frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*} > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}.$$  \hfill (59)

Since (53) implies (29), it follows for this case that, in the presence of liquidity policy, $B_i$ continues as the borrower and $B_j$, as the lender. □

**Proof of Proposition 3**

Denote by $\pi_{ij}^*$ the profits of $B_i$ with a large project when it borrows from $B_j$, and by $\pi_i$, the profits with a small one. The $L$ superscripted variables denote the presence of liquidity policy. From Proposition 1, liquidity policy does not decrease the number of interbank loans, whereas from Proposition 2, if (56) holds:

$$r_i^* - (r_i + r_j) \geq \frac{\rho - \rho^*}{\omega_i - e_i} r_j^* - (r_j + r_i) \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]},$$ \hfill (60)

then in the absence of liquidity policy $B_i$ borrows from $B_j$, with the later strictly preferring to either lend or invest in a small project, i.e., $\pi_{ij}^* > \pi_i$ and $\pi_j > \pi_{ji}^*$. However, in the presence of liquidity policy, the scenario changes and now both banks prefer an investment in a large project - $\pi_{ij}^* L > \pi_i^L$ and $\pi_{ji}^* L > \pi_j^L$. If $\pi_{ji}^* L > \pi_{ij}^* L$, i.e.:

$$[r_i^* - (1 - \omega_i) - e_i - r_j] - [r_j^* - (1 - \omega_j) - e_j - r_i] < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{[\rho^* + \gamma^* (1 - \rho^*)]},$$ \hfill (61)

then from (30) it follows that the role of banks is switched - $B_i$ becomes the lender and $B_j$, the borrower. Since $\rho^* + \gamma^* (1 - \rho^*) > \rho^*$, (61) implies that:

$$[r_i^* - (1 - \omega_i) - e_i - r_j] - [r_j^* - (1 - \omega_j) - e_j - r_i] < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*},$$ \hfill (62)
and thus, in the absence of liquidity policy, the profit of $B_j$ with a large project is larger than that of $B_i$ with an analogous asset, i.e., $\pi^*_ji > \pi^*_ij$. Hence:

$$\pi_j > \pi^*_ji > \pi^*_ij > \pi_i,$$

whereas in the presence of liquidity policy:

$$\pi^*_ji > \max\{\pi^*_ij, \pi^*_Lij\} \quad \text{and} \quad \pi^*_ij > \pi^*_ij.$$  \hspace{1cm} (64)

Liquidity policy then leads to a higher combined capitalization of $B_i$ and $B_j$ if:

$$\pi^*_ji > \pi^*_ij + \pi^*_Lij > \pi^*_ij + \pi_j.$$  \hspace{1cm} (65)

From $\pi^*_ji > \pi^*_ij$ and $\pi^*_ij > \pi^*_ij$, it follows that $\pi^*_ji > \pi^*_ij$. If $\pi^*_Lij > \pi^*_ij$, then $\pi^*_Lij > \pi^*_ij > \pi^*_ij$ implies $\pi^*_Lij > \pi_j$ and, therefore, (65) is satisfied - liquidity policy leads to an increase in the combined capitalization of banks. \hspace{1cm} □