Modelling Ireland’s exchange rates: from EMS to EMU

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Purchasing Power Parity: The Irish Experience Revisited

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ECB Website
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MODELLING
IRELAND’S
EXCHANGE RATES
FROM EMS TO EMU

by Derek Bond, Michael J. Harrison and Edward J. O’Brien
Non-stationarity and nonlinearity are closely related. Perron (1989) and Harrison and Bond (1992)

It is difficult to statistically distinguish between difference stationary series and nonlinear but stationary series.

Recent work in the area include: Lee (2004) and Hong (2005).
The Aims of Presentation

To explore the use two recent developments in econometric theory

- Dolado(2002) Fractional Augmented Dickey Fuller test (FADF)

To the behaviour of time series characteristics of financial series.
\{y_t\}_{t=0}^{\infty} \text{ is said to be integrated of order } d \text{ denoted by } I(d) \text{ if the series is differenced } d \text{ times before it is stationary.}

Classical analysis } d \text{ is integer. Normally either:}

\[ \Delta y_t = y_t - y_{t-1} \]

or

\[ y_t \]

is stationary
The restriction that $d$ is integer is relaxed.

Thus:

$$\Delta^d y_t = y_t - dy_{t-1} + \frac{1}{2!} d(d-1)y_{t-2} - \frac{1}{j!} d(d-1)\ldots(d-j+1)y_{t-j} \ldots$$
When $0 < d < 1$ it follows that long memory exists

If $0 < d < 0.5$ then $\{y_t\}_t^\infty = 0$ is stationary

if $0.5 \leq d < 1.0$ then the series $\{y_t\}_t^\infty = 0$ is non-stationary.
Testing for Fractional Integration

- Traditional methods use semi-parametric spectral estimates.
- Dolado (2002) approach is based on the distribution of the 't' statistic on $\phi$ from the generalised ADF regression:

\[
\Delta^{d_0} y_t = \phi \Delta^{d_1} y_{t-1} + \sum_{i=1}^{p} \zeta_i \Delta y_{t-i} + \nu_t
\]

where $\nu_t$ is a hypothesised white noise error.
- Dolado (2002) set $d_0$ equal to 1.
Increasingly Smooth Transition Regression being used
- rather arbitrary in choice of transition variable
An alternative is random fields regression.
Random field approach has relatively better small sample performance than a wide range of parametric and non-parametric alternatives including LSTR and ESTR.
Random Fields

- The basic model is of the form:
  \[ y_t = \mu(x_t) + \epsilon_t \]
  where the functional form \( \mu(x_t) \) is unknown and assumed to be the outcome of a random field.

- Hamilton’s conditional mean function \( \mu(x_t) \) as
  \[ \mu(x_t) = \alpha_0 + \alpha' x_t + \lambda m(\tilde{x}_t) \]
  where \( \tilde{x}_t = g \odot x_t \), \( g \) is a kx1 vector of parameters and \( \odot \) denotes the Hadamard product of matrices.
Gaussian random field

- Defined fully by its first two moments 0 and $H_k$.
- So it follows that

$$y_t = \alpha_0 + \alpha_1 x_t + u_t$$

where $u_t = \lambda m(\bar{x}_t) + \epsilon_t$ or in matrix form

$$y = X\beta + u$$  \hspace{1cm} (1)$$

where $\beta = (\alpha_0, \alpha_1)$ so it follows that:

$$u \sim N(0, \lambda^2 H_k + \sigma^2 I_T)$$  \hspace{1cm} (2)$$
Random Field Inference

- Can be viewed as a generalised least squares problem
- The profile maximum likelihood function can be obtained and estimated.
- Only problem is that the form of the covariance matrix is unknown.
- Hamilton derives $H_k$ as a simple moving average representation of the random field based on $g$ using a $L_2$ — norm measure.
Random Field Non-Linearity Tests

- Simple method of testing is to check if $\lambda$ is zero or not.
- MLE $\bar{\lambda}^2$ for fixed $g$ is consistent and asymptotically normal. Computationally complex.
- Simpler method is to construct Lagrange Multiplier test under the assumption of normality and linearity
- Provided the covariance function of the random field can be derived, for a fixed $g$ this only requires a single linear regression to be estimated.
Hamilton' Test

- Hamilton(2001) used covariance function based on the $L_2$ – norm derived the appropriate score vectors of first derivatives, for up to $k=5$, and the associated information matrix and proposed a form of the LM test for practical application.

- $\lambda^E_H$, the test statistic is distributed as $\chi_1^2$ under the null hypothesis.

- Problem of nuisance parameters under $H_1$
Dahl’s alternatives

- $\lambda_{OP}^E$, covariance matrix based on the $L_1 - norm$.
- $\lambda_{OP}^A$, covariance function depicted by a Taylor expansion.
- $g - test$ makes no assumption about either the covariance function or $\lambda$.
- $\lambda_{OP}^A$ and the $g - test$ have, in many circumstances, better power than other tests of nonlinearity.
estimating $\lambda$ and $g$ gives useful insights.

need to estimate reparameterised likelihood

$$
\eta(y, X : g, \zeta) = -\frac{T}{2} \ln(2\pi) - \frac{T}{2} \ln \hat{\sigma}^2_T(g, \zeta) - \frac{1}{2} \ln |W(X : g, \zeta)| - \frac{T}{2}
$$

$$\hat{\beta}_T(g, \zeta) = [X' W(X : g, \zeta)^{-1} X]^{-1} [X' W(X : g, \zeta)^{-1} y]
$$

$$\hat{\sigma}^2_T(g, \zeta) = \frac{1}{T} [y - X \hat{\beta}_T(g, \zeta)]' W(X : g, \zeta)^{-1} [y - X \hat{\beta}_T(g, \zeta)]
$$

where $\zeta = \lambda/\sigma$ and $W(X : g, \zeta) = \zeta^2 H_k + I_T$.

maximised with respect to $(g, \zeta)$
The theory of purchasing power parity (PPP) has become a major area of research in applied econometrics. Hypothesis is that national price levels should be equal when expressed in a common currency. If $s_t$ is the logarithm of the nominal exchange rate, $p_t$ and $p_t^*$ are the logarithms of the domestic and foreign price levels respectively and $q_t$ the logarithm of the real exchange rate in period $t = [1, 2, ..., T]$, then:

$$q_t = s_t - p_t + p_t^* \quad \forall t$$
The empirical analysis

- $q_t$ must be stationary for long-run PPP to hold
- Empirical work has been concerned with
  
  1. whether $q_t$ has a mean reversion tendency overtime or
  2. whether $s_t$ and $p_t, p_t^*$ move together overtime.

- Often the second has involved the equation:

  $$s_t = \alpha_0 + \beta_1 p_t + \beta_2 p_t^* + \epsilon_t$$
Exploratory Results

- Standard co-integration results do not support PPP
- Fractional analysis inconclusive
- Non-linear test generally support non-linear causal model
- Confusing results for real exchange rate
Modelling Results

- Indication that prices are non-linear
- Possibility that interest rates are also important
- Analysis suggests regime switching
Conclusions

- Non-linearity in relationships is important
- Discovering and Modelling is difficult
- Much more research needed
  - Parametric inference
  - Semi Parametric: e.g. relationship between ’d’ and frequency domain