

**University of California, Berkeley**

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**From the Selected Works of David D Nolte**

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# Historical Timelines in the Development of Phase Space

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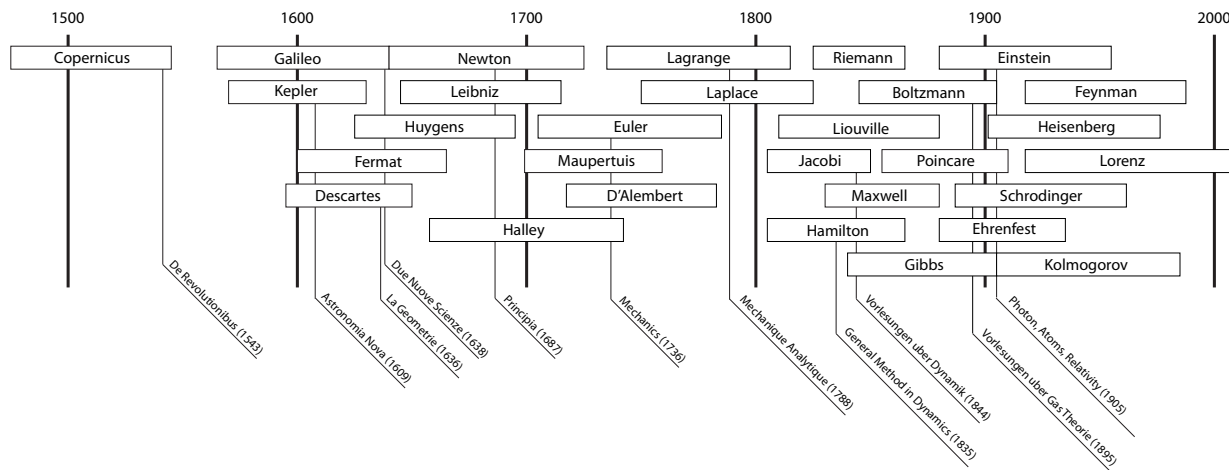


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# Historical Timelines of the Theory of Trajectories and Phase Space

David D. Nolte

Dynamics TimeLine (1500 - 2000)



More stories behind the development of phase space and its role in modern dynamics can be found at the WordPress Blog Site

<https://galileo-unbound.blog/>

## Chapter 2: A New Scientist

- 1564 Galileo born
- 1581 Enters University of Pisa
- 1585 Leaves Pisa without a degree
- 1586 Invents hydrostatic balance
- 1588 Receives lecturship in mathematics at Pisa

- 1592 Chair of mathematics at Univeristy of Padua
- 1595 Theory of the tides
- 1595 Invents military and geometric compass
- 1596 *Le Meccaniche* and the principle of horizontal inertia
  
- 1600 Bruno Giordano burned at the stake
- 1601 Death of Tycho Brahe
- 1609 Constructs his first telescope, makes observations of the moon
- 1610 Discovers 4 moons of Jupiter, *Starry Messenger (Sidereus Nuncius)*, appointed chief philosopher and mathematician of the Duke of Tuscany, moves to Florence, observes Saturn, Venus goes through phases like the moon
- 1611 Travels to Rome, inducted into the Lyncean Academy, name “telescope” is first used (47 years old)
- 1611 Scheiner discovers sunspots
- 1611 Meets Barberini, a cardinal
- 1613 Letters on sunspots published by Lincean Academy in Rome
- 1614 Denounced from the pulpit
- 1615 (April) Bellarmine writes an essay against Copernicus
- 1615 Investigated by the Inquisition
- 1615 Writes Letter to Christina, but does not publish it
- 1615 (December) travels to Rome and stays at Tuscan embassy
- 1616 (January) Francesco Ingoli publishes essay against Copernicus
- 1616 (March) Decree against copernicanism
- 1616 Publishes theory of tides, Galileo meets with Pope Paul V, Copernicus’ book is banned, Galileo warned not to support the Copernican system, Galileo decides not to reply to Ingoli, Galileo proposes eclipses of Jupiter’s moons to determine longitude at sea
- 1618 Three comets appear, Grassi gives a lecture not hostile to Galileo
- 1618 Galileo, through Mario Guiducci, publishes scathing attack on Grassi
- 1619 Jesuit Grassi (Sarsi) publishes attack on Galileo concerning 3 comets
- 1619 Marina Gamba dies, Galileo legitimizes his son Vincenzio
- 1619 Kepler’s third law
- 1623 Barberini becomes Urban VIII, *The Assayer* published (response to Grassi)
- 1624 Galileo visits Rome and Urban VIII
- 1629 Birth of his grandson Galileo
- 1630 Death of Johannes Kepler
- 1632 Publication of the *Dialogue Concerning the Two Chief World Systems*, Galileo is indicted by the Inquisition (68 years old)
- 1633 (February) Travels to Rome
- 1633 Convicted, abjurs, house arrest in Rome, then Siena, then home to Arcetri
- 1638 Blind, publication of *Two New Sciences*
- 1642 Dies (77 years old)

### Chapter 3: Galileo’s Trajectory

- 1583 Galileo Notices isochronism of the pendulum
- 1588 Receives lecturship in mathematics at Pisa
- 1589 – 1592 Work on projectile motion in Pisa
- 1592 Chair of mathematics at Univeristy of Padua
- 1596 *Le Meccaniche* and the principle of horizontal inertia
  
- 1600 Guidobaldo shared technique of colored ball
- 1602 Discovered isochronism of the pendulum (experimentally)
- 1604 First experiments on uniformly accelerated motion
- 1604 Wrote to Scarpi about the law of fall ( $s \approx t^2$ )
- 1607-1608 Identified trajectory as parabolic
- 1609 Velocity proportional to time
- 1632 Publication of the *Dialogue Concerning the Two Chief World Systems*, Galileo is indicted by the Inquisition (68 years old)
- 1636 *Letter to Christina* published in Augsburg in Latin and Italian
- 1638 Blind, publication of *Two New Sciences*
- 1641 Invented pendulum clock (in theory)
- 1642 Dies (77 years old)

#### Chapter 4: On the Shoulders of Giants

- 1644 Descartes' vortex theory of gravitation
- 1662 Fermat's principle
- 1669 – 1690 Huygens expands on Descartes' vortex theory
- 1698 Maupertuis born
  
- 1729 Maupertuis entered University in Basel. Studied under Johann Bernoulli
- 1736 Euler publishes *Mechanica sive motus scientia analytice exposita*
- 1737 Report on expedition to Lapland. Earth is oblate. Attacks Cassini.
- 1744 Maupertuis Principle of Least Action. Euler Principle of Least Action.
- 1745 Maupertuis becomes president of Berlin Academy. Paris Academy cancels his membership after a campaign against him by Cassini.
- 1746 Principle of Least Action for mass
- 1749 du Chatelet dies
- 1751 Samuel König disputes Maupertuis' priority
- 1756 Cassini dies. Maupertuis reinstated in the French Academy
- 1759 Maupertuis dies
- 1759 du Chatelet's French translation of Newton's *Principia* published posthumously
- 1760 Euler 3-body problem (two fixed centers and coplanar third body)
- 1760-1761 Lagrange, Variational calculus published in *Miscellanea Taurinensia*
- 1762 Beginning of the reign of Catherine the Great of Russia
- 1763 Euler colinear 3-body problem
- 1765 Euler publishes *Theoria motus corporum solidorum* on rotational mechanics
- 1766 Euler returns to St. Petersburg
- 1766 Lagrange arrives in Berlin

- 1772 Lagrange equilateral 3-body problem, *Essai sur le problème des trois corps*, 1772, Oeuvres tome 6
- 1775 Beginning of the American War of Independence
- 1776 Adam Smith *Wealth of Nations*
- 1781 William Herschel discovers Uranus
- 1783 Euler dies in St. Petersburg
- 1787 United States Constitution written
- 1787 Lagrange moves from Berlin to Paris
- 1788 Lagrange, *Mécanique analytique*
- 1789 Beginning of the French Revolution
- 1799 Pierre-Simon Laplace *Mécanique Céleste* (1799-1825)

## Chapter 5: Geometry on My Mind

- 1629 Fermat described higher-dim loci
- 1637 Descartes's *Geometry*
- 1649 van Schooten's commentary on Descartes *Geometry*
- 1694 Leibniz uses word "coordinate" in its modern usage
- 1697 Johann Bernoulli shortest distance between two points on convex surface
- 1732 Euler geodesic equations for implicit surfaces
- 1748 Euler defines modern usage of function
- 1801 Gauss calculates orbit of Ceres
- 1807 Fourier analysis
- 1807 Gauss arrives in Göttingen
- 1827 Karl Gauss establishes differential geometry of curved surfaces, *Disquisitiones generales circa superficies curvas*
- 1830 Bolyai and Lobachevsky publish on hyperbolic geometry
- 1834 Jacobi n-fold integrals and volumes of n-dim spheres
- 1836 Liouville-Sturm theorem
- 1838 Liouville's theorem
- 1841 Jacobi determinants
- 1843 Arthur Cayley systems of n-variables
- 1843 Hamilton discovers quaternions
- 1844 Hermann Grassman n-dim vector spaces, *Die Lineale Ausdehnungslehre*
- 1846 Julius Plücker *System der Geometrie des Raumes in neuer analytischer Behandlungsweise*
- 1848 "Vector" coined by Hamilton
- 1854 Riemann's habilitation lecture
- 1861 Riemann n-dim solution of heat conduction
- 1868 Publication of Riemann's Habilitation
- 1868 Boltzmann distribution in momentum space
- 1869 Christoffel and Lipschitz work on multiple dimensional analysis
- 1871 Betti refers to the n-ply of numbers as a "space".

- 1871 Klein publishes on non-euclidean geometry
- 1872 Jordan “Essay on the geometry of n-dimensions”
- 1872 Felix Klein’s “Erlangen Programme”
- 1872 Weierstrass’ Monster
- 1872 Dedekind cut
- 1872 Cantor paper on irrational numbers
- 1872 Cantor meets Dedekind
- 1874 Cantor beginning of set theory
- 1877 Cantor one-to-one correspondence between the line and n-dimensional space
- 1881 Gibbs codifies vector analysis
- 1883 Cantor set and staircase *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*
- 1884 Abbott publishes *Flatland*
- 1887 Peano vector methods in differential geometry
- 1890 Peano space filling curve
- 1891 Hilbert space filling curve
- 1888 Darboux discusses least action as geodesic
- 1889 Darboux vol. 2 treats dynamics as a point in d-dimensional space. Applies concepts of geodesics for trajectories.
- 1898 Ricci-Curbastro *Lessons on the Theory of Surfaces*
  
- 1902 Lebesgue integral
- 1904 Hilbert studies integral equations
- 1904 von Koch snowflake
- 1906 Frechet thesis on square summable sequences as infinite dimensional space
- 1908 Schmidt *Geometry in a Function Space*
- 1910 Brouwer proof of dimensional invariance
- 1913 Hilbert space named by Riesz
- 1914 Hilbert space used by Hausdorff
- 1915 Sierpinski fractal triangle
- 1918 Hausdorff non-integer dimensions
- 1918 Weyl’s book *Space, Time, Matter*
- 1918 Fatou and Julia fractals
- 1920 Banach space
- 1927 von Neumann axiomatic form of Hilbert Space
- 1935 Frechet full form of Hilbert Space
- 1967 Mandelbrot coast of Britain
- 1982 Mandelbrot’s book *The Fractal Geometry of Nature*

## Chapter 6: The Tangled Tale of Phase Space

- 1804 Jacobi born (1804 – 1851) in Potsdam
- 1804 Napoleon I Emperor of France
- 1806 William Rowan Hamilton born (1805 – 1865)
- 1807 Thomas Young describes “Energy” in his lecture notes on physics
- 1808 Bethoven performs his Fifth Symphony

- 1809 Joseph Liouville born (1809 – 1882)
- 1821 Hermann Ludwig Ferdinand von Helmholtz born (1821 – 1894)
- 1824 Carnot published *Reflections on the Motive Power of Fire*
- 1834 Jacobi n-fold integrals and volumes of n-dim spheres
- 1834-1835 Hamilton publishes his principle.
- 1836 Liouville-Sturm theorem
- 1837 Queen Victoria begins her reign as Queen of England
- 1838 Liouville develops his theorem on products of n differentials satisfying certain first-order differential equations. This becomes the classic reference to *Liouville's Theorem*.
- 1847 Helmholtz Conservation of Energy (force)
- 1849 Thomson makes first use of “Energy” (From reading Thomas Young’s lecture notes)
- 1850 Clausius establishes First law of Thermodynamics: Internal energy. Second law: Heat cannot flow unaided from cold to hot. Not explicitly stated as first and second laws
- 1851 Thomson names Clausius’ First and Second laws of Thermodynamics
- 1852 Thomson describes general dissipation of the universe (“energy” used in title)
- 1854 Thomson defined absolute temperature. First mathematical statement of 2<sup>nd</sup> law. Restricted to reversible processes
- 1854 Clausius stated Second Law of Thermodynamics as inequality
- 1857 Clausius constructs kinetic theory, Mean molecular speeds
- 1858 Clausius defines mean free path, Molecules have finite size. Clausius assumed that all molecules had the same speed
- 1860 Maxwell publishes first paper on kinetic theory. Distribution of speeds. Derivation of gas transport properties
- 1865 Loschmidt size of molecules
- 1865 Clausius names entropy
- 1868 Boltzmann adds (Boltzmann) factor to Maxwell distribution
- 1872 Boltzmann transport equation and H-theorem
- 1876 Loschmidt reversibility paradox
- 1877 Boltzmann  $S = k \log W$
- 1890 Poincare: Recurrence Theorem. Recurrence paradox with Second Law (1893)
- 1896 Zermelo criticizes Boltzmann
- 1896 Boltzmann posits direction of time to save his H-theorem
- 1898 Boltzmann *Vorlesungen über Gas Theorie*
- 1905 Boltzmann kinetic theory of matter in Encyklopädie der mathematischen Wissenschaften
- 1906 Boltzmann dies
- 1910 Paul Hertz uses “Phase Space” (Phasenraum)
- 1911 Ehrenfest’s article in Encyklopädie der mathematischen Wissenschaften
- 1913 A. Rosenthal writes the first paper using the phrase “phasenraum”, combining the work of Boltzmann and Poincaré. “Beweis der Unmöglichkeit ergodischer Gassysteme” (Ann. D. Physik, 42, 796 (1913))

- 1913 Plancheral, “Beweis der Unmöglichkeit ergodischer mechanischer Systeme” (Ann. D. Physik, 42, 1061 (1913). Also uses “Phasenraum”).

## Chapter 7: The Lens of Gravity

- 1697 Johann Bernoulli was first to find solution to shortest path between two points on a curved surface (1697).
- 1728 Euler found the geodesic equation.
- 1783 The pair 40 Eridani B/C was discovered by William Herschel on 31 January
- 1783 John Michell explains infalling object would travel faster than speed of light
- 1796 Laplace describes “dark stars” in *Exposition du system du Monde*
- 1827 The first orbit of a binary star computed by Félix Savary for the orbit of Xi Ursae Majoris.
- 1827 Gauss curvature *Theoriem Egregum*
- 1844 Bessel notices periodic displacement of Sirius with period of half a century
- 1844 The name “geodesic line” is attributed to Liouville.
- 1845 Buys Ballot used musicians with absolute pitch for the first experimental verification of the Doppler effect
- 1854 Riemann’s habilitationsschrift
- 1862 Discovery of Sirius B (a white dwarf)
- 1868 Darboux suggested motions in n-dimensions
- 1872 Lipshitz first to apply Riemannian geometry to the principle of least action.
- 1895 Hilbert arrives in Göttingen
- 1902 Minkowski arrives in Göttingen
- 1905 Einstein’s miracle year
- 1906 Poincaré describes Lorentz transformations as rotations in 4D
- 1907 Einstein has “happiest thought” in November
- 1907 Einstein’s relativity review in Jahrbuch
- 1908 Minkowski’s Space and Time lecture
- 1908 Einstein appointed to unpaid position at University of Bern
- 1909 Minkowski dies
- 1909 Einstein appointed associate professor of theoretical physics at U of Zürich
- 1910 40 Eridani B was discovered to be of spectral type A (white dwarf)
- 1910 Size and mass of Sirius B determined (heavy and small)
- 1911 Laue publishes first textbook on relativity theory
- 1911 Einstein accepts position at Prague
- 1911 Einstein goes to the limits of special relativity applied to gravitational fields
- 1912 Einstein’s two papers establish a scalar field theory of gravitation
- 1912 Einstein moves from Prague to ETH in Zürich in fall. Begins collaboration with Grossmann.
- 1913 Einstein EG paper
- 1914 Adams publishes spectrum of 40 Eridani B



- 1915 Sirius B determined to be also a low-luminosity type A white dwarf  
 1915 Einstein Completes paper  
 1916 Density of 40 Eridani B by Ernst Öpik  
 1916 Schwarzschild paper  
 1919 Eddington expedition to Principe  
 1920 Eddington paper on deflection of light by the sun  
 1922 Willem Luyten coins phrase “white dwarf”  
 1924 Eddington found a set of coordinates that eliminated the singularity at the Schwarzschild radius  
 1926 R. H. Fowler publishes paper on degenerate matter and composition of white dwarfs  
 1931 Chandrasekhar calculated the limit for collapse to white dwarf stars at 1.4MS  
 1933 Georges Lemaitre states the coordinate singularity was an artefact  
 1934 Walter Baade and Fritz Zwicky proposed the existence of the neutron star only a year after the discovery of the neutron by Sir James Chadwick.  
 1939 Oppenheimer and Snyder showed ultimate collapse of a 3M<sub>s</sub> “frozen star”  
 1958 David Finkelstein paper  
 1965 Antony Hewish and Samuel Okoye discovered "an unusual source of high radio brightness temperature in the Crab Nebula". This source turned out to be the Crab Nebula neutron star that resulted from the great supernova of 1054.  
 1967 Jocelyn Bell and Antony Hewish discovered regular radio pulses from CP 1919. This pulsar was later interpreted as an isolated, rotating neutron star.  
 1967 Wheeler’s “black hole” talk  
 1974 Joseph Taylor and Russell Hulse discovered the first binary pulsar, PSR B1913+16, which consists of two neutron stars (one seen as a pulsar) orbiting around their center of mass.
- 2015 LIGO detects gravitational waves on Sept. 14 from the merger of two black holes  
 2017 LIGO detects the merger of two neutron stars

## Chapter 8: On the Quantum Footpath

- 1885 Balmer Theory:  $\frac{1}{\lambda} = R_H \left( \frac{1}{4} - \frac{1}{n^2} \right)$   
 1897 J. J. Thomson discovered the electron  
 1904 Thomson plum pudding model of the atom  
 1911 Bohr PhD thesis filed. Studies on the electron theory of metals. Visited England.  
 1911 Rutherford nuclear model  
 1911 First Solvay conference  
 1911 “ultraviolet catastrophe” coined by Ehrenfest  
 1913 Bohr combined Rutherford’s nuclear atom with Planck’s quantum hypothesis:  
 1913 Bohr model  
 1913 Ehrenfest adiabatic hypothesis  
 1914-1916 Bohr at Manchester with Rutherford

- 1916 Bohr appointed Chair of Theoretical Physics at University of Copenhagen: a position that was made just for him
- 1916 Schwarzschild and Epstein introduce action-angle coordinates into quantum theory
- 1920 Heisenberg enters University of Munich to obtain his doctorate
- 1920 Bohr's Correspondence principle: Classical physics for large quantum numbers
- 1921 Bohr Founded Institute of Theoretical Physics (Copenhagen)
- 1922-1923 Heisenberg studies with Born, Franck and Hilbert at Göttingen while Sommerfeld is in the US on sabbatical.
- 1923 Heisenberg Doctorate. The exam does not go well. Unable to derive the resolving power of a microscope in response to question by Wien. Becomes Born's assistant at Göttingen.
- 1924 Heisenberg visits Niels Bohr in Copenhagen (and met Einstein?)
- 1924 Heisenberg Habilitation at Göttingen on anomalous Zeeman
- 1924 – 1925 Heisenberg worked with Bohr in Copenhagen, returned summer of 1925 to Göttingen
- 1924 Pauli exclusion principle and state occupancy
- 1924 de Broglie hypothesis extended wave-particle duality to matter
- 1924 Bohr Predicted Hafnium (72)
- 1924 Kronig's proposal for electron self spin
- 1924 Bose (Einstein)
- 1925 Heisenberg paper on quantum mechanics
- 1925 Dirac, reading proof from Heisenberg, recognized the analogy of noncommutativity with Poisson brackets and the correspondence with Hamiltonian mechanics.
- 1925 Uhlenbeck and Goudschmidt: spin
- 1926 Born, Heisenberg, Kramers: virtual oscillators at transition frequencies: Matrix mechanics (alternative to Bohr-Kramers-Slater 1924 model of orbits). Heisenberg was Born's student at Göttingen.
- 1926 Schrödinger wave mechanics
- 1927 de Broglie hypothesis confirmed by Davisson and Germer
- 1927 Complementarity by Bohr: wave-particle duality "*Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects.*"
- 1927 Heisenberg uncertainty principle (Heisenberg was in Copenhagen 1926 – 1927)
- 1927 Solvay Conference in Brussels
- 1928 Heisenberg to University of Leipzig
- 1928 Dirac relativistic QM equation
- 1929 de Broglie Nobel Prize
- 1930 Solvay Conference
- 1932 Heisenberg Nobel Prize
- 1932 von Neumann operator algebra
- 1933 Dirac Lagrangian form of QM (basis of Feynman path integral)
- 1933 Schrödinger and Dirac Nobel Prize
- 1935 EPR paper and Bohr's response.

- 1935 Schrodinger's cat  
 1939 Feynman graduates from MIT  
 1941 Heisenberg (head of German atomic project) visits Bohr in Copenhagen  
 1942 Feynman PhD at Princeton, "The Principle of Least Action in Quantum Mechanics"  
 1942 – 1945 Manhattan Project, Bethe-Feynman equation for fission yield  
 1943 Bohr escapes to Sweden in a fishing boat. Went on to England secretly.  
 1945 Pauli Nobel Prize  
 1945 Death of Feynman's wife Arline (married 4 years)  
 1945 Fall, Feynman arrives at Cornell ahead of Hans Bethe  
 1947 Shelter Island conference: Lamb Shift, did Kramer's give a talk suggesting that infinities could be subtracted?  
 1947 Fall, Dyson arrives at Cornell  
 1948 Pocono Manor, Pennsylvania, troubled unveiling of path integral formulation and Feynman diagrams, Schwinger's master presentation  
 1948 Feynman and Dirac. Summer drive across the US with Dyson  
 1949 Dyson joins IAS as a postdoc, trains a cohort of theorists in Feynman's technique  
 1949 Karplus and Kroll first g-factor calculation  
 1950 Feynman moves to Cal Tech  
 1965 Schwinger, Tomonaga and Feynman Nobel Prize  
 1967 Hans Bethe Nobel Prize

## Chapter 9: From Butterflies to Hurricanes

- 1760 Euler 3-body problem (two fixed centers and coplanar third body)  
 1763 Euler colinear 3-body problem  
 1772 Lagrange equilateral 3-body problem  
 1881-1886 Poincare memoires "Sur les courbes de 'finies par une equation differentielle"  
 1890 Poincare "Sur le probleme des trois corps et les equations de la dynamique". First-return map, Poincare recurrence theorem, stable and unstable manifolds  
 1892 – 1899 Poincare Celestial Mechanics  
 1892 Lyapunov *The General Problem of the Stability of Motion*  
 1899 Poincare homoclinic trajectory  
  
 1913 Birkhoff proves Poincaré's last geometric theorem, a special case of the three-body problem.  
 1927 van der Pol and van der Mark  
 1937 Coarse systems, Andronov and Pontryagin  
 1938 Morse theory  
 1942 Hopf bifurcation  
 1945 Cartwright and Littlewood study the van der Pol equation (Radar during WWII)  
 1954 Kolmogorov A. N., *On conservation of conditionally periodic motions for a small change in Hamilton's function.*  
 1960 Lorenz: 12 equations  
 1962 Moser *On Invariant Curves of Area-Preserving Mappings of an Annulus.*

- 1963 Arnold *Small denominators and problems of the stability of motion in classical and celestial mechanics*
- 1963 Lorenz: 3 equations
- 1964 Arnold diffusion
- 1965 Smale's horseshoe
- 1969 Chirikov standard map
- 1971 Ruelle-Takens (Ruelle coins phrase "strange attractor")
- 1972 "Butterfly Effect" given for Lorenz' talk (by Philip Merilees)
- 1975 Gollub-Swinney observe route to turbulence along lines of Ruelle
- 1975 Yorke coins "chaos theory"
- 1976 Robert May writes review article of the logistic map
- 1977 New York conference on bifurcation theory
- 1987 James Gleick *Chaos: Making a New Science*

## Chapter 10: Darwin in the Clockworks

- 1202 Fibonacci
- 1766 Thomas Robert Malthus born
- 1776 Adam Smith *The Wealth of Nations*
- 1798 Malthus "An Essay on the Principle of Population"
- 1817 Ricardo *Principles of Political Economy and Taxation*
- 1838 Cournot early equilibrium theory in duopoly
- 1848 John Stuart Mill
- 1848 Karl Marx *Communist Manifesto*
- 1859 Darwin *Origin of Species*
- 1867 Karl Marx *Das Kapital*
- 1871 Darwin *Descent of Man, and Selection in Relation to Sex*
- 1871 Jevons *Theory of Political Economy*
- 1871 Menger *Principles of Economics*
- 1874 Walrus *Éléments d'économie politique pure, or Elements of Pure Economics* (1954)
- 1890 Marshall *Principles of Economics*
- 1908 Hardy constant genetic variance
- 1910 Brouwer fixed point theorem
- 1910 Alfred J. Lotka autocatalytic chemical reactions
- 1913 Zermelo determinacy in chess
- 1922 Fisher dominance ratio
- 1922 Fisher mutations
- 1925 Lotka predator-prey in biomathematics
- 1926 Vita Volterra published same equations independently
- 1927 JBS Haldane (1892—1964) mutations
- 1928 von Neumann proves the minimax theorem

- 1930 Fisher ratio of sexes  
 1932 Wright Adaptive Landscape  
 1932 Haldane *The Causes of Evolution*  
 1933 Kolmogorov *Foundations of the Theory of Probability*  
 1934 Rudolph Carnap *The Logical Syntax of Language*  
 1936 John Maynard Keynes, *The General Theory of Employment, Interest and Money*  
 1936 Kolmogorov generalized predator-prey systems  
 1938 Borel symmetric payoff matrix  
 1942 Sewall Wright *Statistical Genetics and Evolution*  
 1943 McCulloch and Pitts *A Logical Calculus of Ideas Immanent in Nervous Activity*  
 1944 von Neumann and Morgenstern *Theory of Games and Economic Behavior*  
 1950 Prisoner's Dilemma simulated at Rand Corporation  
 1950 John Nash *Equilibrium points in n-person games* and *The Bargaining Problem*  
 1951 John Nash *Non-cooperative Games*  
 1952 McKinsey *Introduction to the Theory of Games* (first textbook)  
 1953 John Nash *Two-Person Cooperative Games*  
 1953 Watson and Crick DNA  
 1955 Braithwaite's *Theory of Games as a Tool for the Moral Philosopher*  
 1961 Lewontin *Evolution and the Theory of Games*  
 1962 Patrick Moran *The Statistical Processes of Evolutionary Theory*  
 1962 Linus Pauling molecular clock  
 1968 Motoo Kimura neutral theory of molecular evolution  
 1972 Maynard Smith introduces the evolutionary stable solution (ESS)  
 1972 Gould and Eldridge Punctuated equilibrium  
 1973 Maynard Smith and Price *The Logic of Animal Conflict*  
 1973 Black Scholes  
 1977 Eigen and Schuster *The Hypercycle*  
 1978 Replicator equation (Taylor and Jonker)  
 1982 Hopfield network  
 1982 John Maynard Smith *Evolution and the Theory of Games*  
 1984 R.Axelrod *The Evolution of Cooperation*

## Chapter 11: The Measure of Life

- 1642 Galileo dies  
 1649 Vincenzo Gamba dies  
 1656 Huygens invents pendulum clock  
 1665 Huygens observes "odd kind of sympathy" in synchronized clocks  
 1673 Huygens publishes *Horologium Oscillatorium sive de motu pendulorum*  
  
 1736 Euler Seven Bridges of Königsberg  
  
 1845 Kirchhoff's circuit laws  
 1852 Guthrie four color problem  
 1857 Cayley trees  
 1858 Hamiltonian cycles

- 1887 Cajal neural staining microscopy
  
- 1913 Michaelis Menten dynamics of enzymes
- 1924 Berger, Hans: neural oscillations (Berger invented the EEG)
- 1926 van der Pol dimensionless form of equation
- 1927 van der Pol periodic forcing
- 1943 McCulloch and Pits mathematical model of neural nets
- 1948 Wiener cybernetics
  
- 1952 Hodgkin and Huxley action potential model
- 1952 Turing instability model
- 1956 Sutherland cyclic AMP
- 1957 Broadbent and Hammersley bond percolation
- 1958 Rosenblatt perceptron
- 1959 Erdős and Renyi random graphs
- 1962 Cohen EGF discovered
- 1965 Sebeok coined zoosemiotics
- 1966 Mesarovich systems biology
- 1967 Winfree biological rhythms and coupled oscillators
- 1969 Glass Moire patterns in perception
- 1970 Rodbell G-protein
- 1971 phrase “strange attractor” coined (Ruelle)
- 1972 phrase “signal transduction” coined (Rensing)
- 1975 phrase “chaos theory” coined (Yorke)
- 1975 Werbos backpropagation
- 1975 Kuramoto transition
- 1976 Robert May logistic map
- 1977 Mackey-Glass equation and dynamical disease
- 1982 Hopfield network
- 1990 Strogatz and Murillo pulse-coupled oscillators
- 1997 Tomita systems biology of a cell
- 1998 Strogatz and Watts Small World network
- 1999 Barabasi Scale Free networks
  
- 2000 Sequencing of the human genome