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A Theory of Military Dictatorships

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Abstract

We investigate how nondemocratic regimes use the military and how this can lead to the emergence of military dictatorships. Under some conditions the elite build a strong military that behaves as a perfect agent and, in other circumstances, the military may turn against them. Once transition to democracy takes place, a strong military poses a threat against the nascent democratic regime until it is reformed. The role of income inequality and natural resources on the political equilibrium in nondemocracies and in nascent democracies is analyzed. We show how a greater national defense role of the military may facilitate democratic consolidation.

Keywords: coups, democracy, military, nondemocracy, political economy, political transitions.

JEL Classification: H2, N10, N40, P16.

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"The class that bears the lance or holds the musket regularly forces its rule upon the class that handles the spade or pushes the shuttle." Gaetano Mosca (1939 p. 228).

Nondemocratic regimes almost always rely on some degree of repression against competing groups. This repression is often exercised by a specialized branch of the state, *the military*.[1] Despite the prevalence of nondemocratic regimes throughout history and the important role played by the military in such regimes, the typical assumption is that the military is a "perfect agent" of some social group, such as the elite. There has been little systematic analysis of why and how the military uses its coercive powers to support a nondemocratic regime rather than setting up a regime more in line with its own preferences. This question is relevant since, while many nondemocratic regimes survive with the support of the military, there are also numerous examples of *military dictatorships* that have emerged either as a result of a coup against a nondemocratic regime or against the subsequent democratic government.

In this paper, we take a first step in the analysis of the role of the military in nondemocratic regimes and develop a theory of military dictatorships. At the center of our approach is the agency relationship between the elite in oligarchic regimes and the military.[2] The main idea is that creating a powerful military is a double-edge sword for the elite. On the one hand, a more powerful military is more effective in preventing transitions to democracy. On the other hand, a more powerful military necessitates either greater concessions to the military or raises the risk of a military takeover. We investigate the conditions under which the military will act as the agent of the elite in nondemocratic regimes (oligarchies) and the conditions under which oligarchies will turn into military dictatorships. Our approach also sheds light on the role of the military in coups against democracy. If the elite create a powerful military to prevent democratization, then the military also plays an important role in democratic politics until it is reformed, and such reform is not instantaneous.[3] In particular, we show that faced with a powerful military, a newly-emerging democratic regime will either need to make costly concessions or face a high probability of a coup. This coup threat disappears once the military is reformed. Interestingly, however, it is the anticipation that the military will be reformed

[1] Throughout the paper, the military may be thought to include the secret police and other law-enforcement agencies. We also use the terms the "military" and the "army" interchangeably.

[2] Since we draw a distinction between military dictatorships and nondemocratic regimes controlled by the elite, we use the term *oligarchy* to refer to the latter and nondemocracy as a general term encompassing both military dictatorships and oligarchies.

[3] This assumption is consistent with Guillermo A. O'Donnell and Philipp C. Schmitter's (1986) emphasis that the power of the army plays an important role at the early stages of democratic transitions. For instance, a relatively weak army may be easier to reform than a stronger one. Our model will show that this is generally true, but also that weaker militaries may sometimes be more difficult to control.

as soon as the opportunity arises that makes it difficult to control the military during the early phases of a democratic regime—because this creates a *commitment problem*, making it impossible for democratic governments to make credible promises to compensate soldiers for not taking actions against democracy.

More specifically, our model economy consists of two groups, the rich elite and the citizens, distinguished by their incomes (endowments). Democracy leads to redistributive policies, in particular, to the provision of public goods, which are beneficial for the citizens and costly for the rich elite. Consequently, starting from an *oligarchy* in which they hold power, the elite are unwilling to allow a transition to democracy. The only way they can prevent this is by creating a specialized unit of the state, the military, responsible for using force and repressing demands for democratization.[4] A powerful military, however, is not only effective in preventing a transition to democracy but also creates a *political moral hazard problem* because it can turn against the elite and take direct control of the government (for example, in order to create greater redistribution towards its own members). Consequently, the elite have three potential strategies in oligarchy: (1) no repression, thus allowing a rapid (*smooth*) transition to democracy; (2) repression, while also paying soldiers an *efficiency wage* so as to *prevent* military takeovers; (3) repression without significant concessions to soldiers, thus opting for *non-prevention* or facing the risk of a military takeover.

We characterize the equilibria in this environment and analyze the role of the military in politics. The presence of a large (strong) military changes both democratic and nondemocratic politics. If democracy inherits a large military from the previous nondemocratic regime, then it will also be confronted with a choice between making concessions to the military and facing a coup threat. The decisive voter in democracy always wishes to prevent coups but this may not be possible. In particular, soldiers realize that when the opportunity arises, democracy will reform the military reducing their rents. Since democracy cannot commit to not reforming the military when it has the chance to do so, it can only make current concessions to soldiers (since promises of future concessions are not credible) and current concessions may not be sufficient to compensate the soldiers for the prospect of a military dictatorship. Consequently, societies in which nondemocratic regimes in the past have chosen large militaries may have

[4] In principle, the elite may prevent democratization by using some combination of "carrot" and of "stick," that is, not only by using repression but also by making concessions and promising redistribution of income to (some of) the poor. The scope and limitations of such promises of redistribution have been analyzed in previous work (see Daron Acemoglu and James A. Robinson, 2000, 2006, 2008) and here we ignore them for simplicity. We also ignore the possibility of cooptation of some subset of the citizens using various means, such as the distribution of public jobs (see, for example, Acemoglu, Davide Ticchi and Andrea Vindigni, 2006).

difficulty consolidating democracy and may instead end up with military dictatorships. This result is not only intuitive but also provides us with a particular reason why social conflict in nondemocracy may create costs for (future) democracies. More specifically, the desire of the rich to prevent democratization by bequeathing a large army to democracy may lead to worse economic performance during democracy because of the conflict between citizens and soldiers that this induces.

In oligarchy, whether the elite prefer to set up a large military depends on the effectiveness of the military and on the extent of inequality. When the military is not very effective or inequality is limited, the elite prefer to allow a smooth transition to democracy, because such a regime will not be highly redistributive (while repression is likely to fail). When military repression is likely to be effective and there is sufficient inequality, the elite may prefer to build a large military for repression and deal with the political agency problem by paying the military an efficiency wage. This equilibrium configuration will therefore correspond to a situation in which the military is (effectively) an agent of the elite, which is the presumption in the existing literature. However, we also show that under certain circumstances the elite may prefer to use repression but not pay high wages to soldiers, thus allowing military coups against their own regime to take place along the equilibrium path. In this case, nondemocratic regimes persist due to the repression of the citizens, though the military undertaking the repression is not an agent of the elite and acts in its own interests (and attempts a coup against the elite when there is an opportunity for doing so).

Our model also highlights a new interaction between inequality and the size and composition of government spending. When inequality is low, the society is likely to become democratic rapidly; but the amount of redistributive spending is relatively low, and most of it is in the form of public goods. As the level of inequality increases and the society remains democratic, the size of the government (the amount of public good provision) increases. In societies with very high levels of inequality, however, the society is more likely to be nondemocratic (either oligarchic or a military dictatorship), and in these cases, there will be little spending on public goods and greater (perhaps substantial) spending on the military.

We then enrich our baseline model by introducing natural resources. Natural resources increase the *political stakes* because soldiers will be able to capture the natural resource rents if they take power. As a result, natural resource abundance makes democracies more likely to fall to military coups. The effect of natural resources on oligarchic regimes is ambiguous, however. On the one hand, the oligarchic regime has a stronger preference for repression and may be able to use the income from natural resources in order to buy off the military. On the other, the military is more tempted to undertake coups against the oligarchic regime.

Our baseline model abstracts from the national defense role of the military, so that the only use of military coercion is in domestic politics. At the end of the paper, we also use our model to investigate a potentially important interaction between international and domestic politics. In particular, we investigate the national defense role of the military on democratization. Somewhat paradoxically, we find that when the army is more important for national defense, democratic consolidation becomes more likely. The reason is that, in the absence of an international role for the military, the citizens are unable to commit to maintaining a strong military. The presence of international threats makes the promises of the citizens more credible, because democracy also needs the military, and facilitates democratic consolidation.

The two building blocks of our approach are that the military should be considered as a potentially self-interested body—or in fact, a collection of self-interested individuals—and that there should be a distinction between nondemocratic regimes controlled by the economic elite ("oligarchies") and military dictatorships.[5] The political science literature provides support to both and also highlights the important role that the military plays in politics. The self-interest of soldiers and the corporate self-interest of the army are major themes of the political science literature on military dictatorships, emphasized by, among others, Samuel A. Finer (1976), Eric A. Nordilinger (1977) and Martin C. Needler (1987).[6] Nordilinger (1977 p. 78), for example, argues that

> "The great majority of coups are partly, primarily, or entirely motivated by the defense or enactment of the military's corporate interests."

Similarly, Needler (1987 p. 59) observes:

> "... the military typically intervenes in politics from a combination of motives in which defense of the institutional interests of the military itself predominates, although those interests are frequently construed so as to be complementary to the economic interests of the economic elite."

[5] This contrasts with existing models of democratic transitions or coups, such as those in Acemoglu and Robinson (2001, 2006), where the military is assumed to be a perfect agent of the elite. This same perspective is often adopted in Much over the political science and sociology literatures on comparative development (e.g., Barrington Moore, 1966, Gregory Luebbert, 1991).

[6] In our model, "corporate self-interest" of the military corresponds to high wages for all soldiers. In practice, it is more likely to correspond to high income for high-level officers as well as higher levels of defense spending and foreign policies in line with the preferences of the military.

The distinction between oligarchic regimes and the military dictatorships is also well rooted in a large political science literature (see for example the survey in Paul Brooker, 2000, and the references there). Samuel P. Huntington (1968) and Finer (1976), for example, emphasize the prevalence of authoritarian elite-controlled regimes supported by the military, which are similar to our oligarchic regimes, and contrast these with military dictatorships. Examples of the former include the dictatorship that Getulio Vargas established in Brazil in 1937, which was a mainly civilian authoritarian regime, relying on the support of the military for its political survival, and other Central and South American regimes formed at roughly the same time. More recent examples include Marcos's long lasting regime in the Philippines and President Fujimori's regime's in Peru, which was established following his de facto coup to extend his rule and powers beyond their constitutional limits. Both of these regimes were backed by the army, but the military establishment did not have important decision-making powers.

Perhaps more common in practice are military dictatorships, where the military or a subset of officers are in direct control. Such military dictatorships are studied in detail by Brooker (2000), Claude E. Jr. Welch and Arthur K. Smith (1974), Amos Perlmutter (1977, 1981), and Nordlinger (1977). Contemporary examples include the regimes established in Pakistan by General Ayub Khan, by General Muhammad Zia-ul-Haq and by General Pervez Musharraf, the regimes established in Turkey after the coups in 1960, 1971 and 1980, in Guatemala after the coup of 1954 under the leadership Carlos Catrillo Armas, in El Salvador in 1956 with Oscar Osorio's government, in Brazil after the overthrow of President Joao Goulart's government in 1964, and in Greece after the military coup of 1967. The military has also been the dominant political force in Thailand since the 1932 coup and has repeatedly intervened in politics whenever it perceived a threat to its own power by nascent civilian political institutions.[7]

The contrast between Costa Rica and other Central American countries highlights the role of the military in the emergence and consolidation of democracy (or lack thereof). During the 19^{th} century, Costa Rica did not experience predatory caudillos (who were typically influential in the political and economic life in much of the rest of Latin America). Between 1891 and 1948, there was a single coup in Costa Rica, followed by a brief dictatorship. After this

[7] It is also possible to give a slightly different (somewhat more speculative) interpretation to the possibility of non-prevention in oligarchy. According to this interpretation, the case where the elite allow a powerful military to form but do not take steps to prevent coups can be viewed as an implicit support for military dictatorships by the elite. An example of this is the experience of Peru in the early 1930s. The rise of the Alianza Popular Revolucionaria Americana (a violent revolutionary movement) led the Peruvian elites to support a coup d'état by Colonel Sanchez Cerro. Alain Roquié (1987 p. 115) describes this as follows: "The ruling classes that had long been civilian in their orientation put aside their distrust of the military and supported the colonel's coup. The massacres in Trujillio in 1932 involving the army and the APRA were to establish a long-lasting defensive alliance between the military and the upper bourgeoise."

episode, in 1949, the Costa Rican military was demobilized and essentially disbanded. The successful democratic consolidation of Costa Rica since the mid-twentieth century stands in contrast to the different political development paths pursued by other Central American nations. For example, highly repressive military dictatorships were established in the 1950s both in Guatemala and in El Salvador. Consistent with our emphasis in this paper, the militaries, which were initially created by the elite for repression of the lower strata in these highly polarized societies, ultimately became strong enough to seize power and established their own dictatorships. In Honduras and Nicaragua, instead, traditional oligarchic regimes led by the civilian elite, but supported by a significant military element, emerged during this time period (e.g., Richard Millet, 1977, Harry E. Vanden and Gary Prevost, 2002).

Our paper is a contribution to a number of distinct literatures. First, there is now a substantial literature on political transitions, but to the best of our knowledge, no paper in this literature models the relationship between the elite and the military (see Acemoglu and Robinson, 2006, for an overview of this literature). Consequently, this literature does not distinguish between oligarchic regimes and military dictatorships discussed above.

Second, there is a large literature on political agency in democracies, where citizens try to control politicians using elections and other methods (e.g., Robert Barro, 1973, John Ferejohn, 1986, Torsten Persson, Gérard Roland and Guido Tabellini, 1997, Acemoglu, 2005, Acemoglu, Michael Golosov and Aleh Tsyvinski, 2008, Alberto Alesina and Tabellini, 2007, Timothy Besley, 2006). In contrast, the principal-agent relationship between the elite and the military has not been investigated. While several recent works, most notably Acemoglu, Robinson and Thierry Verdier (2004), Besley and Robinson (2007), Georgy Egorov and Konstantin Sonin (2004), Alexandre Debs (2007), and Ticchi and Vindigni (2003), study certain aspects of the internal organization of nondemocratic regimes, they neither provide a framework for the analysis of the emergence of military dictatorships nor investigate whether the military will act as an agent of the elite.

Third, the recent literature in comparative politics of public finance (e.g., Persson and Tabellini, 2003) investigates the influence of different types of democratic institutions on fiscal policy and economic outcomes, but does not investigate the impact of different types of nondemocratic regimes, and in particular, the contrast between oligarchic regimes and military dictatorships. Our paper adds to this literature by modeling the impact of different types of nondemocratic institutions on fiscal policy and economic outcomes. In particular, a distinctive implication of our theory in this respect is that nondemocracies should typically have more military spending than democracies.[8]

Finally, as mentioned above, there is a substantial political science literature on military dictatorships, though this literature does not provide formal models of the relationship between the military and the elite, nor does it approach it as an agency problem. Some of the important contributions in this literature have been cited above.

The rest of the paper is organized as follows. Section 1 presents our basic model. Section 2 characterizes the equilibria and presents our main results. Section 3 presents a number of extensions, in particular for the analysis of the implications of natural resources and international threats. Section 4 concludes. Some of proofs omitted from the text are contained in Appendix A. Appendix B, which presents a glossary of notation used in the model of Section 1, and Appendix C, which contains other omitted proofs, are available online.

1 Basic Model

We consider an infinite horizon economy in discrete time with a unique final good. Each agent j at time $t = 0$ maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} + \chi_{j,t} G_t \right), \tag{1}$$

where $\mathbb{E}_0$ is the expectation at time $t = 0$, $\beta \in (0, 1)$ is the discount factor, $c_{j,t} \geq 0$ denotes the consumption of the agent in terms of the final good, $G_t \geq 0$ is the amount of public good provided at time t, and $\chi_{j,t} \in \{0, 1\}$ is an indicator function denoting the occupational choice of the agent. This variable determines whether the individual benefits from the public good.

The total population of the society is normalized to 1. Of those $1 - n > 1/2$ have low skills and can produce $A^L \geq 0$, while the remaining n agents are highly skilled and can produce $A^H \geq A^L$. We will often refer to the (rich) high-skill agents as the "elite" since at the beginning they will be in control of the political system, and we will refer to low-skill agents as the "citizens." There are two occupations: producer and soldier. With a slight abuse of notation, we use the subscript $j \in \{L, M, H\}$ to also denote low-skill producers, military (soldiers) and high-skill producers. Soldiers do not produce any output, while producers generate income A^L or A^H depending on their skill level. To simplify the analysis and reduce notation, we assume throughout that only low-skill agents can become soldiers.[9]

[8] This result is consistent with the evidence presented in Casey B. Mulligan, Ricard Gil and Xavier Sala-i-Martin (2004), that nondemocratic regimes spend more on the military than democracies.

[9] This is natural, since military wages are more attractive to low-skill than to high-skill agents. We could introduce an additional constraint to ensure that the income of rich agents is greater than those of soldiers whenever the military is recruiting, though we do not do so to simplify notation.

Furthermore, we also assume that $\chi_{j,t} = 1$ for $j = L$ and $\chi_{j,t} = 0$ otherwise. This implies that only production workers benefit from the public good, for example, because the public good corresponds to services that rich agents and the military receive through other means (such as health care or schooling) or because the public good is associated with roads and other infrastructures mostly used by low-skill production workers. This assumption is adopted to simplify the expressions and has no effect on our qualitative results.

The size of the military at time t is denoted by x_t. We normalize the size of the military necessary for national defense to 0, thus the only reason for $x_t > 0$ is repression in domestic politics. We simplify the analysis by assuming that x_t takes one of two values, $x_t \in \{0, \bar{x}\}$, where $\bar{x} > 0$ is size of the military necessary for repression. We denote the decision to build an army ("strong" or "large" military) at time t, so that $x_t = \bar{x}$, by $a_t \in \{0, 1\}$, with $a_t = 1$. The government (social group) in power chooses $a_t \in \{0, 1\}$, except that, as will be described below, it may take a while for a newly-emerging democracy to be able to reform (disband) an already-existing army by choosing $a_t = 0$ (that is, there will be further constraints on the choice of a_t in democracy). We also assume throughout that

$$\bar{x} < 1/2 - n,$$

so that low-skill producers are always the absolute majority in the population. This implies that, given the policy instruments specified below, in majoritarian elections the median voter will always be a low-skill producer.

Aggregate (pre-tax) output at time t will be

$$Y_t = \varphi_t \left((1 - n - x_t) A^L + n A^H \right), \tag{2}$$

where $\varphi_t \in \{1 - \phi, 1\}$ captures potential distortions from coups with $\phi \in (0, 1)$. When there is no coup attempt (against oligarchy or democracy), we have $\varphi_t = 1$. Instead, when there is a coup attempt, a fraction ϕ of production is lost due to the disruption created by the coup, thus $\varphi_t = 1 - \phi < 1$. Let us also denote

$$Y \equiv (1 - n) A^L + n A^H, \tag{3}$$

as the *potential output* of the economy, which will apply when the size of the military is equal to 0 and there are no disruptions from coups.

Aggregate output can be taxed at the rate $\tau_t \in [0, 1]$ to raise revenue for public good provision and to pay the salaries of soldiers. We model the distortion of the costs of taxation in a simple reduced-form manner: when the tax rate is τ_t, a fraction $C(\tau_t)$ of the output (thus a total of $C(\tau_t)Y_t$) will be lost due to tax distortions. These may result from distortions resulting because taxes discourage labor supply or savings, or because of the administrative costs of collecting taxes. This fiscal technology implies that when the tax rate is equal to τ_t, government revenues per unit of production will be

$$\tau_t - C(\tau_t).$$

We assume that $C : [0,1] \to \mathbb{R}_+$ is a continuously differentiable and strictly convex function that satisfies the following Inada-type conditions (which will ensure interior solutions): $C(0) = 0$ (so that there are no distortions without taxation), $C'(0) = 0$ and $C'(1) > 1$. Let us also define $\hat{\tau} \in (0,1)$ as the level of taxation at which fiscal revenues are maximized (i.e., the peak of the Laffer curve), which is uniquely defined by

$$\hat{\tau} \equiv \left(C'\right)^{-1}(1), \tag{4}$$

where $(C')^{-1}$ is the inverse of the derivative of the C function. This tax rate $\hat{\tau}$ is strictly between 0 and 1 because of the Inada conditions.

Finally, without loss of any generality, we parameterize A^L and A^H as

$$A^L \equiv \frac{1-\theta}{1-n}Y \text{ and } A^H \equiv \frac{\theta}{n}Y,$$

for some $\theta \in (n,1)$. This parameterization implies that a higher θ corresponds to greater inequality. We do not make the dependence of A^L and A^H on θ explicit unless this is necessary for emphasis.

We will represent the economy as a dynamic game between soldiers, low-skill producers (citizens) and the elite. As explained further below, given the policy instruments, there is no conflict within the groups, so we can suppose without any loss of generality that a representative agent from each group (e.g., the commander of the army, a decisive voter in democracy, or a representative agent in oligarchy) makes the relevant policy choices.[10]

In principle, there are two state variables in this game. The first one is the size of the military from the previous period, $x_{t-1} \in \{0, \bar{x}\}$. The second is the political regime, denoted by s_t, which takes one of three values, democracy D, oligarchy (elite control) E, or military M. The size of the military from the previous period, x_{t-1}, only matters because immediately

[10] Alternatively, we could consider a citizen-candidate model (e.g., Besley and Stephen Coate, 1997) in which in democracy all agents vote, in military only the soldiers vote, and in oligarchy only the rich elite vote. The results in this case are identical to those presented below, but the analysis requires additional notation.

following a transition to democracy (without a coup attempt) it may not be possible for the democratic government to choose $a_t = 0$ and reform the military. Instead, we assume that they will have to wait for one period before being able to reform the military. This assumption captures the realistic feature that once a large army is in place, a new and potentially weak democratic government may not be able to disband the army immediately. The fact that it takes only one period for it to be able to do so is only for simplicity and in subsection 3.3, we consider the case where the opportunity to reform the military arises stochastically.

This structure simplifies not only the analysis but also the notation. In particular, instead of carrying x_{t-1} as a state variable we can define an additional regime, "transitional" democracy TD, which occurs if, and only if, $x_{t-1} = \bar{x}$, $s_{t-1} = E$, and there has been no coup attempt. Loosely speaking, transitional democracy corresponds to a situation in which the majority (low-skill producers) has *de jure* political power but this is constrained by the *de facto* political power of the military. The only difference between $s_t = TD$ and $s_t = D$ is that with $s_t = TD$, $a_t = 0$ is not possible. Otherwise, in both regimes, a representative low-skill producer is in power. Since in our baseline model transitional democracy lasts for one period, $s_t = TD$ is immediately followed by $s_{t+1} = D$, unless there is a successful military coup, in which case it is followed by $s_{t+1} = M$. Given this description, we represent the state of the dynamic game by $s_t \in \mathcal{S} \equiv \{D, E, M, TD\}$.

Key policy decisions are made by the government in power. The policy decisions are a linear tax rate $\tau_t \in [0, 1]$ on the income of the producers, the level of public good provision, $G_t \geq 0$, wage for soldiers, $w_t \geq 0$, and the decision regarding the size of the army, $a_t \in \{0, 1\}$. In addition to a_t, τ_t, G_t and w_t, the military (the military commander) decides whether or not to undertake a coup against the regime in power (if the military is active), which is denoted by $\psi_t \in \{0, 1\}$, with $\psi_t = 1$ corresponding to a coup attempt, and, if $\psi_t = 0$, it also decides whether or not to repress the citizens in oligarchy, which is denoted by $\rho_t \in \{0, 1\}$, with $\rho_t = 1$ corresponding to repression. (The glossary in Appendix B summarizes the notation). In addition, individuals should have a decision of whether to apply to an army position and also soldiers should have a decision to quit the military. Both of these decisions translate into a *participation constraint* that the value to a low-skill agent in the military should be higher than the value to a low-skill producer. Assumption 1 below ensures that this is always the case and to simplify notation, we do not introduce these additional decisions explicitly.[11]

If in oligarchy the elite choose $a_t = 0$, then there is a *smooth transition* to democracy, in

[11] In other words, it will always be a best response for low-skill producers to apply to the military and it is never a best response for soldiers to quit the military.

particular, $s_{t+1} = D$ following $s_t = E$. The important point here is that when the elite choose smooth transition, there is no transitional democracy since the oligarchic regime has not set up a military. In contrast, if in oligarchy, the military is present but chooses not to repress ($\rho_t = 0$), then the regime transitions to transitional democracy, that is, $s_{t+1} = TD$. Finally, if $a_t = 1$ and $\rho_t = 1$, so that the military is present and chooses to repress the citizens, then transition to democracy takes place with probability $\pi \in [0, 1]$. Therefore, π represents an inverse measure of the effectiveness of the military repression, with $\pi = 0$ corresponding to the case in which military repression is fully effective.

We also assume that when the military attempts a coup against either regime, which, in both cases, is denoted by $\psi_t = 1$, it succeeds with probability $\gamma \in [0, 1]$. If the coup succeeds, then a military dictatorship, $s_{t+1} = M$, emerges. To simplify the analysis, we assume that $s = M$ is *absorbing*, so the society will remain as a military dictatorship if a coup ever succeeds (and in equilibrium, $s_t = D$ will also be absorbing). However, if a military coup fails, then we immediately have $s_{t+1} = D$ regardless of the regime at time t; if the regime at time t is $s_t = TD$, then the transitional period will be over and the army will be reformed at $t+1$; if, on the other hand, $s_t = E$, then the conflict between the military and the elite implies that there is no effective repression and a consolidated democratic regime emerges as democracy can use the window of opportunity resulting from the failure of the coup to reform the military. This description thus implies that the transitional democracy regime, $s = TD$, only arises following failed repression.[12] If there is no army, there is no repression, so democracy is automatically consolidated (and thus $s = D$). Finally, recall that when the military attempts a coup ($\psi_t = 1$), the society suffers an income loss and $\varphi_t = 1 - \phi < 1$ in equation (2). Figure 1 depicts the game forms starting from oligarchy ($s_t = E$) and transitional democracy ($s_t = TD$).

Throughout we adopt the convention that fiscal policies enacted at time t are implemented even if there is regime change and the new regime starts enacting policies from $t + 1$ onwards. Moreover, the military wage w_t announced by any non-military regime is conditional on both repression and no coup attempt. These wages are withheld if there is a coup attempt.

The government budget constraint at time t can therefore be written as

$$w_t x_t + G_t \leq (\tau_t - C(\tau_t)) Y_t, \tag{5}$$

where Y_t is aggregate income at time t given in (2), $(\tau_t - C(\tau_t)) Y_t$ is total tax revenue resulting from the linear tax τ_t, and the left-hand side is the total outlays of the government, consisting

[12] This regime can also arise if there is a military but it does not undertake a repression, which is equivalent to repression failing with probability 1.

of spending on the military, $w_t x_t$, and public good expenditures, G_t, which are determined as residual (see footnote 13).

We now summarize the timing of events. At time t, the economy starts with the state variable $s_t \in \mathcal{S}$, which determines the group in power.

1. Unless $s_t = TD$, the group in power chooses $a_t \in \{0, 1\}$ and announces a fiscal policy vector (τ_t, w_t, G_t) that satisfies the government budget constraint (5). If $s_t = TD$, then $a_t = 1$ and the group in power only chooses (τ_t, w_t, G_t).

2. In oligarchy, if there is no military ($a_t = x_t = 0$), then there is a transition to fully-consolidated democracy and $s_{t+1} = D$.

3. When $x_t = \bar{x}$, in democracy and in oligarchy, the military commander decides whether or not to attempt a coup $\psi_t \in \{0, 1\}$. If a coup is attempted, it is successful with probability γ and a military dictatorship is established (and $s_{t+1} = M$). If a coup fails, a consolidated democratic regime, $s_{t+1} = D$, emerges next period.

4. If the elite have formed a military ($x_t = \bar{x}$), and the military does not attempt a coup $\psi_t = 0$, then it also decides whether or not to repress the citizens, $\rho_t \in \{0, 1\}$. If repression fails (probability π) or if the military chooses not to repress, then $s_{t+1} = TD$.

5. Taxes are collected and wages are paid according to the announced policy vector (τ_t, w_t, G_t) if there is no military coup attempt. If there is such an attempt, then $w_t = 0$.[13]

Finally, we assume that the society starts with $s = E$, i.e., an oligarchic regime, and with $x_{-1} = 0$, so the elite are free to form a military of size $\bar{x}$ or leave $x_0 = 0$ in the initial period.

2 Characterization of Equilibria

We now characterize the Markov Perfect Equilibria of the game described in the previous section. Markov Perfect Equilibria are both simple and natural in the current context. In subsection 2.7, we show that our main results generalize to Subgame Perfect Equilibria.

2.1 Definition of Equilibrium

We first focus on pure strategy Markov Perfect Equilibria (MPE). Let $h^{t,k}$ denote the history of the dynamic game described above up to time t and stage k of the stage game of time t, and let $H^{t,k}$ be the set of such histories. Strategies assign actions for any history in $H^{t,k}$. Markovian

[13] This description of the timing of events makes it clear that when a particular vector of policies, (τ_t, w_t, G_t), is announced, it is not known for sure whether this will satisfy the government budget constraint, (5), because there might be a coup attempt reducing income and tax revenues, and also removing the burden of military expenditures. In such cases, we assume that the amount of public good provision, G_t, is the "residual claimant," so that if (τ_t, w_t, G_t) does not satisfy (5) as equality, G_t adjusts up or down to ensure this.

strategies, instead of conditioning on the entire history, condition only on the payoff-relevant state variables, here $s_t \in \mathcal{S}$, and on the prior actions within the same stage game, denoted by $k_t \in \mathcal{K}$. Consequently, a MPE is defined as a set of Markovian strategies that are best responses to each other given every possible history $h^{t,k} \in H^{t,k}$. In the context of the game here, MPE is a natural equilibrium because it directly introduces the commitment problems that are central to our analysis. However, we will see that the same commitment problems are present in a very similar fashion when we focus on subgame perfect equilibria.

More formally, let σ be a Markovian strategy mapping, that is,

$$\sigma : \mathcal{S} \times \mathcal{K} \rightarrow [0,1] \times \mathbb{R}_+^2 \times \{0,1\}^3 ,$$

which assigns a value for each of the actions, the tax rate $\tau_t \in [0,1]$, the military wage $w_t \in \mathbb{R}_+$, the level of the public good $G_t \in \mathbb{R}_+$, the decision of whether to create or reform the military $a_t \in \{0,1\}$, and the coup and repression decisions of the military, $\psi_t \in \{0,1\}$ and $\rho_t \in \{0,1\}$, for each value of the state variable $\mathcal{S}$ and each combination of prior moves in the stage game given by $\mathcal{K}$. An MPE is a mapping σ^* that is a best response to itself at every possible history $h^{t,k} \in H^{t,k}$. To characterize the dynamics of political institutions, we define the one-step transition probability of s_t conditional on its past value and the limiting distribution of s induced by the MPE strategy profile σ^* as

$$p\left(s_t \mid s_{t-1}\right) : \mathcal{S} \times \mathcal{S} \rightarrow [0,1] \text{ and } q\left(s\right) : \mathcal{S} \rightarrow [0,1] .$$

These concepts will be useful in describing how regimes change in equilibrium and the likelihood of different regimes in the long run.

We next proceed to characterizing the MPE by first determining (the net present discounted) values of different individuals (groups) under different regimes.

2.2 Values in Democracy

We start with the values of the three groups in democracy, $s_t = D$. Decisions in democracy are made by majoritarian voting and the median voter will always be a low-skill producer. Therefore, democracy will be an absorbing state as long as $a_t = 0$ is chosen in all future periods, and clearly, $a_t = 0$ is always a dominant strategy for a low-skilled producer in democracy. In view of this, we obtain the following proposition (recall that Y is defined in (3)).

Proposition 1 *The unique MPE starting in any subgame with $s = D$ (i.e., a consolidated democracy) involves $\left(a_t = 0, \tau_t = \tau^D, w_t = \cdot, G_t = G^D\right)$ at each date t, where the democratic* *tax rate τ^D is given by*

$$\tau^D \equiv \left(C'\right)^{-1}\left(1 - \frac{A^L}{Y}\right), \tag{6}$$

and the level of public good is given by

$$G^D = \left(\tau^D - C\left(\tau^D\right)\right) Y. \tag{7}$$

Moreover, consolidated democracy is an absorbing state so that $p\,(D \mid D) = 1$ and $q\,(D) = 1$.

Proof. Clearly, the median voter in democracy would never choose $a_t = 1$, since this would reduce the tax base and potentially create a coup threat against democracy. Therefore, democracy is an absorbing state and the optimal policy can be characterized by the solution to the following static maximization problem

$$u^L\,(D) \equiv \max_{\tau \in [0,1], G \in \mathbb{R}_+} (1-\tau)\,A^L + G \tag{8}$$
$$\text{subject to } G \leq (\tau - C\,(\tau))\,Y.$$

Given the Inada conditions and the convexity of $C\,(\tau)$, (8) has a unique interior solution. The first-order condition of this problem then gives the unique equilibrium tax rate as (6), and the corresponding level of public good as (7). ■

The values of low- and high-skill producers are then given by

$$V^L\,(D) = \frac{u^L\,(D)}{1-\beta} \equiv \frac{\left(1-\tau^D\right) A^L + G^D}{1-\beta} \tag{9}$$

and

$$V^H\,(D) = \frac{u^H\,(D)}{1-\beta} \equiv \frac{\left(1-\tau^D\right) A^H}{1-\beta}. \tag{10}$$

Here and throughout, D (or E, M or TD) in parentheses denotes the regime, while superscripts denote the identity of the agent. In addition, u denotes per period returns and V denotes “values,” that is, the net present discounted values.

The value of an ex soldier in this regime is also $V^M\,(D) = V^L\,(D)$, since former soldiers will now work as low-skill producers. At this point, it is also useful to define

$$a^L \equiv \left(1-\tau^D\right) A^L + G^D \tag{11}$$

as the net per period return to a low-skill producer in democracy. This expression will feature frequently in the subsequent analysis.

Finally, recall that according to our parameterization $A^L \equiv (1-\theta)\,Y/\,(1-n)$. It is then evident that τ^D does not depend on Y and is a strictly increasing function of θ. This last result is due to the well-known effect of inequality on redistribution in models of majority voting on linear taxes (e.g., Thomas Romer, 1975, Kevin W.S. Roberts, 1977, Allan H. Meltzer and Scott F. Richard, 1981). We note this as a corollary for future reference (proof omitted).

Corollary 1 *The democratic tax rate τ^D, given by (6), is strictly increasing in the extent of inequality parameterized by θ.*

2.3 Values under Military Rule

A military regime, $s = M$, can only occur when $x_t = \bar{x}$, and as noted above, it is an absorbing state. Since $x_t \in \{0, \bar{x}\}$, the military government has no option to change its size without disbanding itself (and obviously it would not want to expand the military even if it could, since this would dissipate the rents captured by the military among a greater number of soldiers). Consequently, the military government will simply maximize the utility of soldiers subject to the government budget constraint. Moreover, we assume that there is a natural seniority system in the military, so that current soldiers are not fired in order to hire new applicants. Then, provided that the participation constraint of soldiers is satisfied (see below), current soldiers will remain in the army forever and receive the military wage. Finally, because in state M the military can never lose power, this problem boils down to that of maximizing the static utility of a representative soldier subject to the government budget constraint.

At this point, consider the maximization problem of the military government:

$$\begin{aligned} u^M(M) &\equiv \max_{\tau\in[0,1], w\in\mathbb{R}_+, G\in\mathbb{R}_+} w \\ &\text{subject to } w\bar{x} + G \leq (\tau - C(\tau))\left(Y - \bar{x}A^L\right), \end{aligned} \tag{12}$$

where the objective function incorporates the fact that soldiers do not benefit from the public good (i.e., $\chi_{M,t} = 0$ in (1)). Since τ does not feature in the objective function, the solution to (12) involves taxing at rate $\hat{\tau}$ defined in (4) to maximize tax revenues (thus maximizing the constraint set) and also setting to zero the public good, $G_t = 0$. This generates a unique per soldier wage of w^M given by

$$w^M \equiv \frac{(\hat{\tau} - C(\hat{\tau}))\left(Y - \bar{x}A^L\right)}{\bar{x}}, \tag{13}$$

where Y again denotes the potential output defined in (3). Evidently, as long as the military government is in power it will set the tax rate $\hat{\tau}$ extracting as much revenue from the producers as possible and will redistribute all the proceeds to the soldiers. As noted above, the interesting case for the current paper is the one where w^M is sufficiently high that entering the military and undertaking a coup is an attractive option for low-skill producers. We next impose Assumption 1, which will ensure this. Recall that the per period utility of the low-skill agents in democracy is a^L (which is simply a function of the underlying parameters, given by (11), with τ^D and G^D defined uniquely in (6) and (7)). The following assumption ensures that w^M is sufficiently greater than a^L so that the participation constraint of soldiers (of being in the military both during military dictatorships, democratic regimes and oligarchies) is always satisfied.

Assumption 1

$$\gamma \geq \frac{1-\beta}{\beta} \frac{a^L}{w^M - a^L} \equiv \bar{\gamma}.$$

This assumption implies that the expected value of a coup for soldiers is always greater than the value of a low-skill producer in permanent democracy, that is,

$$V^M(\text{coup}) \equiv \beta \left[\gamma V^M(M) + (1-\gamma) V^L(D)\right] \geq V^L(D),$$

which is sufficient to guarantee the participation constraint in all regimes. Note that Assumption 1 is not very restrictive. For example, as the size of the military, $\bar{x}$, becomes small, any $\gamma > 0$ satisfies this assumption. Assumption 1 is maintained throughout the paper.

The following proposition describes the equilibrium under a military rule (proof omitted).

Proposition 2 *The unique MPE in any subgame starting with $s = M$ (i.e., military dictatorship) involves the following policy vector at each date: $\tau^M = \hat{\tau}$ as defined in (4), $G^M = 0$, and w^M given in (13). Moreover, military dictatorship is an absorbing state so that $p(M \mid M) = 1$ and starting with $s = M$ at any point, we have $q(M) = 1$.*

Given the unique continuation equilibrium in Proposition 2, the (net present discounted) values for, respectively, current members of the military, low-skill non-military agents and high-skill elite are given by

$$V^M(M) = \frac{u^M(M)}{1-\beta} \equiv \frac{1}{1-\beta} \frac{(\hat{\tau} - C(\hat{\tau}))\left(Y - \bar{x}A^L\right)}{\bar{x}}, \tag{14}$$

$$V^L(M) = \frac{u^L(M)}{1-\beta} \equiv \frac{(1-\hat{\tau})A^L}{1-\beta}, \tag{15}$$

and

$$V^H(M) = \frac{u^H(M)}{1-\beta} \equiv \frac{(1-\hat{\tau})A^H}{1-\beta}. \tag{16}$$

Notice that the expressions for $u^L(M)$ and $V^L(M)$ do *not* include a term for the option value of low-skill producers becoming soldiers. This is because even though each low-skill producer would like to become a soldier, the military will not be hiring any more soldiers, since $x_t = \bar{x}$ already and there are no quits from the army.

2.4 Values in Transitional Democracy

We now turn to the analysis of transitional democracy, where $s_t = TD$. Recall that this regime will emerge when $s_{t-1} = E$, $x_{t-1} = \bar{x}$, $\rho_{t-1} = 1$ and repression fails (probability π). Moreover, this regime is indeed "transient"; if there is no coup attempt or the coup attempt fails at this point, then $s_{t+1} = D$, and if there is a successful coup attempt, then $s_{t+1} = M$. Clearly, depending on the subsequent regime, either the equilibrium of Proposition 2 or that of Proposition 1 will apply. We now investigate policy choices and the reaction of the military during the transitional period.

Suppose that the democratic government during the transitional period has announced the policy vector $\left(\tau^{TD}, w^{TD}, G^{TD}\right)$. To start with, let us also ignore the *participation constraint* of soldiers, which ensures that they prefer not to quit the military (see footnote 15). If the military chooses $\psi_t = 0$ (that is, no coup attempt), then the value of a typical soldier is

$$V^M(TD \mid \text{no coup}) = w^{TD} + \beta V^L(D), \tag{17}$$

which incorporates the fact that the soldiers will receive the military wage w^{TD} today and do not receive utility from the public good. We have also substituted for the continuation value to the soldiers, which, from Proposition 1 and equation (9), is given as the value in a consolidated democracy $V^M(D) = V^L(D)$. In particular, in line with Proposition 1, at the next date the government will choose to reform the army and all former soldiers will become low-skill producers, accounting for the continuation value of $V^L(D)$.

In contrast, if $\psi_t = 1$, the value of a soldier is

$$V^M(TD \mid \text{coup}) = \beta\left[\gamma V^M(M) + (1-\gamma)V^L(D)\right], \tag{18}$$

which incorporates the fact that when the military undertakes a coup, the soldiers do *not* receive the wage w^{TD} and the coup succeeds with probability γ. Following a successful coup, a military dictatorship is established and the continuation value of the soldiers is given by $V^M(M)$ as in (14) (cf. Proposition 2). If the coup fails (probability $1-\gamma$), soldiers become low-skill producers and simply receive the continuation value of the low-skill producer in a consolidated democracy, $V^L(D)$ as in (9).[14] Comparing these two expressions, we obtain the *no-coup constraint*:

$$V^M(TD \mid \text{no coup}) \geq V^M(TD \mid \text{coup}).$$

Using (17) and (18), the no-coup constraint can be written as

$$w^{TD} \geq \frac{\beta}{1-\beta}\gamma\left(w^M - a^L\right), \tag{19}$$

where w^M is defined in (13) and a^L in (11). Constraint (19) defines the minimum military wage that democracy must offer to soldiers in order to prevent a coup attempt. In what follows, we will use w^{TP} for the ("transitional prevention") wage that makes this constraint hold as equality and τ^{TP} for the corresponding tax rate. This wage level can be thought of as an *efficiency wage* for the military to induce them to take the right action (that is, not to undertake a coup). Clearly, this wage depends on the success probability of the coup γ, and the gap between the value that soldiers will receive in a military dictatorship, $V^M(M) = w^M/(1-\beta)$, and their value in democracy, $V^L(D) = a^L/(1-\beta)$.

The question is whether a democratic government would pay this minimum military wage to prevent a coup attempt. This will depend on two factors. The first is whether it is *feasible* to pay such a wage (and satisfy the budget constraint, (5)). The second is whether it is *desirable* for low-skill producers to pay this wage. The feasibility condition requires this minimum wage times the number of soldiers to be less than the maximum revenue that can be raised, that is,

$$w^{TP}\bar{x} \leq (\hat{\tau} - C(\hat{\tau}))\left(Y - \bar{x}A^L\right). \tag{20}$$

The right-hand side is the maximum revenue that can be raised, since it involves taxation at the revenue-maximizing rate $\hat{\tau}$ and the tax base consists of the entire population except the $\bar{x}$ soldiers. Using the expression for w^M in (13), the condition (20) can be seen to be equivalent to $w^{TP} \leq w^M$. Alternatively, using the expression for w^{TP} from (19), we find that the feasibility constraint, (20), is satisfied if and only if

$$\gamma \leq \frac{1-\beta}{\beta}\frac{w^M}{w^M - a^L} \equiv \hat{\gamma}. \tag{21}$$

This condition states that preventing a coup attempt is feasible only if γ is less than some critical threshold $\hat{\gamma}$. Otherwise, only very high wages will discourage soldiers from attempting a coup and such high wages cannot be paid without violating the government budget constraint. Note also that Assumption 1 ensures $a^L < w^M$ and thus $\bar{\gamma} < \hat{\gamma}$.[15]

[14] There is no differential taxation or further punishments that are possible on former soldiers.

[15] This also implies that the participation constraint of soldiers in this case, $V^M(TD) \geq V^L(D)$, is always satisfied.

Next, suppose that it is feasible for democracy to pay the wage w^{TP}. Is it beneficial for low-skill producers to pay this wage or is it better for them to face the risk of a military coup? To answer this question, suppose that a wage of w^{TP} can indeed be paid—i.e., (21) is satisfied—and compare the low-skill producers' utilities under the two scenarios. When they pay the necessary efficiency wage, the value of low-skill producers is

$$V^L(TD \mid \text{no coup}) = \left(1 - \tau^{TP}\right) A^L + G^{TP} + \beta V^L(D), \tag{22}$$

where τ^{TP} is the utility-maximizing tax rate for the low-skill producer subject to the no-coup constraint, (19), and $G^{TP} \geq 0$ is the utility-maximizing level of public good spending during the transitional phase. Alternatively, if there is no coup prevention during the transitional democracy, the value of a low-skill producer is

$$V(TD \mid \text{coup}) = \left(1 - \tau^{TN}\right)(1 - \phi) A^L + G^{TN} + \beta \left[\gamma V^L(M) + (1 - \gamma) V^L(D)\right], \tag{23}$$

since in this case there are no payments to the military, and now τ^{TN} and G^{TN} denote the utility-maximizing policy choices when coup attempts are not prevented.[16] The rest of the expression incorporates this fact. Current output is a fraction $1 - \phi$ of potential output (3) because of the disruption caused by the coup attempt and there is a probability γ that the coup is successful and the regime from tomorrow on will be a military dictatorship, giving value $V^L(M)$ to the representative low-skill producer. Combining the previous two expressions, we obtain that a low-skill producer will prefer to prevent coups during transitional democracy if

$$V^L(TD \mid \text{no coup}) \geq V^L(TD \mid \text{coup}). \tag{24}$$

The next proposition shows that whenever (21) is satisfied, (24) is also satisfied, so that coups against democratic governments are always prevented when prevention is fiscally feasible. Assumption 1 is assumed to hold throughout, thus we limit attention to $\gamma \geq \bar{\gamma}$.

Proposition 3 *Let $\hat{\gamma}$ be defined by (21). Then, the unique MPE in any subgame starting with $s = TD$ is as follows.*

- *If $\gamma \in [\bar{\gamma}, \hat{\gamma}]$, then the transitional democracy chooses the policy vector $\left(\tau^{TP}, w^{TP}, G^{TP}\right)$ and prevents a military coup. At the next date, $s' = D$ (i.e., $p(D \mid TD) = 1$), the military is reformed and the policy vector in consolidated democracy characterized in Proposition 1 is implemented. The long-run equilibrium in this case involves democracy with probability 1, i.e., $q(D) = 1$.*

[16] The full maximization program when the low-skill producer chooses not to prevent coups is given in the proof of Proposition 3.

- *If $\gamma \in (\hat{\gamma}, 1]$, then transitional democracy chooses the policy vector $\left(\tau^{TN}, w^{TN}, G^{TN}\right)$ and the military attempts a coup, i.e., $\psi = 1$. Consequently, we have $p(D \mid TD) = 1 - \gamma$ and $p(M \mid TD) = \gamma$, and thus starting with $s = TD$, $q(D) = 1 - \gamma$ and $q(M) = \gamma$.*

Proof. See Appendix A. ■

The essence of Proposition 3 is that condition (24) is always satisfied, so low-skill producers are better off when coups are prevented. However, this may not be possible because coup prevention may require excessive efficiency wages. In particular, this will be the case when condition (21) is not satisfied. There is a clear *inefficiency* in equilibria involving coups (because of the economic disruption that they cause). The source of this inefficiency is in the commitment problem; if the democratic regime could promise high wages to soldiers in the future, both low-skill producers and soldiers could be made better off. But, as shown in Proposition 1, the unique MPE after $s = D$ involves reform of the military and thus no efficiency wages for the soldiers. Thus the inability of the democratic regime to commit to future rewards to soldiers is the source of coup attempts. We will see in subsection 2.7 that the restriction to Markovian strategies is not important here. Instead, the commitment problem emerges from the underlying economics of the interaction between democracy and the military—the fact that the military, whose main function here is repression, is not needed in democracy.

A number of other features related to this result are worth noting. First, if (21) is satisfied, then transitional democracy may pay even higher wages (and thus make greater concessions) to the military than an oligarchic regime would (which we will study in the next subsection). This again reflects the commitment problem; because democracy has no use for a large military, it cannot commit to not reforming it and thus it needs to make greater concessions today in order to prevent coup attempts. Second, the extent of income inequality influences whether there will be coup attempts against democracy. In particular, the threshold $\hat{\gamma}$ in (21) depends on the inequality parameter θ via its effects on a^L and w^M. It can be verified easily that greater inequality—a higher θ—increases w^M and reduces a^L, thus reducing $\hat{\gamma}$. This makes coups more likely in more unequal societies. Intuitively, this is because in a more unequal society, $V^L(D)$ is lower, and thus the prospect of becoming a low-skill producer is less attractive for the current soldiers, who are more tempted to undertake a coup to secure a military dictatorship. The rents that soldiers can appropriate in the military regime are also greater in a more unequal society because net output is greater (a smaller fraction of the potential output Y given in (3) is foregone as a result of the fact that some of the potential producers are joining the army). We state this result in the next corollary (proof in the text):

Corollary 2 *Higher inequality (higher θ) reduces $\hat{\gamma}$ and makes coups in transitional democracy more likely.*

Finally, it is also useful for future reference to compute the values to soldiers and the elite in a transitional democracy. First, the value to soldiers in transitional democracy does *not* depend on whether there is coup prevention or not. This is because in both cases soldiers receive the value of a coup against democracy, either as expected return for undertaking a coup or as a result of the efficiency wages paid by the democratic government to satisfy their no-coup constraint. This value is given by

$$V^M(TD) = \beta\left[\gamma V^M(M) + (1-\gamma)V^L(D)\right]. \tag{25}$$

In contrast, the value to the elite depends on whether γ is greater or less than $\hat{\gamma}$, which determines whether transitional democracy can prevent coups. When $\gamma > \hat{\gamma}$, there will be a coup attempt in transitional democracy and the elite's value is given by

$$V^H(TD \mid \text{coup}) = (1-\phi)\left(1-\tau^{TN}\right)A^H + \beta\left[\gamma V^H(M) + (1-\gamma)V^H(D)\right]. \tag{26}$$

In contrast, when $\gamma \leq \hat{\gamma}$,

$$V^H(TD \mid \text{no coup}) = \left(1-\tau^{TP}\right)A^H + \beta V^H(D), \tag{27}$$

where τ^{TN} and τ^{TP} refer to the tax rates defined in Proposition 3.

2.5 Values in Oligarchy

We now turn to the analysis of subgames starting with $s = E$. The key economic insight here is that unlike democracy, an oligarchic regime may benefit from having a military used for repression (thus preventing democratization). Counteracting this is the *political moral hazard* problem mentioned in the Introduction, whereby the military may turn against the elite and try to establish a military dictatorship.

In oligarchy, the elite have three possible strategies:

1. choose $x_t = 0$ and allow immediate democratization (recall that $x_{-1} = 0$). We denote this strategy by S, "smooth transition;"

2. choose $x_t = \bar{x}$ and allow coups. We denote this strategy by N for "non-prevention;"

3. choose $x_t = \bar{x}$ and pay high enough military wages to prevent coups. We denote this strategy by P for "prevention."

The third strategy would not be attractive for the elite if the military chooses not to repress, since they would obtain no benefit from having the military and pay both direct (financial) costs and indirect costs (in terms of the risk of a military dictatorship). Lemma 1 shows that the military always prefers repression, that is, $\rho = 1$ whenever $s = E$, thus the third strategy for the elite is indeed viable. Throughout this subsection we make use of the result of this lemma, which will be stated and proved at the end.

We next compute the values to the elite corresponding to these three strategies. In all cases, the elite always supply no public good, since this is costly for them in terms of taxes and they do not obtain any benefit from public good. Consequently, if they choose the first strategy, that of smooth transition, they will set the lowest possible tax rate ($\tau^S = 0$) and accept the fact that $p(D \mid E) = 1$. This will give them a value of

$$V^H(E, S) = A^H + \beta V^H(D), \tag{28}$$

where $V^H(D)$ is the value of the high-skill (elite) individuals in consolidated democracy given by (10).

The second strategy for the elite, non-prevention, is to create an army, but not to prevent military coups. In this equilibrium, soldiers attempt a coup against the oligarchic regime and therefore receive zero wages. Consequently, zero taxes are again feasible and optimal for the elite, and their value from this strategy can be written as

$$V^H(E, N) = (1 - \phi) A^H + \beta \left[\gamma V^H(M) + (1 - \gamma) V^H(D)\right], \tag{29}$$

where $(1 - \phi) A^H$ is the flow payoff to the elite (net of the disruption caused by the coup), $V^H(M)$, given by (16), is the value to the elite under military dictatorship, which occurs if the coup attempt by the military is successful (probability γ), and finally $V^H(D)$, given by (10), is the value to the elite in consolidated democracy, which occurs if the coup attempt by the military fails (probability $1 - \gamma$). (Here we made use of the assumption that if a coup attempt against oligarchy fails, this immediately leads to a consolidated democracy, see Figure 1).

Finally, if the elite set up an army to repress the citizens and also pay the required efficiency wage to prevent military coups, then their value can be written recursively as

$$V^H(E, P) = \left(1 - \tau^P\right) A^H + \beta \left[(1 - \pi) V^H(E, P) + \pi V^H(TD)\right], \tag{30}$$

where $V^H(TD)$ is given by (26) or (27) in the previous subsection depending on whether γ is greater than or less than $\hat{\gamma}$. (30) also incorporates that to prevent military coups the elite have to impose a tax rate of τ^P on all incomes (to finance the military efficiency wage), and the state of oligarchy with prevention recurs next period with probability $1-\pi$, whereas with probability π, repression fails and the political state switches to transitional democracy, giving the elite the value $V^H(TD)$. Rearranging (30), we obtain

$$V^H(E,P\mid\pi)=\frac{\left(1-\tau^P\right)A^H+\beta\pi V^H(TD)}{1-\beta(1-\pi)}, \tag{31}$$

where, for future reference, we have written this value as a function of the probability that repression fails, π, i.e. as $V^H(E,P\mid\pi)$. The important observation here is that $V^H(E,P\mid\pi)$ is continuous and strictly decreasing in π. The easiest way to see that it is decreasing is from (30), by noting that $V^H(E,P\mid\pi)=V^H(E,P)>V^H(TD)$.

To characterize the equilibrium starting in a subgame with $s=E$, we need to compute the value of the tax rate τ^P necessary to allow for prevention, and then compare the values to the elite from the three possible strategies outlined above.

Let us first look at the values to the military. The military has three possible strategies. They decide whether to attempt a coup against oligarchy and, if they do not undertake a coup, whether or not to repress the citizens. The participation constraints of soldiers in oligarchy will make sure that the last option is not chosen. We refer to the military's strategies as "coup", "repression" (to denote no coup and repression), and "non-repression" (to denote no coup and non-repression). Let us first consider the value to the soldiers when they attempt a coup ($\psi=1$). Their value can then be written as:

$$V^M(E\mid\text{coup})=\beta\left[\gamma V^M(M)+(1-\gamma)V^L(D)\right]. \tag{32}$$

This expression incorporates the fact that when they attempt a coup, soldiers will not receive a wage and that the coup will succeed with probability γ giving them a value of $V^M(M)$ and fail with probability $1-\gamma$, in which case the regime will transition to a fully consolidated democracy with a value of $V^L(D)$ for the soldiers.[17]

The value to the soldiers when they do not attempt a coup and choose repression ($\psi=0$ and $\rho=1$) satisfies the recursion

$$V^M(E\mid\text{repression})=w^P+\beta\left[\pi V^M(TD)+(1-\pi)V^M(E\mid\text{repression})\right].$$

[17] Notice that the value to soldiers from a coup against the oligarchy is the same as the value from transitional democracy, i.e. $V^M(E\mid\text{coup})=V^M(TD)$, since a coup gives them the same value regardless of which regime it is attempted against. Furthermore, recall also that in the MPE of the subgame beginning in transitional democracy, soldiers have the same value regardless of whether they attempt a coup (since when a coup is prevented, the no-coup constraint is binding).

This expression reflects the fact that soldiers receive the wage w^P today because they have successfully carried out the necessary repression. The continuation value then accounts for the fact that the same state will recur tomorrow with probability $1-\pi$ (i.e., as long as repression succeeds). If instead repression fails (probability π), they will receive the continuation value of transitional democracy, $V^M(TD)$, given by (25) (recall that this expression applies regardless of whether γ is greater than $\hat{\gamma}$ or not). Solving the recursion above we obtain that

$$V^M(E \mid \text{repression}) = \frac{w^P + \beta\pi V^M(TD)}{1-\beta(1-\pi)}. \tag{33}$$

The *no-coup constraint* in oligarchy is therefore

$$V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup}) \tag{34}$$

with $V^M(E \mid \text{repression})$ and $V^M(E \mid \text{coup})$ given in (32) and (33).[18] The expressions in (33) and (32) immediately imply that the constraint (34) is equivalent to $w^P \geq \beta\gamma w^M + \beta(1-\gamma)a^L$. The minimum military wage consistent with coup prevention is

$$w^P = \beta\gamma w^M + \beta(1-\gamma)a^L. \tag{35}$$

To finance this military wage, the elite will impose a tax rate of τ^P, which must satisfy the government budget constraint (5), thus

$$w^P\bar{x} = \left(\tau^P - C\left(\tau^P\right)\right)\left(Y - \bar{x}A^L\right). \tag{36}$$

Combining (36) with (35) and using (13), we find that this tax rate is implicitly and uniquely defined by

$$\tau^P = \beta\gamma(\hat{\tau} - C(\hat{\tau})) + \beta(1-\gamma)\frac{a^L\bar{x}}{Y-\bar{x}A^L} + C\left(\tau^P\right), \tag{37}$$

and moreover $\tau^P \in (0,\hat{\tau})$. The uniqueness of τ^P follows from the fact that $C(\cdot)$ is strictly convex and satisfies $C(0) = 0$, and the remaining terms on the right-hand side are strictly positive. Thus at most one value of τ^P can satisfy (37) and moreover, this unique solution is strictly between 0 and $\hat{\tau}$.[19]

[18] We can also verify at this point that the participation constraint of soldiers in oligarchy is always satisfied. This participation constraint is given by $V^M(E \mid \text{repression}) \geq V^L(E,S)$, since, in view of Lemma 1, the military always prefers repression to no repression. Without the military, there will be smooth transition to democracy. Using the fact that $V^L(E,S) = A^L + \beta V^L(D)$ together with the no-coup constraint (34) and Assumption 1, ensures that the above inequality and thus the participation constraint is always satisfied.

[19] That $\tau^P > 0$ follows from the fact that $C(0) = 0$. To establish that $\tau^P < \hat{\tau}$, note from (37) that τ^P is strictly increasing in γ and that, by the definition of $\hat{\tau}$, $\tau - C(\tau)$ is strictly increasing in τ for all $\tau < \hat{\tau}$, and is maximized at $\hat{\tau}$. Moreover, when $\gamma = 1$, we have $\tau^P - C\left(\tau^P\right) = \beta(\hat{\tau} - C(\hat{\tau}))$, which implies that $\tau^P < \hat{\tau}$ when $\gamma = 1$; since τ^P is strictly increasing in γ, the same conclusion holds for all $\gamma \in [0,1]$.

Differentiation of (37) with respect to θ also establishes that the tax rate τ^P is strictly decreasing in inequality. Therefore, when inequality is greater, a lower tax rate is sufficient for the elite to prevent coup attempts. This result is the outcome of two counteracting effects of inequality on the no-coup constraint. First, as inequality increases democracy becomes less attractive for the soldiers because in democracy they will become low-skill producers. A consolidated democracy is more likely when the military attempts a coup against the oligarchic regime and thus this "outside option" effect of inequality reduces w^P and τ^P. Counteracting this is the "greed" effect resulting from the fact that as inequality increases, the value to soldiers in military dictatorship, $V^M(M)$, increases (recall (14)). However, the fiscal revenues collected for any level of τ when $x_t = \bar{x}$ are also increasing in θ and in fact proportional to the effect of inequality on $V^M(M)$. Consequently, the outside option effect always dominates the greed effect and τ^P is decreasing in θ (in the extent of inequality).

Finally, it can be verified that the participation constraint of soldiers in oligarchy is always satisfied. This constraint is given by

$$V^M(E \mid \text{repression}) \geq V^L(E, S) = A^L + \beta V^L(D). \tag{38}$$

The no-coup constraint, (34), implies that $V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup})$, while Assumption 1, combined with (32), implies $V^M(E \mid \text{coup}) \geq V^L(D)$, where $V^L(D)$ is defined in (11). Since $V^L(D) > V^L(E, S)$, (38) is always satisfied.

We end this subsection by showing that the military never chooses no coup and non-repression ($\psi = 0$ and $\rho = 0$) in oligarchy. The remainder of the analysis of equilibrium in oligarchy is presented in the next subsection in the context of the complete characterization of the MPE.

Lemma 1 *In any MPE starting in a subgame with the elite in power ($s_t = E$), the military never simultaneously chooses not to undertake a coup and not to repress the citizens (i.e., $\psi_t = 0$ and $\rho_t = 0$ is not an equilibrium).*

Proof. The value to the soldiers when they choose not to attempt a coup and not repress, $\psi_t = 0$ and $\rho_t = 0$, is $V^M(E \mid \text{no repression}) = \beta V^M(TD)$, since, without repression, soldiers receive no wage and the regime in the next period will be transitional democracy where they receive the value $V^M(TD)$ given in (25). When they choose to attempt a coup, $\psi_t = 1$, they obtain the value $V^M(E \mid \text{coup})$ given in (32), and since this value is equal to $V^M(TD)$ (see footnote 17), we have that $V^M(E \mid \text{coup}) > V^M(E \mid \text{no repression})$. This implies that in oligarchy there exists a strategy giving the soldiers a strictly higher equilibrium payoff than no coup and non-repression. ■

2.6 Equilibrium Dynamics and Interpretation

We now combine the results from the previous four subsections, complete the analysis of the elite's decisions in oligarchy, and present a full characterization of MPE.

First notice that a direct comparison of $V^H(E, S)$ in (28) and $V^H(E, N)$ in (29) shows that $V^H(E, S)$ is always greater, so the elite will never choose the non-prevention strategy. Thus the choice of the elite in oligarchy boils down to allowing smooth transition to democracy versus building a strong military and dealing with the political moral hazard problem by paying the required efficiency wage. The following proposition provides a full characterization of MPE.

Proposition 4 *Let $\hat{\pi} \in [0, 1]$ be defined as the solution between 0 and 1 to*

$$V^H(E, S) = V^H(E, P \mid \pi = \hat{\pi}). \tag{39}$$

When a solution exists, it is uniquely defined. When such a solution to (39) does not exist, then either we have $V^H(E, S) > V^H(E, P \mid \pi)$ for all $\pi \in [0, 1]$, or $V^H(E, S) < V^H(E, P \mid \pi)$ for all $\pi \in [0, 1]$. In the former case, we set $\hat{\pi} = 0$ and in the latter case, $\hat{\pi} = 1$. Then, provided that $\pi \neq \hat{\pi}$, the political game described above has a unique MPE, such that:

1. *if $\pi \in [0, \hat{\pi})$, then whenever $s = E$ the elite build an army for repression (i.e., $a = 1$), set $\tau = \tau^P$ and $w = w^P$, and prevent military coups. The military chooses $\psi = 0$ and $\rho = 1$ (no coup and repression). Transitional democracy arises with probability $p(TD \mid E) = \pi$, while oligarchy persists with probability $p(E \mid E) = 1 - \pi$. Proposition 3 characterizes the unique MPE of starting in any subgame $s = TD$, so that we have $q(D) = 1$ when $\gamma \in [\bar{\gamma}, \hat{\gamma}]$ and $q(D) = 1 - \gamma$ and $q(M) = \gamma$ when $\gamma \in (\hat{\gamma}, 1]$;*

2. *if $\pi \in (\hat{\pi}, 1]$, the elite do not build an army (i.e., $a = 0$). Transition to consolidated democracy occurs with probability $p(D \mid E) = 1$, and in the long run consolidated democracy obtains with probability $q(D) = 1$ with allocations as described in Proposition 1.*

When $\pi = \hat{\pi}$, configurations in parts 1 and 2 are MPE.

Proof. The argument preceding the proposition establishes that the elite never choose non-prevention, so their choice depends on the comparison of S and P (i.e., between $V^H(E, S)$ and $V^H(E, P \mid \pi)$). As noted above, $V^H(E, P \mid \pi)$ defined in (31) is a continuous and strictly decreasing function of π, whereas $V^H(E,S)$ does not depend on π. Therefore, with the conventions adopted in the definition of (39), there exists a unique $\hat{\pi} \in [0,1]$ such that we have $V^H(E,P \mid \pi) \gtreqless V^H(E,S)$ whenever $\pi \lesseqgtr \hat{\pi}$ (that is, there will be repression only when $\pi < \hat{\pi}$). Moreover, $\hat{\pi} \in (0,1)$ if and only if $V^H(E,P \mid \pi = 0) > V^H(E,S)$ *and* $V^H(E,P \mid \pi = 1) < V^H(E,S)$. The first condition is equivalent to $\tau^P < \beta\tau^D$. The second condition is equivalent to $\tau^P > \beta\left[V^H(TD) - V^H(D)\right]/A^H$ (when there is no coup in the subgame obtaining in state $s = TD$, this condition is equivalent to $\tau^P > \beta\left(\tau^D - \tau^{TP}\right)$ with τ^{TP} given in Proposition 3). It can also be verified that if $V^H(E,P \mid \pi = 0) \leq V^H(E,S)$, i.e., if $\tau^P \geq \beta\tau^D$, then $V^H(E,P \mid \pi) < V^H(E,S)$ for any $\pi > 0$, which implies that $\hat{\pi} = 0$. ■

Remark 1 When $\hat{\pi}$ is not interior—i.e., $\hat{\pi} = 0$ or $\hat{\pi} = 1$—only one of the two parts of Proposition 4 is relevant. The necessary and sufficient conditions that guarantee that $\hat{\pi}$ is interior, i.e., $\hat{\pi} \in (0,1)$, are specified in the proof of Proposition 4. It can be verified that the set of parameters that satisfies both of these conditions is nonempty (for example, both conditions are satisfied when A^L is sufficiently small, so that inequality is very high).

Note also that the threshold implicitly defined in (39) can be written as

$$\hat{\pi} = \frac{1-\beta}{\beta} \frac{\left(\beta\tau^D - \tau^P\right)A^H}{\left(1-\beta\tau^D\right)A^H - (1-\beta)V^H(TD)},$$

which means that $\hat{\pi} = \left(\beta\tau^D - \tau^P\right)/\beta\tau^{TP}$ when there are no coups in transitional democracy (i.e., when $\gamma \leq \hat{\gamma}$), and $\hat{\pi} = (1-\beta)\left(\beta\tau^D - \tau^P\right)/\beta\left[\phi(1-\beta) + (1-\beta)(1-\phi)\tau^{TN} + \beta\gamma\left(\hat{\tau} - \tau^D\right)\right]$ when there are coups (i.e., when $\gamma > \hat{\gamma}$).

Proposition 4 shows that the elite will choose to set up an army only when repression is sufficiently effective, i.e., $\pi < \hat{\pi}$. When this happens, oligarchy will persist, recurring with probability $1-\pi$ in each period. Moreover, as long as oligarchy persists, the military receives high (efficiency) wages so that it aligns itself with the elite. When repression fails (probability π), the society becomes a transitional democracy. Proposition 3 then implies that, in this case, the democratic government pays the cost of the oligarchy having established a powerful military previously. It either has to make significant concessions to the military or risk a coup by the military. In contrast, when repression is not very effective, i.e., $\pi > \hat{\pi}$, the elite do not find it profitable to set up an army and pay the high wages necessary to co-opt the soldiers. In this case there is a smooth transition to a consolidated democracy.

Income inequality has two counteracting effects on the decision of the elite on whether to build a military to repress the citizens. Greater inequality encourages repression because it reduces the tax rate τ^P under prevention (see the discussion above) and increases the tax rate τ^D in democracy (see Corollary 1). Both effects increase the threshold $\hat{\pi}$ defined in (39) and therefore the region where repression takes place. However, greater inequality also makes coups against transitional democracy more likely (see Corollary 2), which is costly for the elite. This makes the effect of income inequality on repression ambiguous in general. Nevertheless, this last effect becomes less relevant when the effectiveness of the military in repressing the citizens is high (i.e., when π is low) because the probability that an oligarchic regime will transition to democracy is low in this case. Moreover, it is also straightforward to see that while the effect of inequality is in general ambiguous, repression is never optimal when inequality is sufficiently low, and conversely, when the society is sufficiently unequal, the set of parameters for which the elite prefer repression is necessarily nonempty (see Remark 1).

Proposition 4 implies that income inequality also has interesting effects on fiscal policies. When inequality is low, a consolidated democracy is likely to emerge, and thus a small increase in inequality starting from a low base leads to higher taxes and greater public good provision (see Corollary 1). When inequality increases further, oligarchy tends to persist for longer because of repression, and this leads to lower taxes, but in this case, all tax revenue is spent on the military (and there is no public good provision). High income inequality also makes military coups against nascent democracies more likely (Corollary 2), and military regimes will set high taxes and spend all the proceeds for themselves (again with no public good provision). Therefore, the effect of inequality on taxes and public good provision is nonmonotonic. These implications might account for the lack of a monotonic relationship between inequality and redistribution across countries (e.g., Roberto Perotti, 1996, Roland Bénabou, 2000).

The role of inequality in the emergence of military dictatorships highlighted by our analysis is broadly consistent with the contrast of Costa Rica to other Central American countries briefly discussed in the Introduction. Costa Rica lacks a large indigenous population, and likely as a consequence, smallholder production plays a more important role in Costa Rica than the rest of Central America.[20] The relative equality and homogeneity of Costa Rica's society are often cited as the major reason why Costa Rican ruling elites have shown limited interest in the creation of a large and powerful military.[21] The tradition of low militarization of Costa

[20] An indication of these differences is provided by the average number of economically active individuals in the agricultural sector relative to the number of farms around 1950 (see Needler, 1987, Table 5, p. 98). This ratio was 10.9 in Costa Rica, 38.2 in El Salvador and 48.1 in Guatemala. This comparison therefore suggests a much more equal distribution of income in the agricultural sector in Costa Rica than in El Salvador and Guatemala.

[21] For instance, describing the economic and institutional reforms promoted during the "liberal" era (the period of Latin American history roughly going from 1870 to 1930, following the phase of caudillo politics),

Rica culminated with the formal demobilization of the armed forces in 1949, after these were decimated during a short civil war. This decision has not been reversed ever since. Consistent with the predictions of our model, the successful demobilization of the Costa Rican army appears to have contributed significantly to the stability of its nascent democratic institutions. Consequently, Costa Rica has become the most stable democratic country in Latin America. The successful consolidation of democracy in Costa Rica contrasts with the experiences in Guatemala, El Salvador, Honduras and Nicaragua.

2.7 Subgame Perfect Equilibria

We end this section by showing that in our political game the MPE and the Subgame Perfect Equilibria (SPE) coincide. Although MPE, which makes strategies depend only on the payoff-relevant states, is a natural equilibrium concept in this context and highlights the political-economic commitment problem that is at the heart of our theory in a clear manner, it is useful to show that implicit promises between social groups that may be possible in SPE do not prevent the inefficiencies identified above or change the qualitative results. In particular, the next proposition shows that in our political game the MPE and the SPE coincide, thus there was no loss of generality in focusing on Markovian equilibria.

Proposition 5 *Suppose that $\pi \neq \hat{\pi}$ (as defined in (39)). The political game described above has a unique SPE identical to the MPE described in Proposition 4.*

Proof. See Appendix A. ■

3 Extensions

In this section we provide a number of extensions. First, we extend the environment so that military dictatorships emerge not only during transitions to democracy, but also as a result of coups against oligarchic regimes (such as those discussed in footnote 22 below). Second, we investigate the implications of rents from natural resources on regime dynamics. Third, we show how the national defense role of the military makes it easier for a democracy to commit to not reforming the military and facilitates democratic consolidation. Finally, we

James Mahoney (2001 p. 266) writes that, "In Costa Rica, where the reform period was launched at the time of independence, liberals were not faced with the kinds of political threats that led reformers elsewhere to build large standing armies... The pattern of reformist liberalism that had developed by the early twentieth century saw neither the creation of a powerful military coercive branch that commanded a prominent position in the state nor an associated rural economy marked by polarized class structures and a high potential for lower-class agrarian revolts."

briefly illustrate that stronger oligarchic regimes may sometimes choose less repression and make democracy more likely.

Throughout the rest of this section we simplify the notation and the analysis by focusing on MPE and also adopting a simpler form for the tax distortion function C. In particular, we assume that there exists $\hat{\tau} > 0$ (not the same as $\hat{\tau}$ defined in (4)) such that $C(\tau) = 0$ for all $\tau \leq \hat{\tau}$ and $C(\tau) = 1$ for all $\tau > \hat{\tau}$. This implies that there are no costs of taxation until $\tau = \hat{\tau}$ and that taxation above $\hat{\tau}$ is prohibitively costly. Furthermore, we assume the following restriction on the fundamental parameters of the model

$$Y > (1 + \bar{x}) A^L. \tag{40}$$

We maintain these assumptions throughout without stating them explicitly.

Faced with the new cost schedule for taxation, the elite still prefer zero taxes (except for paying the military wages when they have to); moreover, by condition (40), both democratic and military regimes would set taxes equal to $\hat{\tau}$. Consequently, the value functions now take simpler forms. For example, the value to the elite in military dictatorship, in consolidated democracy, and in transitional democracy are

$$V^H(M) = V^H(D) = \frac{(1-\hat{\tau}) A^H}{1-\beta}, \tag{41}$$

and

$$V^H(TD) = \frac{((1-\beta)\varphi_t + \beta)(1-\hat{\tau}) A^H}{1-\beta}. \tag{42}$$

The military wage and the instantaneous payoffs to low-skill producers in consolidated democracy are also modified similarly and become

$$w^M = \frac{\hat{\tau}\left(Y - \bar{x}A^L\right)}{\bar{x}} \text{ and } a^L = (1-\hat{\tau}) A^L + \hat{\tau} Y. \tag{43}$$

Also throughout these extensions, Assumption 1 is in effect and is sufficient to ensure that the participation constraint of soldiers is always satisfied (except with non-prevention in the next subsection, which is discussed separately).

3.1 Coups Against Oligarchy

We now present a modified environment in which there may be equilibrium coups against oligarchy.[22] Suppose that when the oligarchic regime chooses to create a military, there are

[22] Examples of military dictatorships that have resulted from coups against oligarchic/nondemocratic regimes include the majority of the military regimes that have emerged in Peru between the 1820s and the 1930s, the army-backed regime created in Thailand after the 1932 coup that ended the rule of the Thai absolutist monarchy, the regime created in Egypt after Major Nasser's coup against the monarchy, and the junta in Panama in 1968 that followed the coup by the National Guard under the leadership of Omar Torrijos.

two additional states of the world, denoted by the variable $\eta_t \in \{\eta^I, \eta^{NI}\}$. In state η^I, the elite are insulated from both the threat of a coup and from possible transitions to democracy. In contrast, in state η^{NI}, both types of transitions away from oligarchy can occur. In particular, the military can attempt a coup and, if it does not, transition to democracy is possible and the probability of this event depends on the repression decision of the military as in the baseline model. The additional state variable η_t evolves according to an exogenous stochastic process, whose realizations are identically and independently distributed over time with $\Pr[\eta_t = \eta^{NI}] = \mu$. The elite have to decide military wage at time t before the realization of η_t and this wage cannot be conditioned on η_t. We also assume that the soldiers can leave the military only before the realization of η_t.

The following proposition provides a summary of the results in this case and a more complete statement is contained in Appendix C.

Proposition 6 *Consider the extended model presented in this subsection. Then there exist $\overline{\mu}$, $\underline{\mu} < \overline{\mu}$ and $\bar{\pi}$ such that for $\mu \in (\underline{\mu}, \overline{\mu})$, the unique MPE is as follows:*

1. *If $\pi \in [0, \bar{\pi})$, then whenever $s = E$, the elite build an army for repression (i.e., $a = 1$), set $\tau = \tau^P$ and $w = w^P$, and prevent military coups. The military chooses $\psi = 0$ and $\rho = 1$ (no coup and repression). Transitional democracy arises with probability $p(TD \mid E) = \mu\pi$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu\pi$. Proposition 3 characterizes the unique MPE starting in any subgame $s = TD$, so that $q(D) = 1$ when $\gamma \in [\bar{\gamma}, \hat{\gamma}]$, and $q(D) = 1 - \gamma$ and $q(M) = \gamma$ when $\gamma \in (\hat{\gamma}, 1]$.*

2. *If $\pi \in (\bar{\pi}, 1]$, the elite build an army for repression (i.e., $a = 1$), set $\tau = 0$ and $w = 0$, and do not prevent coups. The military chooses $\psi = 1$ (coup) in state μ^{NI}. Military dictatorship arises with probability $p(M \mid E) = \mu\gamma$, consolidated democracy arises with probability $p(D \mid E) = \mu(1 - \gamma)$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu$. Consequently, the long-run likelihood of regimes are given by $q(D) = (1 - \gamma)$ and $q(M) = \gamma$.*

If $\mu \notin (\underline{\mu}, \overline{\mu})$, the MPE is identical to that in Proposition 4.

The important additional result in Proposition 6 is that now coups against oligarchy also arise along the equilibrium path. Previously, the threat of coups against oligarchy affected the equilibrium allocations, but such coups never took place in equilibrium (and military dictatorships always emerged from coups against democracy). The introduction of such coups is important for two reasons. First, coups against oligarchy provide a clearer demonstration of the *political moral hazard* problem facing the elite in building a strong army—that the army can turn against them. Second, the model can now potentially account for why the military dictatorships we observe in practice have different origins; some resulting from coups against democracy, while others are preceded by non-military oligarchy regimes.

3.2 Natural Resources

In this section, we extend the baseline model by assuming that there is a natural resource, which generates income equal to $R \geq 0$ in each period. The natural resources are owned by the elite (high-productivity agents) and all of the natural resource rents initially accrue to them. Since there are now two sources of income, we allow for two different fiscal instruments; a tax rate on income at the rate τ (with tax distortions specified as in the beginning of this section with the function C, so that there are no distortions until $\tau = \hat{\tau}$) and a tax rate on income from natural resources $\zeta \in [0, 1]$. We assume that the taxation of natural resources generates no distortions. This is reasonable since natural resources are typically supplied inelastically.

The characterization of the MPE is similar to before (in particular, the elite never choose non-prevention N since it gives a smaller equilibrium payoff than smooth transition S), and we provide fewer details. The main observation that simplifies the analysis is that both a military regime and a democratic regime will choose $\zeta = 1$, thus taxing all income from natural resources. Consequently, an analysis identical to that in Section 2 implies that military wages in a military dictatorship and instantaneous payoffs to low-skill agents in democracy are

$$\tilde{w}^M = \frac{\hat{\tau}\left(Y - \bar{x}A^L\right) + R}{\bar{x}}, \text{ and} \tag{44}$$

$$\tilde{a}^L \equiv (1 - \hat{\tau})\, A^L + \tilde{G}^D, \tag{45}$$

where $\tilde{G}^D = \hat{\tau}Y + R$. These expressions exploit the fact that $\tau = \hat{\tau}$ and $\zeta = 1$, and thus they incorporate the additional revenues coming from natural resource rents either directly in the military wage or in the amount of public good provided in democracy.

Let us start with a subgame where $s = TD$. An analysis identical to that leading to Proposition 3 immediately implies that low-skill producers are always better-off by preventing coups. In fact, now retaining political power has become more valuable because of the additional source of revenues coming from natural resources. However, natural resources also make military coups more attractive for the soldiers. In particular, as in Section 2, a military coup can be prevented only if given the fiscal instruments and natural resource rents, democracy can raise enough revenue to pay soldiers a sufficient amount to satisfy the *no-coup constraint*. This feasibility constraint now takes the form

$$w^{TD}\bar{x} \leq \hat{\tau}\left(Y - \bar{x}A^L\right) + R. \tag{46}$$

In this expression, the revenues include the income raised by taxing production at rate $\hat{\tau}$ and from the taxes on natural resource rents at the rate $\zeta = 1$. The military wage necessary to prevent coups is now

$$\tilde{w}^{TP} = \frac{\beta}{1-\beta}\gamma\left(\tilde{w}^M - \tilde{a}^L\right), \tag{47}$$

with $\tilde{w}^M$ and $\tilde{a}^L$ now defined by (44) and (45). It is straightforward to verify that $\tilde{w}^{TP} > w^{TP}$, making coup prevention more difficult. In particular, the combination of (46) and (47) shows that the prevention of coups is now possible if γ satisfies the following condition, which is a simple generalization of (21),

$$\gamma \leq \frac{1-\beta}{\beta}\frac{\tilde{w}^M}{\tilde{w}^M - \tilde{a}^L} \equiv \tilde{\gamma}(R). \tag{48}$$

With this threshold replacing $\hat{\gamma}$, Proposition 3 continues to apply.[23] The proof of Proposition 7 in Appendix C shows that the threshold $\tilde{\gamma}(R)$ is strictly decreasing in R. This is intuitive; greater natural resource rents raise the *political stakes* and make military coups more attractive for soldiers (and the additional revenue available to democracy is not sufficient to compensate soldiers for the prospect of dividing natural resources among themselves). Hence, transitional democracy is less likely to consolidate in natural resource abundant societies.

We next turn to subgames starting with $s = E$. Natural resources will again raise political stakes, though in this case their effects will be somewhat more complex. Recalling that the value to the elite in the subgames starting with $s = M$ and $s = D$ are given by (41) above, we obtain the value to the elite from smooth transition as

$$\tilde{V}^H(E,S) = \frac{(1-\beta)\left(A^H + R/n\right) + \beta(1-\hat{\tau})A^H}{1-\beta}, \tag{49}$$

which is identical to (28) in Section 2 above, except that the elite enjoy the rents from natural resources for one period. If, on the other hand, they build a strong army and choose coup prevention, their value, as a function of π, will be given by the solution to:

$$\tilde{V}^H(E,P \mid \pi) = \max_{\tilde{\tau}\in[0,\hat{\tau}],\zeta\in[0,1]} \frac{(1-\tilde{\tau})A^H + (1-\zeta)R/n + \beta\pi\tilde{V}^H(TD)}{1-\beta(1-\pi)}, \tag{50}$$

[23] It can again be verified that the participation constraint of soldiers is satisfied in this case as well. In particular, this constraint in transitional democracy with natural resources is $\tilde{V}^M(TD) \geq \tilde{V}^L(D)$, where $\tilde{V}^M(TD) = \tilde{V}^M(TD \mid \text{coup}) = \beta\gamma\tilde{V}^M(M) + \beta(1-\gamma)\tilde{V}^L(D)$, $\tilde{V}^L(D) = \tilde{a}^L/(1-\beta)$ and $\tilde{V}^M(M) = \tilde{w}^M/(1-\beta)$, and holds under Assumption 1.

subject to the government budget constraint, which written as an equality takes the form

$$\tilde{\tau}\left(Y-\bar{x}A^{L}\right)+\zeta R=\tilde{w}^{P}\left(R\right)\bar{x}. \tag{51}$$

Expression (50) is obtained from the solution of a recursive equation analogous to (30), but uses (42) and incorporates the fact that to prevent coups the elite have to pay the now higher efficiency wage $\tilde{w}^{P}(R)$. This efficiency wage has the same expression as in (35), except that w^{M} and a^{L} are now replaced by their counterparts $\tilde{w}^{M}$ and $\tilde{a}^{L}$ in (44) and (45), so that

$$\tilde{w}^{P}\left(R\right)=\beta\left(\gamma\tilde{w}^{M}+\left(1-\gamma\right)\tilde{a}^{L}\right). \tag{52}$$

Let us also denote the tax levels that solve the maximization problem in (50) by $\tilde{\tau}^{P}$ and ζ^{P}. Comparing the maximized value of this program to (49), we obtain a threshold $\tilde{\pi}(R)$ replacing $\hat{\pi}$ in Proposition 4. The rest of this proposition continues to apply.[24] Consequently, we have the following characterization of equilibrium in the presence of natural resource rents.

Proposition 7 *The extended model with natural resources has a unique MPE, identical to that in Proposition 4 (except that $\tilde{\gamma}(R)$ replaces $\hat{\gamma}$ and $\tilde{\pi}(R)$ replaces $\hat{\pi}$). Moreover:*

- *a higher level of R makes democratic consolidation starting in state $s=TD$ less likely;*
- *there exists $R^{*}>0$ and $\hat{x}>0$, such that an increase in natural resources makes repression, starting in state $s=E$, more likely if $\bar{x}>\hat{x}$, for any initial $R>R^{*}$.*

Proof. See Appendix C. ■

The new results in this proposition are the comparative statics with respect to the size of natural resource rents. Greater natural resource abundance increases the political stakes and makes democratic consolidation more difficult, because the military has more to gain from taking control and democracy may not be able to compensate soldiers for forgoing these returns.

The second part of the proposition shows that greater natural resource abundance also affects the likelihood of repression in oligarchy. Nevertheless, the effect is, in general, ambiguous, because of two opposing effects. On the one hand, greater natural resources make the elite more willing to use repression in order to prevent a transition to democracy again because of the greater political stakes. On the other hand, natural resources also intensify the

[24] The participation constraint of soldiers in oligarchy under prevention is now given by $\tilde{V}^{M}(E\mid\text{repression})\geq\tilde{V}^{L}(E,S)$, where $\tilde{V}^{M}(E\mid\text{repression})=\tilde{V}^{M}(E\mid\text{coup})$, $\tilde{V}^{M}(E\mid\text{coup})=\beta\gamma\tilde{V}^{M}(M)+\beta(1-\gamma)\tilde{V}^{L}(D)$, and $\tilde{V}^{L}(E,S)=A^{L}+\beta\tilde{V}^{L}(D)$. Assumption 1 again ensures that this constraint is satisfied.

political moral hazard problem because the military, once formed, will have stronger incentives to undertake a coup. Which effect dominates depends on the size of the army and the size of natural resource rents. With a larger army, per soldier rents in military dictatorship are lower and thus when the size of the army is larger than a threshold $\hat{x} > 0$ and $R > R^*$, an increase in natural resource rents makes repression more likely.[25]

Consistent with the emphasis in this subsection, natural resources appear to have played an important role in the emergence of several military dictatorships. Terry L. Karl (1997) argues that the two oil price hikes in the early 1970s and 1980s exacerbated political instability in many "petro-states," in particular, in Iran, Nigeria, Venezuela and Algeria. Nigeria, for instance, experienced growing economic and ethnic tensions after the oil price increases and witnessed the reemergence of military rule in 1983. This was followed by a new transition to a fragile democracy between 1986 and 1991, but was again interrupted by a military coup in 1993. Algeria experienced a severe crisis in the early 1990s, which lead to the assassination of the president, Mohammed Boudiaf, to the cancellation of elections, and to frequent switches between military rule and weak civilian rule.[26]

3.3 National Defense and Democratic Consolidation

The baseline model analyzed in Sections 1 and 2 was simplified by two assumptions; firstly and more importantly, the only role of the military was repression in domestic politics; and secondly, the transitional phase in democracy lasted for one period, i.e., the military could be reformed after one period. We now relax both of these assumptions. Our substantive objective is to investigate the impact of international threats and defense role of the military on democratic politics.

The model is identical to our baseline model in Section 1, except that we are still using the simplified fiscal technology introduced at the beginning of this section, and more substantively, we assume that there is an international threat (e.g., a threat of invasion from another country).

[25] Here the threshold R^*, provided in Appendix C, is such that when $R > R^*$, the elite will use both taxes on production income and natural resource rents to finance military wages.

[26] Further evidence that oil and abundant endowments of other natural resources may adversely affect democratic consolidation is offered by the empirical evidence presented in Michael Ross (2001), and by Nathan Jensen and Leonard Wantchekon (2004). In particular, Jensen and Wantchekon (2004) show that in African countries, the abundance of natural resources tends to make both the transition to, and the consolidation of, democracy less likely. In fact, many of the more successful examples of democratic transitions in sub-Saharan Africa are by relatively natural resource poor nations such as Benin, Madagascar and Mali. Instead, natural resource abundant countries have experienced greater political turmoil and have not been successful in establishing democratic regimes. Examples include Gabon, Cameroon, Togo, Zambia, Algeria, Nigeria, Congo and Sierra Leone.

If the army is present, that is, $x_t = \bar{x}$, such an invasion is not possible. If, on the other hand, $x_t = 0$, the invasion would succeed and all agents (including low-skill producers) receive a lower payoff than even in the military dictatorship. This implies that all agents would like to prevent an invasion if possible. To capture the role of international threats in the simplest possible way, we assume that the foreign power posing the threat of invasion may reform its own military and when this happens, the international threat disappears. We do not model the behavior of the foreign power explicitly, and instead assume that such reform happens with probability $\lambda \in (0,1)$ in each period. Until the foreign threat disappears (due to reform of the foreign military), a transitional democratic government cannot reform the military. After this threat disappears, the transitional democratic government will choose to disband the military (since there is no longer any foreign threat). In this light, the current model is also a generalization of the baseline setup in Section 1, since reforming the military takes potentially longer than one period. Finally, returning to the discussion in footnote 3 and to O'Donnell and Schmitter's (1986) conjecture, λ can be interpreted as another (inverse) measure of the relative strength of the army, since when λ is low, there is greater need for the military because of national defense purposes and the military is stronger.[27]

The next proposition states the main result of this subsection.

Proposition 8 *In any subgame beginning with $s = TD$, there exists a unique MPE such that the transitional democratic government prevents coup attempts if and only if*

$$\gamma \leq \frac{(1-\beta)(1-\beta(1-\lambda))}{\beta\lambda} \frac{w^M}{w^M - a^L} \equiv \hat{\gamma}(\lambda). \tag{53}$$

Moreover, $\hat{\gamma}(\lambda)$ is strictly decreasing in λ.

Proof. See Appendix C. ■

This proposition has two interesting implications, both following from the fact that $\hat{\gamma}(\lambda)$ is decreasing in λ (which implies that transitional democracy is more likely to prevent coups and consolidate when λ is low). The first and more important implication is that when there is a stronger foreign threat and thus a more important role of the army in national defense, which here corresponds to lower λ, democratic consolidation is more likely. This is intimately related to the key economic force emphasized in this paper: when the army has an important national defense role, it is less threatened by reform in transitional democracy and this translates into more credible commitments by transitional democracy to compensate soldiers for not

[27] Naturally, an even starker measure is whether $x_t = 0$ or $\bar{x}$, and we have already investigated the implications of this measure of military strength on democratic consolidation.

undertaking a coup. In contrast, when λ is close to 1 (meaning that there are no serious foreign threats and no important role of the army in national defense), concessions by the transitional democratic regime are not credible for the military because they foresee imminent reform and are more willing to attempt coups. This result therefore highlights a new and potentially important interaction between international and domestic politics.

The second implication of Proposition 8 is that, somewhat paradoxically, the model suggests that over a certain range, democratic consolidation may be more likely when the military is stronger (since a lower λ corresponds to a stronger military). Therefore, the logic of commitment emphasized by our approach shows that O'Donnell and Schmitter's (1986) conjecture about stronger militaries always making the survival of transitional democracies less likely need not be true. Note that a version of this conjecture was true in our baseline model, when comparing $x_t = 0$ to $x_t = \bar{x}$. However, our extended model here shows why this conjecture may not capture the complete set of interactions between the strength of the military and transitional democracies because commitments to a strong military are more credible.

3.4 Persistence in Oligarchy

Finally, we allow for persistence of oligarchy without repression and study whether better organized democratic movements (or less strong oligarchies) are likely to lead to more rapid democratization. Our main result in this section shows why this may not be the case.

Suppose that in the absence of repression, oligarchy persists with probability $\alpha \in (0,1)$. If, instead, there is repression and it fails, democratization happens only with probability $(1-\alpha)\pi < \pi$. Thus, in this extension α represents a direct measure of the degree of consolidation of the power of the elite in oligarchy. The analysis of the interactions between the elite and the military in oligarchy are similar to our analysis in Section 2. Briefly, the value to be an elite agent from smooth transition can be computed using the expressions in (42) as

$$V^H(E,S) = \frac{(1-\beta)A^H + \beta(1-\alpha)(1-\hat{\tau})A^H}{(1-\beta)(1-\beta\alpha)}, \tag{54}$$

and the value from prevention, as a function of π, is

$$V^H(E,P \mid \pi) = \frac{\left(1-\tau^P\right)A^H + \beta\pi(1-\alpha)V^H(TD)}{1-\beta(1-\pi(1-\alpha))}. \tag{55}$$

A similar analysis again shows that non-prevention is dominated by smooth transition (in fact, the persistence of oligarchy without repression makes smooth transition more desirable for the elite relative to non-prevention). Consequently, we immediately obtain the analog of

Proposition 4: the elite choose prevention if π is below some threshold $\breve{\pi}$ and smooth transition otherwise, and it is straightforward to verify that $\breve{\pi} \in (0,1)$. The main new result, described in the next proposition, concerns the comparative statics of regime transitions with respect to the persistence parameter α.

Proposition 9 *Let $\breve{\pi}$ be defined by $V^H(E,S) = V^H(E,P \mid \breve{\pi})$. If no coups take place in the subgame beginning in state $s = TD$, then $\breve{\pi}$ is strictly decreasing in α. If coups take place in the subgame beginning in state $s = TD$, then $\breve{\pi}$ is strictly decreasing in α for any $\phi < \phi^*$, and $\breve{\pi}$ is strictly increasing in α for any $\phi > \phi^*$, where $\phi^* \in (0,1]$.*

Proof. See Appendix C. ■

Proposition 9 implies that the elite are *less* likely to choose repression when their power is *more* consolidated (greater α) under two related conditions: first, when coups do not happen in transitional democracy; second, when coups take place in transitional democracy but they do not cause too much income disruption (ϕ small). This result is intuitive; as α increases, both the value to the elite from smooth transition and from prevention increase. Whether the threshold $\breve{\pi}$ increases (and thus whether repression becomes more likely) depends on two opposing forces. Whether coups take place in transitional democracy and what their costs for the elite are (ϕ) determine the balance of these two forces. When coups do not take place, they are more likely to choose repression when their power is less consolidated (corresponding to a lower value of α). When coups are possible after democratization, the trade-off for the elite depends on how disruptive these coups are. When they are not very disruptive ($\phi < \phi^*$), a lower α (less consolidation of elite power in oligarchy) encourages repression. However, when coups are highly disruptive ($\phi > \phi^*$), then a lower α makes transitional democracy more likely after repression and the elite prefer smooth transition to avoid the potential costs of coups in the future.

The substantive implication of Proposition 9 is that, somewhat paradoxically, democratic regimes are *not* necessarily more likely to emerge in societies where the citizens are better organized politically and the oligarchy is weaker. In particular, a lower value of α in this model corresponds to a weaker oligarchy (and thus to a stronger democratic movement). Proposition 9 shows that smooth transition to democracy may be more likely when α is high, which corresponds to societies where the democratic movement is weak and the power of the elite is sufficiently consolidated, so that they feel less need to create an army for additional repression.

4 Concluding Remarks

In this paper, we presented a first analysis of the emergence of military dictatorships and the conditions under which the military will act as an agent of the elite (as opposed to acting in its own interests and against those of the elite). These questions are relevant for research in political economy for a number of reasons. First, most nondemocratic regimes survive with significant support from the military, so understanding the objectives of the military is important in the study of political transitions. Second, many nondemocracies in practice are military regimes, and we need to understand whether military dictatorships emerge and persist for different reasons than oligarchic regimes and their economic consequences.

An investigation of these questions necessitates a model in which the military consists of a set of individuals who act in their own interests (though they can be convinced to align themselves with the elite if this is consistent with their interests). We introduced this feature by assuming that the means of violence in the society are in the monopoly of the military, and if the elite decide to form a strong military, then they have to live with the political moral hazard problem that this causes. In particular, a strong military may not simply work as an agent of the elite, but may instead turn against them in order to create a regime more in line with their own objectives. One immediate implication of the political moral hazard problem is that the cost of using repression in nondemocratic regimes is now higher, because the elite need to pay "efficiency wages" or make concessions to soldiers to prevent coups.

An important consequence of the presence of a strong military is that once transition to democracy takes place, the military poses a coup threat against the nascent democratic regime until it is reformed. The anticipation that the military will be reformed in the future acts as an additional motivation for the military to undertake coups against democratic governments. Consequently, societies where the elite form a strong military in order to prevent democratization are more likely to later lapse into military dictatorships because the military retains some of its power during transitional democracy and can attempt a successful coup against democracy. This leads to a specific (and to the best of our knowledge, novel) channel for the emergence of military dictatorships, which appears to be consistent with the historical evidence. It also highlights how repression during a nondemocratic era can have important effects on the economic and political success of a later democratic regime.

Our analysis also showed how, under certain circumstances, military coups against nondemocratic elites are also possible, thus creating another channel for the emergence of military dictatorships. In light of these results, one might wish to distinguish between three different types of nondemocratic regimes. The first is oligarchies where the rich elite are in power and the military acts as an agent of the elite. This type of regime emerges endogenously in our model depending on the technology and the incentives of the elite. The second is a military dictatorship that emerges as a result of a coup against a democratic regime. The third is a military dictatorship that results from coups against oligarchic regimes. The examples of all three types of regimes were discussed in the Introduction.

Our model also provides a range of comparative statics about when such dictatorships are more likely. Greater inequality makes democratic consolidation more difficult, and generally increases the likelihood of repression in nondemocracy, though there are also countervailing effects, making this last result ambiguous. Greater natural resource rents make military coups against (unconsolidated) democracies more likely, and also have ambiguous effects on the political equilibrium in nondemocracies, which become more valuable for the elite, but at the same time more expensive to maintain because of the more severe political moral hazard problem resulting from the high natural resource rents. More importantly, the model also implies that democratic consolidation is more likely when there is a potential foreign threat, making the military necessary for national defense. This is a new and interesting link between international politics and domestic politics. The logic of the result is very related to the main economic force in our model; when there is an international threat, concessions from democratic regimes to the military become more credible, because democracy also needs the military.

We view our paper as a first step in the study of military dictatorships and the political agency problems that are ubiquitous between branches of the state that control the means of violence and the economic elite, especially in nondemocracies. Several of the assumptions used in this paper were adopted to ensure tractability and a more systematic investigation of political dynamics may require relaxing these assumptions. These include: (i) the assumption that there is no conflict of interest among soldiers, among poor non-soldiers, and among the elite; relaxing this assumption is important for an analysis of whether the elite could co-opt a subset of the poor without using repression and also to study the formation of coalitions between subsets of the elite and other social groups; (ii) the assumption that inequality is exogenous; relaxing this assumption would enable a study of how different regimes affect the distribution of income and thus future political attitudes in society, and how the elite may be formed endogenously as a function of the prevailing political regime; (iii) the assumption that military dictatorship is an absorbing state; relaxing this assumption would be essential for studying transitions from military regimes to democracy.

Finally, several other topics highlighted by our paper deserve further study. First, a systematic empirical analysis of policy and economic performance differences between different types of nondemocratic regimes is necessary. Second, the current framework can be extended so that an alliance between the military and the elite can be formed during democratic periods as well. Third, the current framework already highlighted the important interactions between international and domestic politics. However, we did not endogenize the political economy equilibrium in other countries. A fruitful area for future research appears to be the international relations aspects of the interactions between the military and democratic regimes. In particular, it would be interesting to investigate how military or democratic reforms in one country affect politics in other countries.

Appendix A: Proofs of Proposition 3 and Proposition 5

Proof of Proposition 3

By Assumption 1, $\gamma \geq \bar{\gamma}$. If $\gamma > \hat{\gamma}$, then the feasibility constraint, (20), is necessarily violated and there will be a coup attempt in equilibrium ($\psi = 1$). Thus we only have to show that when $\gamma \in [\bar{\gamma}, \hat{\gamma}]$, (24) is satisfied.

When transitional democracy chooses prevention, then $s_{t+1} = D$ and thus the soldiers must be paid w^{TP} that satisfies (19) as equality. Thus the only relevant decisions concern the choice of τ^{TP} and G^{TP}, which can be determined as the solution to:

$$\begin{aligned} u^L(TD \mid \text{no coup}) \equiv & \max_{\tau\in[0,1], G\in\mathbb{R}_+} (1-\tau) A^L + G \\ & \text{subject to } G \leq (\tau - C(\tau)) \left(Y - \bar{x}A^L\right) - w^{TP}\bar{x}. \end{aligned} \tag{56}$$

Next, using the expressions (13) and (19), w^{TP} can be written as

$$w^{TP} = \frac{\beta}{1-\beta}\gamma \left[\frac{(\hat{\tau} - C(\hat{\tau}))\left(Y - \bar{x}A^L\right)}{\bar{x}} - a^L\right]. \tag{57}$$

Provided that the solution to (56) involves $G^{TP} > 0$, taxes are set at the level τ^{TP} defined implicitly by the first-order condition of the program, which implies

$$A^L = \left(1 - C'(\tau)\right)\left(Y - \bar{x}A^L\right), \tag{58}$$

and moreover the utility of low-skill producers during the transitional period will be

$$\left(1-\tau^{TP}\right)A^L + \left(\tau^{TP} - C\left(\tau^{TP}\right)\right)\left(Y - \bar{x}A^L\right) - w^{TP}\bar{x}.$$

If, on the other hand, the solution to (56) involves $G^{TP} = 0$, then taxes are determined by the government budget constraint (5) as

$$\tau^{TP} = \frac{w^{TP}\bar{x}}{Y - \bar{x}A^L} + C\left(\tau^{TP}\right), \tag{59}$$

and the utility of low-skill producers in the transitional period will be $\left(1-\tau^{TP}\right)A^L$.

If, instead, coups are not prevented, the tax rate and the level of public good provision are chosen to maximize the utility of a representative low-skill producer in the transitional period, with no additional constraint, but taking into account the output disruption caused by the coup, which will be forthcoming in this case. In particular, the tax τ^{TN} and the level of public good provision G^{TN} in question are the solution to:

$$\begin{aligned} u^L\left(TD \mid \text{ coup}\right) &\equiv \max_{\tau\in[0,1],G\in\mathbb{R}_+} \left(1-\tau\right)\left(1-\phi\right)A^L + \ G \\ &\text{subject to } G \leq \left(1-\phi\right)\left(\tau - C\left(\tau\right)\right)\left(Y - \bar{x}A^L\right). \end{aligned}$$

The low-skill producers benefit from prevention when $V^L\left(TD \mid \text{no coup}\right) \geq V^L\left(TD \mid \text{coup}\right)$, where $V^L\left(TD \mid \text{no coup}\right) = u^L\left(TD \mid \text{no coup}\right) + \beta V^L\left(D\right)$ and $V^L\left(TD \mid \text{coup}\right) = u^L\left(TD \mid \text{ coup}\right) + \beta\left[\gamma V^L\left(M\right) + \left(1-\gamma\right)V^L\left(D\right)\right]$ as given by (22) and (23) above. Rearranging these expressions, we can write the condition for prevention to be preferred, when $G^{TP} > 0$, as

$$\begin{aligned} &\left(1-\beta\right)\left[\left(1-\tau^{TP}\right)A^L + \left(\tau^{TP} - C\left(\tau^{TP}\right)\right)\left(Y - \bar{x}A^L\right)\right] - \left(1-\beta\right)w^{TP}\bar{x} \\ \geq\ &\left(1-\beta\right)u^L\left(TD \mid \text{ coup}\right) - \beta\gamma\left(a^L - \left(1-\hat{\tau}\right)A^L\right). \end{aligned} \tag{60}$$

Now the result follows from three observations. First, from (57), $\left(1-\beta\right)w^{TP}$ is linear and increasing in β. This observation, and the facts that $u^L\left(TD \mid \text{ coup}\right)$ does not depend on β, and that τ^{TP} does not depend on β when $G^{TP} > 0$, implies that both the right-hand-side and the left-hand-side of (60) are strictly decreasing linear functions of β. Therefore, there is at most one value of β, β', such that the left- and the right-hand sides are equal. Second, (60) is satisfied at $\beta = 1$, since in this case this condition can be written as

$$\left(\hat{\tau} - C\left(\hat{\tau}\right)\right)Y - \left(\hat{\tau} - C\left(\hat{\tau}\right)\right)\bar{x}A^L - a^L\bar{x} \leq a^L - \left(1-\hat{\tau}\right)A^L,$$

where $\left(\hat{\tau} - C\left(\hat{\tau}\right)\right)Y \leq a^L - \left(1-\hat{\tau}\right)A^L$. Third, condition (60) also holds when $\beta = 0$, because in this case $w^{TP} = 0$ and thus prevention is for free. Therefore, there exists no $\beta' \notin [0,1]$ such that the right-hand-side and the left-hand-side of (60) are equal, thus this condition is always satisfied and we have $V^L\left(TD \mid \text{no coup}\right) > V^L\left(TD \mid \text{coup}\right)$ for any value of $\beta \in [0,1]$. This establishes that, if $G^{TP} > 0$, coup prevention is always better for transitional democracy.

We next consider the case where $G^{TP} = 0$. It is straightforward to show that this case applies when $\beta \in [\beta^*, 1]$, where β^* is defined as the minimum value of β such that the constraint $G \geq 0$ in problem (56) is binding (this constraint is implicit in $G \in \mathbb{R}_+$). Observe that w^{TP} is a strictly increasing function of β and the tax rate defined by (58) does not depend on β, hence the constraint $G \geq 0$ is slack for $\beta \leq \beta^*$ and binds for $\beta > \beta^*$. The equivalent of condition (60) in this case can be rewritten as

$$\tau^{TP} A^L \leq A^L - u^L(TD \mid \text{ coup}) + \frac{\beta}{1-\beta}\gamma\left(a^L - (1-\hat{\tau})A^L\right). \tag{61}$$

We now show that (61) holds for any $\beta \in [\beta^*, 1]$. The last term on the right-hand side is positive by the definition of a^L in (11) and $A^L - u^L(TD \mid \text{ coup})$ does not depend on β. Therefore, the right-hand side is linear (increasing) in $b \equiv \beta/(1-\beta)$, whereas from (59) and (57) τ^{TP} is a strictly convex function of b. Clearly, condition (61) holds as $\beta \to 1$. Moreover, since the payoff to the citizens is a continuous function of β over $[0, 1]$ by Berge's Maximum Theorem, condition (60) also holds at β^*, where the set of active constraints in program (56) changes. Finally, observe that if a convex function is less than a linear function at two end points of an interval in the extended real line, $b^* \equiv \beta^*/(1-\beta^*)$ and $b^\infty \equiv \infty$, then it is also less than the same function at any $b \in (b^*, b^\infty)$. This establishes that (61) also holds for any $\beta \in [\beta^*, 1]$ and completes the proof. ■

Proof of Proposition 5

Recall that $h^{t,k}$ is the history of play of the game up to time t and stage k within the stage game. A strategy profile for all the players in the game can be represented by a mapping $\tilde{\sigma} : H^{t,k} \to [0, 1] \times \mathbb{R}_+^2 \times \{0, 1\}^3$, where the range of the strategy profiles again refers to the tax rate $\tau_t \in [0, 1]$, the military wage $w_t \in \mathbb{R}_+$, the level of the public good $G_t \in \mathbb{R}_+$, the decision of whether to create or reform the military $a_t \in \{0, 1\}$, and the coup and repression decisions of the military, $\psi_t \in \{0, 1\}$ and $\rho_t \in \{0, 1\}$. A strategy profile $\tilde{\sigma}^*$ is a SPE if it is a best response to itself for all $h^{t,k} \in H^{t,k}$ (i.e., if it is *sequentially rational*).

First consider some history $h^{t,0}$ (i.e., at the beginning of the stage game at t) where $s_t = M$. Since military rule is absorbing, it is clear that the unique sequentially rational play will involve the military maximizing their own utility, thus Proposition 2 applies. Next consider a similar history where $s_t = D$. Now democracy is absorbing and the same argument implies that the unique sequentially rational play after this history is identical to that described in Proposition 1. Next consider a similar history with $s_t = TD$. Since there are no strategies that can be used to punish former soldiers in democracy and the continuation play after a successful coup is already pinned down uniquely, this implies that the unique sequentially rational play after any history involving $s_t = TD$ will be the same as that characterized in subsection 2.4, and in particular, soldiers will undertake a coup if the *no-coup constraint*, (19) is violated. Given this behavior, the unique best response of low-skill agents is provided by Proposition 3. Now since sequentially rational play after any history $h^{t,0}$ involving either of $s_t = M$, D and TD is uniquely pinned down, the behavior after histories where $s_t = E$ is also unique. In particular, if the no-coup constraint in this case, (34), is not satisfied, it is a unique best response for soldiers to undertake a coup. If it fails, they receive exactly the same value as in (9), since the regime will be $s_t = D$, and if it succeeds, they receive the continuation value associated with subgames starting with $s_t = M$. Finally, the value to the elite from S is also unchanged, so the results of Proposition 4 hold as the unique subgame perfect equilibrium in this case. ■

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Appendix B: Key Notation for Section 1 (Not for Publication)

$\beta \in (0,1)$: discount factor.

$\chi_{j,t} \in \{0,1\}$: indicator function denoting the occupational choice of the agent and determining whether or not the individual benefits from the public good.

n: size of high-skill agents ($1-n$ is size of low-skill agents).

A^H: income of high-skill producers.

A^L: income of low-skill producers.

$Y \equiv (1-n)\, A^L + nA^H$: potential output of the economy.

$\theta \in (n,1)$: parameter of income inequality (higher θ corresponds to greater inequality).

$x_t \in \{0, \bar{x}\}$: size of the military. $\bar{x}$: size of the military necessary for repression.

$a_t \in \{0,1\}$: decision regarding the size of the military at time t. $a_t = 1$ corresponds to $x_t = \bar{x}$, $a_t = 0$ corresponds to $x_t = 0$.

τ_t: tax rate on the income of the producers.

$C(\tau_t)$: fraction of the output lost due to tax distortions.

G_t: level of public good provision.

w_t: wage for soldiers.

ϕ: fraction of production lost due to the disruption created by the coup.

$\varphi_t \in \{1-\phi, 1\}$: fraction of output that remains after a coup attempt $(1-\phi)$. $\varphi_t = 1$ when there is no coup.

$s_t \in \{D, E, M, TD\}$: political regime at time t. $s_t = D$: democracy. $s_t = E$: oligarchy. $s_t = M$: military dictatorship. $s_t = TD$: transitional democracy.

$\psi_t \in \{0,1\}$: decision of the military whether or not to undertake a coup against the regime in power, with $\psi_t = 1$ corresponding to a coup attempt.

$\rho_t \in \{0,1\}$: decision of the military whether or not to repress the citizens in oligarchy, with $\rho_t = 1$ corresponding to repression.

$\pi \in [0,1]$: probability that repression fails.

$\gamma \in [0,1]$: probability that the coup attempt is successful.

τ^D: tax rate in democracy.

G^D: level of public good in democracy.

$a^L \equiv \left(1-\tau^D\right) A^L + G^D$: net per period return to a low-skill producer in democracy.

$\hat{\tau}$: tax rate maximizing government revenues (peak of the Laffer curve).

w^M: soldiers' wage in a military dictatorship.

S, N, P: strategies of the elite corresponding to "smooth transition", "non-prevention" and "prevention," respectively.

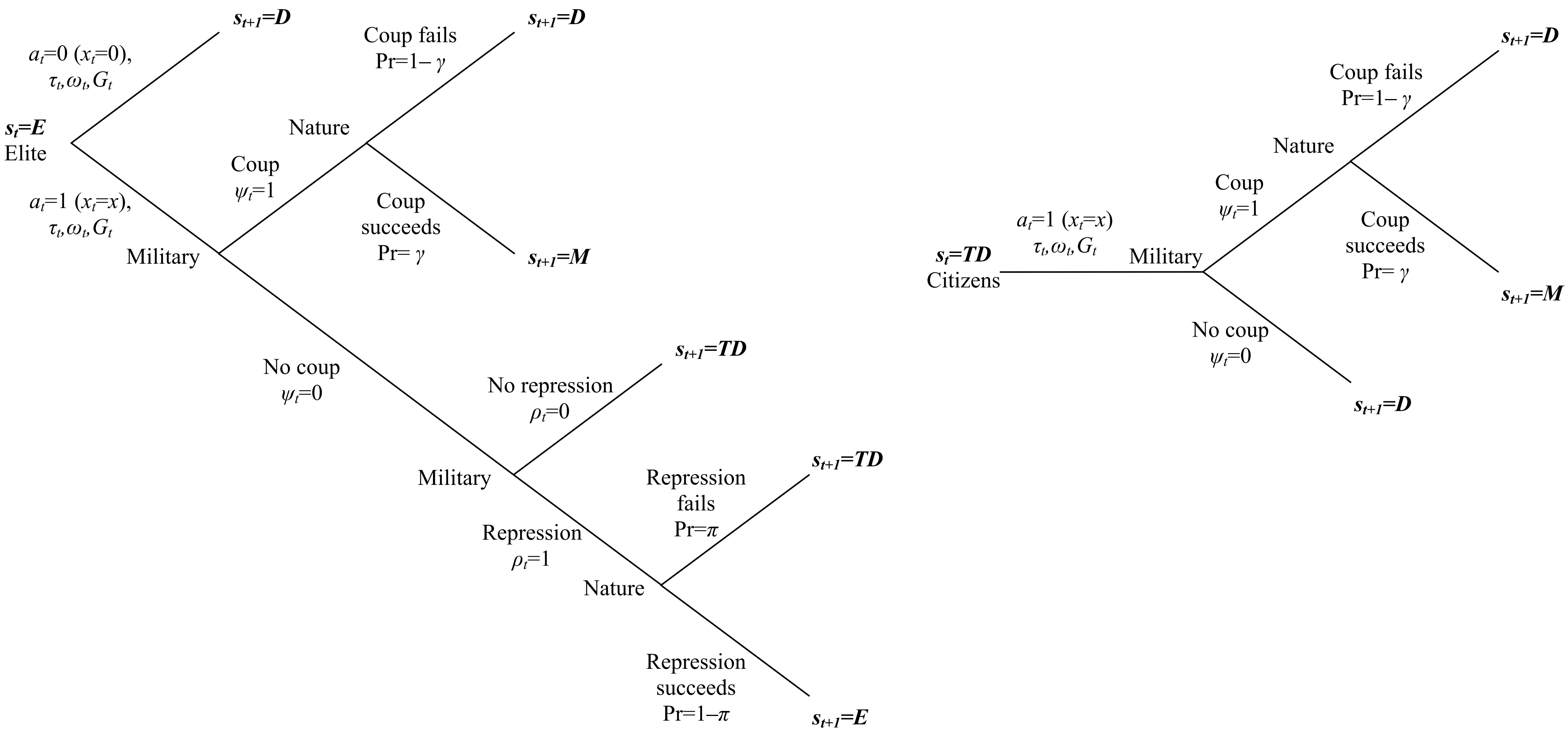

Fig. 1. The game forms starting from oligarchy (s_t=***E***) and transitional democracy (s_t=***TD***).

Appendix C: Additional Material and Proofs (Not for Publication)

4.1 Details of the Model of Subsection 3.1

It can be verified that Lemma 1 applies without any change in this modified model. In particular, in any MPE starting in a subgame with $s_t = E$ and $\eta_t = \eta^{NI}$, the military never chooses $\psi_t = 0$ and $\rho_t = 0$.

As in the baseline model, coups against oligarchy can be prevented only if the appropriate no-coup constraint is satisfied. This constraint is binding only when $\eta_t = \eta^{NI}$ because in state $\eta_t = \eta^{I}$ the elite are insulated from coups and transitions to democracy and it is still given by (34), that is, by $V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup})$, where $V^M(E \mid \text{coup})$ is still defined in (32) and $V^M(E \mid \text{repression})$ is the value of a soldier when the state variable η_t takes the value η^{NI} and the military repress. This is a consequence of the fact that coups are only possible when $\eta_t = \eta^{NI}$ and conditional on this event, they succeed with probability γ as in the baseline model. The value of a typical soldier when $\eta_t = \eta^{NI}$ and the military repress is

$$V^M(E \mid \text{repression}) = w^P + \beta\left[\pi V^M(TD) + (1-\pi)V^M(E,P)\right] \tag{62}$$

where $V^M(TD)$ is still given by (25) and $V^M(E,P)$ is the value of soldiers under prevention before they know the realization of η_t. This value is defined recursively as

$$V^M(E,P) = w^P + \beta\left\{(1-\mu)V^M(E,P) + \mu\left[\pi V^M(TD) + (1-\pi)V^M(E,P)\right]\right\}$$

and it is therefore given by

$$V^M(E,P) = \frac{w^P + \beta\mu\pi V^M(TD)}{1-\beta(1-\mu\pi)}. \tag{63}$$

Combining (62) and (63), we obtain

$$V^M(E \mid \text{repression}) = \frac{(1-\beta\pi(1-\mu))w^P + \beta\pi(1-\beta(1-\mu))V^M(TD)}{1-\beta(1-\mu\pi)}. \tag{64}$$

From (64), (32) and the no-coup constraint (34),[28] the military wage consistent with coup prevention has the same expression of the efficiency wage w^P necessary to prevent coups against oligarchy as in the baseline model, (35), with the only difference that, because of the change in the fiscal technology, the expressions for w^M and a^L are now given by (43). With

[28] Recall that $V^M(TD) = V^M(E \mid \text{coup}) = \beta\gamma V^M(M) + \beta(1-\gamma)V^L(D)$ (see footnote 17 for details).

these changes, the tax rate for the oligarchy to be able to finance the military wages necessary to prevent coups becomes:[29]

$$\tau^P = \beta\gamma\hat{\tau} + \beta(1-\gamma)\frac{a^L\bar{x}}{Y - \bar{x}A^L}. \tag{65}$$

The net present discounted value of the elite from prevention, starting with $s_t = E$, is recursively defined as

$$V^H(E,P) = \left(1-\tau^P\right)A^H + \beta\left[(1-\mu\pi)V^H(E,P) + \mu\pi V^H(TD)\right],$$

and can then be rewritten as

$$V^H(E,P) = \frac{\left(1-\tau^P\right)A^H + \beta\mu\pi V^H(TD)}{1-\beta(1-\mu\pi)} \tag{66}$$

where $V^H(TD)$ is given by (42).

The net present discounted value of the elite from non-prevention, starting with $s_t = E$, is then given by

$$V^H(E,N) = \hat{a}^H + \beta\left[(1-\mu)V^H(E,N) + \mu V^H(\text{coup})\right].$$

The first term in this expression, $\hat{a}^H \equiv (1-\mu\phi)A^H$, is the expected flow payoff to the elite, which takes into account that with non-prevention there will be coups when possible and thus income disruption. The probability that a coup will take place is μ, because a coup can take place only when $\eta_t = \eta^{NI}$. In addition, $V^H(\text{coup}) \equiv (1-\gamma)V^H(D) + \gamma V^H(M)$ denotes the expected *future* value to the elite in case there is a coup. From (41), we have that $V^H(\text{coup}) = (1-\hat{\tau})A^H/(1-\beta)$. In addition, with probability $(1-\mu)$, the elite are insulated from political change today and the same political state recurs tomorrow, i.e. $s_{t+1} = E$. Therefore,

$$V^H(E,N) = \frac{\hat{a}^H + \beta\mu V^H(\text{coup})}{1-\beta(1-\mu)}. \tag{67}$$

The following conditions on the set of parameters are useful for the characterization of the equilibrium of the model.

Condition 2 $\mu < \overline{\mu} \equiv \beta\hat{\tau}/(\phi + \beta\hat{\tau})$.

Condition 2 ensures that the elite strictly prefer non-prevention to smooth transition, that is, $V^H(E,N) > V^H(E,S)$, where $V^H(E,S)$ is defined in (28), with $V^H(D)$ now given by

[29] The participation constraint of soldiers under prevention is $V^M(E,P) \geq V^L(E,S)$ and it is always satisfied. This follows by combining the no-coup constraint, (34), $V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup})$, which implies $w^P \geq (1-\beta)V^M(E \mid \text{coup})$, and Assumption 1, which implies that $V^M(TD) \geq V^L(D)$.

(41). If this condition did not hold, the elite would prefer S to N for any value of π (or would be indifferent between them when $\mu = \overline{\mu}$). This follows since both $V^H(E,N)$ and $V^H(E,S)$ are independent of π. Therefore, when Condition 2 fails to hold, the MPE in any subgame starting in $s = E$ would be identical to that in Proposition 4 in the previous section and would not feature coups against oligarchy.[30]

The participation constraint of soldiers under non-prevention is

$$V^M(E,N) \geq V^L(E,S), \tag{68}$$

where $V^M(E,N)$ is the value of soldiers under non-prevention before the realization of the variable η_t. This value is defined recursively as

$$V^M(E,N) = (1-\mu)\beta V^M(E,N) + \mu V^M(E \mid \text{coup})$$

where $V^M(E \mid \text{coup})$ is still defined in (32), and it is equal to

$$V^M(E,N) = \frac{\mu}{1-\beta(1-\mu)} V^M(E \mid \text{coup}). \tag{69}$$

Using (69) and $V^L(E,S) = A^L + \beta V^L(D)$ in (68), we obtain that the participation constraint of soldiers under non-prevention is satisfied if and only if the following condition is satisfied.

Condition 3

$$\mu \geq \underline{\mu} \equiv \frac{1-\beta}{\beta} \frac{(1-\beta) A^L + \beta a^L}{\gamma(w^M - a^L) + (1-\beta)(a^L - A^L)}.$$

If this condition does not hold, the participation constraint of soldiers under non-prevention is violated and this means that this strategy is not feasible for the elite. Moreover, notice that this condition is always satisfied when β is high enough, and it can be easily verified that the set of parameters where Conditions 2 and 3 both hold is not empty.

Let us next define $\bar{\pi} \in [0,1]$ in a similar fashion to $\hat{\pi}$ in subsection 2.6. In particular, let $\bar{\pi}$ be the solution to the equation

$$V^H(E,P \mid \pi = \bar{\pi}) = V^H(E,N), \tag{70}$$

when such a solution exists, with the value of prevention for the elite defined as in (66). By the same argument as in the proof of Proposition 4, $V^H(E,P \mid \pi)$ is strictly decreasing in π

[30] Notice that $\overline{\mu} < 1$ and also that, except the simplification in the fiscal technology, the baseline framework is a special case of this extended model with $\mu = 1$.

and $V^H(E,N)$ is independent of π, so that when a solution $\bar{\pi} \in (0,1)$ to (70) exists, it is uniquely defined and $V^H(E,P \mid \pi) \gtreqless V^H(E,N)$ for any $\pi \lesseqgtr \bar{\pi}$. When (70) does not have any solution $\bar{\pi} \in [0,1]$, then we set $\bar{\pi} = 0$ (when $V^H(E,P \mid \pi = 0) < V^H(E,N)$) or $\bar{\pi} = 1$ ($V^H(E,P \mid \pi = 1) > V^H(E,N)$).

Finally, notice also that the MPE in transitional democracy is still given by Proposition 3 (again with the only difference that the threshold $\hat{\gamma}$ in (21) now features w^M and a^L defined by (43)). Using these observations, we obtain a more complete version of Proposition 6.

Proposition 10 *Consider the extended model presented in subsection 3.1 and suppose that $\mu \neq \overline{\mu}$, where $\overline{\mu}$ is defined in Condition 2. If Condition 2 or 3 does not hold, the MPE is identical to that in Proposition 4. If Conditions 2 and 3 hold and $\pi \neq \bar{\pi}$, where $\bar{\pi}$ is defined above, then there exists a unique MPE as follows:*

1. *If $\pi \in [0,\bar{\pi})$, then whenever $s = E$, the elite build an army for repression (i.e., $a = 1$), set $\tau = \tau^P$ and $w = w^P$, and prevent military coups. The military chooses $\psi = 0$ and $\rho = 1$ (no coup and repression). Transitional democracy arises with probability $p(TD \mid E) = \mu\pi$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu\pi$. Proposition 3 characterizes the unique MPE starting in any subgame $s = TD$, so that $q(D) = 1$ when $\gamma \in [\bar{\gamma}, \hat{\gamma}]$, and $q(D) = 1 - \gamma$ and $q(M) = \gamma$ when $\gamma \in (\hat{\gamma}, 1]$.*

2. *If $\pi \in (\bar{\pi}, 1]$, the elite build an army for repression (i.e., $a = 1$), set $\tau = 0$ and $w = 0$, and do not prevent coups. The military chooses $\psi = 1$ (coup) in state μ^{NI}. Military dictatorship arises with probability $p(M \mid E) = \mu\gamma$, consolidated democracy arises with probability $p(D \mid E) = \mu(1-\gamma)$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu$. Consequently, the long-run likelihood of regimes are given by $q(D) = (1-\gamma)$ and $q(M) = \gamma$.*

Proof. First, note that, because both $V^H(E,S)$ and $V^H(E,N)$, given in (28) and in (67) respectively, are independent of π, either $V^H(E,S) > V^H(E,N)$ or $V^H(E,S) < V^H(E,N)$ for any value of π (the case where $V^H(E,S) = V^H(E,N)$ is ruled out by the assumption that $\mu \neq \overline{\mu}$). If Condition 2 does not hold, then $V^H(E,S) > V^H(E,N)$ and non-prevention is never chosen by the elite. In this case, the equilibrium is the same as in Proposition 4. When Condition 3 does not hold, the participation constraint of soldiers under non-prevention cannot be satisfied, thus this strategy is not feasible and again the equilibrium from Proposition 4 applies.

Let us then focus on the case where both Conditions 2 and 3 hold. In this case, $V^H(E,S) < V^H(E,N)$ and non-prevention is feasible and preferred by the elite to smooth transition. By the argument in the text and the definition of $\bar{\pi}$, we have that $V^H(E,P \mid \pi) \gtrless V^H(E,N)$ for any $\pi \lesseqgtr \bar{\pi}$, which establishes the result. ∎

Remark 2 The requirement that $\pi \neq \bar{\pi}$ plays an identical role to the assumption that $\pi \neq \hat{\pi}$ in Proposition 4. When π is equal to $\bar{\pi}$, then the elite will have two best responses, so that the equilibrium is not unique, but its nature is unchanged from that described in Proposition 10. Also, the case where $V^H(E,N \mid \pi = 0) = V^H(E,P \mid \pi = 0)$ is not covered in Proposition 10 since it emerges when $\pi = \bar{\pi} = 0$, which is ruled out by the restriction $\pi \neq \bar{\pi}$. Finally, the requirement that $\mu \neq \overline{\mu}$ rules out the case where the elite obtain the same value from non-prevention and smooth transition.

4.2 Proof of Proposition 7

We begin by showing that the threshold $\tilde{\gamma}(R)$ defined in (48) is strictly decreasing in R. Straightforward differentiation of $\tilde{\gamma}(R)$ gives

$$\tilde{\gamma}'(R) = \frac{1-\beta}{\beta}\frac{\bar{x}\tilde{w}^M - \tilde{a}^L}{\bar{x}\left(\tilde{w}^M - \tilde{a}^L\right)^2},$$

where $\tilde{w}^M$ and $\tilde{a}^L$ are defined in (44) and in (45). Next, observe that taking into account the expressions of w^M and a^L defined in (43), we obtain

$$\bar{x}\tilde{w}^M - \tilde{a}^L = \bar{x}w^M - a^L = -\hat{\tau}\bar{x}A^L - (1-\hat{\tau})A^L \leq 0.$$

Therefore, $\tilde{\gamma}'(R) \leq 0$, with equality if and only if $A^L = 0$.

Next consider the decision of the elite. First, define R^* as the level of natural resources such that

$$\tilde{w}^P(R^*)\bar{x} = \hat{\tau}\left(Y - \bar{x}A^L\right). \tag{71}$$

In other words, R^* is the level of natural resources such that when $R = R^*$, total military wages necessary for coup prevention can be financed by taxing production income *only* at the maximum possible rate $\hat{\tau}$. By substituting for $\tilde{w}^M$ and $\tilde{a}^L$ in (52), we obtain

$$\tilde{w}^P(R) = w^P + \beta\frac{\gamma + (1-\gamma)\bar{x}}{\bar{x}}R. \tag{72}$$

Combining this expression with (71), we have

$$R^* = \bar{x}\frac{w^M - w^P}{\beta\left(\gamma + (1-\gamma)\bar{x}\right)}. \tag{73}$$

Claim 1 *Suppose that R^* is given by (73) and $R > R^*$. Then in any MPE with coup prevention, the elite set $\tilde{\tau}^P = \hat{\tau}$ and choose $\zeta^P \geq 0$ to balance the government budget constraint, which implies*

$$\zeta^P = \beta\left(\gamma + (1-\gamma)\,\bar{x}\right) - \bar{x}\frac{w^M - w^P}{R}. \tag{74}$$

Proof. The expression of the government budget constraint provided by (51) implies that

$$\tilde{\tau}^P = \frac{\tilde{w}^P(R)\,\bar{x} - \zeta R}{(Y - \bar{x}A^L)}.$$

Using this expression, the per period utility of the elite in oligarchy can be written as

$$\left(1 - \frac{\tilde{w}^P(R)\,\bar{x} - \zeta R}{Y - \bar{x}A^L}\right)A^H + (1-\zeta)\,R/n.$$

This expression is everywhere decreasing in ζ provided that $nA^H + \bar{x}A^L < Y$, which is always the case, since $\bar{x} < (1-n)$ by assumption, and since $Y \equiv nA^H + (1-n)\,A^L$. Therefore, $\tilde{\tau}^P$ will be set at the maximum possible level $\hat{\tau}$, and ζ will be determined to satisfy the government budget constraint, that is, ζ^P as given in (74). ■

Using the fact that in equilibrium $\tilde{\tau}^P = \hat{\tau}$, and that ζ^P is given by (74), we have that (50) can be written as

$$\tilde{V}^H(E, P \mid \pi) = \frac{(1-\beta)\left((1-\hat{\tau})\,A^H + \left(1-\zeta^P\right)R/n\right) + \beta\,(1-\beta)\,\tilde{V}^H(TD)\,\pi}{(1-\beta)\,(1-\beta\,(1-\pi))}.$$

We also have

$$\tilde{V}^H(E, S) = \frac{(1-\beta)\left(A^H + R/n\right) + \beta\,(1-\hat{\tau})\,A^H}{1-\beta}.$$

Moreover, using (49), the threshold $\tilde{\pi}(R)$ at which $\tilde{V}^H(E,S) = \tilde{V}^H(E, P \mid \pi)$ is given by

$$\tilde{\pi}(R) = \frac{1-\beta}{\beta}\frac{\left(1-\zeta^P\right)R/n - \left(\Delta - (1-\hat{\tau})\,A^H\right)}{\Delta - (1-\beta)\,\tilde{V}^H(TD)},$$

where $\Delta \equiv (1-\beta)\left(A^H + R/n\right) + \beta\,(1-\hat{\tau})\,A^H$. Now since $\partial\left(\left(1-\zeta^P\right)R\right)/\partial R = 1 - \beta\left(\gamma + (1-\gamma)\,\bar{x}\right)$, we have

$$\begin{aligned}
\tilde{\pi}'(R) \quad &= \quad \frac{1-\beta}{\beta}\frac{\left[1 - \beta\left(\gamma + (1-\gamma)\,\bar{x}\right) - (1-\beta)\right]\left[\Delta - (1-\beta)\,\tilde{V}^H(TD)\right]}{n\left[\Delta - (1-\beta)\,\tilde{V}^H(TD)\right]^2} \\
&\quad - \frac{1-\beta}{\beta}\frac{\left[\left(1-\zeta^P\right)R/n - \left(\Delta - (1-\hat{\tau})\,A^H\right)\right](1-\beta)}{n\left[\Delta - (1-\beta)\,\tilde{V}^H(TD)\right]^2}.
\end{aligned}$$

The numerator of this expression is decreasing in $\tilde{V}^H(TD)$ and $\tilde{V}^H(TD) \leq (1-\hat{\tau}) A^H / (1-\beta)$. Therefore,

$$[1-\beta(\gamma+(1-\gamma)\bar{x})]\left(\Delta-(1-\hat{\tau}) A^H\right) > (1-\beta)\left(1-\zeta^P\right) R/n \tag{75}$$

is sufficient for $\tilde{\pi}'(R) > 0$. Using the fact that $\Delta - (1-\hat{\tau}) A^H = (1-\beta)\left(\hat{\tau} A^H + R/n\right)$, substituting for ζ^P and rearranging terms, (75) is equivalent to

$$n[1-\beta(\gamma+(1-\gamma)\bar{x})]\hat{\tau} A^H > \bar{x}\left(w^M - w^P\right),$$

which in turn is the same as the following condition:

$$\bar{x} > \frac{\hat{\tau}(1-\beta\gamma)(1-n)}{\hat{\tau}(1-\beta\gamma)+\beta(1-\gamma)(1-n\hat{\tau})} \equiv \hat{x}.$$

This establishes that when $R > R^*$ and $\bar{x} > \hat{x}$, $\tilde{\pi}'(R) > 0$ and thus higher resource rents make repression more likely. This completes the proof of the proposition. ■

4.3 Proof of Proposition 8

The analysis of the MPE in this case is very similar to that in Section 1, except that whether the foreign threat is still active is now an additional state variable. Let us start in a subgame with $s = TD$ and with the foreign threat active (the analysis of the case where there is no foreign threat is identical to that in Section 1 and is omitted). The value to the military from attempting a coup is now given by[31]

$$V^M(TD \mid \text{coup}) = \beta\left\{\gamma V^M(M) + (1-\gamma)\left[\lambda V^L(D) + (1-\lambda) V^M(TD \mid \text{coup})\right]\right\}.$$

This expression differs from the version in the baseline model, (18), because when the coup is not successful (with probability $1-\gamma$) the military can now be reformed only with probability λ, while with probability $1-\lambda$ the external threat does not disappear, it is not optimal to reform the military and the political system remains in transitional democracy. This value can also be rewritten as

$$V^M(TD \mid \text{coup}) = \frac{\beta\gamma w^M + \beta\lambda(1-\gamma) a^L}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}.$$

The return to the military from not attempting a coup is

$$V^M(TD \mid \text{no coup}) = w^{TP} + \beta\left[\lambda V^L(D) + (1-\lambda) V^M(TD \mid \text{no coup})\right],$$

[31] It is again straightforward to verify that Assumption 1 ensures that the military participation constraint is satisfied.

where we again use w^{TP} to denote the military wage in transitional democracy when there is coup prevention. Note, however, that the expression for this wage will be slightly different than the one in Section 2 (see below). This value function also takes into account that the same state will recur with probability $1-\lambda$ (when the foreign threat remains active and there is no opportunity to reform the military). Rearranging this expression, we obtain

$$V^M(TD \mid \text{no coup}) = \frac{(1-\beta)\, w^{TP} + \beta\lambda a^L}{(1-\beta)(1-\beta(1-\lambda))},$$

where a^L is now defined in (43). The expression for w^{TP} in this extended environment can be obtained by solving the incentive compatibility equation, $V^M(TD \mid \text{coup}) = V^M(TD \mid \text{no coup})$, as

$$w^{TP} = \frac{1-\beta(1-\lambda)}{(1-\beta)[1-\beta(1-\gamma)(1-\lambda)]}\beta\gamma w^M - \frac{\lambda}{(1-\beta)[1-\beta(1-\gamma)(1-\lambda)]}\beta\gamma a^L, \tag{76}$$

where w^M is the soldiers' wage in a military dictatorship given by (43).

As in our analysis in Section 2, transitional democracies will prevent coups if two conditions are satisfied: first, low-skill producers should prefer to prevent coups; second, they should be able to pay high enough wages to the military to achieve this. Let us start with the second requirement. The necessary condition for the transitional democracy to pay high enough wages to the military again takes the form $w^{TP} \leq w^M$. Using the expressions for these two wage levels, the condition for the prevention of coups in transitional democracy leads to (53), that we rewrite

$$\gamma \leq \frac{(1-\beta)(1-\beta(1-\lambda))}{\beta\lambda}\frac{w^M}{w^M-a^L} \equiv \hat{\gamma}(\lambda).$$

Condition (53) is the generalization of condition (21) and shows that transitional democracies can prevent coups as long as the probability that coup attempts will be successful is not too high. Moreover, it can be verified that $\hat{\gamma}(\lambda)$ is a strictly decreasing function of λ and that $\hat{\gamma}(\lambda) \to \hat{\gamma}$ as $\lambda \to 1$. This implies the interesting result that condition (53) becomes more difficult to satisfy as λ increases (in the limit as $\lambda \to 1$, this condition coincides with (21)).

We next verify that low-skill producers prefer to prevent coups when this is feasible. If they prevent coups, their value in transitional democracy is

$$V^L(TD \mid \text{no coup}) = (1-\hat{\tau})\, A^L + G^{TP} + \beta\left[\lambda V^L(D) + (1-\lambda)\, V^L(TD,P)\right], \tag{77}$$

where $V^L(TD,P) = V^L(TD \mid \text{no coup})$ and $G^{TP} = \hat{\tau}\left(Y - \bar{x}A^L\right) - w^{TP}\bar{x}$ incorporates the fact that taxes will be equal to $\hat{\tau}$ (given the tax distortion technology adopted at the beginning of Section 3), and whatever is left over from paying soldiers the efficiency wage goes into public good expenditures.[32] Using the fact that $V^L(D) = a^L/(1-\beta)$, $V^L(TD,P) = V^L(TD \mid \text{no coup})$, that $G^{TP} = \hat{\tau}\left(Y - \bar{x}A^L\right) - w^{TP}\bar{x} \geq 0$, and that w^{TP} is given by (76), the value to low-skill producers when they prevent coups $V^L(TD \mid \text{no coup})$, (77), can be rewritten as

$$\begin{aligned} V^L(TD \mid \text{no coup}) \;=\;& \frac{(1-\hat{\tau})A^L + \hat{\tau}\left(Y - \bar{x}A^L\right)}{1-\beta(1-\lambda)} + \frac{\beta\lambda a^L}{(1-\beta)(1-\beta(1-\lambda))} + \\ & + \frac{\beta\gamma\lambda\bar{x}a^L}{(1-\beta)(1-\beta(1-\lambda))(1-\beta(1-\gamma)(1-\lambda))} + \\ & - \frac{\beta\gamma\bar{x}w^M}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}. \end{aligned} \tag{78}$$

Alternatively, without prevention, the value to low-skill producers is

$$\begin{aligned} V^L(TD \mid \text{coup}) \;=\;& (1-\hat{\tau})(1-\phi)A^L + G^{TN} + \\ & + \beta\left\{\gamma V^L(M) + (1-\gamma)\left[\lambda V^L(D) + (1-\lambda)V^L(TD \mid \text{coup})\right]\right\}, \end{aligned} \tag{79}$$

where $G^{TN} \equiv \hat{\tau}(1-\phi)\left(Y - \bar{x}A^L\right)$, since in this case zero wages are paid to soldiers (i.e., $w^{TN} = 0$). This expression also takes into account that, as before, when a coup attempt fails, the military can be reformed, and therefore there is a transition to a fully consolidated democracy, only with probability λ. Using the expressions of $V^L(D) = a^L/(1-\beta)$, of $V^L(M)$ in (15), and the fact that $G^{TN} \equiv \hat{\tau}(1-\phi)\left(Y - \bar{x}A^L\right)$, the value to low-skill producers when they do not prevent coups $V^L(TD \mid \text{coup})$, given by (79), can be rewritten as

$$V^L(TD \mid \text{coup}) = \frac{(1-\beta)\left[(1-\hat{\tau})(1-\phi)A^L + \hat{\tau}(1-\phi)\left(Y - \bar{x}A^L\right)\right] + \beta\gamma(1-\hat{\tau})A^L + \beta\lambda(1-\gamma)a^L}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}. \tag{80}$$

Assuming that coup prevention is a feasible strategy, namely that condition (53) holds and therefore the wage w^{TP} defined in (76) can be offered to the military, low-skill producers prefer to prevent coups if $V^L(TD \mid \text{no coup}) \geq V^L(TD \mid \text{coup})$. Combining (78) and (80), and taking into account that w^M and a^L are given by (43), this condition is equivalent to

$$\beta\gamma\lambda\hat{\tau}\bar{x}A^L + \beta\gamma\lambda\bar{x}a^L + \phi(1-\beta)(1-\beta(1-\lambda))\left[(1-\hat{\tau})A^L + \hat{\tau}\left(Y - \bar{x}A^L\right)\right] > 0. \tag{81}$$

The left-hand-side of this inequality is always positive and, therefore $V^L(TD \mid \text{no coup})$ is always greater than $V^L(TD \mid \text{coup})$, which means that the low-skill producers always prefer to prevent coups.

The rest of the proposition, including the fact that $\hat{\gamma}(\lambda)$ is strictly decreasing in λ, follows immediately from the arguments in the text. ■

[32] The hypothesis that coup prevention is possible, that is, $w^{TP} \leq w^M$, ensures that $G^{TP} \geq 0$.

4.4 Proof of Proposition 9

Using (54) and (55), the threshold $\breve{\pi}$ (defined as $V^H(E,S) = V^H(E,P \mid \breve{\pi})$) can be written as

$$\breve{\pi} = \frac{1-\beta}{\beta(1-\alpha)} \frac{\left(\tau^P - \beta\hat{\tau} + \alpha\beta\left(\hat{\tau} - \tau^P\right)\right) A^H}{\left[(1-\beta) V^H(TD) - (1-\beta\hat{\tau}) A^H\right] + \alpha\beta\left[(1-\hat{\tau}) A^H - (1-\beta) V^H(TD)\right]}.$$

If there is no coup in transitional democracy, (42) implies $V^H(TD) = (1-\hat{\tau}) A^H / (1-\beta)$ and

$$\breve{\pi} = 1 - \frac{(1-\alpha\beta)\tau^P}{(1-\alpha)\beta\hat{\tau}},$$

which is strictly decreasing in α.

If coups take place along the equilibrium path in transitional democracy, then (42) yields

$$V^H(TD) = (1-\hat{\tau})(1-\phi) A^H + \frac{\beta}{1-\beta}(1-\hat{\tau}) A^H.$$

Using this expression, the threshold $\breve{\pi}$ becomes

$$\breve{\pi} = \frac{\tau^P - \beta\hat{\tau} + \alpha\beta\left(\hat{\tau} - \tau^P\right)}{\beta(1-\alpha)\left[\alpha\beta\phi(1-\hat{\tau}) - \hat{\tau} - \phi(1-\hat{\tau})\right]}.$$

Since τ^P does not depend on α, we obtain that the derivative of this expression $d\breve{\pi}/d\alpha$ is proportional to

$$B(\phi) \equiv \left[\alpha(1-\beta)\tau^P - (1-\alpha)\left(\alpha\beta\left(\hat{\tau} - \tau^P\right) - \left(\beta\hat{\tau} - \tau^P\right)\right)\right](1-\hat{\tau})\beta^2\phi - (\hat{\tau} + (1-\hat{\tau})\phi)(1-\beta)\beta\tau^P,$$

where $B(\phi)$ is the numerator of $d\breve{\pi}/d\alpha$. This expression is linear in ϕ, and is negative when $\phi = 0$. Therefore, it has at most one root $\phi = \phi^*$ over the interval $[0,1]$. This implies that for any $\phi < \phi^*$, $B(\phi) < 0$ (so that $\breve{\pi}$ is strictly decreasing in α) and for any $\phi > \phi^*$, $B(\phi) > 0$ (so that $\breve{\pi}$ is strictly increasing in α). Moreover, if $B(\phi)$ has no root in $[0,1]$, then $B(\phi) < 0$ for all $\phi \in [0,1]$ and we set $\phi^* = 1$. This establishes all the claims in the proposition. ■