A simplified unbalanced bidding model

David William Cattell
Paul Anthony Bowen
Ammar P. Kaka

Available at: https://works.bepress.com/david_cattell/3/
A simplified unbalanced bidding model

Published:

Much research effort to date has focused on the development and use of bidding models in optimizing contractors’ bid prices in competitive tendering environments. The objective of such models, incorporating all the complexity associated with modelling a project’s revenue and its costs, has been to maximize a project’s profit. Unbalanced bidding models, permitting differentiated mark-up to all of a project’s items of work, have also been unnecessarily complicated by incorporating consideration of a project’s item costs. It is shown here that unbalanced bidding models have been unnecessarily complicated by incorporating consideration of a project’s item costs. Bidding models can be significantly simplified by having the objective of maximizing a project’s top-line revenue rather than maximizing bottom-line profit. A new model, incorporating all three standard effects of item price loading: namely, front-end loading, individual-rate loading, and back-end loading, is proposed that gives effect to determining the optimum pricing for a project’s component items.

*Keywords:* Bidding, competitive advantage, cost modelling, revenue, mark-up.
Introduction

Bidding models are mathematical techniques designed for use by building contractors, amongst others, to assist them with optimising their bid prices in competitive tenders (Cattell, 1985). This area of research has been led by that of Friedman (1956) and Gates (1959) and more than 1000 papers have been published since then with much of the debate focussed on the underlying mathematics. The debate has gone on for 40 years and it is only recently that Skitmore et al. (2007) has provided a proof that the mathematics advocated by Gates is to be preferred over that which was instead proposed by Friedman. However, Skitmore (2004) warns that both methods are problematic.

Bidding models are focussed on the determination of a project’s overall bid price. Contractors are also, however, required to submit prices for each component item such that the summation of these component prices equates to the overall project price. These item prices are then used as the basis by which these contracts are then administered.

Gates (1959) was the first to identify the role of item price loading as a strategy. This approach entails allocating different mark-ups to individual items within a project so as to realise advantages that are not likely to be accomplished by way of allocating a universally constant mark-up to all of a project’s items. Further research (as reviewed by Cattell et al., 2007) led to the development of a variety of mathematical techniques by which to optimise this and these have become known as unbalanced bidding models.

These models are typically designed for use by building or engineering contractors and are often utilized in the oil and forestry industries. This study is, however, concerned
primarily with the use of these models by contractors in the construction industry although much of this research is relevant to the full spectrum of the potential use of these models.

Common to these models (such as those advocated by Gates, 1959, 1967; Stark, 1968, 1972, 1974; Ashley and Teicholz, 1977; Teicholz and Ashley, 1978; Diekmann et al., 1982; Cattell, 1987; and, Tong and Lu, 1992) is that their objective is to maximize a project’s profit. In the process they are all constructed so that they model a project’s bottom-line profit, taking account that this is the difference between a project’s top-line revenue and its costs. These models therefore incorporate all of the complexity and effort associated with modelling both a project’s revenue as well as its costs. It is shown here that a model can be constructed that it accomplishes the same effect regardless of whether it incorporates this aspect of cost. This simplified model has the objective of maximizing the top-line revenue, rather than the bottom-line profit. This approach is simpler, easier and more practical to use, and yet it performs the same role to the same effect.

The effects of item price loading

Each item of work within a project has largely different characteristics to other such items. Some relate to work that has to be done early in the construction schedule; others to activities scheduled later. Many fall within different escalation workgroups in terms of contract price adjustment provisions. Some items have an initial quantity attached to them that the contractor can be fully confident will not differ from the final quantity. Some others describe work that is expected to vary in quantity when it is built: some of these items may be expected to finally be allocated a higher quantity and others a lower
quantity. Some items’ final quantities are easier to estimate than others, and thus some enjoy a higher degree of confidence as regards their variability than others. And so on. Thus, if one considers that each item incorporates many different characteristics in different proportions; most items are different in their overall character from any other item.

Item price loading as a theory (see Cattell et al., 2004, 2007) relies on this reality that most items are very different from others as regards their character. By allocating some items higher mark-ups than other items, item price loading is seeking to take advantage of each item’s uniqueness in character.

Consider the following examples (taken from Cattell, 1987):

- If high prices are allocated to items scheduled to arise early in the contract’s project plan, the contractor will receive larger amounts of money for the first few interim payments which will aid their initial cash flow for the contract – a practice known as ‘front-end loading’;

- If the contractor was to allocate high prices to items that are scheduled to occur late in the project plan and to those that fall into workgroups that have a high expected escalation, the contractor will receive larger amounts in escalation in compensation for inflation – perhaps more than the cost to them of escalation – but, more importantly, most certainly more than if they were to allocate lower prices to such items – a practice known as ‘back-end loading’;
• If a contractor was able to predict variations in the contract’s design or identify mistakes in the measured quantities, they may take advantage by allocating high prices to items for which they expect the quantity to be adjusted upwards and low prices to items that they expect will be reduced – a practice known as ‘quantity error exploitation’.

Pursuing any one of these opportunities (as examples) in isolation is intuitively very simple. However, the reality is a lot more complex when one considers that each single item within a project does not only have one such characteristic, but instead, they all have a largely unique and complex blend of many such characteristics. Each item cannot simply be described, for instance, as being an ‘early’ or ‘late’ item without also having to recognize that it will have other characteristics as well. Each of these characteristics calls for a different treatment as regards item price loading. For instance, a ‘late’ item might fall into an escalation workgroup that has a high expected rate of escalation. In this instance, it is not obvious whether this item should be allocated a high or low mark-up (Cattell, 1984). The objectives of front-end loading would suggest heuristically that ‘late’ items should be allocated low prices and yet the objectives of back-end loading suggest that a ‘late’ item in a ‘high-escalation’ workgroup should be allocated a high price. Which of these two is, in this instance, of greater significance?

To add further to the complexity, consider that a project typically comprises many hundreds or thousands of items, most of which are unique in character. Thus, when viewed holistically, with all of the complexity that this has inherent in it, it becomes a lot less obvious how best to pursue the advantages of item price loading. Nevertheless, not only does it remain obvious that these advantages remain intact regardless of this
complexity, but also, it is hypothesized that the more complex a project is, the more there are opportunities for item price loading, and the more should be the advantages of this practice.

The objective for any unbalanced bid is to determine the optimum distribution of a project’s overall tender price amongst its multitude of component items. To satisfy this objective, one might look to maximize a project’s profit assuming that one is able to determine such profit from each possible item-price combination. To accomplish this suggests that one must be able to quantify the profit contribution from each item, if assigned a particular price, such that the project’s profit will amount to a summation of all of the contributions of profit from each of these items. We advocate that each item’s benefit can be determined by way of its profit contribution relative to its price (and this will later be shown to be a linear relationship).

This concept is similar to what Teicholz and Ashley (1978) referred to as each item’s ‘Desirability Index’. This approach was advocated by Cattell (1987) and is also similar to the approach that Tong and Lu (1992) adopted as regards their maximum-minimum technique. The measure of each item’s profit relative to price shall be referred to hereafter as the ‘Profitability Responsiveness Index’ (‘PRI’) and the ranking of a project’s items in terms of this index, as the ‘Profitability Responsiveness Item Ranking’ (‘PRIR’).

Having the PRI facilitates a contractor knowing whether the combined effects of any item’s characteristics should rank as being more ‘worthy’ of being allocated a higher price than other items. This (PRIR) ranking of items should therefore list the items in the
sequence that they promise to reward the contractor with added profit, in response to being allocated a higher price. Thus, the item with the highest PRIR is the item that has been identified as most likely to generate the best return for being allocated a unit of currency. If the contractor’s objective was only that of profitability, then it is the one item that a contractor should most wish to price as high as possible. The PRI comprises a summation of similar values that are derived from each of the practices of front-end loading, individual rate loading and back-end loading.

**Front-end loading**

If front-end loading were to be pursued in isolation, one would desire to mark-up as high as possible the items scheduled to be built early on in the project. The objective of this practice is for the contractor to generate as much cash as possible, as quickly as possible. One might make the following assumptions as the basis by which to initially simplify the building of this model, namely: that there is no practice of retention; that there is no practice of escalation; that there is no possibility of any quantity variation; that the costs of any item \( j \) are incurred simply simultaneously with the receipt of the interim payment for that same month; and that the costs when incurred are exactly as they were estimated to be and are fixed (in the sense of not being subject to inflation) at the time of the estimate.

Some of these assumptions may seem unreasonable (because they do not describe reality) but they are intended to serve two purposes, namely:
1. so that the model initially appears as simple as is possible, holding back on some of
   the complexities so that they are added only later, once the basic model has been
   formulated; and

2. because it will become obvious later that some of the variables in the initial model
   become irrelevant as later phases of the modelling process are introduced. It is
   therefore not necessary to have to burden any elegant simplicity of the initial phases
   of the modelling process with superfluous complexity that will only later be discarded
   when found to be redundant.

On this basis, considering only the purpose of pursuing front-end loading, a contractor
could determine the present value $PV$ for any item $j$ (that shall later form the basis of the
PRI for that same item) by way of the following equation:

$$PV_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \lambda_{nj} Q_j \left( P_j - C_j \right) \right]$$

Equation (1)

where

- $j$ = item number
- $n$ = month number
- $N$ = duration of project in months
- $r_j$ = monthly discount rate appropriate to the risk of item $j$
- $\lambda_{nj}$ = proportion of $Q_j$ to be built in month $n$

$$\sum_{n=1}^{N} \lambda_{nj} = 1.0 \text{ for any item } j$$
\[ Q_j = \text{bill quantity of item } j \]
\[ P_j = \text{bill price per unit of item } j \]
\[ C_j = \text{unit cost of item } j \]

Items that are scheduled to be built early on in the project will have high values of \( \lambda_{nj} \) when \( n \) is of a low value. For these (‘early’) items, when \( n \) tends to \( N \), \( \lambda_{nj} \) is most probably 0, or at very least of very low positive value (i.e. \( \to 0 \)). One can see from this equation that these early items will generate a higher \( PV_j \) than the equivalent item scheduled to be built later in the project. This corresponds to the principle of front-end loading in that it clearly recognizes that early items priced highly will generate higher \( PV_j \) s than later items priced similarly.

Furthermore, it obviously follows that the higher the discount rate \( r_j \) the greater will be the differential between the \( PV_j \) generated by early and late items.

**Individual rate loading**

The loading of the rates of individual items is a practice otherwise referred to as quantity error exploitation (see Tong and Lu, 1992). This practice amounts to allocating high prices to items whose initially-measured quantities are thought likely to be increased, while allocating low prices to items whose initially-measured quantities are thought likely to decrease.
This practice arises in those forms of contract where the initial contract quantities are not fixed as final but instead are subject to review and adjustment as the realities of the project unfold. These adjustments are typically necessitated by initial uncertainties in the design, or by the soil and rock conditions on site being found to be different from that which was initially expected.

This practice was first commented on by Gates (1959) and has subsequently been referred to many times (see, for instance, Gates, 1967; Diekmann et al., 1982; Cattell, 1984, 1987; Tong and Lu, 1992) as something that is very common and widespread amongst contractors.

The benefit to a contractor from individual rate loading is derived from their ability to shift their margin onto items where, when the consequently high prices (with high margins built in) are applied to increases in these items’ quantities, the contractor enjoys far greater compensation for their extra work than is reflected in their increased cost of the added work. Furthermore, a contractor can use the opportunity of a prediction that an item’s final quantity will be less than its initial quantity depicted in the bills of quantities, by allocating such items a low price. If the prediction is correct, the ultimate reduction in the payment made to the contractor will be less than if they were to have priced such an item any higher.

On the basis of the same assumptions as made above (with the obvious exception of quantity variations) and considering only the purpose of pursuing individual rate loading (i.e. the pursuit of the exploitation of any errors or other adjustments in the initial contract quantities), a contractor could quantify the present value $PV$ for any item $j$ (that shall later
form the basis by which to determine the PRI for that same item) by way of the following
equation:

\[ PV_j = \sum_{n=1}^{N} \left( \frac{1}{1+r_j^n} \right) \lambda_{nj} \left( Q_j + Q'_j \right) \left( P_j - C_j \right) \]  

Equation (2)

where \( Q'_j \) = additional quantity of item \( j \) due to variation

Items that have a high \( Q'_j \) will generate a high \( PV_j \) - especially when combined with the allocation of a high price \( P_j \). Thus, any model that has the objective of maximising the summation of the \( PV_j \)’s will give cause for high prices to be allocated to those items that have relatively high \( Q'_j \)’s. The reverse is true of items that have relatively low (typically negative) \( Q'_j \)’s and hence these items create the “funding” differential by which the margin can be shifted from those items of low \( Q'_j \)’s to those items of high \( Q'_j \)’s.

**Back-end loading**

The opportunity with back-end loading is for a contractor to be over-compensated for the inflationary increases in their expenses. This opportunity arises in contracts that incorporate the practice of escalation payments in terms of contract price adjustment provisions. In such situations an estimate is made of the contractor’s actual cost of inflation with the objective being that they should be compensated for this added expense. The concept is such that the contractor should neither profit nor make a loss from inflation but rather that any risk that comes from inflation should be passed over from the contractor to the project’s developer.
In such contracts, the contractor cannot simply provide proof of their actual costs of inflation. Contractors’ initial estimates of their costs constitute confidential information and while they are known only to them, it is not appropriate for the developer or their professional agents to have access to this. Neither is it appropriate for developers or their agents to have access to knowledge of the final cost that is incurred by the contractor. Moreover, the sheer volume of paperwork involved would render it impractical to employ a ‘proven cost’ method of dealing with inflation. This nature of contract therefore incorporates the concept of ‘escalation’ by which an estimate is made of the actual increase in the contractor’s costs. This escalation estimate provides the mechanism by which an amount roughly representing the actual increase in costs can be borne by the developer, rather than the contractor. Published indices are usually used for this purpose.

The opportunity for back-end loading stems from the fact that the values of escalation is, in such instances, determined from estimates based on the contractor’s gross item prices, rather than being based on his actual costs. For instance, in South Africa the escalation calculation is done in terms of the “Haylett” contract price adjustment provisions (JBCC, 2005), with a non-adjustable element of 15% i.e. the adjustment factor is 0.85. This implies that it is being assumed that a contractor’s cost is 85% of any item’s price. Thus, if a contractor’s price is high, the assumed cost is also high, regardless of the actual cost. One simple way for a contractor to practice back-end loading is therefore to apply high prices to items that are scheduled to occur late in the contract. These high prices will give the impression that these items have high costs and therefore they will enjoy high levels of escalation adjustment.
Another opportunity to benefit from back-end loading is derived from the system by which items of work are typically categorized into escalation workgroups (Cattell, 1987). This system is structured so that inflation is monitored in each of the workgroups and indices published specific to each of them. This method facilitates that if, for example, the cost of iron-ore ‘sky-rockets’, that the work that entails the use of this material will be adequately compensated for. The potential for loading arises from the ability for a contractor to make predictions of the escalation rates in each of the workgroups. They could allocate higher prices to those items that fall within the workgroups that are expected to have higher than average rates of escalation.

Both of the above two techniques could be combined. Contractors practicing back-end loading should ideally allocate the highest prices to those items that are scheduled for late in the contract and also which are categorized as falling into the workgroups that are expected to have the highest rates of escalation. The ‘funding’ for these high prices obviously needs to be sourced from the use elsewhere of relatively low prices, the lowest of which should be allocated to the items that are scheduled for completion early in the project and which furthermore are categorized into the workgroups that have the lowest expected rates of escalation.

The following formulation describes the basis by which an item’s present value $PV_j$ can be quantified:

$$PV_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \left( \lambda_{nj} \left( Q_j + Q_j' \right) \right) \left[ \Upsilon_{nj} f P_j - C_{nj}' \right] \right]$$

Equation (3)

where
\[ \Upsilon_{nj} = \text{adjustment for escalation} = \frac{\text{index}_n - \text{index}_0}{\text{index}_0} \]

\[ f = \text{adjustable factor (e.g. 0.85 for “Haylett” contracts)} \]

\[ C'_{nj} = \text{actual increase in the unit cost of item } j \text{ in month } n \]

The assumptions made to keep this formulation simple continue to include the following:
that there is no practice of retention; and that the costs of any item \( j \) are incurred simply
simultaneously with the receipt of the interim payment for that same month.

**Complex composite loading**

Equations 1, 2 and 3 can be combined as one which is presented as Equation 4 below.
Equation 2 can be thought of as the same as Equation 1 with the addition that the one
assumption as regards quantities remaining fixed being addressed. Thus, for these
purposes Equation 2 can be regarded as an enhanced version of Equation 1. To combine
these three equations, one therefore only has to combine Equations 2 and 3. These can be
added together as follows:

\[
P V_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \lambda_{nj} (Q_j + Q'_j) \left( P_j - C_j \right) + \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ (\lambda_{nj} (Q_j + Q'_j)) \left( \Upsilon_{nj} f P_j - C'_{nj} \right) \right]
\]

Thus

\[
P V_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \lambda_{nj} (Q_j + Q'_j) \left( P_j - C_j + \Upsilon_{nj} f P_j - C'_{nj} \right)
\]
which is

\[ PV_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \lambda_{nj} (Q_j + Q'_j) \left( (1 + \gamma_{nj}) P_j - C'_{nj} \right) \right] \]

Equation (4)

where \( C''_{nj} = C_j + C'_{nj} \)

\[ C''_{nj} = \text{actual, inflated unit cost of item } j \text{ in month } n \]

This formulation still incorporates the assumptions that applied to equation 3 above.

Equation 4 can now be adjusted in order to incorporate the effects of retention:

\[ PV_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \lambda_{nj} (Q_j + Q'_j) \left( (1 + \gamma_{nj}) P_j - C''_{nj} \right) + R'_n \right] \]

Equation (5)

where \( R_n = \text{proportion retained in month } n \)

\[ R'_n = \text{the amount (if any) released from the retention fund in month } n \]

including any interest earned (if applicable)

Notice that Equation 5 takes the form of a linear equation:

\[ PV_j = \alpha_j + \beta_j P_j \]

where \( \alpha_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ -C''_{nj} \lambda_{nj} (Q_j + Q'_j) + R'_n \right] \)

and \( \beta_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \left( \lambda_{nj} (Q_j + Q'_j) \right) \left( (1 + \gamma_{nj}) (1 - R_n) \right) \right] \)
with slope \( \beta_j = \frac{\Delta PV_j}{\Delta P_j} \)

and with \( \Delta PV_j \) as the change in \( PV_j \)

\[
\Delta PV_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \left[ \left( \lambda_{nj} (Q_j + Q_j') \right) \left[ (1 + Y_{nj} f) (1 - R_n) \Delta P_j \right] \right]
\]

It is noteworthy that item price loading can do nothing to change \( \alpha_j \), which is the fixed intercept regardless of \( P_j \). (The interest on retained funds may be marginally affected by front-end loading, in particular, but it is suggested that this is of such small consequence that it should be considered irrelevant.) The focus of attention for any item price loading model needs therefore to be on the slope \( \beta_j \).

The significance of this observation is that this aspect of the overall item price loading model can ignore all considerations of costs (as represented by \( C_j \) and \( C_j' \)). This furthermore goes to imply that the effect of having the objective of maximising a project’s revenue will be the same as having the objective of maximising its profit.

The slope \( \beta_j \) is equal to \( \Delta PV_j \) if we let \( \Delta P_j = 1 \), thus

\[
\beta_j = \sum_{n=1}^{N} \left( \frac{1}{1 + r_j} \right)^n \lambda_{nj} (Q_j + Q_j') \left( 1 + Y_{nj} f \right) (1 - R_n)
\]

where \( \beta_j \) is the sensitivity of an item’s contribution \( PV_j \) to an item’s price \( P_j \).
This composite formulation (incorporating the standard forms of unbalanced bidding, as dealt with individually above) provides an important means to identify the relative worth of each of the unbalanced bidding techniques. For instance, it provides a basis by which to determine whether $\beta_j$ for a particular early item should be relatively high for reason that it is likely to be a valuable contributor to a front-end loading strategy, or instead relatively low for reason that back-end loading should wish to allocate it as low a price as possible. This single, composite formulation presents the means to measure all items, regardless of their unique character, on the same common basis with respect to their potential contribution to a project’s overall return.

Let $\beta_j$ going forward be referred to as the ‘Profitability Responsiveness Index’ ($PRI_j$). It indicates the sensitivity of an item in terms of the extent to which this item’s profit contribution ($PV_j$) can be improved by way of an increase to its unit price ($P_j$). If one ignores the issue of risk, the item $j$ having the highest $PRI_j$ should be the one that has the greatest cause to be allocated the highest price.

Note that it is the relativity of the items’ $PRI_j$ that is significant, not the underlying absolute values of the $PRI_j$ s themselves. The $PRI_j$ s facilitate that a contractor can rank their items in order of their $PRI_j$ values (giving them their $PRIR_j$ relative ranking). This ranking identifies the relative significance of a project’s items in terms of which items will contribute the most profit for any increase in unit price.

To apply this equation, contractors who have the objective of maximizing their profit should look to allocate as high a price as possible to the item with the highest $PRIR_j$. 
Having priced this item as high as possible, they should then look to price the item with the next highest \( PRIR_j \) with the highest price possible, and so on, working their way through to the item with the lowest \( PRIR_j \) ranking.

**Tender price constraint**

The objective of maximizing a contractor’s profit suggests that, if there were not any constraints, the most profit (in particular, an infinite amount of profit) could be accomplished if a contractor were free to price the item with the highest \( PRIR_j \) with an infinitely high price. An infinitely high profit could, in fact, be accomplished by pricing any of the items with an infinitely high price. There are, however, obvious constraints that govern this situation that need to be incorporated into any such mathematical model. Clearly, it is obvious that one constraint is needed to ensure that the summation of the priced items is the same as the project’s overall tender price. This constraint can be expressed by way of the following formulation:

\[
\sum_{j=1}^{J} Q_j P_j = TP
\]

This type of constraint has similarly been incorporated into the models proposed by Stark (1968, 1972, 1974) and Diekmann *et al.* (1982).

**Discussion**

Cattell *et al.* (2007) identified that the most recent unbalanced bidding models recognize that the practice of unbalanced bidding provides two benefits for building contractors: it
not only can improve their profits but it can also reduce their risks. Diekmann et al. (1982) and Cattell (1987) have both provided models that incorporate both of these aspects. By comparison, earlier models were only focussed on maximizing a project’s profit. In common with these other models, the model proposed here does not address the aspect of risk. This should not be interpreted to suggest that risk should be ignored. Instead, the intention is that this model should not be applied on its own but rather that it should applied together with another model that has the sole purpose of quantifying the combined risks that are generated by way of different item price combinations. These two models should then be used together so that, in combination, they will maximize profit and minimize risk. It is proposed that these two models should be combined using modern portfolio theory (‘MPT’), together with indifference mapping and expected utility theory. Ultimately, the (combined) model identifies the optimum item prices best suited to the contractor’s objectives (being both profitability and risk reduction). The risk model and the method of combining these two models are not the subject of this paper.

The risks that need to be considered include those associated with the use of the Present Value method of accounting for the timing of the various cash flows. In the above model an interest rate $r_j$ has to be chosen, which is at risk of being wrong. The interest rate is also unlikely to remain a constant for the entire duration of the project. This choice of interest rate will have an effect on all calculations and a relatively small error in the estimate of this one variable can have wide-reaching repercussions. When applying this model, this aspect of the risk should be considered and incorporated into the risk versus return combination as described in the paragraph above.
Price rate constraints

It has been commented on by Stark (1968, 1972, 1974), Diekmann et al. (1982), Teicholz and Ashley (1978), and Tong and Lu (1992) that individual item prices should be bound by constraints. They all expressed these constraints in the following format:

\[ L_j \leq P_j \leq U_j \]

where \( L_j \) and \( U_j \) are specified lower and upper bounds, respectively, for the unit price of item \( j \).

Heuristics certainly support the view that some nature of constraint is needed to avoid such extreme situations as having one single item being allocated such a high price that all other items are given the price of nil. However, it is questionable whether the best way to formulate the objective of these constraints is with the use of \( L_j \) and \( U_j \) as fixed, non-negotiable, constants.

The use of \( L_j \) and \( U_j \) as fixed constants suggests that a contractor would never consider pricing item \( j \) at the price \( L_j - 0.01 \) or at \( U_j + 0.01 \) regardless of the merits of doing so. This is patently unrealistic. It is surely more practical to imagine that a contractor may consider using a price of \( L_j - a \) or \( U_j + a \) where \( a \) is a small positive number, should there be sufficient cause for doing so. On this basis, it is impossible to contemplate the justification for the use of any constant values for \( L_j \) and \( U_j \) no matter how extreme, or how far apart, they are.
These constraints have considerable significance. In the instances of the models formulated by Stark (1968, 1972, 1974), Diekmann et al. (1982), Teicholz and Ashley (1978), and Tong and Lu (1992) these constraints have the effect of almost all items being allocated prices at either their upper \(U_j\) or lower \(L_j\) bound limits.

In all of these models the researchers felt that contractors should have some intuition that can guide them so as to decide the constant values assigned to \(U_j\) and \(L_j\). None of them advocated any scientific basis by which a contractor could decide the values of \(U_j\) and \(L_j\) and yet it is hypothesized that it is these decisions that are the most significant, and yet the most difficult to accomplish, in any of these item price loading strategies.

The model proposed here instead advocates that there should be no need for such item price constraints. It is instead suggested that the underlying rationale for such constraints is one of risk. If an item were to be priced exceptionally high (i.e. beyond what any of the above-mentioned models might have capped with an upper limit of \(U_j\)) or else exceptionally low (below what these models might have capped by way of \(L_j\)) then, it is argued, this would give cause for extraordinary and unacceptable levels of risk. Such risks might include the overall tender being rejected by the client or their quantity surveyor (on the grounds that the item pricing is unacceptable). Another risk will be from variation orders where an exceptionally high-priced item may be reduced in scale, or else totally eliminated, while a low-priced item may be vastly increased in scale.

As discussed above, this model only considers revenue and not risk. This was also the case with the models proposed by Stark (1968, 1972, 1974), Teicholz and Ashley (1978),
and Tong and Lu (1992), but while they felt there to be a need for item price constraints within their models, we advocate that these are only required when having to take risk into account. It has instead been shown that any unbalanced bidding model need not necessarily include the complexity that is intrinsic when modelling a project’s bottom-line profit and that rather it is possible that a simpler model of a project’s top-line revenue is sufficient to accomplish the same effect.

**Conclusions**

We have provided a comprehensive basis by which to quantify an item’s potential contribution to a project’s overall profitability. The basis proposed incorporates all three standard effects of item price loading: namely, front-end loading, individual-rate loading, and back-end loading.

The model proposed here is formulated so as to maximize a project’s revenue and not its profit. It has been shown that the maximization of a project’s revenue accomplishes the same effect as when the objective is instead to maximize a project’s profit. Earlier models are instead structured to take a project’s costs into account and hence to maximize the profit. It has been shown that this is more difficult to accomplish and that there is no benefit to be derived from this added complexity.

**Recommendations**

Based on the findings, it is proposed that future work focus on the development of a complementary risk model, the purpose of which is the quantification of the combined risks that are generated by way of different item price combinations. This model, when
used in combination with the model proposed here, can then be employed to maximize profit and minimize risk. It is offered that these two models should be combined using modern portfolio theory (‘MPT’), together with indifference mapping and expected utility theory.

References


