Collusion and the "Old Boys Club"

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Abstract: This paper begins with the assumption that collusion can be sustained by means of sincere group loyalty among members of a cartel, and that such loyalty can be inculcated into others. We show that inculcating loyalty into potential entrants can increase profits for both the incumbents and for the (loyal) entrants, particularly if the experiences that serve to inculcate loyalty also bestow valuable industry-specific human capital on the recipient. This can allow the incumbents to blockade non-loyal entry and can also allow incumbents to profitably expand the cartel even absent an entry threat. It appears that, in the presence of genuine loyalty, activities that are typically associated with an “Old Boys Club” can increase collusive profits.

JEL Codes: L1, Z1
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I. Introduction:

In standard models of collusion, it is only fear of future punishment that motivates the members of a cartel not to defect. In this paper we make the alternative assumption that cartels are sustained by means of group loyalty. That is, we assume that some or all participants in an industry regard themselves as members of a group to which loyalty is due and behave collusively because they sincerely prefer to do so.¹

Frank (1987) shows that an innate desire to be loyal to others in one’s group can be adaptive insofar as it generates difficult-to-mimic signals that one can be relied upon to behave loyally, which makes it easier to enter into beneficial cooperative arrangements with others. There is now quite a large empirical literature showing that members of an ethnic group or social class often can sustain cooperation that would not otherwise be in the interests of the individual members.²

The specific idea for this paper came from Genesove & Mullin (2001). In that paper, the authors analyze the minutes of meetings of the Sugar Institute, a cartel management organization for a sugar cartel that existed in the U.S. from 1927-1936. The institute did not present itself as merely a joint framework for advancing the interests of the individual members. Rather, it presented itself as a moral community. It had a “Code of Ethics” (under which competition was held to be unethical), which members seemed to take quite seriously. In one striking example, a cartel member refers to a particular competitive practice as “deplorable and expensive.”

The assumption that group loyalty sustains collusive behavior leads trivially to the result that, absent non-loyal entry, collusion will succeed just as if all of the firms in the cartel had merged.

¹ Group loyalty may in part arise from fear of non-market pressure such as social ostracism. The mechanism by which this pressure works (i.e., whether it works because it induces genuine remorse or it only because it involves a non-market material punishment) does not matter for our purposes. Our paper only requires that loyal group members can be relied upon to behave collusively.

² For examples, see Greif (1993), Podolny & Scott Morton (1999), Fisman (2003), Gil & Hartmann (2006), and Alesina & La Ferrara (2005) and the references therein.
It also leads trivially to the result that the ability of the cartel to make super-normal profits will depend on the entry costs of non-loyal potential entrants. The purpose of this paper is to make the further point that there are actions that loyal incumbents can take to increase their profits, and that these actions are familiar from informal discussions of collusion, but are generally absent from the treatment of collusion in the economic literature. These include activities whose purpose is to cement group loyalty and also to make employees better at their jobs, and low-paid “apprenticeships” in which employees accept below market wages for a period of time as a way to pay their dues in exchange for the opportunity to move up the ladder later on. Simply put, this is a paper about collusion and the Old Boys Club.

The key assumption of the paper is that loyal incumbents have the ability to inculcate loyalty into potential entrants. That is, loyal incumbents can create an environment in which potential future entrants come to sincerely feel a bond with the group, and therefore behave loyally when they enter. This can come about as a result of conscious effort on the part of the incumbents (military training, team building exercises, company-sponsored social activities, etc.), or can come about spontaneously through time spent with other members of the group. Allowing oneself to undergo this process must be incentive compatible for the potential entrants: those who do not yet feel the group loyalty will only agree to place themselves in an environment where they can expect to develop such loyalty if there will ultimately be a positive payoff from doing so.

Our first result is that a loyal entrant may or may not displace a non-loyal entrant when entry costs of both types of entrant are equal and are high enough that there is only room in the market for one entrant. The reason for the ambiguity is that total industry profits are larger if the entrant is loyal, but a non-loyal entrant pricing under the umbrella of the colluding loyal firms earns a
larger fraction of industry profits, so the entrant with higher profits (and hence higher willingness-to-pay to be the entrant) may be of either type.

But what if the process that inculcates loyalty also confers industry-specific human capital that gives an entry cost advantage to loyal entrants? That is, what if the people who are loyal to the cartel and the people who know how to run a firm in that industry tend to be the same people? This increases the profits that a loyal entrant can make relative to those that a non-loyal entrant can make, and this difference in profits makes it more likely that it will be a loyal firm that actually enters, as the loyal firm will be more willing expend resources as necessary to become the entrant. This reduces the degree to which non-loyal entry can force prices down to the non-collusive level. In other words, the process by which group loyalty is inculcated and the cost advantage conferred amounts to manipulating the joint distribution of group loyalty and industry-specific human capital so that the desire to competitively disrupt the cartel and the ability to do so tend not to reside in the same individuals.

The existence of loyal potential entrants with a cost advantage can benefit the incumbents even absent the threat of non-loyal entry. Admitting a loyal entrant reduces the profits of the incumbents, but in the differentiated products framework of this paper it increases total industry profits. The incumbents will therefore allow entry only if there is a mechanism through which some of the entrants’ profits can be transferred to the incumbents. One way for this to happen is for future loyal entrants to have an apprenticeship period (in which they earn below market wages) in exchange for the right to enter when there is a vacancy. As long as such transfers are possible, a stable cartel will have an incentive to admit additional loyal entrants to the cartel until the total increase in cartel profits no longer exceeds the entry costs of a loyal entrant.

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3 In Schmalensee (1978), incumbents proliferate brands in order to crowd the product space so that entry is unprofitable. Though the context is very different, the result in this paper has a similar flavor.
II. Model:

A. Setup.

There are \( n \) firms in the industry, each selling one variety of a differentiated good. Following Cellini, Lambertini, and Ottaviano (2004), the demand for variety \( i \) is:

\[
q_i = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{p_i}{\beta - \gamma} + \frac{\gamma(p_1 + p_2 + \cdots + p_n)}{(\beta - \gamma)[\beta + (n-1)\gamma]}
\]

The parameters \( \alpha, \beta, \) and \( \gamma \) are positive constants. The parameter \( \gamma \) measures the substitutability between the different varieties. It ranges from 0 to \( \beta \); if \( \gamma = 0 \) then the varieties are not substitutes at all, and as \( \gamma \) approaches \( \beta \) the varieties approach being perfect substitutes. This demand function is attractive because it allows for an arbitrary number of differentiated products that can be closer or more distant substitutes for each other, but at the same time is tractable.

There is a cartel consisting of perfectly loyal firms 1 through \( m \), alongside non-loyal firms \( m+1 \) through \( n \). The \( n \) firms play Bertrand, with the cartel firms pricing cooperatively and the independent firms pricing independently. Marginal costs are equal to zero, so variable profits are equal to total revenue. Variable profits for the cartel are equal to:

\[
\Pi = \pi_1 + \pi_2 + \cdots + \pi_m
\]

\[
= \frac{\alpha p_1}{\beta + (n-1)\gamma} - \frac{p_1^2}{\beta - \gamma} + \frac{\gamma(p_1^2 + p_2 + \cdots + p_{m+1} + \cdots + p_n p_{m+1} + \cdots + p_n p_m)}{(\beta - \gamma)[\beta + (n-1)\gamma]}
\]

\[
+ \frac{\alpha p_2}{\beta + (n-1)\gamma} - \frac{p_2^2}{\beta - \gamma} + \frac{\gamma(p_1 p_2 + p_2^2 + \cdots + p_{m+1} p_{m+2} + \cdots + p_n p_{m+1} + \cdots + p_n p_{m+2})}{(\beta - \gamma)[\beta + (n-1)\gamma]}
\]

\[
+ \cdots
\]

\[
+ \frac{\alpha p_m}{\beta + (n-1)\gamma} - \frac{p_m^2}{\beta - \gamma} + \frac{\gamma(p_1 p_m + p_2 p_{m+1} + \cdots + p_{m+1} p_{m+2} + \cdots + p_n p_{m+1} + \cdots + p_n p_{m+2})}{(\beta - \gamma)[\beta + (n-1)\gamma]}
\]

The first-order condition (FOC) for Firm 1 is:
\[
\frac{\partial \Pi}{\partial p_i} = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{2p_i}{\beta - \gamma} + \frac{\gamma[(2p_i + p_2 + \cdots + p_m + p_{m+1} + \cdots + p_n) + (p_2 + \cdots + p_m)]}{(\beta - \gamma)[\beta + (n-1)\gamma]} = 0
\]

The FOCs for the other cartel firms are similar. Variable profits for Firm \(m+1\) are equal to:

\[
\pi_{m+1} = \frac{\alpha p_{m+1}}{\beta + (n-1)\gamma} - \frac{p_{m+1}^2}{\beta - \gamma} + \frac{\gamma(p_1p_{m+1} + p_2p_{m+1} + \cdots + p_mp_{m+1} + p_{m+1}^2 + \cdots + p_{m+1}p_m)}{(\beta - \gamma)[\beta + (n-1)\gamma]} = 0
\]

The FOC for Firm \(m+1\) is:

\[
\frac{\partial \pi_{m+1}}{\partial p_{m+1}} = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{2p_{m+1}}{\beta - \gamma} + \frac{\gamma(p_1 + p_2 + \cdots + p_m + 2p_{m+1} + \cdots + p_n)}{(\beta - \gamma)[\beta + (n-1)\gamma]} = 0
\]

The FOCs for the other independent firms are similar. Imposing symmetry so that \(p_1 = p_2 = \ldots = p_m = p_L\) (where \(p_L\) is the price charged by all of the loyal firms), and \(p_{m+1} = p_{m+2} = \ldots = p_n = p_{NL}\) (where \(p_{NL}\) is the price charged by all of the non-loyal firms), the FOCs are:

\[
\frac{\partial \Pi}{\partial p_L} = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{2p_L}{\beta - \gamma} + \frac{\gamma[(m+1)p_L + (n-m)p_{NL} + (m-1)p_L]}{(\beta - \gamma)[\beta + (n-1)\gamma]} = 0
\]

\[
\frac{\partial \pi_{NL}}{\partial p_{NL}} = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{2p_{NL}}{\beta - \gamma} + \frac{\gamma[mp_L + (n-m+1)p_{NL}]}{(\beta - \gamma)[\beta + (n-1)\gamma]} = 0
\]

The equilibrium prices \(p_L^*\) and \(p_{NL}^*\) are:

\[
p_L^* = \frac{\alpha(\beta - \gamma)[2\beta + (2n-3)\gamma]}{4\beta^2 - 2(5 + m - 3n)\beta \gamma - [m^2 + m(n-4) - 2(n-3)(n-1)] \gamma^2}
\]

\[
p_{NL}^* = \frac{\alpha(\beta - \gamma)[2\beta + (2n-2)\gamma]}{4\beta^2 - 2(5 + m - 3n)\beta \gamma - [m^2 + m(n-4) - 2(n-3)(n-1)] \gamma^2}
\]

The equilibrium per-variety profits \(\pi_L^*\) and \(\pi_{NL}^*\) are

\[
\pi_L^* = \frac{\alpha^2(\beta - \gamma)[\beta + (n-m-1)\gamma][2\beta + (2n-3)\gamma]^2}{[\beta + (n-1)\gamma][-4\beta^2 + 2(5 + m - 3n)\beta \gamma + [m^2 + m(n-4) - 2(n-3)(n-1)] \gamma^2]^2}
\]

\[
\pi_{NL}^* = \frac{(\beta - \gamma)[\beta + (n-2)\gamma][-2\alpha \beta + (2 + m - 2n)\alpha \gamma]^2}{[\beta + (n-1)\gamma][-4\beta^2 + 2(5 + m - 3n)\beta \gamma + [m^2 + m(n-4) - 2(n-3)(n-1)] \gamma^2]^2}
\]
Define $\pi_{L}^{m,n}$ as the profits earned by a loyal firm when there are $m$ loyal firms and $n$ total firms in the industry. Define $\pi_{NL}^{m,n}$ similarly.

B. Inculcating Loyalty.

We assume that loyalty is not confined to current members of the group, but rather can be inculcated into new members. Inculcating loyalty may or may not be costly to the loyal incumbents or to the potential loyal entrant. Throughout the paper, we assume that it is costless. This will be closer to the truth the greater the extent to which loyalty is inculcated naturally as a result of time spent in the industry. If it were costly, then incumbents and potential entrants would make optimizing decisions of how much of it to engage in. The more costly it is, the weaker the results in this paper would become, and if it were sufficiently costly, no one would ever do it and so it would not matter whether or not it was possible.

We assume that loyal entrants do not enter unless they are “invited” to do so by the incumbents. That is, we assume that there are no entrants who enter against the wishes of the incumbents, but who then behave collusively once they have entered. There are also no renegades who accept the industry-specific human capital but who then do not behave loyally. This assumption will hold if the inculcation of loyalty is so effective that it almost never fails to work. It will also hold if the industry-specific knowledge, to be effectively employed, requires continuing loyal relationships with other cartel members. The rare entrant who accepted the industry-specific human capital, but was willing and able to employ that ability while not behaving loyally could be punished, accommodated, or possibly acquired by the loyal firms as circumstances warranted.
III. Results:

A. Preserving the Cartel.

Recall that $m$ is the number of loyal firms and $n$ is the total number of firms. Consider a cartel with $x$ loyal firms and no non-loyal firms ($m = n = x$). Let $K_L$ be the entry cost of a loyal firm and $K_{NL}$ be the entry cost of a non-loyal firm. Suppose there is “room” in the industry for one more firm, whether it is an additional loyal firm ($m = n = x + 1$), or a non-loyal firm ($m = x$, $n = x + 1$). That is, suppose that $\pi_{L}^{x+1,x+1} > K_L$ and $\pi_{NL}^{x,x+1} > K_{NL}$ (so either a loyal firm or a non-loyal firm would find it profitable to enter); but $\pi_{L}^{x+1,x+2}$ and $\pi_{L}^{x+1,x+2}$ are both less than $K_L$, and $\pi_{NL}^{x+1,x+2}$ and $\pi_{NL}^{x+1,x+2}$ are both less than $K_{NL}$ (so only one firm can profitably enter). This “vacancy” exists because of the exogenous exit of one loyal incumbent, because of human capital that resides in an individual who retires or dies. The incumbent firms can inculcate loyalty into a potential entrant as long as the loyal entrant will be allowed to enter and will on net benefit from doing so. Assume that if there is only going to be one entrant, the one who will earn the higher profits will be the one to enter, as that will be the firm that is willing to expend more resources to win any contest to determine who the entrant will be.

Proposition 1: If entry costs for loyal firms are the same as for non-loyal firms, then the profits of a non-loyal entrant can be larger or smaller than those of a loyal entrant.

Proof: See appendix.

Holding the total number of firms constant, total industry profits are higher when all firms are loyal. However, the profits for a non-loyal entrant can be higher than those for a loyal entrant, as the non-loyal entrant is able to earn high profits by non-collusively cutting its price and stealing business from loyal firms. That is, Proposition 1 shows that when the entry costs of loyal and
non-loyal firms are equal, a vacancy created by the departure of a loyal firm may or may not be filled by another loyal firm.\footnote{Non-loyal entrant profits are higher than loyal entrant profits when products are very close substitutes for each other (i.e., when $\gamma$ approaches $\beta$), and vice-versa. When products are perfect substitutes, loyal entry does not expand the cartel’s profits, which means that a loyal entrant only gets a share of a fixed profit, whereas a non-loyal entrant can capture a larger share of that profit. In contrast, when products are not close substitutes, loyal entry significantly increases total cartel profits (of which the loyal entrant gets a share), whereas this is not true for a non-loyal entrant.}

Now suppose that loyal entrants also have industry-specific human capital that makes their entry costs lower than those of a non-loyal entrant (i.e., suppose that $K_L < K_{NL}$). This leads to the following proposition:

**Proposition 2:** The profits of a loyal entrant will be larger than those of a non-loyal entrant if $K_L - K_{NL}$ is sufficiently large.

**Proof:** By assumption, $K_L$ and $K_{NL}$ are such that there will be only one entrant. Let $K_{NL}$ be large enough that a non-loyal entrant would make infinitesimally small profits, and let $K_L$ be small enough that a second loyal entrant would make infinitesimally small losses. This means that a single loyal entrant will make profits that are not infinitesimally small, and so will make higher profits than will a non-loyal entrant.

Proposition 2 shows that if the process that inculcated the loyalty also conferred a sufficiently large cost advantage, then loyal entrants will have higher profits than non-loyal entrants, and so will out-compete them in any contest that determines who the entrant will be. That is, if inculcating loyalty reduces entry costs by enough, it can be a means of blockading non-loyal entry even absent the ability to directly influence non-loyal entry costs.

**B. Expanding the Cartel.**

Now consider a stable cartel with $m = x$ loyal incumbents. It is stable in the sense that, if the incumbents do nothing, there will be no non-loyal entry. Would the incumbents ever have an incentive to permit additional loyal entrants? To answer this, we begin with the following lemma:
Lemma 1: When all firms are loyal, total industry variable profits are increasing in the number of firms, but variable profits of the incumbents are decreasing in the number of firms.

Proof: See appendix.

Lemma 1 means that if entry costs are sufficiently low, total industry profits increase with loyal entry. (Note that this is due to the assumption of differentiated products, and would not be true of a homogenous product industry.) However, entry causes the profits of the incumbents to fall. This means that the incumbents will only allow loyal entry if they can get a share of the entrants’ profits. One way for this profit-sharing to happen (one that avoids the problems associated with the entrant directly paying the incumbents), is for future entrants to accept employment or internships at a wage that is lower than what the incumbent firms would otherwise have to pay.\(^5\) Workers are willing to accept these arrangements as long as it is sufficiently likely they will one day be allowed to enter, and will be left with enough profits upon entering to make it worthwhile to accept the employment (they always have the option of not accepting and earning their outside option).

Proposition 3: If loyal entrants can use low-paid employment to transfer surplus to incumbent cartel members, then a stable cartel can profitably expand to include more loyal members. Lower loyal entry costs \(K_L\) weakly increase the size of the cartel.

Proof: The ability of loyal entrants to use low-paid employment to transfer surplus to incumbents means that any entry that increases total industry profits will make both the incumbents and loyal entrants better off. Lemma 1 shows that loyal entry cases total industry variable profits to increase, which means that loyal entry will happen if \(K_L\) is sufficiently low. Lower \(K_L\) weakly increases the number of entrants for whom the increase in total industry variable profits exceeds the entry costs.

\(^5\) The required discount off of the market wage will be smaller if the inculcated loyalty increases the future entrant’s value as an employee.
The assumption in this section is that the cartel is proof against non-loyal entry with \( m = x \) loyal incumbents. So the entry costs \( K_{NL} \) are not relevant except that they must be high enough to make this condition hold. Conditional on this, the entry costs that would be observed empirically are \( K_L \), which means a predicted negative relationship between entry costs and cartel size.

C. Contrast with Standard Model of Collusion.

Proposition 2 shows that lower loyal entry costs \( K_L \) increase the chances that the cartel will be proof against non-loyal entry. Proposition 3 shows that lower \( K_L \) will lead to a larger cartel even if the cartel was originally stable. That is, in our model lower observed entry costs (the entry costs of non-loyal firms \( K_{NL} \) would not be observed) make successful cartels both more likely to exist and larger than they otherwise would be. This is different from the results of the traditional dynamic collusion model (see Friedman, 1971). In that model, lower entry costs mean a larger cartel to be supported, which increases the threshold level of the discount factor required for collusion to be sustained.\(^6\) In those cases where the discount factor is high enough that collusion could still be sustained with a larger cartel, one would predict an inverse relationship between entry costs and the size of the cartel, just as in our model. But sufficiently low entry costs would also cause the cartel to collapse. That is, both our model and the standard model predict an inverse relationship between observed entry costs and cartel size, but the two models differ in the predicted relationship between entry costs and the probability of successful cartelization.

\(^6\) It is straightforward to show that this standard result, which was originally proven for a homogeneous product industry, holds for the demand function used in this paper.
IV. Discussion and Conclusion

Firms have strong incentives to defect from collusive arrangements. Any model in which collusion arises as an equilibrium outcome must address this fact. The standard economic model of collusion does so by allowing defection in one period to be punished in future periods. If the future punishment is sufficiently severe relative to the current benefit of defection, and if the agents are patient enough, and if the punishers can credibly commit to follow through with the punishment, then collusion can be sustained.

This model of collusion (and its many descendants) makes sense, and it almost certainly has some validity. It is, however, based on somewhat strong assumptions, for example it prohibits successful collusion in any instance where the collusive arrangement must end no later than a certain date. In this paper we develop a model in which collusion is held together by genuine group loyalty, rather than by fear of future punishment. We believe that the evidence on within-group cooperation is sufficiently strong to warrant examination of models in which such group loyalty plays a role, and we do not believe that there are any a priori grounds to believe that such a model is less plausible than the standard one, other than the fact that the standard one is older.

The assumption of group loyalty among incumbent firms leads trivially to the result that collusion will succeed if non-loyal entry costs are high enough: a static model in which group loyalty is assumed to exist will look just like a standard oligopolistic market in which all the incumbent firms are allowed to merge. While this is not the essence of our paper (our main theoretical results, while not very surprising, are not as trivial as this), it is worth noting that even this result has interesting empirical implications. In the standard model, collusion is more likely when there are market characteristics such that firms either cannot detect signs of cheating or cannot be cer-
tain that those signs were caused by cheating and not by something else. In contrast, if collusion is based on loyalty, then it should be more likely in those instances where loyalty is present (say due to a shared family, ethnic, geographic, military, or educational background).

We assume that loyalty can be inculcated, and that the processes by which loyalty is inculcated can at the same time confer a cost advantage. Our first main result is that under this assumption, an existing cartel can profitably admit a loyal entrant as a means of blockading a non-loyal entrant. The further assumption that loyal entrants can transfer surplus from themselves to the loyal incumbents (say by means of low-paid employment relationships such as apprenticeships) leads to our second main result, which is that in a differentiated product industry such as the one modeled here, a stable loyalty-based cartel (i.e., one that is not threatened by entry of non-loyal firms) can profitably expand itself. That is, our main results show that setting up an “Old Boys Club,” in which new members are acculturated into the norms of a group, can lead to a situation in which the group can both exclude non-loyal entrants and profitably expand without any need for an explicit conspiracy.

These results lead to additional empirical implications. They suggest that collusion will be more prevalent the more true it is that the processes that inculcate loyalty also lower costs, and the resulting cartels will be larger in instances where the inculcation process involves some mechanism, such as a low-paid apprenticeship, by which the loyal entrants can transfer surplus to the incumbents.

Any model of collusion must deal with the prospect of entry: if entry costs are too low, then collusion cannot work. The standard model deals with this by implicitly assuming that entry costs are high enough that the number of collusive firms is not unsustainably high. In contrast,

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8 In a homogeneous product industry, additional entrants would not increase total industry profits.
we assume not that entry costs are high, but rather that there is a large difference in entry costs between loyal and non-loyal entrants. That is, in our story collusion is preserved not through the assumption that entry is objectively very difficult, or that the incumbents can do something to directly blockade non-loyal entry, but rather through the assumption that the advantage enjoyed by loyal firms is large enough that those (non-loyal) firms who are inclined to disrupt the collusion via entry are precisely the ones who are not equipped to do so. This result arises not through any kind of conspiracy, but rather falls directly out of our assumption regarding the process of inculcating loyalty. While the empirical importance of this has not been established, we believe that it is a provocative possibility, and that it might mean that in some circumstances collusion is substantially easier to sustain than is commonly believed.
References:


Proof of Proposition 1: \( \pi^{x,x+1}_{nl} - \pi^{x+1,x+1}_{l} \) is equal to:

\[
(A1) \quad \frac{x\alpha^2 \gamma^2 \{4(x-2)\beta^2 + 4[2 + (x-4)x]\beta \gamma - (4x-3)xy^2\}}{4(\beta + xy)[4\beta^2 + 4(x-1)\beta \gamma - xy^2]^2}
\]

This has the sign of the expression in the curly brackets. Dividing that term through by \( \gamma^2 \) gives the following:

\[
(A2) \quad 4(x-2)\left(\frac{\beta}{\gamma}\right)^2 + 4[2 + (x-4)x]\frac{\beta}{\gamma} - (4x-3)x
\]

This expression is quadratic in \( \beta/\gamma \). It is straightforward to show that one of the roots of this expression must be negative, and the other must be greater than one. The latter root is the relevant one, as by definition \( \beta/\gamma \) must be greater than or equal to one. That root is equal to:

\[
(A3) \quad \frac{-2 + (4-x)x + \sqrt{(x-1)^2(4 + (x-2)x)}}{2(x-2)} > 1
\]

The derivative of (A2) with respect to \( \beta/\gamma \), evaluated at the root in (A3) is equal to:

\[
(A4) \quad 4\sqrt{(x-1)^2(4 + (x-2)x)} > 0
\]

This expression must be positive. By continuity, the expression is positive at a value of \( \beta/\gamma \) slightly above that in (A3), and is negative at a point slightly to the left. ■

Proof of Lemma 1: Total industry variable profits in an industry with \( m = x \) loyal firms and no non-loyal firms are equal to \( x^* \pi^{x,x}_{L} \). The derivative of this with respect to \( x \) is:

\[
(A5) \quad \frac{dx^* \pi^{x,x}_{L}}{dx} = \frac{\alpha^2(\beta - \gamma)}{4[\beta + (x-1)\gamma]^2} > 0
\]

This derivative is positive. Variable profits for each of the \( x \) incumbents are \( \pi^{x,x}_{L} \). The derivative of this with respect to \( x \) is:

\[
(A6) \quad \frac{d\pi^{x,x}_{L}}{dx} = -\frac{\alpha^2\gamma}{4[\beta + (x-1)\gamma]^2} < 0
\]

This derivative is negative, which proves the result. ■