High-frequency radio wave absorption in the D-region

David Alan Smith, Utah State University
High-frequency Radio Wave Absorption

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David Alan Smith

Utah State University
Physics Department
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1. Introduction

My PhD research focuses on Polar Cap Absorption (PCA) events. These events are triggered by highly-energized particles of solar origin gaining access to upper regions of the atmosphere at high latitudes. The effect of PCA events is to significantly attenuate high-frequency radio signals propagating through the ionosphere via polar paths (Odenwald, 2010). The scope of this review is to focus on one specific mechanism that is greatly enhanced during PCA events, leading to detrimental effects on communications systems: high-frequency radio wave absorption.

2. Radio Wave Propagation

Communication via radio waves requires certain equipment and conditions. Basic equipment required includes some type of transmitter, transmission lines, and an antenna. The antenna radiates the electromagnetic signal into the environment. At the other end there must of necessity be a receiving antenna and a receiver. This review is concerned with what takes place as the electromagnetic waves propagate through space from transmitting antenna to receiving antenna via the ionosphere.

2.1 Sky Waves

Radio waves propagate generally via two types of waves: ground waves and sky waves. ARRL [1991] provides an excellent definition for ground waves as any wave that stays close to the Earth, reaching the receiving point without leaving Earth's lower atmosphere. We generally think of ground waves as providing line-of-sight communication. Sky waves, on the other hand, generally leave Earth's lower atmosphere. Sky waves allow radio signals to travel great distances, enabling world-wide communication via the ionosphere. The ionosphere is that area of Earth's atmosphere that lies between 50 - 1000 km in altitude. What makes the ionosphere extraordinary is the relatively high number of free electrons that exist within this ionized region.

2.2 Ionospheric Properties

The ionosphere has been defined as a region of Earth's atmosphere that has a high enough concentration of free electron such that it affects radio waves (Moen, 2004). Because of the very light mass of the free electrons, they are easily moved by the oscillating electric field of the passing radio wave (Lied, 1967). The distribution of these free electrons is not uniform.

Earth's neutral atmosphere is divided into regions based on temperature profile. The ionosphere, however, is subdivided based on electron density. In both cases the critical parameter can be defined as a function of height. The layers or regions of the ionosphere are loosely defined as follows:

<table>
<thead>
<tr>
<th>Height Range</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 - 100 km</td>
<td>D-region</td>
</tr>
<tr>
<td>100 - 150 km</td>
<td>E-region</td>
</tr>
<tr>
<td>150 - 500 km</td>
<td>F-region</td>
</tr>
</tbody>
</table>

Table 1. Description of ionospheric regions.

The neutral atmosphere consists mostly of molecular nitrogen and oxygen. However, due to photodissociation, significant amounts of atomic nitrogen and oxygen also exist (Schunk & Nagy, 2009).

2.3 Geometric Optics

One approach to understanding qualitatively how radio waves propagate through the ionosphere is geometric optics. One of the first things we learn in geometric optics is
Snell's Law, also known as the law of refraction,

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]

Here we define \( n_1 \) as the index of refraction of the initial medium, \( n_2 \) as the index of refraction of the new medium, \( \theta_1 \) as the angle of incidence, and \( \theta_2 \) as the refracted angle. When we first learn of Snell's Law we are concerned with macroscopic systems and solids where the index of refraction of the various media are always greater than unity. It can be shown, however, that the index of refraction of a plasma can be less than unity. Since the ionosphere is a weakly-ionized plasma\(^1\), we assume that the index of refraction of the ionosphere can also be less than unity. The importance of this assumption will be made evident.

It can be shown that as monochromatic light passes from a medium with a lower index of refraction into a medium with a higher index of refraction, \((n_1 < n_2)\), the light ray will be refracted toward the normal. In the opposite case \((n_1 > n_2)\) we expect the ray to be bent away from the normal. This would certainly be the case if the light ray was moving from air \((n \sim 1.0)\) into the ionospheric plasma \((n < 1)\).

We can define the index of refraction mathematically in several ways. One is in terms of the phase velocity\(^2\) of a wave through the medium versus the speed of light in a vacuum,

\[ n = \frac{c}{v_p} \]

The above shows that a higher index of refraction would indicate a slower speed through the medium. Another definition of the index of refraction is given in terms of the physical properties of the medium,

\[ n = \sqrt{\varepsilon_r} \]

where \( \varepsilon_r \) is defined as the dielectric constant of the medium.\(^3\) Furthermore, the dielectric constant of a plasma can be defined in terms of the plasma frequency,\(^4\)

\[ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \]

where the angular frequency can be defined in terms of the radio wave frequency, \( \omega = 2\pi f \), and the plasma frequency as,

\[ \omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \]

Here we define \( n_e \) as the electron number density, \( e \) as the elementary charge, \( \varepsilon_0 \) as the permittivity of free space, and \( m_e \) as the electron mass. We now recast the index of refraction in terms of the plasma frequency,

\[ n = \sqrt{\frac{\omega^2}{1 - \frac{\omega_p^2}{\omega^2}}} \]

\[ \Rightarrow n = \sqrt{1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2}} \]

Then putting the index of refraction in terms of the radio wave frequency we see that,

\(^1\) In the accompanying talk, we discuss briefly the difference between a fully-ionized plasma and a weakly-ionized plasma.
\(^2\) It is important at this point to remember the distinction between phase velocity and group velocity. In a nutshell the group velocity is constrained to values less than or equal to the speed of light in vacuum; the phase velocity has no such constraint.
\(^3\) For a detailed discussion of the index of refraction in terms of physical properties derived from Maxwell's equations consult any standard E/M text. Jackson discusses this in Classical Electrodynamics starting on page 396.
\(^4\) Jackson gives an excellent discussion in section 7.5, Frequency Dispersion Characteristics of Dielectrics, Conductors, and Plasmas.
showing that the index of refraction in a plasma depends explicitly on electron density and inversely on the radio wave frequency. Apparently the electron density of the D-region is on the order of \( n_e \sim 10^9 - 10^{10} \) m\(^{-3}\). Figure 1 shows that within the high-frequency spectrum (\( f \sim 3.0 - 30.0 \) MHz) the index of refraction approaches unity.

![Index of Refraction](image1.png)

Figure 1. Index of refraction as function of frequency

From figure 2 we see that as the electron number density increases, the index of refraction decreases, which is the crux of the geometric argument.\(^5\)

![Index of Refraction](image2.png)

Figure 2. Index of refraction as function of electron density.

It should be noted that a medium wherein the index of refraction is frequency-dependent is considered a dispersive media\(^6\) (Lied, 1967).

### 3. High-frequency Absorption

#### 3.1 Absorption Equation

We start our investigation of high-frequency radio wave absorption with a general equation (Davies, 1965) for system loss\(^7\),

\[
L_s = L_{ta} + L_{tp} - G_t + L_p + L_{rp} - G_r + L_{ra}
\]

where \( L_s \) is the total system loss, \( L_{ta} \) the ohmic loss in the transmitting antenna system, \( L_{tp} \) any polarization mismatch of the transmitting antenna, \( G_t \) is the transmitting antenna gain, \( L_p \) the path loss, \( L_{rp} \) is the polarization mismatch of the receiving antenna, \( G_r \) the gain of the receiving antenna, and \( L_{ra} \) the ohmic loss in the receiving antenna system.

Of the several components contributing to system loss, the one relevant to this discussion is path loss \( (L_p) \). We can define the total path loss as the sum of three path loss contributors\(^8\),

\[
L_p = L_d + L_a + L_f
\]

where \( L_d \) is the path loss due to distance, \( L_a \) path loss due to absorption in the ionosphere, and \( L_f \) as path loss due to focusing and/or defocusing \((\mp)\). Of the three path loss terms, path loss due to absorption in the ionosphere is germane to our discussion. Generally the absorption term can be described by\(^9\),

\[^5\] This is a critical part of the geometric optics argument. For a detailed discussion one could consult Lied pp 1-10
\[^6\] As a side note, in section 7.6, Jackson provides a simplified model of propagation in the ionosphere. After showing that the index of refraction is frequency-dependent he describes the ionosphere as, "...a highly dispersive medium," (See Jackson pp 316-319)
\[^7\] Davies equation 5.5
\[^8\] Davies equation 5.8
\[^9\] Davies equation 5.15
\[ L_a = 10 \log \left( \frac{p_r}{p_u} \right) = 20 \log(\rho) \]

Here we define \( p_r \) as the actual power received, \( p_u \) as the power that would have been received in the absence of absorption, and \( \rho \) as the effective amplitude reflection coefficient. Davies [1965] defines this term qualitatively as the ratio of the square of the amplitude of a wave which is reflected once in the ionosphere to the amplitude which would have been received in the absence of dissipative attenuation. Davies [1965] shows this to be\(^{10}\),

\[ \frac{I}{I_0'} = e^{-\int \kappa \, ds} \]

Here, \( I \) is the amplitude of a wave which is reflected once in the ionosphere, \( I_0' \) is the amplitude which would have been received in the absence of dissipative attenuation, and \( \kappa \) is the absorption coefficient. Recasting this equation in terms of the effective amplitude reflection coefficient we find that,

\[ \rho = e^{-\int \kappa \, ds} \]

\[ \Rightarrow \ln(\rho) = -\int \kappa \, ds \]

From the rules of logarithms,

\[ \log(x) = \frac{\ln(x)}{\ln(10)} \]

we find that,

\[ L_a = 20 \log(\rho) = 20 \frac{\ln(\rho)}{\ln(10)} = 8.69 \ln(\rho) \]

\[ \Rightarrow L_a = 8.69 \left( -\int \kappa \, ds \right) \]

The units for the absorption coefficient as defined by Davies [1965] are nepers per meter\(^{11}\). Since there are 8.69 decibels per neper we see that the absorption coefficient is now in units of dB per meter. Since the integral will return units of distance we expect the above equation to yield absorption in dB. However, absorption is frequently reported as decibels per meter.

The absorption coefficient itself can be defined mathematically in several ways, the particular equation depending generally on the type of absorption affecting the radio wave as well as the relation between the ionospheric plasma frequency and the collision frequency.

3.2 Types of Absorption

Before discussing the types of absorption, it is important here to make an assumption that will be discussed thoroughly later. Let us assume that most absorption takes place within the D-region of the ionosphere. This will allow us to further the discussion regarding the types of absorption.

There are two basic types of absorption: deviative absorption and non-deviative absorption (Davies, 1965), (Lied, 1967).

3.2.1 Deviative absorption

Deiative absorption occurs when the frequency of the incoming radio wave is near the plasma frequency of the local ionosphere \((\omega \sim \omega_p)\). In this case the refractive index will be small and the wave

\( ^{10} \) Davies equation 3.2

\( ^{11} \) A neper is the natural log equivalent of a decibel (dB).Apparently one neper is equal to roughly 8.69 dB which will be shown later.
will propagate slowly through the region.\(^{12}\) This can be understood by recalling that the group velocity of a wave propagating through the ionosphere is\(^{13}\),

\[ v_g = cn \]

Hence the ray path of the radio wave may be appreciably deviated. Furthermore, when the index of refraction becomes zero, the group velocity becomes zero and the wave is reflected (Lied, 1967).

### 3.2.2 Non-deviative absorption

Non-deviative absorption occurs when the frequency of the incoming radio wave is greater than the plasma frequency of the local ionosphere \((\omega > \omega_p)\). In this case the refractive index will be near unity and the wave will move through the region at about the speed of light in vacuum. Hence, the ray path of the radio wave is not appreciably deviated. Evidently, for high-frequency waves in the D-region, non-deviative absorption is the norm\(^{14}\).

![Plasma Frequency Height Profile](image)

**Figure 3.** Here we see that within the D-region the plasma frequency is generally less than the radio wave frequency. Hence, non-deviative absorption applies.

### 3.3 Absorption Coefficient

Davies [1965] defines the absorption coefficient in this manner,\(^{15}\)

\[ \kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right) \]

in units of nepers per meter. In the above we introduce \(\nu\) as collision frequency. Since we have shown that non-deviative absorption dominates in the D-region \((\mu \sim 1)\) it follows that,

\[ \kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right) \]

### 3.4 Case Studies

Davies [1965] provides several conditions and derives absorbtion coefficients for each. We examine three. In each case we assume non-deviative absorption. In addition, each version of the absorption coefficient will be given in terms of radians per second as well as cycles per second.

#### 3.4.1 Case 1

In case 1, the radio wave frequency is much greater than the collision frequency \((\omega > \nu)\) and we neglect any geomagnetic effects (mid-latitude).

For case 1 Davies [1965] defines the absorption coefficient as\(^{16}\),

\[ \kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left( \frac{n_e \nu}{\omega^2} \right) \]

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\(^{12}\) Here, \(v_g\) mean much less than \(3.0 \times 10^8\) m/s (the speed of light in vacuum.)

\(^{13}\) This equation is dropped on us by Lied. See pp 6-8. It is derived and discussed thoroughly in Appendix 1.


\(^{15}\) Davies equation 2.98. See Appendix 2 for a detailed discussion of the origins of the absorption coefficient from basic principles.

\(^{16}\) Davies equation 2.99
For case 1 absorption varies inversely as the square of the wave frequency. 

3.4.2 Case 2

For case 2, the radio wave frequency is much less than the collision frequency ($\omega \ll \nu$) and we again neglect geomagnetic effects. Davies [1965] defines the absorption coefficient in this manner, \(^{17}\)

$$\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{\nu} \right]$$

For case 2 absorption varies inversely with the collision frequency.

It is interesting to note that in case 2 as the collision frequency increases, absorption decreases. Apparently this stems from the confinement of electron motions and the small amount of energy extracted from collisions\(^{18}\). Evidently the above equation is intended specifically for polar cap absorption events since electrons could be produced at much lower heights (50 to 60 km). Davies [1965] tells us that at these levels it is necessary to use equation 2.101 to determine $\kappa$. However, under such conditions absorption may be taking place over a range of heights and the exact determination of $\kappa$ may be a complicated process\(^{19}\).

3.4.3 Case 3

For case three the assumption is that the radio wave frequency is about equal to the collision frequency ($\omega \propto \nu$) and we include geomagnetic effects.

Here Davies [1965] defines the absorption coefficient as\(^{20}\)

$$\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{(\omega \pm \omega_H)^2 + \nu^2} \right]$$

$$\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{4\pi^2 (f \pm f_H)^2 + \nu^2} \right]$$

and we define $\kappa$ as the absorption coefficient, $e$ as the elementary charge, $\varepsilon_0$ as the permittivity of free space, $m_e$ as the electron mass, $c$ is the speed of light in vacuum, $n_e$ electron number density, $\nu$ is collision frequency, $f$ is the frequency of radio wave in herz, $f_H$ is the gyrofrequency of the electron, and $f \pm f_H$ is the so-called effective frequency\(^{21}\).

Lied [1967] uses similar versions of Davies' equations for each case. For case 1\(^{22}\)

$$\kappa \sim 1.345 \times 10^{-7} \left[ \frac{n_e \nu}{f^2} \right]$$

and for case 3\(^{23}\)

$$\kappa \sim \frac{e^2}{2\varepsilon_0 m_e c} \left[ \frac{n_e \nu}{4\pi^2 (f \pm f_L)^2 + \nu^2} \right]$$

$$\sim 5.29 \times 10^{-6} \left[ \frac{n_e \nu}{4\pi^2 (f \pm f_L)^2 + \nu^2} \right]$$

Lied [1967] notes that case 1 and case 3 are normally satisfied for high-frequency radio waves propagating through the ionosphere\(^{24}\).

Note also that for case 3 Lied [1967] does not explicitly use gyrofrequency. Rather, he uses what he calls an effective frequency,

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\(^{17}\) Davies equation 2.101

\(^{18}\) See Davies page 82

\(^{19}\) Davies page 231

\(^{20}\) Davies equation 2.103

\(^{21}\) The effective frequency goes by several names. Lied defines it as such on page 16.

\(^{22}\) Lied equation 1.47

\(^{23}\) Lied equation 1.32

\(^{24}\) See Lied page 41
the \( f \pm f_L \) term. \( f_L \) is defined in terms of the gyrofrequency such that \( f_L \equiv f_H \cos(\theta) \)
where \( f_H \) is the gyrofrequency and \( \theta \) is the angle between the direction of propagation of the wave and the geomagnetic field. 
Davies [1965] defines this term as the gyro frequency corresponding to the component of the earth's magnetic field in the direction of phase propagation.\textsuperscript{25}

3.5 Derivation of absorption equation

We now turn our attention to deriving an appropriate equation which will allow us to calculate D-region absorption per unit distance. Restating the important relationships,

\[
L_a = -8.69 \int \kappa \, ds
\]

\[
\kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right)
\]

then inserting the equation for the absorption coefficient into the equation for absorption we find that,

\[
L_a = -8.69 \int \frac{e^2}{2\varepsilon_0 mc} \left( \frac{n_e \nu}{\omega^2 + \nu^2} \right) \, ds
\]

For the moment we focus on case 1 such that,

\[
L_a = -8.69 \int \left[ \frac{e^2}{2\varepsilon_0 m_e c} \left( \frac{n_e \nu}{4\pi^2 f^2} \right) \right] \, ds
\]

As stated previously, we desire an equation that will return units of \( dB \) per unit distance. As will be shown, electron density and collision frequency are height-dependant. Hence, we express them in terms of integrals over height,

\[
L_a = -8.69 \left( \frac{e^2}{8\pi^2 f^2 e_0 m_e c} \right) \int n_e(h) \nu(h) \, dh \int ds
\]

Combining all the constant terms we find that,

\[
L_a = -\frac{1.17 \times 10^{-6}}{f^2} \int n_e(h) \nu(h) \, dh \int ds
\]

and the last integral is over the path. Figure 3 shows the basic geometry of a radio wave propagating along a specific path.

We have shown that within the high-frequency regime nearly all absorption takes place within the D-region\textsuperscript{26} and is non-deviative in nature (Davies, 1965). From figure 4 we see that as the ray propagates along some path it passes through an absorbing region. This would be the D-region.

Since the absorption is non-deviative we assume the ray is propagating along a straight line. Since the ray has some initial elevation angle, we can use the right triangle formed as the ray passes through the D-region and some basic trigonometry to get the path length from the height of the D-region,

\[
sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{s}
\]

with \( \alpha \) defined as the elevation angle, \( h \) as height and \( s \) as the path length through the absorbing region. We then solve this for the path length.

\textsuperscript{25} Davies page 134

\textsuperscript{26} For example Davies page 231 and Lied pages 41-43
\[
\sin \alpha = \frac{h}{s}
\]
\[
\Rightarrow s = \frac{h}{\sin \alpha}
\]

Using the above relationship we can put the differential for path length in terms of height,
\[
s = \frac{h}{\sin \alpha}
\]
\[
\Rightarrow ds = \frac{dh}{\sin \alpha}
\]

\[
\Rightarrow L_a = -\frac{1.17 \times 10^{-6}}{f^2} \int \frac{n_e \nu}{\sin \alpha} dh
\]

From figure 3 we see that the ray passes through the D-region two times per hop. If we let \( p \) be the number of hops then we have a final version of the absorption equation,
\[
\Rightarrow L_a = -\frac{1.17 \times 10^{-6}(2p)}{f^2} \int \frac{n_e \nu}{\sin \alpha} dh
\]
\[
\Rightarrow L_a = -\frac{2.34 \times 10^{-6}p}{\sin \alpha f^2} \int n_e \nu \text{en} dh
\]

where the integral would be over the height of the D-region. Lied [1967] provides a helpful chart on page 42.

![Absorption per Region At 4.0 MHz](image)

Figure 6. We see that most of the absorption at 4.0 MHz occurs within the D-region.

### 4.0 Electron Density

Electron density is important because the electron interactions allow radio waves to propagate through the ionosphere. Lied [1967] tells us that it is the free electrons which on account of their light mass can be moved in resonance with the oscillating electric field of the radio wave and thus control the refracting properties of the ionized medium.\(^{27}\)

But this is only part of the story. Besides free electrons there are ions and neutral...
species in the atmosphere. It is possible, in fact quite likely, that as free electrons move about they will collide with ions and/or neutrals. Neutral species are much more numerous in the lower regions of the ionosphere. Furthermore, electrons may recombine with certain ions, reducing the number of free electrons. It's a crazy process: free electrons are produced as ionization takes place and free electrons are lost as recombination takes place; free electrons gain energy as they are moved by incoming radio waves but they lose energy as they collide with larger ions and neutrals. It is this continuous production and loss of free electrons and re-radiation of radio waves that creates such havoc in the D-region.

A particularly important neutral species in the D-region is nitric oxide (NO), though NO is considered a minor species in the atmosphere overall. Within the D-region NO is ionized by extreme ultraviolet solar radiation. Specifically,

\[ \text{NO} + h\nu \rightarrow \text{NO}^+ + e^- \]

Here we define \( h \) as Planck's constant \( \sim 4.14 \times 10^{-15} \text{eV} \cdot \text{s} \) and \( \nu \) is the photon frequency (about \( 2.2 \times 10^{15} \text{Hz} \) for Lyman-\( \alpha \) during non-storm periods). Hence, a majority of electrons in the D-region are produced via Lyman-\( \alpha \) solar radiation. We make the approximation that in the D-region the number of nitric oxide cations is about equal to the number of electrons \( (\text{NO}^+ \sim e^-) \).

As mentioned previously, electrons are lost via recombination. One specific type of fast recombination is dissociative recombination,

\[ \text{NO}^+ + e^- \rightarrow N + O \]

Here, a nitric oxide cation combines with an electron to produce nitrogen and oxygen atoms. Thus we see that electrons and ions are continually being produced and lost. However, under normal conditions we find that the number density of electrons at a particular altitude remains fairly constant.\(^\text{28}\)

5.0 Collision frequency

As the free electrons collide with neutral particles they lose energy. As they lose energy the intensity of the radio wave is reduced. This is the crux of high-frequency absorption in the D-region. We know that from about 50 – 100 km the electron density range is about \( 10^7 - 10^{11} \text{ m}^{-3} \).

Within that same altitude range\(^\text{29}\) the total density of all neutrals is on the order of \( 10^{18} - 10^{20} \text{ m}^{-3} \). Earlier we made the assumption that the number of ions is about equal to the number of electrons, so the ion density is on the order of \( 10^7 - 10^{11} \text{ m}^{-3} \) as well. The most abundant neutrals in the D-region are \( \text{N}_2 \) and \( \text{O}_2 \) (Hunsucker & Hargreaves, 2003) but as we discussed earlier, \( \text{NO} \) is an important neutral since Lyman-\( \alpha \) photons are able to penetrate down to this altitude, making \( \text{NO}^+ \) a critical

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\(^{28}\) Moen section 4.2.4

\(^{29}\) See Hunsucker page 11. Note that Hunsucker compares well with figure 5.
ion (Zolesi & Cander, 2014). When considering the total collision frequency do we need to account for electron-ion and electron-neutral collisions? The total electron collision frequency can be given as

\[ \nu_e = \nu_{el} + \nu_{en} \]

Here \( \nu_e, \nu_{el}, \) and \( \nu_{en} \) are the total electron, electron-ion, and electron-neutral collision frequencies. Kelley & Heelis [1989] show us that the electron-ion collision frequency is given by

\[ \nu_{el} = [34 + 4.18 \ln(T_e^3 / n_e)] n_e T_e^{-3/2} \text{s}^{-1} \]

Here \( T_e \) is the electron temperature. The electron-neutral collision frequency is given by

\[ \nu_{en} = (5.4 \times 10^{-10}) n_n \sqrt{T_e} \text{s}^{-1} \]

Here \( n_n \) is the neutral number density. Let's look at an example: At an altitude of 100 km,

\[ n_e \sim 1.0 \times 10^5 \text{ cm}^{-3} \]
\[ n_n \sim 6.0 \times 10^{12} \text{ cm}^{-3} \]
\[ T_e \sim 208 \text{ K} \]

We can get a rough estimate of the electron collision frequencies,

\[ \nu_{el} = [34 + 4.18 \ln(T_e^3 / n_e)] n_e T_e^{-3/2} \]
\[ \Rightarrow \nu_{el} = [34 + 4.18 \ln(208^3 / 1.0 \times 10^5)](1.0 \times 10^5)(208)^{-3/2} \]
\[ \Rightarrow \nu_{el} \sim 1.8 \times 10^3 \text{ s}^{-1} \]

\[ \nu_{en} = (5.4 \times 10^{-10}) n_n \sqrt{T_e} \text{s}^{-1} \]
\[ \Rightarrow \nu_{en} = (5.4 \times 10^{-10})(6 \times 10^{12})\sqrt{208} \]
\[ \Rightarrow \nu_{en} \sim 4.7 \times 10^4 \text{ s}^{-1} \]

Figure 8. Total electron collision frequency versus altitude. Note D-region \( \leq 100 \text{ km.} \) (Kelley & Heelis, 1989).

The example shows that \( \nu_{en} \gg \nu_{el} \). Hence, to first-order approximation we can neglect the electron-ion collisions. Recall from figure 5 and from figure 8 that the electron collision rate decreases with increasing altitude. Using only electron-neutral collisions we can recast the absorption equation, substituting for collision frequency,

\[ L = \frac{-2.34 \times 10^{-6} P}{\sin(a)} \frac{f}{f^2} \int_{h_0}^{h_f} n_e(h) \nu_{en}(h) \, dh \]
\[ \Rightarrow L = \frac{-2.16 \times 10^{-15} P}{\sin(a)} \frac{f}{f^2} \int_{h_0}^{h_f} n_e(h) n_n(h) \sqrt{T_e(h)} \, dh \]

The above equation allows us to determine the absorption of high-frequency radio waves propagating through the D-region as a function of height where \( h_0 \sim 50 \text{ km} \) and \( h_f \sim 100 \text{ km} \).

6.0 Example

Suppose we have a transmitter driving an isotropic antenna with an input power...
measured in watts. Assume a perfect system such that there are no losses and the antenna puts out the full power. We can compute the field strength at a distance of 1 km from the antenna (Rawer, 1952),

\[ E_0 = 5470 \sqrt{P} \]

\( P \) is the radiated power in watts and the units of \( E_0 \) are microvolts per meter (\( \mu V/m \)). Since it is helpful to work with decibels, Rawer [1952] shows that we can convert the field strength to decibels above 1 \( \mu V/m \),

\[ E_0 = 74.8 + 10 \log(P) = dB \text{ over } 1 \frac{\mu V}{m} \]

If we assume a radiated power of 100 watts,

\[ E_0 = 5470 \sqrt{100} = 54700 \frac{\mu V}{m} \]

\[ \Rightarrow E_0 = 74.8 + 10 \log(100) = 94.8 \, dB \text{ over } 1 \frac{\mu V}{m} \]

Rawer [1952] then shows that the total field at some distant point is given as,

\[ E = E_0 - [L_d + L_a + L_f] \]

For our example we will neglect \( L_f \) and use information from Rawer [1952] to estimate \( L_d \) for single-hop refraction via the F-region. Rawer [1952] shows that assuming a total distance of about 2000 km one would expect path loss due to distance of roughly 63 db. This is a rough estimate, but Rawer [1952] gives us a very sound footing. See Rawer [1952] figure 4(b).

\[ L_a = -\frac{2.34 \times 10^{-6} \mu V}{\sin \alpha} \frac{f^2}{\sqrt{2}} \int n_e \nu_{en} \, dh \]

For this example, rather than using constant values for \( n_e \) and \( \nu_{en} \) we shall instead use the table of values found in appendix 4 and run them through a simple FORTAN code that will compute the D-region absorption. We assume one hop. Hence, the radio wave will pass through the D-region twice. Table 2 shows the computed values for total path loss due to absorption within the D-region for a range of frequencies within the high-frequency spectrum.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>( L_a ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>56.6</td>
</tr>
<tr>
<td>5</td>
<td>20.4</td>
</tr>
<tr>
<td>7</td>
<td>10.4</td>
</tr>
<tr>
<td>9</td>
<td>6.29</td>
</tr>
<tr>
<td>11</td>
<td>4.21</td>
</tr>
<tr>
<td>13</td>
<td>3.02</td>
</tr>
<tr>
<td>15</td>
<td>2.27</td>
</tr>
<tr>
<td>17</td>
<td>1.76</td>
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<tr>
<td>19</td>
<td>1.41</td>
</tr>
<tr>
<td>21</td>
<td>1.16</td>
</tr>
<tr>
<td>23</td>
<td>0.964</td>
</tr>
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<td>25</td>
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</tr>
<tr>
<td>27</td>
<td>0.699</td>
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<tr>
<td>29</td>
<td>0.606</td>
</tr>
<tr>
<td>31</td>
<td>0.530</td>
</tr>
</tbody>
</table>

Table 2. Comparison of total path loss due to absorption for a variety of frequencies.

Thus we easily see the frequency dependence of absorption within the D-region. We can then define the electric field at the receiving end to be,

\[ E = E_0 - [L_d + L_a + L_f] \]

\[ \Rightarrow E = 94.8 - [63 + L_a] \]

Table 3 shows the calculated received signal strength for a range of frequencies within the high-frequency spectrum. It is
interesting to note that as the upper-end of the HF spectrum is reached very little change in signal strength is manifest.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>dB over 1 µV/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-24.8</td>
</tr>
<tr>
<td>5</td>
<td>11.4</td>
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<tr>
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<td>28.8</td>
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<td>29</td>
<td>31.2</td>
</tr>
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<td>31</td>
<td>31.3</td>
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</tbody>
</table>

Table 3. Comparison of received signal strength over a variety of frequencies within the HF spectrum.

It was previously noted that neither the electron density nor the collision frequency are actually constant. However, this does give a good indication that the equations are working properly.\(^{37}\)

To put table 3 into perspective it is helpful to understand how the received signal strength is generally reported to the user. A typical high-frequency receiver is equipped with an S-meter. In very basic terms, an S-meter provides the operator a reading of the received signal strength. A typical S-meter will have markings ranging from S1-S9. Many models include S9 + 10 dB or higher ranges as well. A change of one S-unit represents a change of 6 dB. Furthermore, by convention each S-unit corresponds to some number of decibels above one microvolts per meter (field strength). Table 4 provides the necessary information.

<table>
<thead>
<tr>
<th>S-unit</th>
<th>dB over 1 µV/m</th>
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</thead>
<tbody>
<tr>
<td>S9+10</td>
<td>44</td>
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<tr>
<td>S9</td>
<td>34</td>
</tr>
<tr>
<td>S8</td>
<td>28</td>
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<td>S7</td>
<td>22</td>
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<tr>
<td>S6</td>
<td>16</td>
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<td>S5</td>
<td>10</td>
</tr>
<tr>
<td>S4</td>
<td>4</td>
</tr>
<tr>
<td>S3</td>
<td>-2</td>
</tr>
<tr>
<td>S2</td>
<td>-8</td>
</tr>
<tr>
<td>S1</td>
<td>-14</td>
</tr>
</tbody>
</table>

Table 4. Typical S-unit values.

Comparing table 4 with table 3 we can determine approximate S-meter readings for various frequencies. For example, based on our assumptions we would expect an S-meter reading of between S8-S9 for the 31 MHz signal. By contrast, we would expect a reading of less than S1 for the 3 MHz signal. One key ingredient we have ignored here is the noise floor. In other words, the background noise. An S3 signal is very readable if the background noise is at S1, but an S8 signal is of no use if the background noise is well above S9. Hence, many factor come in to play.

![Absorption plot](image)

Figure 9. D-region absorption at 4 MHz over vertical path. Note good comparison with derived results. (Lied [1967] figure 1.21, p 24)

Figure 9 shows approximate D-region absorption at 4 MHz over a period of several years and solar cycles.\(^{38}\) Using the previous example we find total D-region absorption over a vertical path at 4 MHz to be about

---

\(^{37}\) The results compare very favorably with Rawer [1952] as well as Lied [1967].

\(^{38}\) The entire plot in Lied [1967] compares sunspot number over the same time period, with maximum sunspot number coinciding with absorption maximum seen at about year 1947.
31.9 dB, which compares favorably with Lied [1967] at solar maximum. Of course there is no way of knowing what sunspot numbers were used by Kelley & Heelis [1989] nor by Bain and Harrison [1972]. All we know is that they assumed a solar maximum.

6.0 Conclusion

During this report much has been learned by the author regarding high-frequency radio wave absorption. As a ham radio operator the author observed D-region effects daily, but without the understanding of the physics involved. The author is pleased to have found a remedy for that situation. However, the bulk of this report has dealt with absorption at mid-latitudes. In order to investigate PCA events it will be necessary to apply what has been learned at high-latitudes. At high latitudes it turns out that absorption is much less sensitive to diurnal variation (Lied, 1967). Energetic solar particles, protons especially, play an increasing role in ionization. In fact, Lied [1967] tells us that ionization due to solar energetic protons is more important than ionization due to solar radiation.

Whereas solar radiation has a typical diurnal variation, the stream of solar energetic protons has the potential to vary widely. Though true that major solar events can trigger ionospheric disturbance at mid-latitudes, small solar disturbances can wreak havoc in the polar regions. Hence, understanding high-frequency absorption in general as well as understanding how solar energetic protons gain access to the upper atmosphere at high latitudes is critical to explaining and understanding polar cap absorption events. With that understanding it will be possible to assist the various stakeholders that rely on high-frequency radio communication via polar paths.

Bibliography


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39 See the section Time-variations in the D-region pp 26-28 of Lied.
40 See Odenwald, p 27.
41 See the section Problems in Polar Regions beginning on page 79 of Lied.

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42 For example, Odenwald references a presentation given by H. Krakowski in 2008 titled Evolving Aviation Technologies and the Need for Space Weather Prediction and Specification wherein Krakowski discusses the significant costs involved in rerouting commercial flights that could encounter radio blackouts due to PCA events.
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Appendix 1

**Appendix 1**

**Group Velocity**

It has been shown that the *phase velocity* of a radio wave propagating through the ionosphere is given as\(^1\),

\[ v_p = \frac{c}{n} \]

Here \(n\) is defined as the *index of refraction*. Since the ionosphere is defined as a *dispersive medium\(^2\)*, we expect the index of refraction to be a function of the frequency of the radio wave\(^3\).

We also expect the *group velocity* to be defined in terms of some index of refraction relative to the group velocity\(^4\),

\[ v_g = \frac{c}{n'} \]

with \(n'\) being the index of refraction of the ionosphere relative to the group velocity. Recasting the above in terms of the indices of refraction we find that,

\[ n = \frac{c}{v_p} \quad \text{and} \quad n' = \frac{c}{v_g} \]

Recalling wave theory from electrodynamics, we again recast the equations for phase and group velocities in terms of wave properties\(^5\),

\[ v_p = \frac{\omega}{k} \quad \text{and} \quad v_g = \frac{d\omega}{dk} \]

Here \(\omega\) is defined as the *angular frequency* of the wave and \(k\) as the *wave number*. Inserting the equation for group velocity into the equation for index of refraction we find that,

\[ n' = \frac{c}{v_g} \Rightarrow n' = c \frac{dk}{d\omega} \]

But we can define the angular frequency of the radio wave in terms of the frequency,

\[ \omega = 2\pi f \]

\[ \Rightarrow n' = c \frac{dk}{2\pi df} \]

Equating the two phase velocity equations gives us an equation for \(k\),

---

1. For example, Jackson p 296 (equation 7.55). See also Lied pp 6-8
2. Again, Jackson makes this clear. See pp 316-319.
3. Jackson states that the index of refraction is generally considered a function of frequency. See Jackson p296. We use the relationship \(k = \frac{2\pi}{\lambda}\) which then allows us to put \(k\) in terms of frequency since \(\lambda = \frac{c}{f}\).
4. Davies takes this approach. See Davies pp 89-93
5. These two equations are a basic staple of electromagnetic wave theory. See for example Jackson section 7.8 pp 325-26 or Griffiths section 9.4.3 pp 417-418
Appendix 1

\[ v_p = \frac{c}{n} \quad \text{and} \quad v_p = \frac{\omega}{k} = \frac{2\pi f}{k} \Rightarrow k = \frac{2\pi f}{c} n(f) \]

And again, we recall that the index of refraction, \( n \), is a function of frequency.

Then taking the derivative of \( k \) with respect to \( f \),

\[ \frac{dk}{df} = \frac{2\pi n}{c} + \frac{2\pi f}{c} \frac{dn}{df} \]

we can solve for the index of refraction relative to the group velocity. Recall that,

\[ n' = \frac{c}{2\pi} \frac{dk}{df} \]

\[ \Rightarrow n' = \frac{c}{2\pi} \left[ \frac{2\pi n}{c} + \frac{2\pi f}{c} \frac{dn}{df} \right] \]

Cancelling terms we find that,

\[ n' = \left[ n(f) + f \frac{dn}{df} \right] \]

Furthermore, it was shown previously that for a weakly-ionized gas the index of refraction is defined by\(^6\),

\[ n = \sqrt{1 - \frac{f_p^2}{f^2}} \]

Here, \( f_p \) and \( f \) are the plasma frequency and radio wave frequency respectively. Inserting the above into the equation for \( n' \) we find that,

\[ n' = \sqrt{\frac{1 - \frac{f_p^2}{f^2}}{f^2} + \frac{f_p^2}{f^2}} \]

\[ = \frac{f^2 \left( 1 - \frac{f_p^2}{f^2} \right) + f_p^2}{f^2 \sqrt{1 - \frac{f_p^2}{f^2}}} \]

\[ \text{\textsuperscript{6} Again, Jackson thoroughly goes through this in section 7.5.} \]
Appendix 1

\[
= \frac{f^2 - f_p^2 + f_p^2}{f^2 \sqrt{1 - \frac{f_p^2}{f^2}}}
\]

\[
= \frac{f^2}{f^2 \sqrt{1 - \frac{f_p^2}{f^2}}}
\]

\[
= \frac{1}{\sqrt{1 - \frac{f_p^2}{f^2}}}
\]

\[
\Rightarrow n' = \frac{1}{\sqrt{1 - \frac{f_p^2}{f^2}}} = \frac{1}{n}
\]

Now we define the group velocity in terms of the index of refraction relative to the phase velocity,

\[
v_p = \frac{c}{n} \quad \text{and} \quad v_g = \frac{c}{n}
\]

\[
\Rightarrow v_g = \frac{c}{\left(\frac{1}{n}\right)} = cn
\]

Hence, the equations for phase velocity and group velocity become\(^7\),

\[
v_p = \frac{c}{n} \quad \text{and} \quad v_g = cn
\]

\(^7\) These two equations are given without proof by Lied pp 6-8. A detailed discussion is given in Davies section 2.4 pp 89-93
Appendix 2

Origins of the absorption coefficient.

Davies defines the *absorption coefficient* generally as\(^1\),

\[
\kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_e v}{\omega^2 + v^2} \right)
\]

But where does this absorption coefficient have its origins? We turn our attention to basic electrodynamics and Maxwell’s equations. Recall that in free space,

\[
\nabla \cdot \mathbf{E} = 0 \quad \nabla \cdot \mathbf{B} = 0
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

These represent a set of first-order, coupled, partial-differential equations. These can be decoupled by applying the curl\(^2\),

\[
\nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

\[
\Rightarrow \nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

and the same can be shown for the magnetic field. Using *separation of variables* we can solve the above *wave equation* showing that,

\[
\mathbf{E} = \mathbf{E}_0 \cos(kx_1 - \omega t)
\]

and \(x_1\) is the direction of propagation. In exponential terms the preceding becomes,

\[
\mathbf{E} = \mathbf{E}_0 e^{i(kx_1 - \omega t)}
\]

In a *dispersive media*\(^3\) the electric field can be defined by a *complex* wave equation,

\[
\nabla^2 \overline{\mathbf{E}} = \varepsilon_0 \mu_0 \frac{\partial^2 \overline{\mathbf{E}}}{\partial t^2}
\]

which allows *complex* solutions of the form,

\[
\overline{\mathbf{E}} = \overline{\mathbf{E}_0} e^{i(kx_1 - \omega t)}
\]

---

\(^1\) Davies equation 2.98
\(^2\) For a formal derivation one could consult Jackson chapter 7
\(^3\) See Jackson pp310 or Griffiths pp 321
Appendix 2

and we note that $\tilde{k}$ is the complex wave number and can be defined as,

$$\tilde{k} = \sqrt{\varepsilon \mu_0} \omega$$

We can recast the complex wave number in terms of its real and imaginary components,

$$\tilde{k} = k + i \frac{\kappa}{2}$$

In the above, $\kappa$ is defined as the absorption coefficient\(^4\). If we define the parameter

$$\alpha = \frac{\kappa}{2}$$

and recall that the intensity of the wave falls off as $e^{-2\alpha x}$, then,

$$\Rightarrow \tilde{E} = E_0 e^{-\kappa x} e^{i(kx_1 - \omega t)}$$

Davies defines the absorption coefficient as "...a measure of the decay of amplitude per unit distance\(^5\),"

$$\kappa = \frac{\omega}{c}$$

$$\Rightarrow E = E_0 e^{i(\omega x_1 \kappa)} e^{i(kx_1 - \omega t)}$$

and $\chi$ is defined as the imaginary part of the complex index of refraction\(^6\). Thus we see that the absorption coefficient has its origins in electrodynamics and is derived from Maxwell's equations.

Davies then makes the connection between the complex index of refraction

$$n^2 = (\mu - i\chi)$$

where $\mu$ is the real component and $\chi$ is the imaginary component, and the absorption coefficient

$$\kappa = \frac{\omega}{c} \chi$$

by combining them to show that\(^7\),

$$\kappa = \frac{e^2}{2\varepsilon_0 mc} \left( \frac{1}{\mu} \right) \left( \frac{n_e v}{\omega^2 + v^2} \right)$$

---

\(^4\) Jackson pp 310, Griffiths pp 422
\(^5\) Davies pp 80
\(^6\) See Davies section 2.3.3.3.
\(^7\) Davies' detailed derivation is found in chapter 2 Theory of Wave Propagation.
Appendix 3

Model data used for absorption calculations

The absorption results are based on 4 critical parameters: electron density, neutral density, electron temp, and collision frequency. However, collision frequency is a function of neutral density and electron temperature.

Electron Density

The electron density profile is based on work done by Bain & Harrison [1972]. The altitude resolution is 1 km and covers the altitude range $50 < h < 100$ km. Hence, it was important to have the same resolution for the other critical parameters. Bain & Harrison [1972] base their model on solar maximum at local noon.

Neutral Density

First we assume that the neutral density is constant throughout the D-region. The data set for neutral density comes from Kelley and Heelis [1989]. For this model we used Table B1, near sunspot maximum. This compares favorably with Bain & Harrison [1972] since their model is based on solar maximum as well. The problem is that Kelley and Heelis [1989] use an altitude resolution of 10 km within the D-region. Therefore, we plotted their data points and plotted a best-fit exponential function that approximates Kelley and Heelis [1989] fairly well,

$$n_n \approx 5.0 \times 10^{20} \exp[-0.182h]$$

Electron Temperature

Again, we used data from Kelley and Heelis [1989], interpolating their results to get a 1 km resolution for altitude,

$$T_e = 0.0012h^3 - 0.147h^2 - 0.2221h + 517.6$$

Collision Frequency.

Here we turned again to Kelley and Heelis [1989], using their equation for the electron-neutral collision frequency,

$$\nu_{en} = [5.4 \times 10^{10}]n_n\sqrt{T_e}$$

The data set follows.
### Appendix 3

<table>
<thead>
<tr>
<th>Height (km)</th>
<th>Neutral ( \text{(cm}^{-3} )</th>
<th>Electron ( \text{(m}^{-3} )</th>
<th>Temp ( K )</th>
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