April, 2014

The Practicality of a Seemingly Simple Project to Learn the Secrets of the Universe. Or How Physics is Like Making a Cake.

David Alan Smith, *Utah State University*

The Practicality of a Seemingly Simple Project to Learn the Secrets of the Universe. Or How Physics is Like Making a Cake

April 14, 2014

A senior seminar presented by

David A. Smith

Outline

- Introduction/Motivation
- Radio Telescope Design
- Data Collection
- Data Analysis
- Final Thoughts

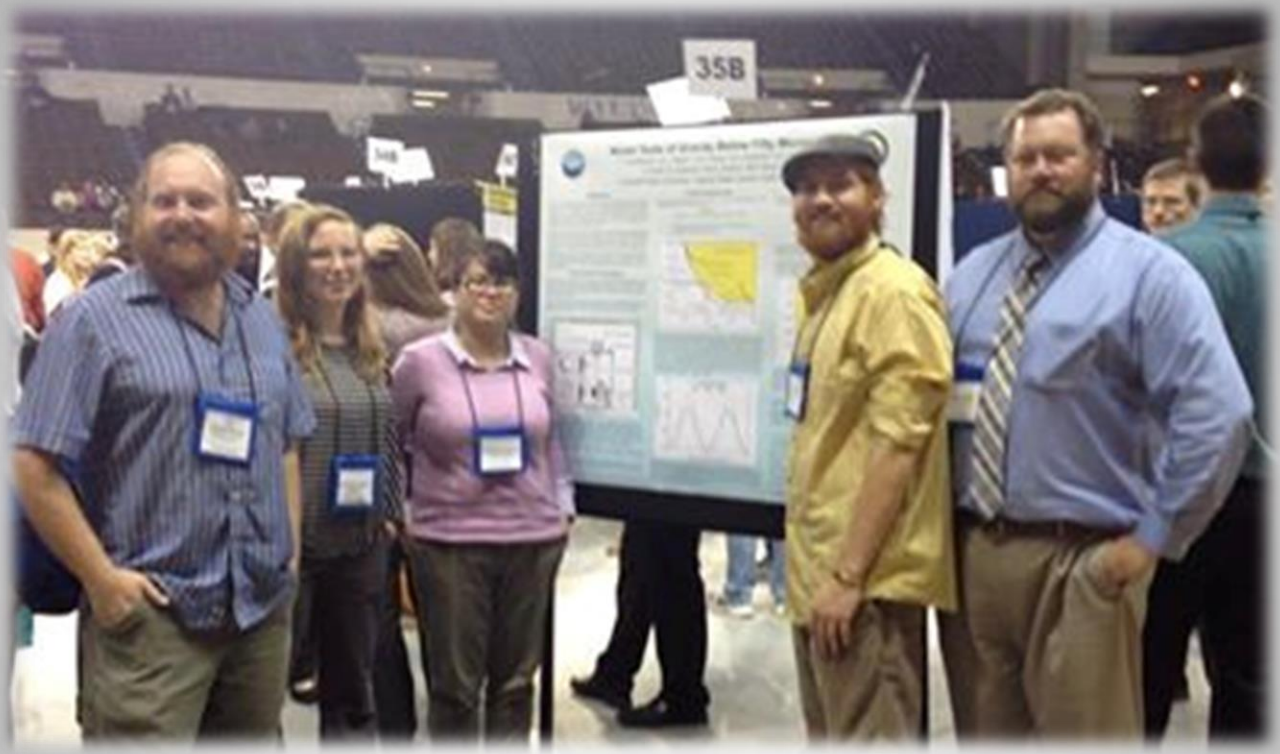
Introduction/Motivation

Purpose of Senior Seminar?

Introduction/Motivation

Demonstrate Learning

Introduction/Motivation



Present Research

…But Why The Title?

…But Why The Title?

- I've built lots of antennas

…But Why The Title?

- I've built lots of antennas
    - Didn't seem too difficult

…But Why The Title?

- I've built lots of antennas…
  - Didn't seem too difficult…
    - Parabolic dish antenna…

…But Why The Title?

- I've built lots of antennas…
  - Didn't seem too difficult…
    - Parabolic dish antenna…
      - Radio telescope just an antenna

…But Why The Title?

- I've built lots of antennas…
  - Didn't seem too difficult…
    - Parabolic dish antenna…
      - Radio telescope just an antenna
        - ARRL Antenna Book…

…But Why The Title?

$$F_c = \frac{6917.26}{d(inches)}$$

"The circular feed can be made of copper, brass, aluminum, or even tin in the form of a coffee or juice can… The circular feed must be within a proper size (diameter) range for the frequency being used. This feed operates in the dominant circular waveguide mode knows as the TE_{11} mode. The guide must be large enough to pass the TE_{11} mode with no attenuation, but smaller than the diameter that permits the next higher TM_{01} mode to propagate."

ARRL Antenna Book, Page 18-20 Published by the American Radio Relay League, Newington, CT 1991, 16th Edition

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20 years ago these words meant nothing to me

…But Why The Title?

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20 years ago these words meant nothing to me

Now I understand what they mean!

…But Why The Title?

$$F_c = \frac{6917.26}{d(inches)}$$

- Where did this equation come from?

…But Why The Title?

$$F_c = \frac{6917.26}{d(inches)}$$

- Where did this equation come from?
- Would it be possible to derive it?

…But Why The Title?

$$F_c = \frac{6917.26}{d(inches)}$$

- Where did this equation come from?
- Would it be possible to derive it?
- Became important part of project

So how is physics like making a cake?

So how is physics like making a cake?

Two ways to make a cake…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

Or…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

Or…

- Learn about the ingredients…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

Or…

- Learn about the ingredients…
- Understand how they work together…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

Or…

- Learn about the ingredients…
- Understand how they work together…
- Master the art of baking a cake…

So how is physics like making a cake?

Two ways to make a cake…

- Buy a box…
- Add some stuff…
- Eat cake!

Or…

- Learn about the ingredients…
- Understand how they work together…
- Master the art of baking a cake…

…From scratch!

Use the box…

$$KE = \frac{1}{2}mv^2$$

$$F_c = \frac{6917.26}{d(inches)}$$

Use the tools…

$$E \equiv \gamma mc^2 \Rightarrow E_{kin} = mc^2 \left(\frac{1}{\sqrt{\left(1 - \left(\frac{u^2}{c^2}\right)\right)}} - 1 \right)$$

???

Radio Telescope Design

"A telescope is an instrument that aids in the observation of remote objects by collecting electromagnetic radiation."

http://en.wikipedia.org/wiki/Telescope

10" Reflector/Dobsonian Mount

Electromagnetic radiation between 400-700 nanometers

Radio Telescope Design

Parabolic Reflector

- 6-foot diameter
- Focal length 17.25 inches
- Dish gain at 4GHz≈35dB
- LNB Gain≈66 dB
- System Gain≈101 dB

3 dB double signal; 10dB increases by 10

1 watt → 1.25×10^{10} watts!

Electromagnetic radiation between 7-8 centimeters

Radio Telescope Design

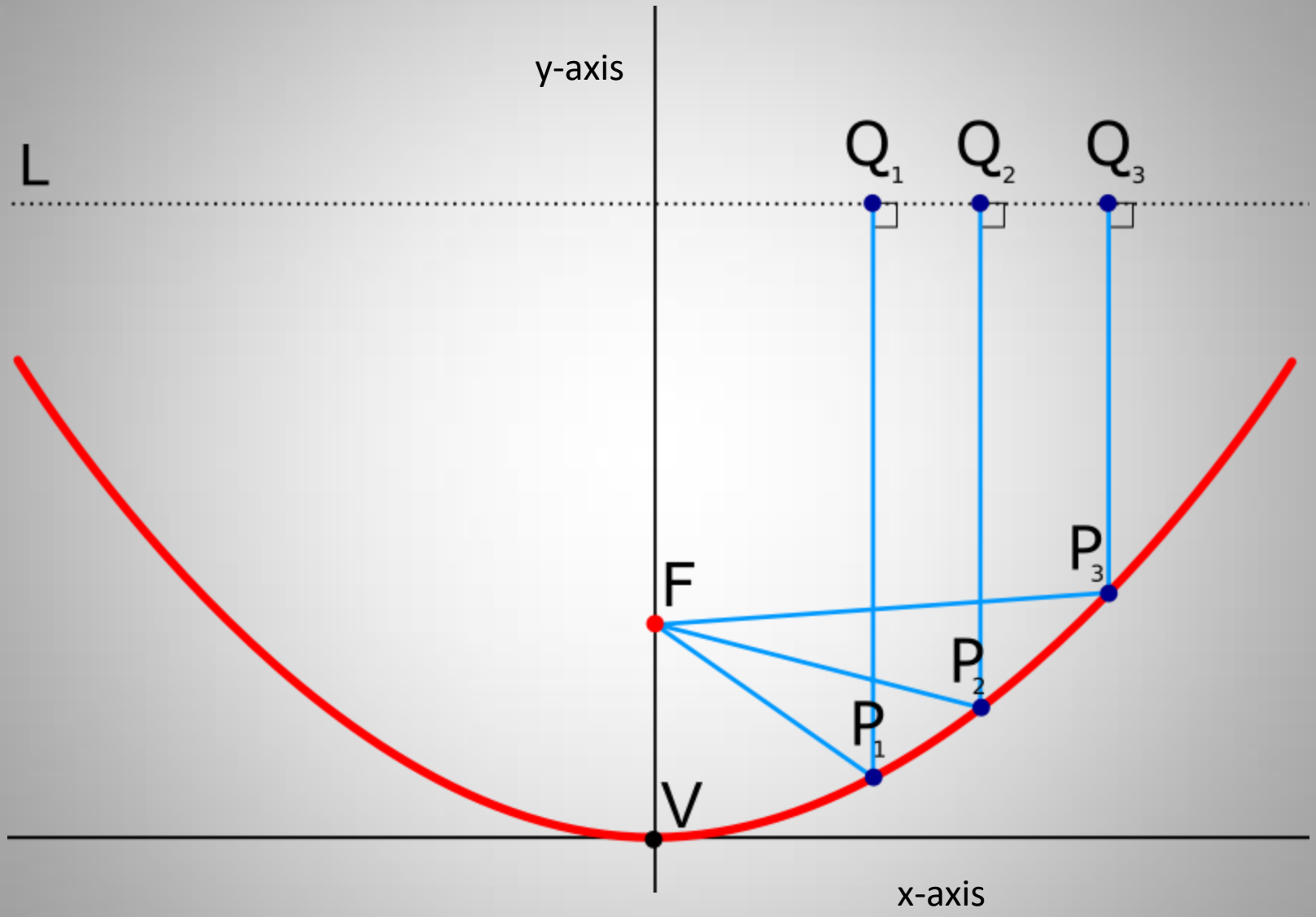


Image: http://en.wikipedia.org/wiki/File:Parabola_with_focus_and_arbitrary_line.svg

Radio Telescope Design

Detail of feed system

- "Diamond" LNB
  - Low Noise Block Down Converter
  - Input 3.7-4.3 GHz
  - Output 950-1450 MHz
- Wave Guide
  - TE_{11} mode
  - Inner diameter 2.10" (5.334 cm)

According to the ARRL formula:

$$TE_{11} \rightarrow F_c = \frac{6917.26}{2.10} = 3.3\ GHz$$

$$TM_{01} \rightarrow F_c = \frac{9034.85}{2.10} = 4.3\ GHz$$

Radio Telescope Design

Let's try to derive the cutoff frequency

Start with the following assumptions:

- Wave guide is perfect conductor
- Inside the material $\mathbf{E} = 0 \; and \; \mathbf{B} = 0$
- At boundary $\Rightarrow \mathbf{E}^{\parallel} = 0 \; and \; B^{\perp} = 0$
- *k* is real
- The area inside the wave guide approximate free space

Then generally,

$$\mathbf{E}(x, y, z, t) = \mathbf{E_0}(x, y)e^{i(kz-\omega t)})$$

$$\mathbf{B}(x, y, z, t) = \mathbf{B_0}(x, y)e^{i(kz-\omega t)})$$

The electric/magnetic fields must satisfy Maxwell's equations in the interior of the wave guide:

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

The goal is to find functions for $\mathbf{E_0}$ and $\mathbf{B_0}$ so the fields obey Maxwell's Equations subject to the boundary conditions. Confined waves are not generally transvers. To fit the BC longitudinal components E_z and B_z are included such that

$$\mathbf{E}_0 = E_{0x}\hat{\boldsymbol{x}} + E_{0y}\hat{\boldsymbol{y}} + E_{0z}\hat{\boldsymbol{z}} \qquad \mathbf{B}_0 = B_{0x}\hat{\boldsymbol{x}} + B_{0y}\hat{\boldsymbol{y}} + B_{0z}\hat{\boldsymbol{z}}$$

Please see "Introduction to Electrodynamics," Fourth Edition, by David J. Griffiths, pp 425-427

Radio Telescope Design

Each of those components are a functions of *x* and *y*. Next the curl of **E** and **B** are evaluated. According to Griffiths, these equations can all be solved for E_x, E_y, B_x, B_y. Griffiths states, "It suffices, then, to determine the longitudinal components. If we knew those, we could quickly calculate all the others, just by differentiating. Inserting equations for E_x, E_y, B_x, B_y into the divergence equations yields uncoupled equations for E_z and B_z" leaving us with,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega}{c}\right)^2 - k^2\right] E_z = 0$$

TE Waves when $E_z = 0$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\frac{\omega}{c}\right)^2 - k^2\right] B_z = 0$$

TM Waves when $B_z = 0$

We want the TE mode, so $E_z = 0$

The proof of the above statement is given in Griffiths, page 427

Radio Telescope Design

Looking back for a moment to Maxwell's Equations then taking the curl of both sides,

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \rightarrow \ \rightarrow \quad \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \rightarrow \ \rightarrow \quad \nabla^2 \mathbf{B_z} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B_z}}{\partial t^2}$$

$$\Rightarrow \mathbf{B_z} = \mathbf{B_{0z}} e^{i(kz-\omega t)}$$

In cylindrical coords B_z is a function of r and φ so the Laplacian is recast. Making the full substitution for $\mathbf{B}_z$ we have

$$\nabla^2 \mathbf{B_z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial \mathbf{B_{0z}} e^{i(kz-\omega t)}}{\partial r}\right] + \frac{1}{r}\frac{\partial^2 \mathbf{B_{0z}} e^{i(kz-\omega t)}}{\partial \varphi^2} + \frac{\partial^2 \mathbf{B_{0z}} e^{i(kz-\omega t)}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B_{0z}} e^{i(kz-\omega t)}}{\partial t^2}$$

Then doing derivatives, cancelling exponentials, and rearranging we see that

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial \mathbf{B_{0z}}}{\partial r}\right] + \frac{1}{r}\frac{\partial^2 \mathbf{B_{0z}}}{\partial \varphi^2} = \mathbf{B_{0z}}(k^2 - \mu_0 \varepsilon_0 \omega^2)$$

We can now use separation of variables…

Radio Telescope Design

$$\mathbf{B_{0z}} = R(r)\Phi(\varphi)$$

$$\Rightarrow \frac{\partial \mathbf{B_{0z}}}{\partial r} = \frac{\partial R}{\partial r}\Phi$$

$$\Rightarrow \frac{\partial^2 \mathbf{B_{0z}}}{\partial \varphi^2} = \frac{\partial^2 \Phi}{\partial \varphi^2}R$$

Subbing in for $\mathbf{B_{0z}}$, multipling by r^2, dividing by $R(r)\Phi(\varphi)$, then moving things around we get

$$\frac{r}{R}\frac{\partial}{\partial r}\left[r\frac{\partial \mathbf{R}}{\partial r}\right] - r^2(k^2 - \mu_0\varepsilon_0\omega^2) = -\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \varphi^2}$$

Depends on $\mathbf{R}$… Depends on Φ.

Since equal, constant. Call the constant m^2

$$\Rightarrow \Phi = A\cos\left(m_\varphi \varphi\right) + B\sin\left(m_\varphi \varphi\right)$$

Radio Telescope Design

$$\frac{r}{R}\frac{\partial}{\partial r}\left[r\frac{\partial \mathbf{R}}{\partial r}\right] - r^2(k^2 - \mu_0\varepsilon_0\omega^2) = m_r^2$$

Two new variables:

$$h = \sqrt{(k^2 - \mu_0\varepsilon_0\omega^2)} \qquad x = r\sqrt{(k^2 - \mu_0\varepsilon_0\omega^2)} \qquad \Rightarrow x \equiv hr \Rightarrow r = \frac{x}{h} \Rightarrow dr = \frac{dx}{h} \Rightarrow \frac{1}{r} = \frac{h}{x}$$

Multiply through by $\frac{R}{r^2}$, inserting the new variables, and using the product rule give us

$$\frac{\partial^2 R}{\partial x^2} + \frac{1}{x}\frac{\partial R}{\partial x} - R\left(1 - \frac{m_r^2}{x^2}\right) = 0$$

This is a Bessel Equation of Order "*n*" with a general solution given as

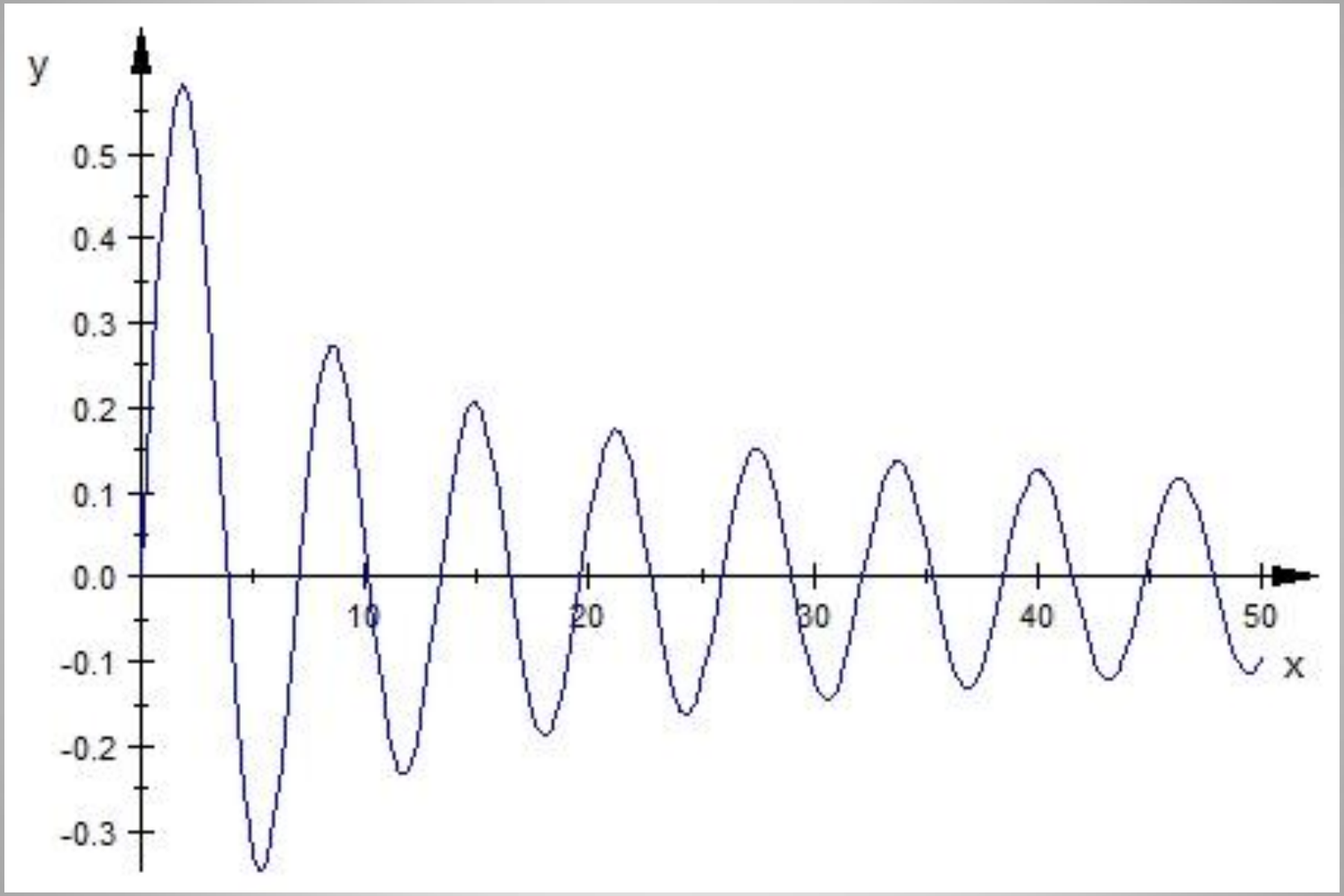
$$R(x) = A_m J_m(x) + B_m Y_m(x)$$

$J_m(x)$ is a Bessel Function of the first kind, $Y_m(x)$ is a Bessel Function of the second kind. Apparently, $Y_m(0) \to \infty$ which would cause the solution to be "unnatural," leaving us with

$$R = A_m J_m(hr)$$

Radio Telescope Design

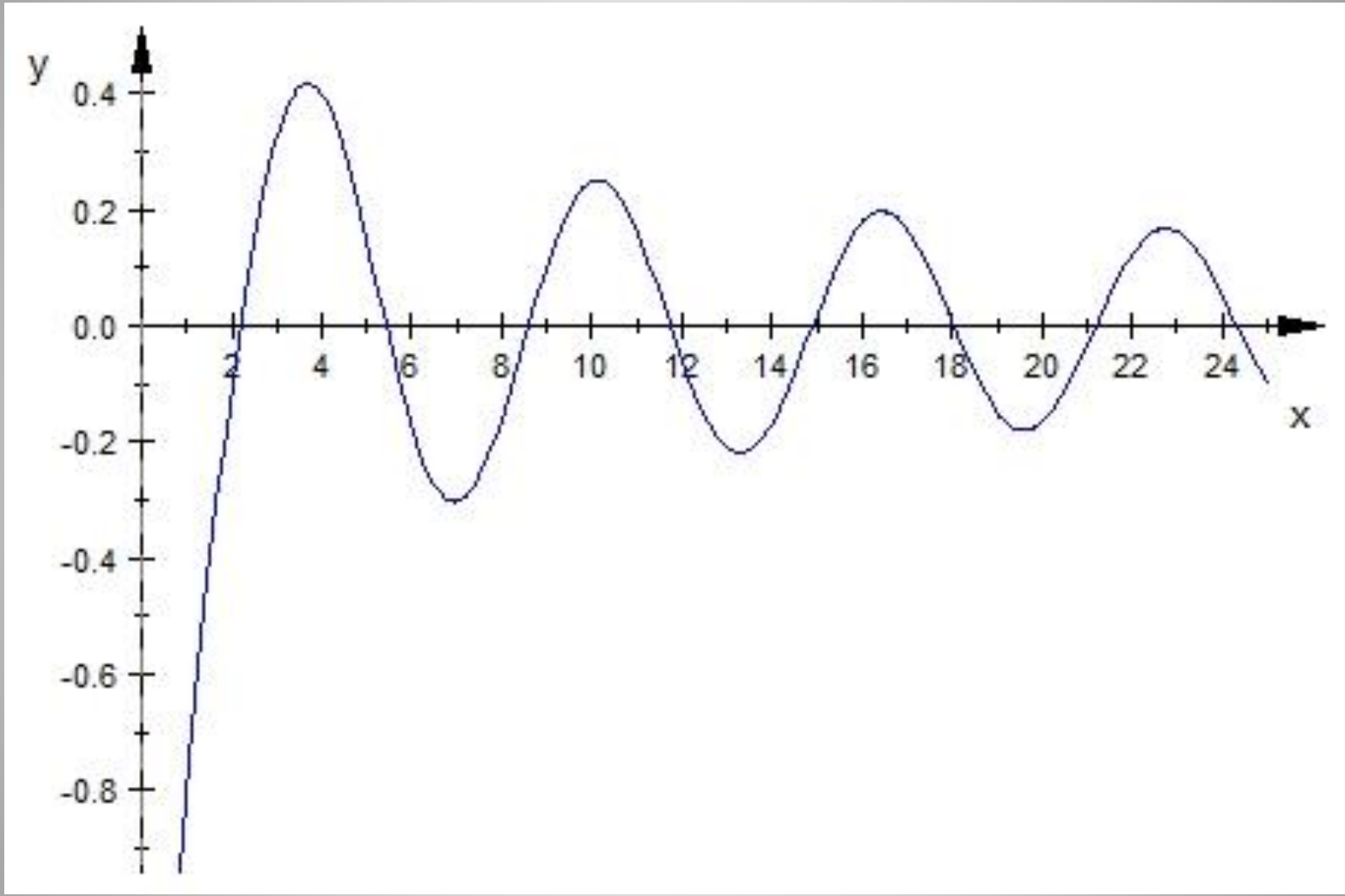
$$R(x) = A_m J_m(x) + B_m Y_m(x)$$



Plot of Bessel Function of the First Kind ($J_m(x)$)

Radio Telescope Design

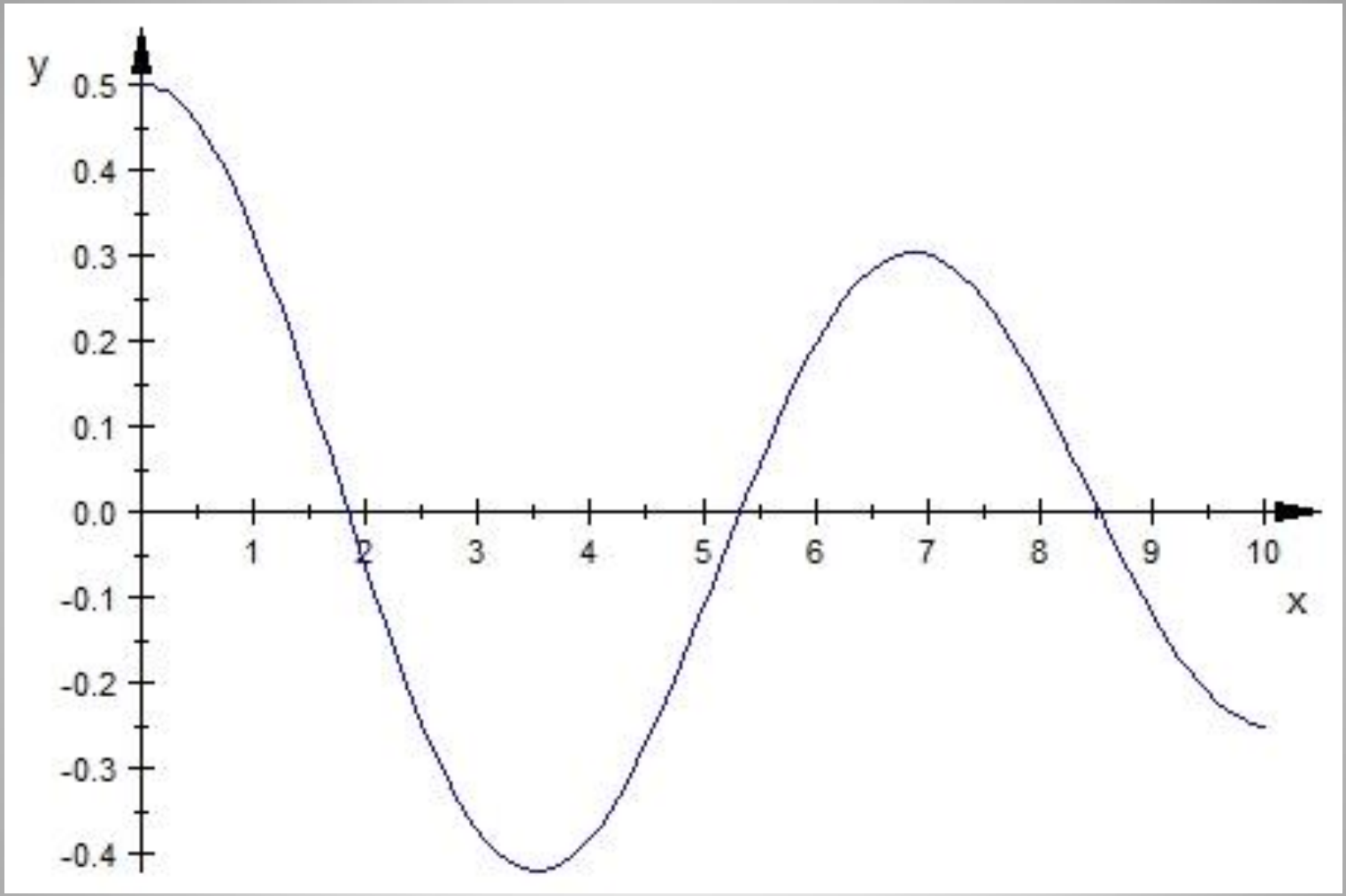
$$R(x) = A_m J_m(x) + B_m Y_m(x)$$



Plot of Bessel Function of the Second Kind ($Y_m(x)$)

…blows up at x=0

Radio Telescope Design



Plot of Derivative of Bessel Function of the First Kind ($J'_m(x)$)

MatLab syntax: numeric::fsolve(diff(besselJ(1, x), x) = 0, x= 1..2)

$x \cong 1.841183781$

Radio Telescope Design

We finally have our solution for B_{0z}. Putting everything together we have,

$$\mathbf{B_z} = A_m J_m(hr)\left[A\cos\left(m_\varphi \varphi\right) + B\sin\left(m_\varphi \varphi\right)\right]e^{i(kz-\omega t)}$$

The decoupled second-order equations for **E** and **B** are identical in form. Hence,

$$\mathbf{E_z} = A_m J_m(hr)\left[A\cos\left(m_\varphi \varphi\right) + B\sin\left(m_\varphi \varphi\right)\right]e^{i(kz-\omega t)}$$

$\mathbf{E_z}$ is a function of r and φ. Plugging $\mathbf{E_z}$ into Maxwell's Equations in cylindrical form allows us to solve for E_φ. Hence,

$$\nabla \times \mathbf{E} = \left[\frac{1}{r}\frac{\partial \mathbf{E_z}}{\partial \varphi} - \frac{\partial \mathbf{E_z}}{\partial z}\right]\hat{\boldsymbol{r}} + \left[\frac{\partial \mathbf{E_z}}{\partial z} - \frac{\partial \mathbf{E_z}}{\partial r}\right]\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left[\frac{\partial (r\mathbf{E_z})}{\partial r} - \frac{\partial \mathbf{E_z}}{\partial \partial \varphi}\right]\hat{\boldsymbol{z}} = -\frac{\partial \mathbf{B_z}}{\partial t}$$

$$\Rightarrow E_\varphi = h_r A_{mn} J'_m(h_r r)\left[A\cos\left(m_\varphi \varphi\right) + B\sin\left(m_\varphi \varphi\right)\right]e^{-ih_z z} = 0$$

Since $J'_m(h_r r)$ is the only term that depends on *r*, and since at the boundary *r*=*R* it follows that $E_\varphi$must equal zero when r=R, then it follows that $J'_m(h_r R) = 0$. Then we can say that

$$h_r R \equiv \chi_{mn} \Rightarrow h_r = \frac{\chi_{mn}}{R}$$

χ_{mn} is the *n*th root of the derivative of the Bessel function of the first kind, of the *m*th order.

So the TE_{11} mode means the 1st root of the Bessel function of the first kind of the 1st order.

See "Advanced Engineering Electromagnetics", by Constantine Balanis, Wiley, New York, 1989, Sect. 9.2.

Radio Telescope Design

$$h^2 = h_r^2 + h_z^2$$

$$\Rightarrow h_z = \sqrt{(h^2 - h_r^2)}$$

$$\Rightarrow h_z = \sqrt{\left(h^2 - \left(\frac{\chi_{mn}}{R}\right)^2\right)}$$

The cutoff frequency is when $h_z = 0 \Rightarrow h = \frac{\chi_{mn}}{R} \Rightarrow h_c = \omega_c\sqrt{\mu_0\mu\varepsilon_0} = \frac{\chi_{mn}}{R}$

$$\Rightarrow \frac{\chi_{mn}}{R} = \omega_c\sqrt{\mu_0\mu\varepsilon_0}$$

$$\Rightarrow \frac{\chi_{mn}}{R} = 2\pi f_c\sqrt{\mu_0\mu\varepsilon_0}$$

$$\Rightarrow f_c = \frac{\chi_{mn}\sqrt{\mu_0\mu\varepsilon_0}}{2\pi R} \Rightarrow f_c = \frac{\chi_{mn}}{2\pi R}c$$

Radio Telescope Design

$$c \cong 186{,}282.397 \frac{miles}{second} = 11{,}802{,}852{,}688 \ \frac{inches}{second}$$

$$\chi_{11} \cong 1.841183781$$

$$f_c = \frac{(1.841183781)(1.18 \times 10^{10})}{2\pi \frac{D}{2}} = \frac{6917262457}{D} \ Hz$$

$$= \frac{6917.26}{D} MHz$$

Radio Telescope Design

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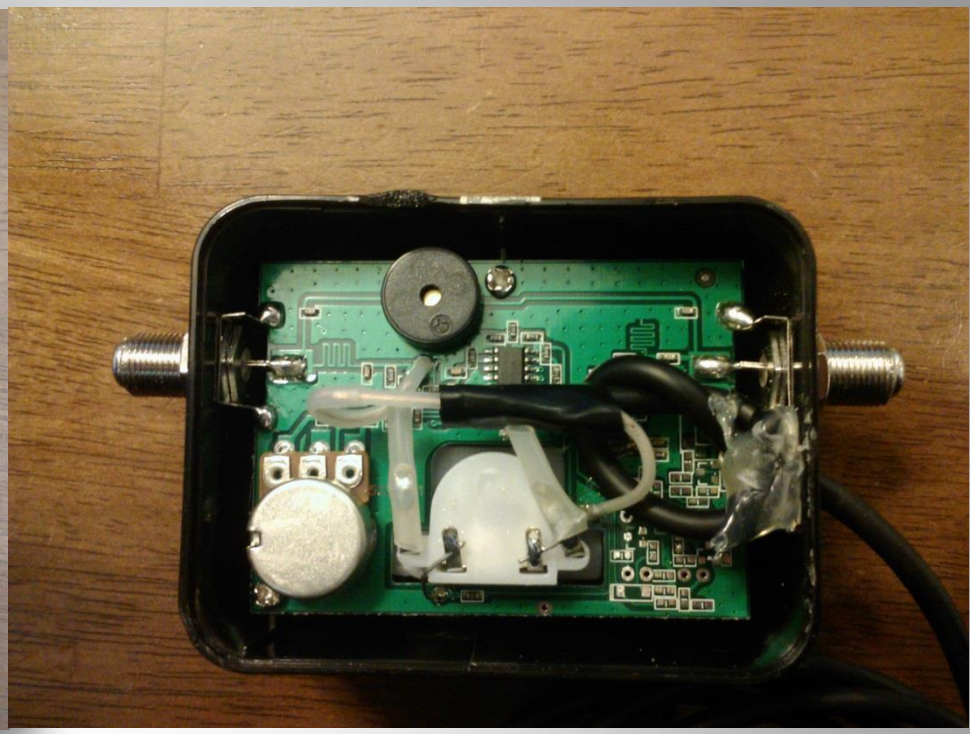
Radio Telescope Design

Radio Telescope Design



Front view

- Freq: 950-2150
- Modified for voltage output
- Mic cable to laptop
- “Radio SkyPipe”

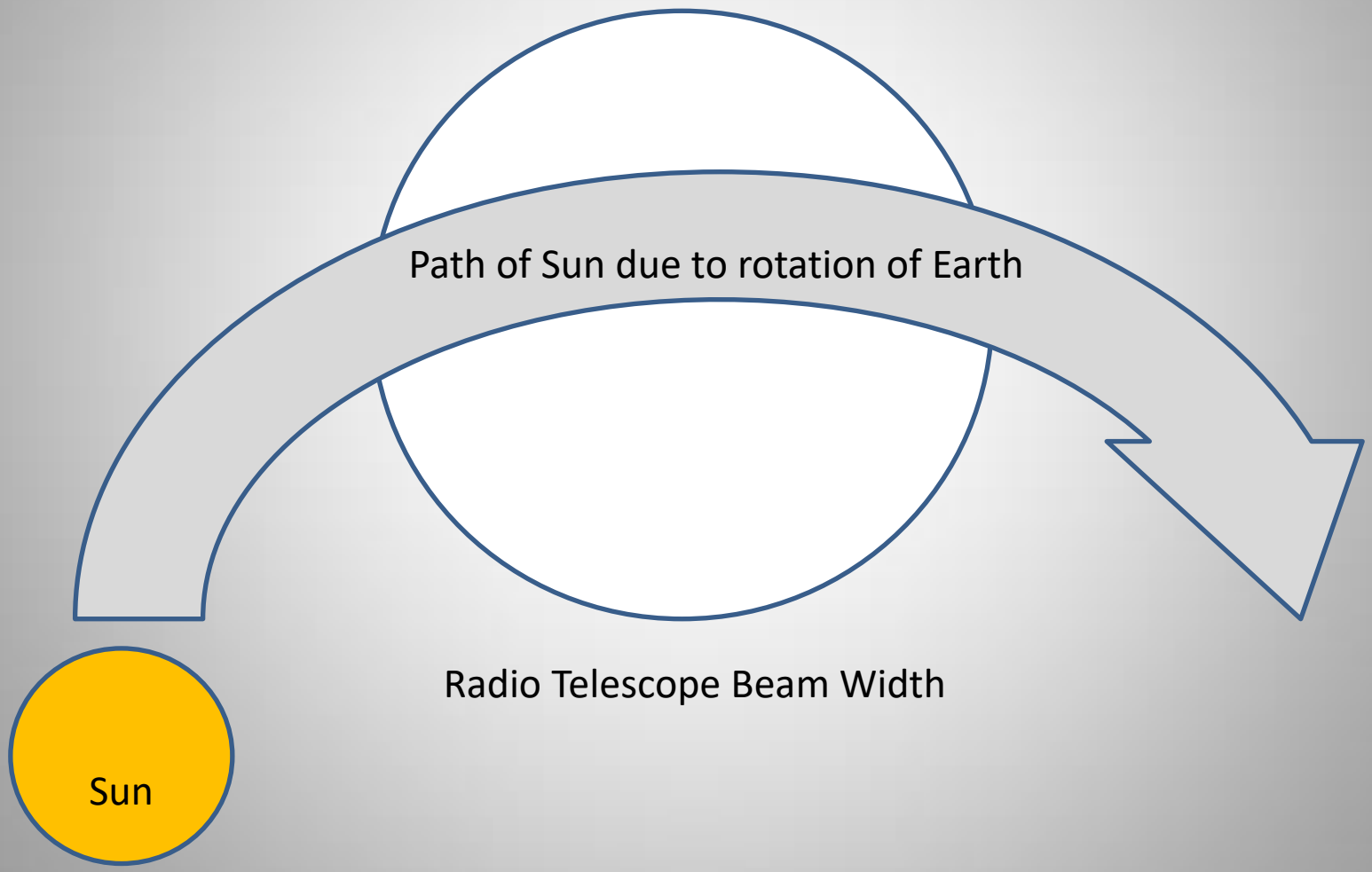
Back view

- Detail of modification
- Thanks Tyler Hooker!

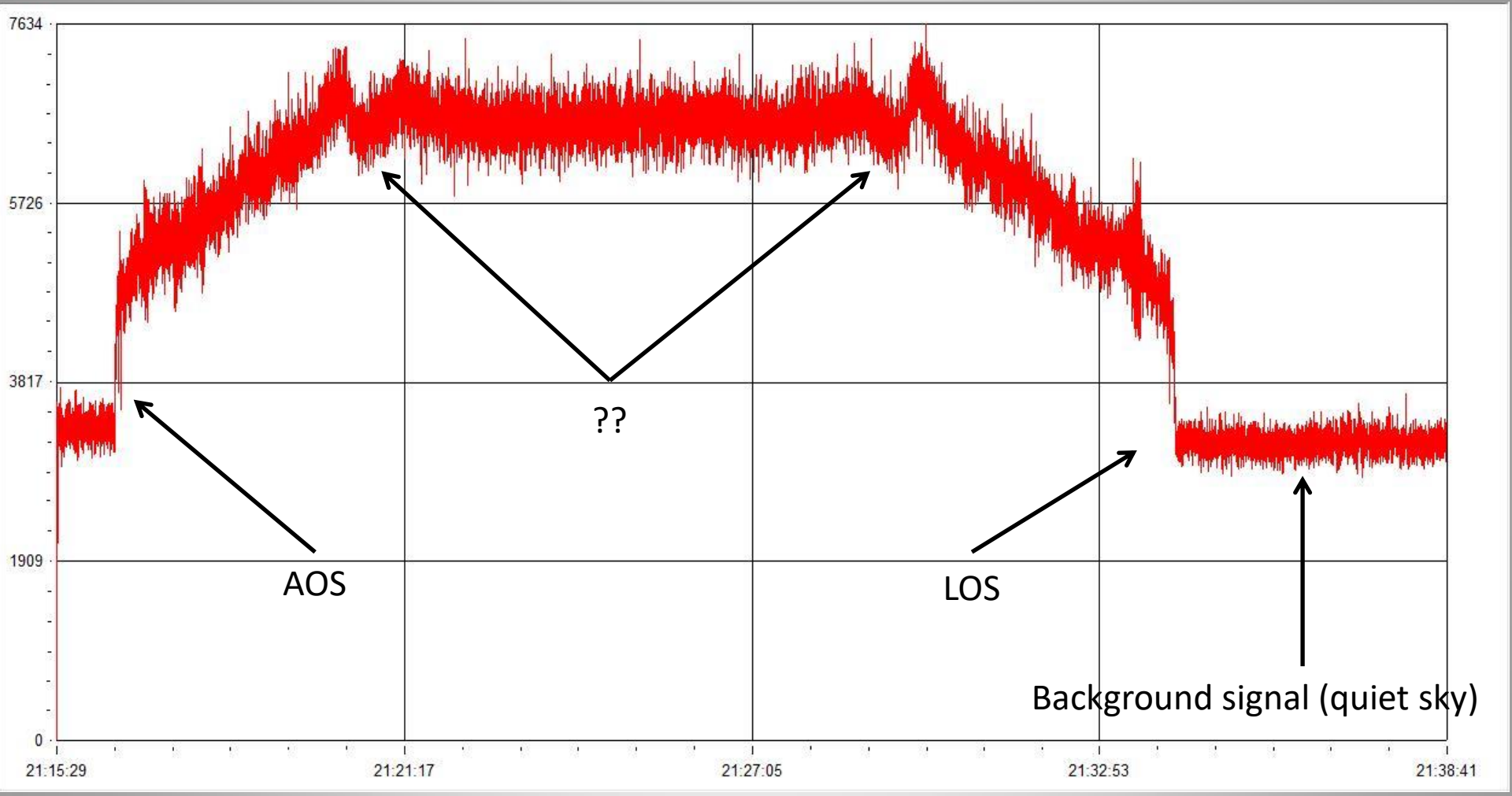
Data Collection

Drift Scan Technique

- Allow Sun to pass through beam of radio telescope
- Difficult to point accurately



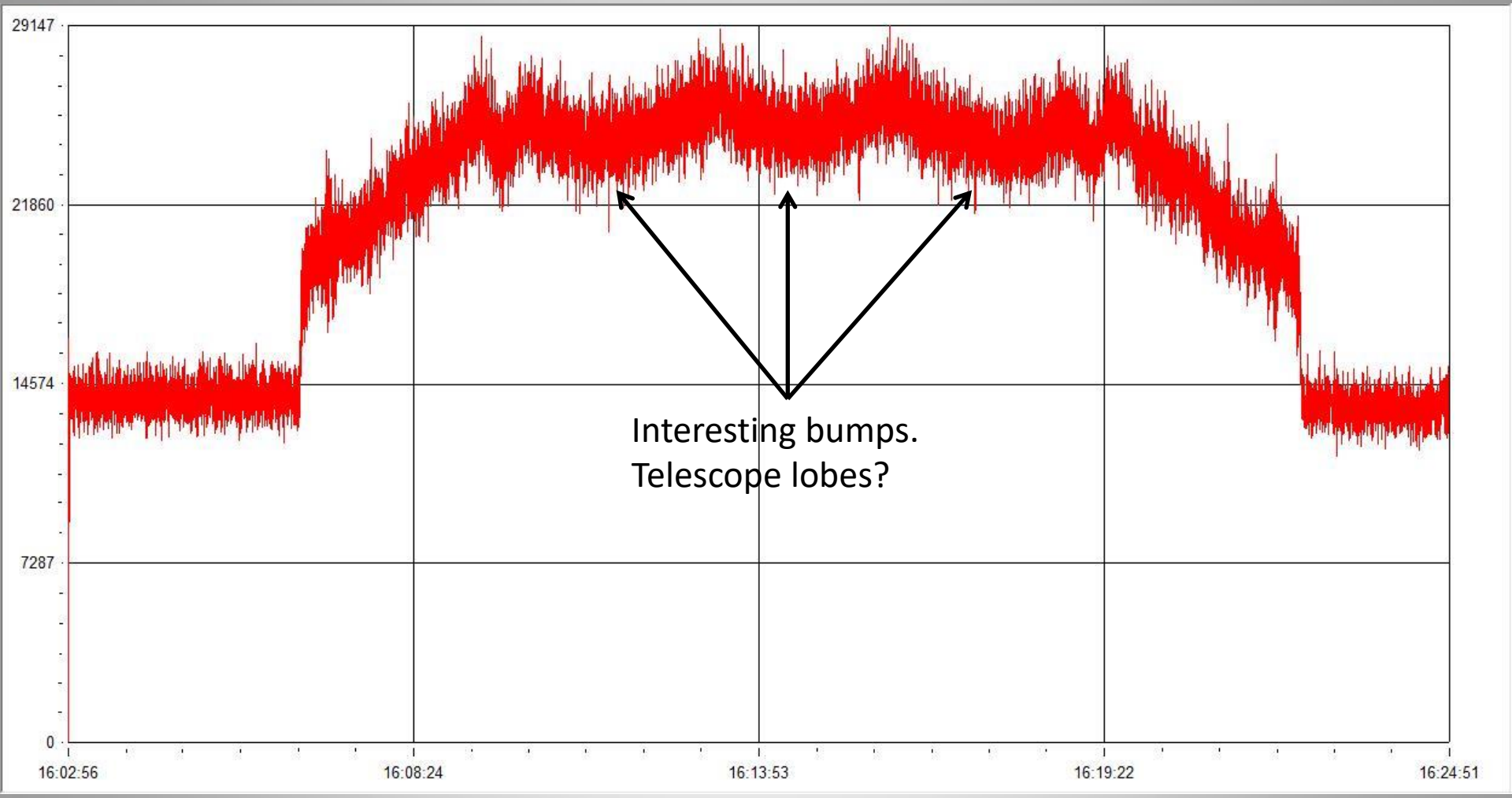
Data Collection



x-axis=time, y-axis=relative signal power

Screen shot of typical Radio SkyPipe chart.

Data Collection



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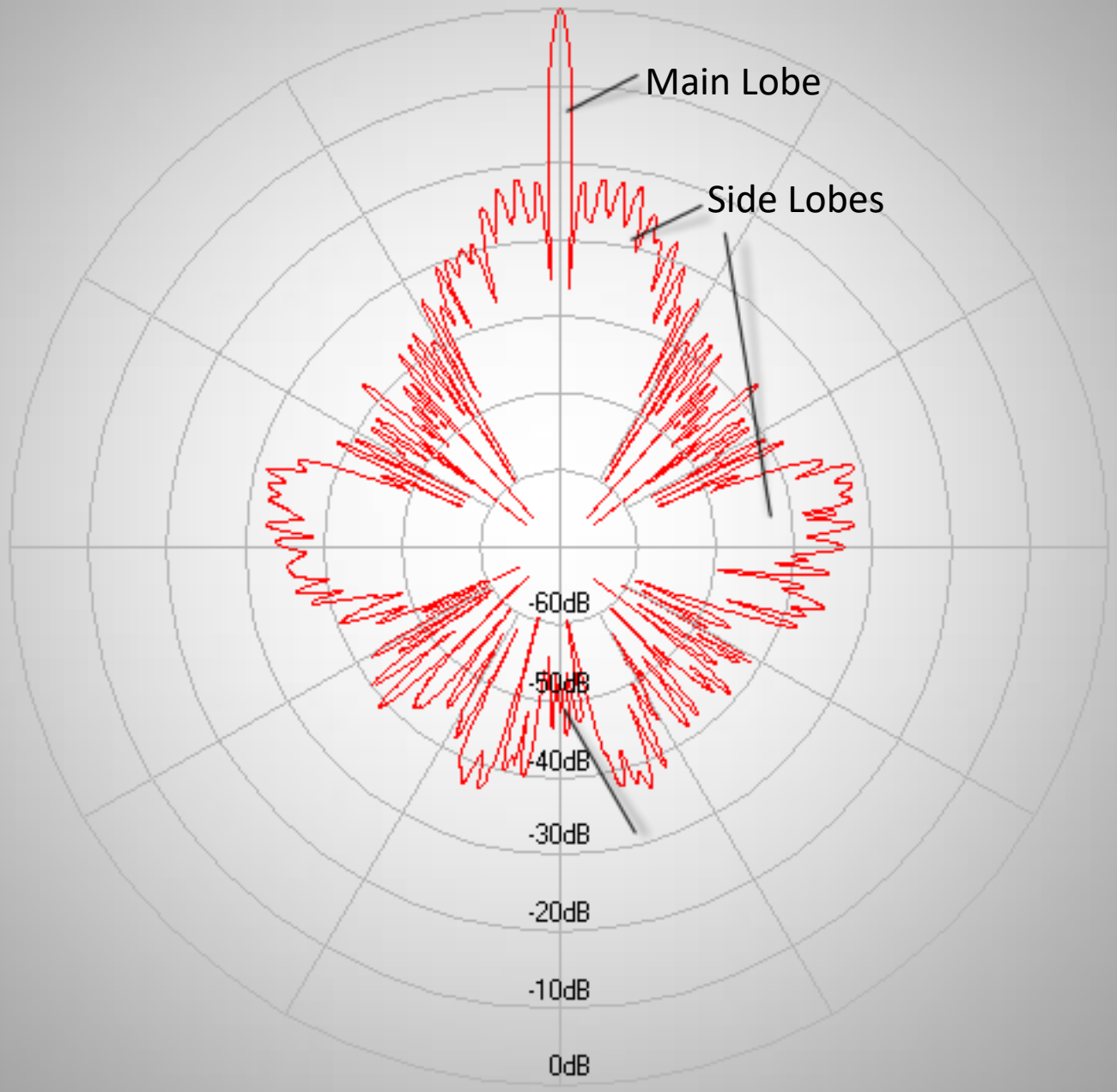

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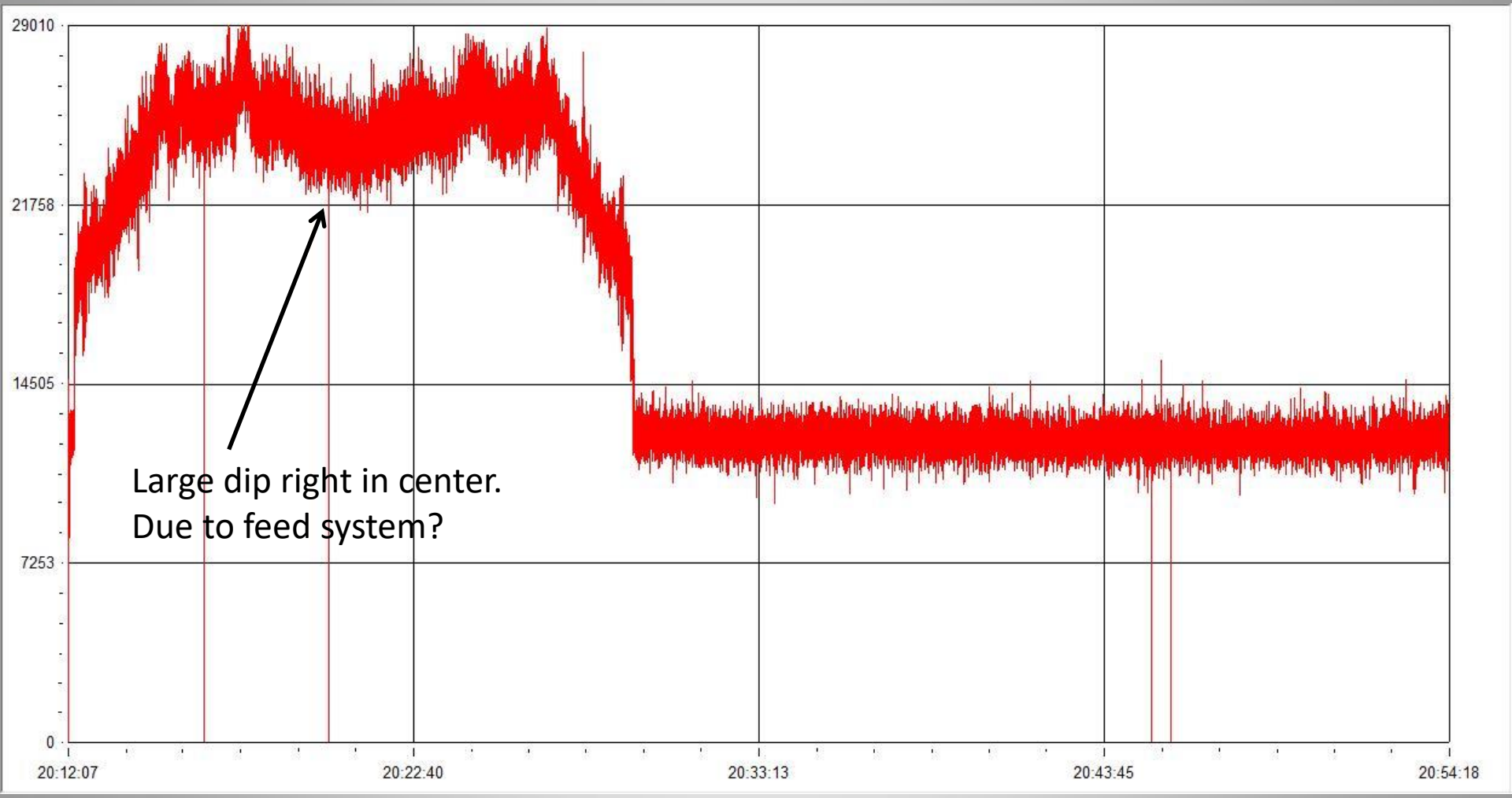
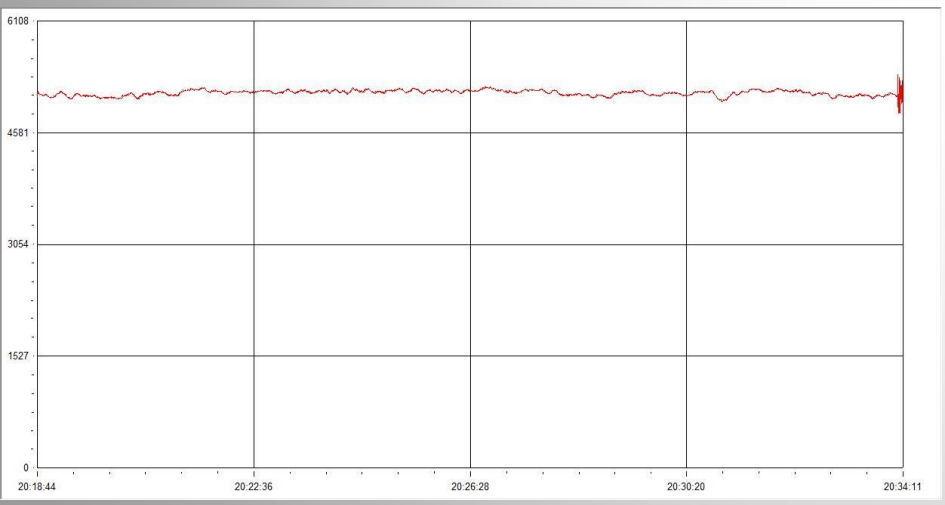


Data Analysis

Analysis Procedure

- Smooth the chart (averaging)
- Get max, min, half-power
- Get calibration signal
  - 300 K building (house)
- Plug numbers into Excel
- Solar Brightness Temp

Smoothed Chart



Smoothed Calibration

Data Points

- Sun Max
- Quiet Sky
- Calibration
- Half-power

Calculations

- True Sun
- True Calibration
- HP elapsed time
- HP beam width
- Antenna Sun Temp
- Solar Temp
- Antenna Temp

Data Analysis

Calculations

True Calibration=$Calibration - Quiet\ Sky$

True Sun=$Sun\ Max - Quiet\ Sky$

Solar Antenna Temp=$\left(\frac{True\ Sun}{True\ Calib}\right) \times Temp$

Half-power=$(Sun\ Max + Quiet\ Sky)\left(\frac{1}{2}\right)$

Half-power elapsed time=$HPt_1 - HPt_2$

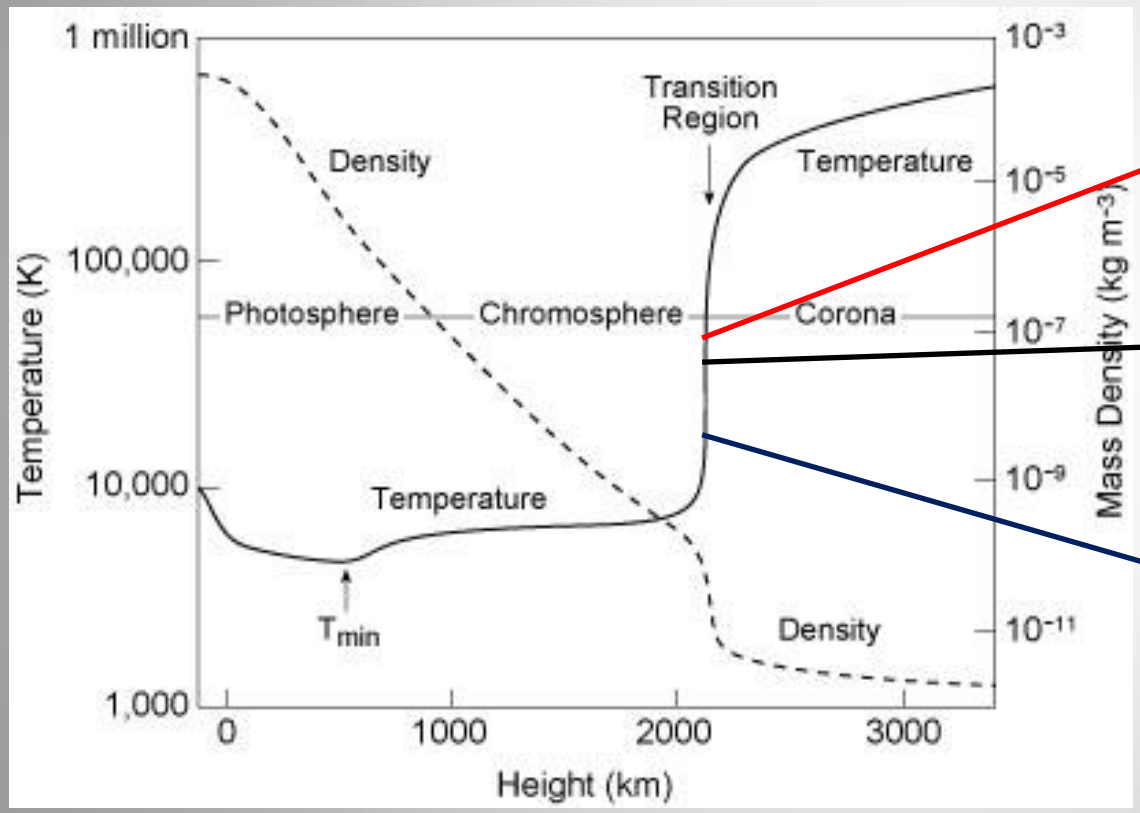
Half-power Beam Width=$\frac{Half-power\ elapsed\ time}{0.25°}$

Solar Temp=$Solar\ Ant\ Temp \times \left(\frac{HPBW}{0.5°}\right)^2$

"Solar Observations with ESA-Haystack," Joachim Köppen, Strasbourg, 2010

Data Analysis

The Solar Atmosphere



Solar Temp at 4.0 GHz

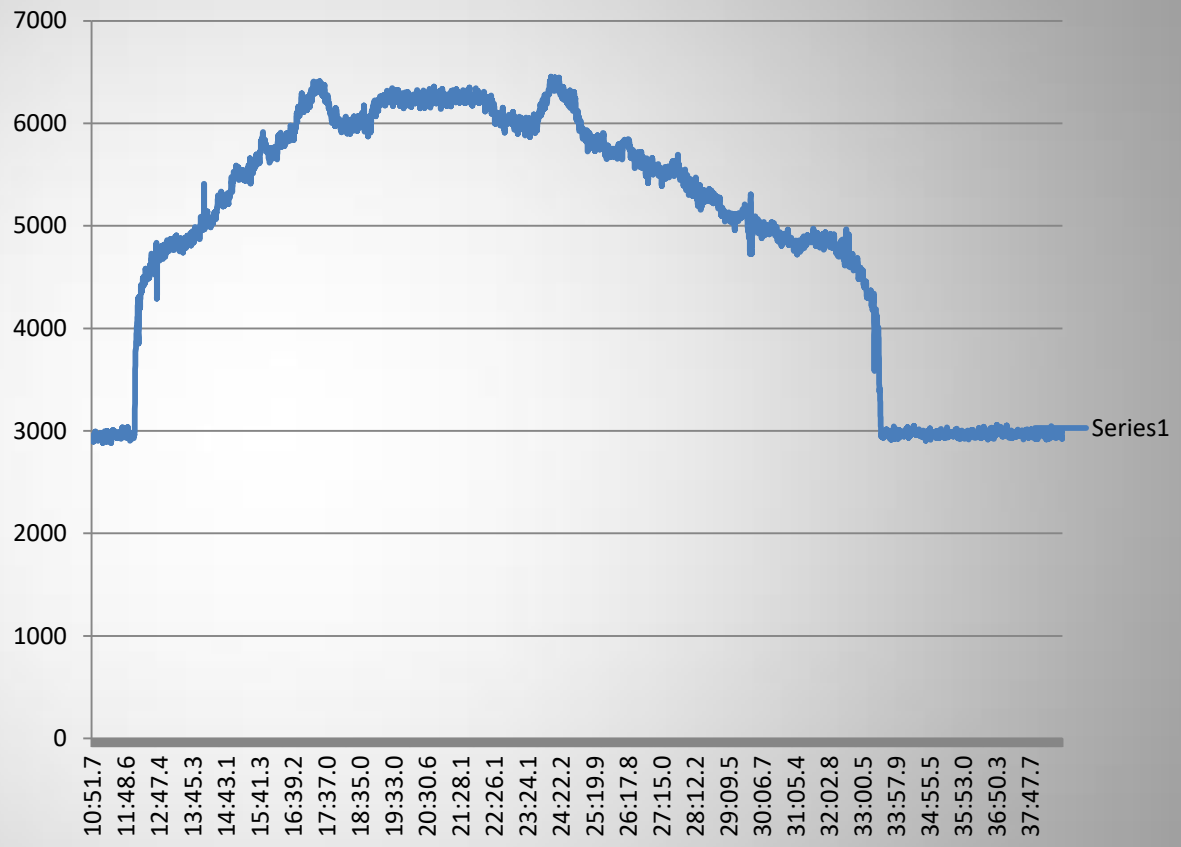
54,111 Wilson and Thursby, 1999

40,000 Kundu, 1965

27,000 Koglin, 1994

Data Analysis

Solar Maximum	6442
Quiet Sky	2985
Calibration Signal	5106
Half-Power Figure	4713.5
Half-Power Time 1	13
Half-Power Time 2	31.2
Elapsed Time (mins)	18.2
HPBW (Degrees)	4.6
True Calibration	2121
True Solar Signal	3457
Signal to Calibration	1.6
Calibration Temp (F)	70
Calibration Temp (K)	325.4
Antenna Temp (K)	530.3214
Solar Temp (K)	43916
Antenna Temp (K)	457.9142



Screen shot from Excel

Data Analysis

Solar Max	6835	6833	5706	6614	5199	5631	6952	7079	6829	6904	5755	5265	6442	6715	6864	5894
Quiet Sky	3444	3247	3227	4134	3201	3409	3168	3179	3007	3039	3402	2967	2985	2918	3286	3274
Calibration	4952	4952	4952	4952	4952	4952	5730	5730	5730	5730	5106	5106	5106	5106	4552	4552
True Calibration	1508	1705	1725	818	1751	1543	2562	2551	2723	2691	1704	2139	2121	2188	1266	1278
True Solar Signal	3391	3586	2479	2480	1998	2222	3784	3900	3822	3865	2353	2298	3457	3797	3578	2620
HPBW	4.4	4.4	3.1	3.3	2.4	2.5	4.2	3.9	4.3	4.3	2.5	2.6	4.6	5.4	3.44	2.27
Solar Max/Quiet Sky Ratio	2.0	2.1	1.8	1.6	1.6	1.7	2.2	2.2	2.3	2.3	1.7	1.8	2.2	2.3	2.1	1.8
Calibration/Quiet Sky Ratio	1.4	1.5	1.5	1.2	1.5	1.5	1.8	1.8	1.9	1.9	1.5	1.7	1.7	1.7	1.4	1.4
Solar Max/Calibration Ratio	1.4	1.4	1.2	1.3	1.0	1.1	1.2	1.2	1.2	1.2	1.1	1.0	1.3	1.3	1.5	1.3
True Solar/True Calibration Ratio	2.2	2.1	1.4	3.0	1.1	1.4	1.5	1.5	1.4	1.4	1.4	1.1	1.6	1.7	2.8	2.1
Estimated Solar Temp	54401	52145	17282	40708	8321	10942	32543	29901	32929	33306	11271	9122	43916	20327	31579	22906

Screen shot from Excel

Final Thoughts

Secrets of the Universe…

- Bessel Functions…
- Limb Darkening…
- Limb Brightening…
- Solar Atmosphere…
- Brightness Temperature…
- HPBW…
- Radio SkyPipe…

Final Thoughts

Thank You…

Dr. C. D. Hoyle – Advisor/Professor/Mentor

Dr. Ken Owens – Professor/Mentor

Dr. Ryan Campbell – Professor/Mentor

Physics Department and Staff

Final Thoughts

Thank You…

Shannon!

Final Thoughts

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