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Sensitivity Considerations for a Short-range Test of the Gravitational Inverse-square Law

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Sensitivity Considerations for a Short-range Test of the Gravitational Inverse-square Law

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Primary Research Areas

*WEP    *ISL
At Short Range

Grant No. 1065697 and 1306783
Gravity is Well Tested

Newtonian Inverse-Square Law for Point Masses

\[ F = -G \frac{m_1 m_2}{r^2} \]

The Weak Equivalence Principle

“The trajectory of a point mass in a gravitational field depends only on its initial position and velocity and is independent of its composition”
...But Lots to Learn

- GR Inconsistent with Standard Model
- String Theory – “Extra Dimensions”
- Dark Energy—Property of Gravity?
- Short-Range Forces—Exotic Particles
- WEP Violations Due to Composition
  - Baryon Number
  - Lepton Number
Newtonian Potential Energy with Yukawa Addition

\[ V(r) = -\frac{Gm_1 m_2}{r} \left( 1 + \alpha e^{-r/\lambda} \right) \]

\[ \lambda = \text{Length Scale of Deviation} \]
\[ \alpha = \text{Dimensionless Strength of Deviation} \]

As \(|\alpha| \to 0 \Rightarrow \text{GR Confirmed}\)
Current Precision/Expectations-ISL

ISL tested to about 55 microns

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Vertical Plate Step Pendulum

Uniform Field—Any Torque May Indicate New Physics

Shaded area of pendulum represents area of different density

- Largely insensitive to Newtonian torque
- Highly sensitive to short-range effects
- Excellent Null Experiment
Gauss's Law for Gravity

\[ \oint g \cdot \hat{n} \, dA = -4\pi GM_A \]

**Spherical Symmetry:**

\[ g = \frac{-GM_A}{r^2} \]

Dependence on distance of separation

- Highly sensitive to Newtonian torque
- Difficult to separate short-range effects

**Linear Symmetry:**

\[ g = \frac{-2GM_A}{Lr} \]

**Infinite Slab/Plane:**

\[ g = \frac{-2GM_A}{A} \implies g \text{ is constant} \]

\[ g = \text{constant} \implies F_g = \text{constant} \]

No dependence on distance of separation

- Attractor mass not infinite plane
- Some Newtonian torque present
- Very small

Where \( M_A \) is the Attractor Mass
Harmonic Torque Amplitudes [fNm]

<table>
<thead>
<tr>
<th></th>
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<th>Yukawa</th>
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<tbody>
<tr>
<td>$1\omega$</td>
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<td>0.0816</td>
</tr>
<tr>
<td>$2\omega$</td>
<td>0.0004</td>
<td>0.0415</td>
</tr>
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</tr>
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Note Harmonic Torque Amplitude of $1\omega$ vs $2\omega$

Assumed Parameters:

\[
\lambda = 100\mu m \quad \alpha = 1
\]

• Blue Curve: Newtonian Torque Due to Finite Plate (Pure Sine Wave)
• Red Curve: Potential Yukawa Torque
Harmonic Torque Amplitudes [fNm]

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1ω: Strong Newtonian and Yukawa Signals
2ω: Strong Yukawa Signal, Weak Newtonian
Current Precision/Expectations-ISL

ISL tested to about 55 microns
Prototypes of Attractor Mass and Step Pendulum
Electrostatic Membrane, AM, Pendulum

ESM Support Structure

Grant No. 1065697 and 1306783
Credits/Sources

Special Thanks to CD Hoyle

end
Torsion Pendulum

- Vary distance between $M_1$ and $M_2$
- Force on $M_1$ due to $M_2$ causes pendulum to twist
- Measure twist angle
- Compare with GR prediction
- Highly sensitive to Newtonian Torque

Grant No. 1065697 and 1306783
Yukawa Force on High-Density Step is Approximated by

\[ F_Y = 2\pi\alpha G \rho_1 \rho_a A \lambda^2 \left[ 1 - e^{-t_1/\lambda} \right] \left[ 1 - e^{-t_a/\lambda} \right] e^{-s/\lambda} \]

\(\rho_1\) = Mass Density of Pendulum Step  
\(\rho_a\) = Mass Density of Attractor Mass  
\(t_1\) = Step Thickness  
\(t_a\) = Attractor Mass Thickness  
\(A\) = Area of Pendulum Step  
\(s\) = Separation Distance, Pendulum Step and Attractor Mass
Short-Range Yukawa Torque on Entire Pendulum is Approximated by

\[ N_Y \approx \pi \alpha G \rho_a R A (\rho_1 - \rho_2) \lambda^2 e^{-s/\lambda} \]

\( \rho_a = \) Mass Density of Attractor Mass
\( \rho_1 = \) Mass Density of Pendulum Step
\( \rho_2 = \) Mass Density of Lighter Step

\( R = \) Width of Step
\( A = \) Area of Pendulum Step
\( s = \) Separation Distance, Pendulum Step and Attractor Mass

Assumed Parameters:

\( \lambda \) very small