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Suburban Employment and Zoning: A General Equilibrium Analysis

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SUBURBAN EMPLOYMENT AND ZONING: A GENERAL EQUILIBRIUM ANALYSIS*

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1. INTRODUCTION

This paper analyzes the long-run impact of a policy of large lot suburban zoning in the context of a two-jurisdiction general equilibrium spatial model of an urban area. Unlike most spatial models, the model allows both the price of land (business and residential) and the wage rate to respond simultaneously to the introduction of a zoning policy. In addition, I include a suburban business area (and a central business district) in the model, thus allowing for an analysis of the impact of zoning on jobs as well as residential location.

One of the limitations of the CBD-oriented urban spatial model becomes apparent when an analysis of large lot zoning is attempted. Because lot size increases with distance from the center of the urban area, it appears that a large lot zoning ordinance may have no impact on the housing market. However, this view becomes suspect when the model is expanded to incorporate the important postwar growth of jobs outside the central business district. As expected, zoning which might have no exclusionary impact in a CBD-oriented model is likely to exclude individuals from residing and possibly working in the suburban portion of the urban area.

Despite several recent analyses of the impact of large lot zoning, there still remains some disagreement over issues relating to the impact of zoning on land values, tax base, and the size of urban areas. A good deal of this disagreement arises because of the limitations involved in the models used, rather than errors in analysis. The paper attempts to resolve some of these difficulties by presenting a spatial model which incorporates the following characteristics. First, all of the results are obtained without assuming particular functional forms for any re-

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1See Kain [3] for a discussion of the growth of suburban employment during the postwar period. For our purposes, Kain’s results are important because they suggest that it is reasonable to assume that the growth of suburban manufacturing jobs tends to attract residents rather than vice versa.

2See, for example, Courant [1], Hamilton [2], Ohls, Weisberg, and White [5], Stull [8] and White [9]. This paper builds most directly on the Courant [1] and White [9] papers because it is spatially oriented and focuses on large lot zoning. [2] and [5] are essentially nonspatial in character, while [8] focuses on externality zoning.

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lationships. Second, by introducing two business areas (in which both land and labor are inputs to the production process), I can analyze the impact of suburban zoning on wages as well as job location. Third, by assuming that capital and land are used as inputs to the housing production process, I can distinguish between the impact of zoning on land values and the impact on property values (and property tax base).

Section 2 of the paper describes the model, which builds closely upon earlier urban spatial equilibrium studies. There are three types of land use possible—a central business district (CBD), a residential area, and a suburban business area (SBA). Businesses produce a composite consumption good (from labor and land), which is sold in a national market. Identical individuals maximize their utility by choosing job location and residential location, while allocating their budgets among housing (produced from land and capital) and the composite consumption good. Long-run equilibrium is attained when households have no desire to move, businesses are making zero profits, and when the supply and demand for land and labor are equilibrated.

Sections 3 and 4 analyze the impact of the introduction of suburban large lot zoning (in the form of a maximum capital-land constraint on the production of housing) in an open model in which labor is mobile among urban areas (3), and in a closed model in which the population of the urban area is assumed fixed (4). Some conclusions and possible extensions of the model are discussed in Section 5 of the paper.

2. THE MODEL

The model differs from the traditional theory of residential location primarily in its labor market assumptions. When clear generalizations of other aspects of the model are available in the literature, I have simplified the corresponding presentation. The model assumptions which appear in many urban spatial models are as follows:

1. The urban area is circular and built on a featureless plain.
2. Housing is produced at each residential location using land and capital and a variable proportions constant returns to scale technology. The residential area is fixed in size.  
3. Land and capital are rented from an absentee landlord. The rental rate on capital is determined in a national capital market, and thus may be taken as exogenous to the urban area.
4. All consumers have identical utility functions, whose arguments are a composite consumption good and housing.

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3 See Polinsky and Rubinfeld [6] for a general equilibrium spatial urban model with an endogenous central business district labor market (as well as references to the residential location in this literature). Some of the model assumptions which are left implicit in this presentation are made explicit in [6].

4 This simplifies our analysis. For a discussion of the effects of zoning on the size of urban areas see White [8].
Each individual works in a business area, to which a fixed number of trips (per unit time) are made.

Transportation to work costs increase monotonically with distance travelled, but are independent of the wage earned.

All markets are perfectly competitive and in long-run equilibrium. To focus on the effect of zoning on the labor market, the following assumptions concerning businesses are added:

Two business areas are located in the urban area, the first a central business district (CBD) on a fixed area of land in the center of the urban area, and the second a suburban business area (SBA) on a fixed ring of land in the suburban portion of the urban area.

Both business areas produce the consumption good from land and labor using identical constant returns to scale technologies with variable factor propositions.

The consumption good is sold in a national market. There are no intraurban or interurban transport costs for this good.

In preparation for a discussion of the fiscal incentives for zoning, it is useful to distinguish explicitly between two local jurisdictions (See Figure 1).

The urban area consists of a center city jurisdiction and a suburban jurisdiction, which are separated spatially. The first boundary, $x^*$, is defined such that initially (in the absence of zoning) all center city residents work in the SBA. However, in general suburban residents may work either in the CBD or the SBA, and $x_t$ represents the endogenous boundary separating CBD and SBA worker residences.

The suburban jurisdiction has the option of putting into effect a zoning ordinance which sets a maximum capital-land ratio in the production of housing.

\footnote{The assumption of identical production functions becomes crucial if we extend the model to include several classes of consumers with varying labor skills. In this case, workers will naturally tend to locate near businesses requiring their particular skills.}

\footnote{We might think of the CBD as sending its output by ship or rail, and the SBA as sending its output by truck.}

\footnote{See Courant [1] for the application of such an assumption in a more simplified urban model.}
Because of the emphasis on the impact of zoning on the labor market, it is helpful to distinguish between two extreme assumptions concerning interurban labor mobility. Thus, I define an urban area as being closed when total population, and therefore total labor supply, is fixed and open when labor is perfectly mobile between urban areas.\(^8\) On the basis of this distinction and the assumptions just made, the important equilibrium conditions of the model can be described. I have chosen to suppress the mathematical form of some conditions, making explicit only those results which are necessary for the analysis which follows.

**Locational Equilibrium**

For an equilibrium to exist the spatial pattern of housing prices must be such that no individual can obtain a higher level of utility by changing either residential or job location. Using the indirect utility function to simplify the analysis, this condition is as follows:

\[(2.1) \quad V^* = V[p_t(x), w_t - t_t(x)] = V[p_1(x), w_1 - t_1(\mid x - x_1)]\]

where the subscript 1 refers to individuals working in the CBD, and the subscript 2 refers to individuals working in the SBA and

- \(x\) represents radial distance from the center of the urban area,
- \(\bar{x}\) is the radial distance from the center of the urban area to the center of the SBA,
- \(p_t(x) = C(r_t(x), s)\) is the price per unit of housing services (per unit time),
- \(r_t(x)\) is the price per unit of land (per unit time),
- \(s\) is the price of capital (per unit time),
- \(C\) is the cost function representing the minimum cost of producing one unit of housing services, given prices \(r_t(x)\) and \(s\),
- \(w_t\) is the wage paid to workers in business area \(i\) (per unit time),
- \(t_t(x)\) is the travel to work cost for workers in business area \(i\) (per unit time)

and

\(V^*\) is a constant level of utility attained by all individuals. \(V^*\) is fixed by assumption in the open model, but may vary in the closed model. (\(V_1 < 0\) and \(V_2 > 0\), where subscripts on \(V\) denote partial derivatives.)

Finally, the consumption good has been chosen as numeraire with a price equal to 1.

**Residential Land Market Equilibrium**

A necessary condition for equilibrium to exist is that the demand and supply of residential land in every ring from \(x\) to \(x + dx\) must be equal, i.e.,

\[(2.2) h(x)L(x)n(x)dx = 2\pi xd\]

\(^8\)The open and closed distinction is made in Polinsky and Shavell [7]. Implicit in the open assumption is the assumption that the urban area is small in relation to the system of urban areas.
where

\( h(x) \) is the demand for housing per capita,
\( L(x) \) is the demand for land per unit of housing,
\( n(x) \) is population density, i.e., the number of individuals residing in a
ring from \( x \) to \( x + ds \) and

\[ 2\pi x dx \] is the approximate supply of residential land in the ring.

It is useful to note that the demand for land per unit of housing varies inversely
with the price of land, or equivalently, the capital-land ratio varies directly with
the price of land. Since with real income (utility) constant, housing demand also
varies inversely with the price of housing, it follows that a compensated increase
in the price of land, which does not alter workplace choice, will result in an
increase in the price of housing and an increase in population density.

**Labor Market Equilibrium**

Since the land area in each business area is fixed, the marginal product of
labor, and thus the wage rate, will vary inversely with the level of employment,
i.e.,

\[ w_i = f_i'(E_i) \quad (i = 1,2) \]

where \( f_i' < 0 \) and \( E_i \) is the level of employment in business area \( i \). Then, for
firms to be in locational equilibrium, the price of land and the wage in each business
area must be such that no firms can obtain a higher level of profits by chang-
ing location. Given that the labor supply to each area is determined in the resi-
dential land market, and given the assumptions of a competitive market and a
constant returns to scale consumption good production technology, land prices
in each business area must adjust (inversely) to wage changes so as to guarantee
that both the CBD and the SBA earn zero profits, i.e.,

\[ r^b_i(x) = g_i(w_i) \quad (i = 1,2) \]

where \( r^b_i(x) \) is the price of land in business area \( i \) and \( g'_i \) < 0.

Finally, the total supply of labor in the urban area must equal the total pop-
ulation, which can be determined by integrating the population density over all
residential locations, i.e.,

\[ E_1 + E_2 = N = \left[ \int_{x_0}^{x_1} n(x) dx \right] + \left[ \int_{x_1}^{x_2} n(x) dx \right] + \left[ \int_{x_2}^{x_3} n(x) dx \right] \]

where \( N \) is the total population of the urban area, and the boundaries of each
residential jurisdiction are illustrated in Figure 1.

This completes the basic description of the model. In the next two sections
it will be used to analyze the effects of zoning under both the open and closed
labor mobility assumptions.

3. **SUBURBAN ZONING IN AN OPEN URBAN AREA**

In this section I examine the effects of the imposition of a maximum capital-
land ratio constraint by the suburban jurisdiction when labor is perfectly mobile.
among urban areas. Specifically, I assume that the maximum rate constraint is binding on at least a portion of the suburban land. This imposition of the constraint is depicted in Figure 2. It is important to note that without the suburban business area, capital-land ratios would decline monotonically with distance from the CBD. As a result, the zoning constraint illustrated in Figure 2 would no longer be binding. This suggests that the impact of a suburban zoning ordinance depends crucially upon the existing labor market structure. Initially, the imposition of a suburban zoning constraint has no effect on individuals residing in the center city, since all center city residents work in the CBD and are guaranteed to maintain their level of utility $V^*$ by maintaining their housing and location choices. However, in the constrained area of the suburban jurisdiction, the ordinance lowers the quantity of housing supplied (land area is fixed and the capital-land ratio has fallen). This forces an initial outmigration of individuals who can maintain their utility level at $V^*$ by residing in a different urban area. If the suburban labor market were unaffected by this outmigration so that the wage rate were fixed, it would follow from the locational equilibrium equation, (2.1), that the price of housing throughout the suburban jurisdiction would remain unchanged (as in Courant [1]). In addition, the price of land would fall in the constrained area, with the magnitude of the decline a function of the extent to which the ordinance forces housing producers to produce inefficiently (i.e., use a suboptimal capital-land ratio). However, the outmigration of individuals employed in the SBA will increase the marginal productivity of suburban workers and thus raise their wage, $w_2$ [Equation (2.3)].

As a result of the open urban area assumption, the price of housing rented by all suburban workers must rise to keep the level of utility constant. Consequently, the price of residential suburban land will rise in the unconstrained portion of the suburban area, but may rise or fall in the constrained area. In addition, capital-land ratios will increase in the unconstrained area, at least until the zoning constraint is reached. Thus, housing production will increase in the unconstrained area, despite the fact that housing demand per capita falls.

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*The interpretation of a maximum capital-land ratio is discussed in Courant [1].*
the price of housing has increased). Population density in the unconstrained area will also increase, since land per unit of housing production (as well as housing demand per capita) has fallen [Equation (2.2)], and real income has remained constant. In the suburban business area land prices will fall as a result of the wage increase [Equation (2.4)].

If individuals were not allowed to alter their workplace choice, the analysis would be complete. However, when we account for the possibility that individuals may switch workplaces, some center city adjustments must occur. This is best seen by considering the work choice of an individual residing just to the right of \( x^* \), the jurisdictional border. Originally, that individual was indifferent between working in the CBD and working in the SBA. After the initial impact of the zoning change, his net wage has increased as has the price of housing. However, by maintaining his suburban work choice, while residing just to the left of \( x^* \), he can pay a lower price for housing and thus increase his utility. This illustrates the fact that the suburban zoning ordinance will cause some suburban workers to reside in the CBD, thus moving the endogenous workplace boundary, \( x_1 \), to the left.

As suburban residents move into the center city, housing prices will rise, forcing some individuals to leave the urban area to maintain their level of utility. With the increased price of housing, capital-land ratios as well as population density will increase. Despite the increased density, the outmigration of CBD workers does imply a reduction in CBD employment and an increased CBD wage, \( w_1 \) [see Equation (2.3)]. At the same time, the migration of SBA workers to the center city will mitigate somewhat the increase in suburban housing prices as well as the suburban wage rate, \( w_3 \). The final outcome is shown in Figure 3, which depicts a typical pattern of prezoning and postzoning residential land prices.

On the basis of this analysis, I reach the following conclusions about the effects of the imposition of a suburban zoning ordinance on the urban area:  

\[ \text{Footnote: To simplify the exposition, we have presented the previous analysis informally. However, all of the stated results can be derived explicitly by means of proof by contradiction. For example, it is not difficult to show that the assumption of a lower wage and lower housing prices is inconsistent with equilibrium in the labor market.} \]
(1) SBA and CBD employment will fall.
(2) The wage paid to both CBD and SBA workers will rise.
(3) Housing production will increase in the unconstrained portions of the urban area, and decrease in the constrained portion of the suburb. However, total housing production will decline unambiguously in the suburbs (fewer individuals are living in the jurisdiction and each is consuming a smaller amount of housing).

(4) Housing prices will rise throughout the urban area.

(5) Residential land prices will rise in the unconstrained portions of the urban area, but may rise or fall in the constrained portion of the suburb.

(6) The aggregate value (price times quantity) of residential land will rise in the unconstrained portions of the urban area, but may rise or fall in the constrained portion. Business land prices, and therefore business land value, will unambiguously fall in both the CBD and the SBA.

(7) The value of housing consumed per capita may rise or fall in each jurisdiction, depending upon the magnitude of the compensated price elasticity of demand for housing. If housing demand is price inelastic, then the value of housing consumption will rise everywhere. However, population density will rise in the unconstrained portions and fall in the constrained portion. Therefore, the aggregate value of property in the unconstrained portions will rise when demand is price inelastic, but may rise or fall in the constrained portion. The aggregate change in property values is clearly indeterminate in both jurisdictions, since it depends in part on the responsiveness of the wage to the employment decline in each area.

The assumption that the urban area is open guarantees that individuals as consumers neither gain nor lose as a result of the introduction of a zoning ordinance. However, conclusions (6) and (7) suggest that there are conditions under which the suburban jurisdiction may wish to adopt a zoning policy, since (a) zoning generates capital gains for a portion of the existing residential landowners and (b) if housing demand is price inelastic, zoning is likely to increase the value of the residential property tax base per capita. However, these considerations must be made in light of the fact that business landowners are made worse off as a result of the zoning ordinance.

4. SUBURBAN ZONING IN A CLOSED URBAN AREA

In this section, I examine the effects of the imposition of a maximum capital-land ratio constraint by the suburban jurisdiction when the population of the urban area is fixed. Once again I will assume that the maximum constraint affects housing production on only a portion of the suburban land.

Preliminary to a more complete analysis, consider the consequences of the zoning constraint when the wages in both business areas are fixed. The zoning or-

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11 No judgment is being made about the social desirability of such a policy. See Margolis [4] and Ohls, Weisberg, and White [5] for a discussion of the fallacy of attempting to maximize the aggregate property tax base in a community.
dinance forces a lower supply of housing in the constrained area. The resulting excess demand for housing will cause the price of housing to rise and the level of the utility to fall throughout the urban area. In addition, individuals will move from the constrained suburban area to the unconstrained suburban area and to the center city. Capital-land ratios, as well as population density, will increase. (At some locations the effective zoning constraint will be reached.) However, the migration from the suburban to the center city jurisdiction implies a change in business employment. Thus, a complete analysis must take into account the labor market impact of the zoning and policy.

Since population density has increased in the center city, CBD employment will rise, while SBA employment will fall. From the labor market equation (2.5), it follows that the CBD wage, $w_1$, will fall, while the SBA wage, $w_2$, will rise. In the new equilibrium, some individuals residing in the center city will be commuting to the SBA for employment. This makes the exclusionary nature of suburban zoning clear. The zoning ordinance not only keeps individuals from residing in the suburban area, but also forces some workers to take jobs in the CBD, when they would have chosen to work in the SBA in the absence of zoning. At the same time, some individuals who work in the SBA are forced by the zoning policy to reside in the center city and to commute outwards to their jobs.

Given the wage adjustment which occurs, the locational equilibrium condition, (2.1), guarantees that the price of housing faced by all SBA workers will rise sharply. However, despite the decline in the wage earned by CBD workers, the price of their housing may rise or fall. If the price of housing were to rise in the center city (Case I), housing demand per capita and land per unit of housing would both fall, (assuming that housing is not a Giffen good) so that population density would unambiguously rise from (2.2)). The resulting pattern of residential land prices is depicted in Figure 3. However, a decrease in the price of housing (Case II) may (for certain parameter values) also be consistent with an increase in population density. To see this, recall that utility has fallen at the same

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12This can be proved by contradiction (if we exclude the labor market from our model and assume that housing is not a Giffen good). If the price of housing were to remain constant (or fall), housing demand per capita (and total housing demand) would rise (or remain constant). On the other hand, the nonincrease in the price of housing would guarantee that land prices, capital-land ratios, and housing supply would fall (or remain constant) on all unconstrained land. Since housing supply falls in the constrained area, aggregate housing supply would fall. The decreased aggregate supply and increased (or constant) aggregate demand for housing are inconsistent, so that the price of housing must rise and utility must fall.

13Since the price of housing and the price of land rise in the unconstrained portions of the urban area, the demand for housing per capita and land per unit of housing must fall (once again we are assuming that housing is not a Giffen good) and consequently population density must rise [Equation (2.5)]. Since total population and residential land area are fixed, population density must fall in the constrained area, despite the fact that housing demand per capita falls [Equation (2.5)].

14If CBD employment were to fall, the increased wage would cause some suburban residents as well as all center city residents to commute to the CBD.

15Consider the individual living in the center city just to the left of the border between two jurisdictions. When the relative wages change, he will find that his loss of utility can be diminished somewhat by maintaining his location, while switching jobs from the CBD to the SBA.
time that the price of housing has fallen (by assumption). As long as the increased demand for housing per capita, resulting from the pure substitution effect, is outweighed by the reduced demand for housing resulting from the decline in real income, housing demand per capita will fall. If at the same time there is a low elasticity of substitution of land for capital in the production of housing, population density may rise, (2.2). The resulting pattern of residential land prices is depicted in Figure 4.

Completing the analysis of the closed model, I reach the following conclusions about the effects of a suburban zoning ordinance on the entire urban area:

1. SBA employment will fall, while CBD employment will rise.
2. The wage paid to suburban workers will rise, while the CBD wage will fall. The level of utility achieved by all individuals will fall.
3. Housing prices will rise throughout the suburban area, but may rise or fall in the center city.
4. Housing production will increase in the unconstrained portion and decrease in the constrained portion of the suburban jurisdiction. Total housing production in the urban area will fall (assuming that housing is not a Giffen good), but total production in the suburban area will be indeterminate.
5. Residential land prices will rise in the unconstrained portion of the suburban area, but may rise or fall in the constrained area. Likewise, residential land prices may rise or fall in the center city.
6. The aggregate value of residential land will be indeterminate in both urban jurisdictions. Business land prices, and therefore business land value, will unambiguously rise in the center city, and fall in the suburban area.
7. The aggregate value of housing consumed per capita may rise or fall in each jurisdiction. Other things equal, to the extent that demand is price inelastic, aggregate property values are likely to rise in the center.

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16This conclusion should be contrasted with that of White [9], who concludes that center city land values rise based on the assumption of a Cobb-Douglas utility function.
city. However, the outmigration of workers to the CBD makes the result indeterminate in the suburban jurisdiction.

Conclusions (6) and (7) suggest that suburban zoning is a poor policy if the welfare of consumers is considered, since utility falls.\textsuperscript{17} However, from the point of view of the central city, suburban zoning may well be a useful policy tool because aggregate center city land values may increase. In addition, the aggregate property tax base is likely to increase if housing demand is price inelastic. Whether the tax base per capita will increase or decrease is not clear. On the other hand, the benefits to be gained by inhabitants of the suburban area are less clear. Aggregate land values will fall in the SBA and may rise or fall in the residential area. Thus, the zoning ordinance is likely to be adopted only if landowners in the area in which land prices rise have sufficient political power. Likewise aggregate property values may rise or fall, as may property values per inhabitant of the suburban area. Thus, to the extent that demand is price inelastic, substantial outmigration takes place, and the SBA wage adjustment is small, the per capita tax base is likely to increase. Whether such a justification for the adoption of zoning exists is an empirical question.

5. SUMMARY AND CONCLUSIONS

I have analyzed the effects of large lot suburban zoning in the context of a general equilibrium urban spatial model with two business areas. The results of the analysis suggest that the extent to which large lot zoning is exclusionary depends, among other things, on the extent to which job opportunities are available in the suburban area. Thus, zoning policies which may have been originally formulated with nonexclusionary objectives, are likely to have become exclusionary as a result of the flow of jobs from the CBD to the suburban areas. This suggests that in considering its choice of fair-share housing plans, policymakers ought to pay close attention to the location of future as well as existing job opportunities.

The results also suggest some of the difficulties involved when one attempts a cross-section empirical study of the impact of zoning on land prices. On the one hand, a stringent zoning policy is likely to create an excess demand for housing and cause housing and land prices to increase (in the constrained area). The extent of this price increase will depend not only on the price elasticity of the demand for housing, but also on the extent to which the suburban wage rate responds to changes in employment. On the other hand, the zoning policy forces housing to be produced inefficiently, i.e., at higher than optimal land-capital ratios. Other things equal, this inefficiency causes land prices to decrease. As a consequence, it is possible for a zoning policy which is constraining in terms of housing and job location to have no net impact on land prices.

In analyzing the impact of zoning on urban structure, I have briefly discussed the gains and losses obtained by renters of housing (as consumers) and land-
owners. The results suggest that, depending upon the labor mobility assumption, there are interest groups which stand to gain from the imposition of a restrictive zoning policy. However, I do not wish to imply that the model contains the sole (or even the best) explanation for the introduction of such a zoning policy. If the analysis were extended to include several important generalizations my conclusion might be less tentative. First, a more general model would include at least two income classes so that the distributional impact of zoning is clarified. Second, the model should be extended to allow for differences in the quality of labor employed in the central and suburban business areas. This would allow for a distinction to be made between exclusion which is related to a lack of income and exclusion which is related to a lack of job skills. In addition, one could model the growth of suburban employment by examining the effects of technical change in the suburban production process. Third, a more general model would consider the impact of zoning when portions of the housing stock are fixed so that one could distinguish between short-run and long-run impacts of zoning. Fourth, a more general model would incorporate the presence of neighborhood externalities, since the existence of such externalities appears to provide a strong motivation for the introduction of an exclusionary zoning ordinance. Finally, a more general model would include an explicit public sector, so that a more complete discussion of the incentives associated with fiscal zoning would be possible.

REFERENCES